Problem 1.1

If 60 C of charge pass through an electric conductor in 30 seconds, determine the current in the conductor.

Suggested Solution

$$i = \frac{\Delta Q}{\Delta t} = \frac{60}{30} = 2\,A$$

$$i = 2\,A$$

Problem 1.2

If the current in an electric conductor is 2.4 A, how many coulombs of charge pass any point in a 30 second interval?

Suggested Solution

$$\Delta Q = i \times \Delta t = 2.4 \times 30 = 72\,C$$

$$\Delta Q = 72\,C$$

Problem 1.3

Determine the time interval required for a 12-A battery charger to deliver 4800 C.

Suggested Solution

$$\Delta t = \frac{\Delta Q}{i} = \frac{4800}{12} = 400\,s$$

$$\Delta t = 400\,s$$

Problem 1.4

A lightning bolt carrying 30,000 A lasts for 50 microseconds. If the lightning strikes an airplane flying at 20,000 feet, what is the charge deposited on the plane?

Suggested Solution

$$\Delta Q = i \times \Delta t = 30,000\left(50 \times 10^{-6}\right) = 1.5\,C$$

$$\Delta Q = 1.5\,C$$

Problem 1.5

Determine the energy required to move 240 C through 6 V.

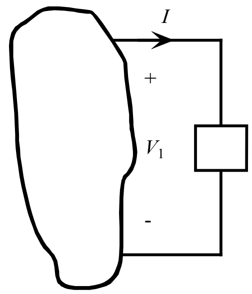
Suggested Solution

$\Delta w = V \times \Delta Q = 6 \times 240 = 1440\,J$

$$\Delta w = 1440\,J$$

Problem 1.6

Determine the amount of power absorbed or supplied by the element shown if
(a) $V_1 = 4V, I = 2A$
(b) $V_1 = -4V, I = -2A$



Suggested Solution

(a) $P = 4 \times 2 = 8W$

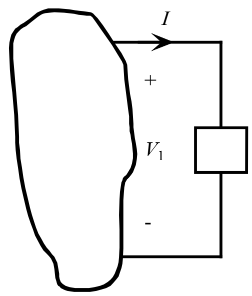
$8W$ absorbed

(b) $P = (-4)(-2) = 8W$

$8W$ absorbed

Problem 1.7

Determine the amount of power absorbed or supplied by the element shown if
(a) $V_1 = -6V, I = 3A$
(b) $V_1 = 6V, I = -3A$



Suggested Solution

(a) $P = (-6) \times 3 = -18W$

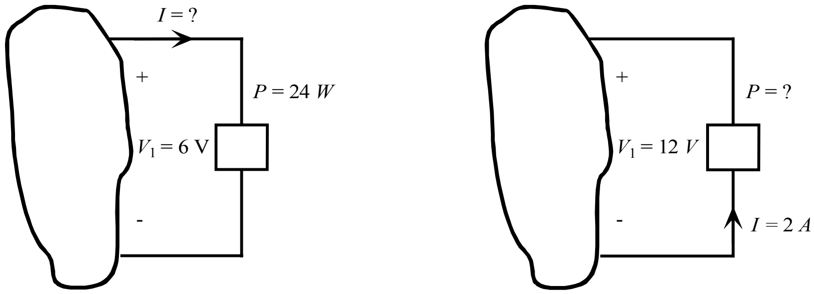
$18W$ supplied

(b) $P = 6 \times (-3) = -18W$

$18W$ supplied

Problem 1.8

Determine the missing quantity in the circuits shown.



(a) (b)

Suggested Solution

(a) $P = 6 \times I = 24 \quad \Rightarrow \quad I = 4\,A$

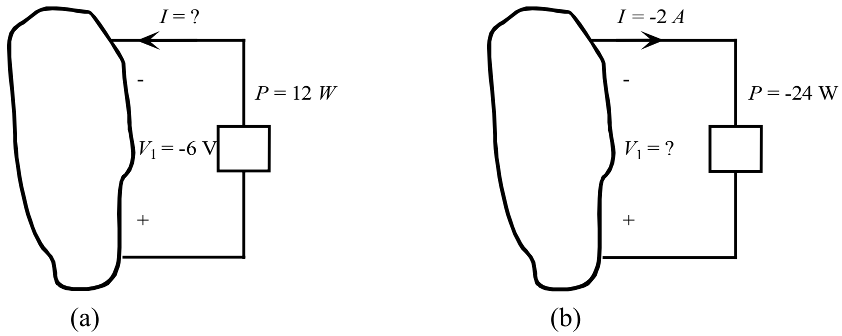
$I = 4\,A$

(b) $P = 12 \times (-2) = -24\,W$

$P = 24\,W$ supplied

Problem 1.9

Determine the missing quantity in the circuits shown.



Suggested Solution

(a) $P = (-6) \times I = 12 \quad \Rightarrow \quad I = -2\,A$

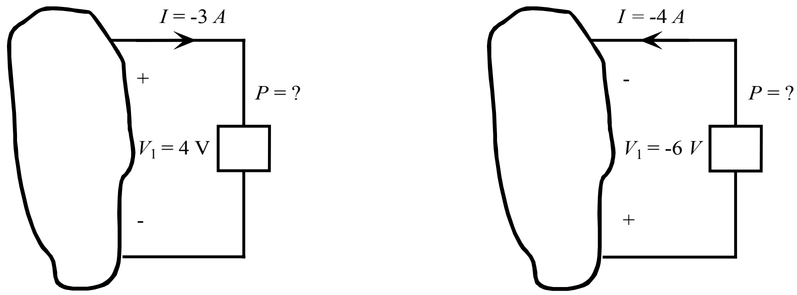
$$I = -2\,A$$

(b) $P = V_1 \times \left[-(-2)\right] = -24 \quad \Rightarrow \quad V_1 = -12\,V$

$$V_1 = -12\,V$$

Problem 1.10

Determine the missing quantity in the circuits shown.



(a) (b)

Suggested Solution

(a) $P = 4 \times (-3) = -12W$

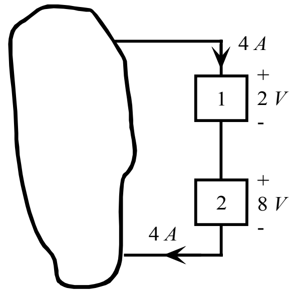
$P = 12W$ supplied

(b) $P = (-6) \times (-4) = 24W$

$P = 24W$ absorbed

Problem 1.11

Determine the power supplied to the elements shown.



Suggested Solution

$$P_1 = 2 \times 4 = 8W$$

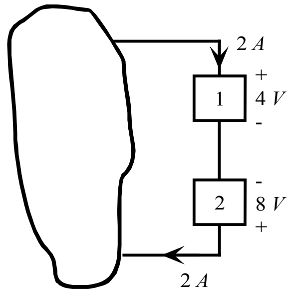
$$P_1 = 8W$$

$$P_2 = 8 \times 4 = 32W$$

$$P_2 = 32W$$

Problem 1.12

Determine the power supplied to the elements shown.



Suggested Solution

$$P_1 = 4 \times 2 = 8W$$

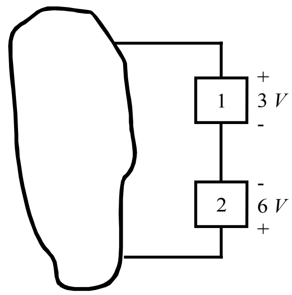
$$P_1 = 8W$$

$$P_2 = (-8) \times 2 = -16W$$

$$P_2 = -16W$$

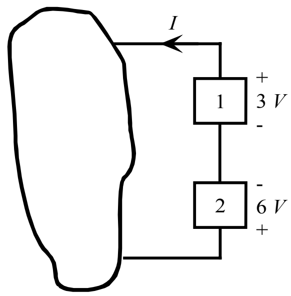
Problem 1.13

Two elements are connected in series, as shown. Element 1 supplies $24W$ of power. Is element 2 absorbing or supplying power, and how much?



Suggested Solution

If element 1 is supplying power, the direction of the current must be as shown.



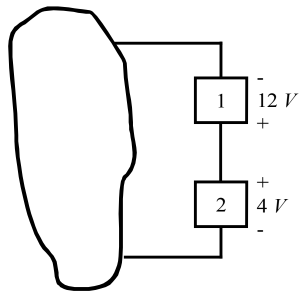
$P_1 = 3 \times (-I) = -24 \quad \Rightarrow \quad I = 8\,A$

Then, $P_2 = 6 \times 8 = 48W$

$P_2 = 48W$ absorbed

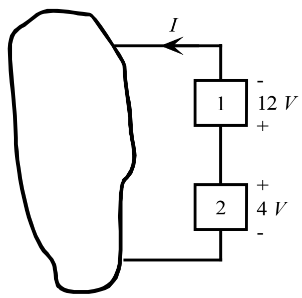
Problem 1.14

Two elements are connected in series, as shown. Element 1 absorbs $36W$ of power. Is element 2 absorbing or supplying power, and how much?



Suggested Solution

If element 1 is absorbing power, the direction of the current must be as shown.



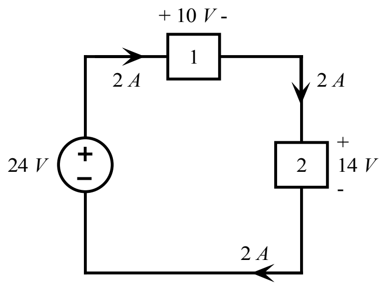
$P_1 = 12 \times I = 36 \quad \Rightarrow \quad I = 3\,A$

Then, $P_2 = 4 \times (-3) = -12\,W$

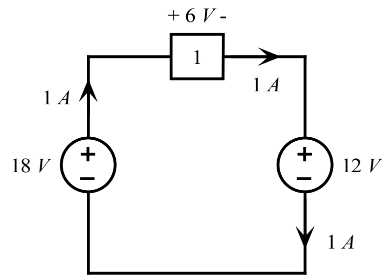
$P_2 = 12\,W$ supplied

Problem 1.15

Determine the power that is absorbed or supplied by the circuit elements shown.



(a)



(b)

Suggested Solution

(a) $P_{24V} = 24 \times (-2) = -48W$

$P_{24V} = 48W$ supplied

$P_1 = 10 \times 2 = 20W$

$P_1 = 20W$ absorbed

$P_2 = 14 \times 2 = 28W$

$P_2 = 28W$ absorbed

(b) $P_{18V} = 18 \times (-1) = -18W$

$P_{18V} = 18W$ supplied

$P_1 = 6 \times 1 = 6W$

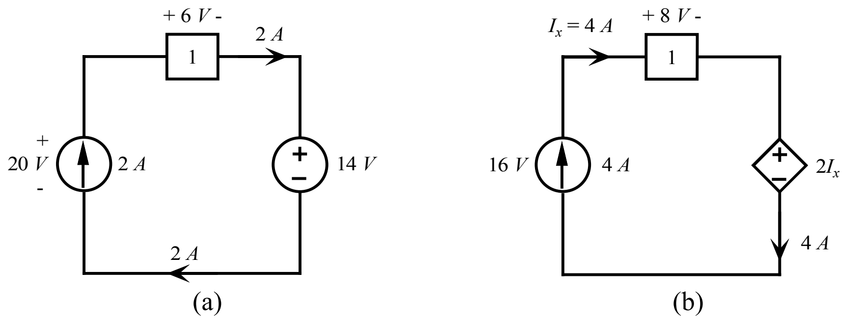
$P_1 = 6W$ absorbed

$P_2 = 12 \times 1 = 12W$

$P_2 = 12W$ absorbed

Problem 1.16

Find the power that is absorbed or supplied by the circuit elements shown.



Suggested Solution

(a) $P_{2A} = 20 \times (-2) = -40W$

$P_{2A} = 40W$ supplied

$P_1 = 6 \times 2 = 12W$

$P_1 = 12W$ absorbed

$P_{14V} = 14 \times 2 = 28W$

$P_{14V} = 28W$ absorbed

(b) $P_{4A} = 16 \times (-4) = -64W$

$P_{4A} = 64W$ supplied

$P_1 = 8 \times 4 = 32W$

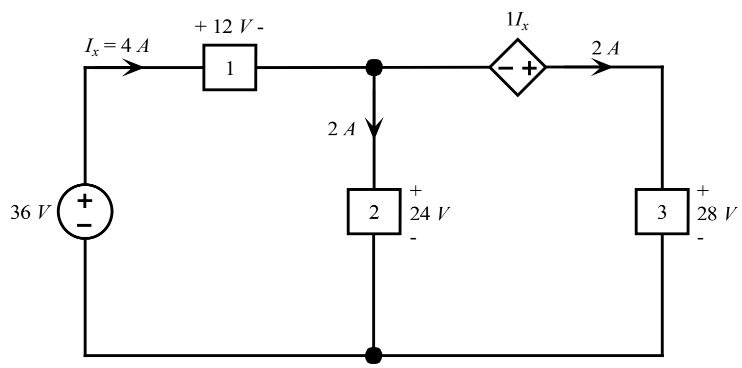
$P_1 = 32W$ absorbed

$P_{2I_x} = (2I_x) \times 4 = 8 \times 4 = 32W$

$P_{2I_x} = 32W$ absorbed

Problem 1.17

Compute the power that is absorbed or supplied by the elements in the network shown.



Suggested Solution

$P_{36V} = 36\times(-4) = -144W$

$P_{36V} = 144W$ supplied

$P_1 = 12\times 4 = 48W$

$P_1 = 48W$ absorbed

$P_2 = 24\times 2 = 48W$

$P_2 = 48W$ absorbed

$P_{1I_x} = (1I_x)\times(-2) = 4\times(-2) = -8W$

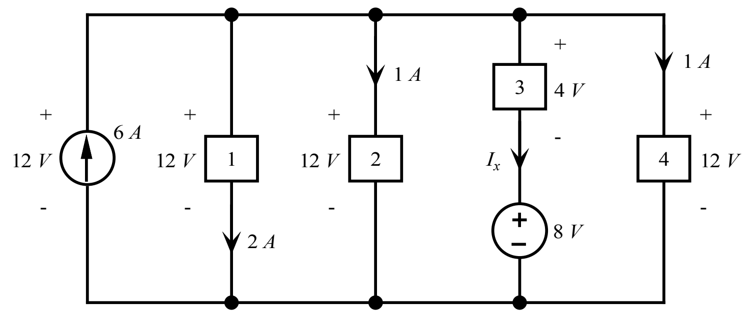
$P_{1I_x} = 8W$ supplied

$P_3 = 28\times 2 = 56W$

$P_3 = 56W$ absorbed

Problem 1.18

Find I_x in the network shown.



Suggested Solution

$P_{6A} = 12 \times (-6) = -72W$ or $72W$ supplied

$P_1 = 12 \times 2 = 24W$ absorbed

$P_2 = 12 \times 1 = 12W$ absorbed

$P_3 = 4 \times I_x = 4I_x W$ absorbed

$P_{8V} = 8 \times I_x = 8I_x W$ absorbed

$P_4 = 12 \times 1 = 12W$ absorbed

Since *Power Supplied = Power Absorbed*, then

$72 = 24 + 12 + 4I_x + 8I_x + 12$

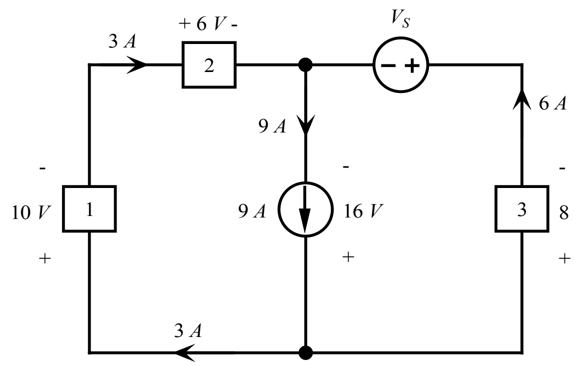
or $12I_x = 24$.

Therefore, $I_x = 2\,A$.

$I_x = 2\,A$

Problem 1.19

Is the source V_S in the network shown absorbing or supplying power, and how much?



Suggested Solution

$P_1 = 10 \times 3 = 30W$ absorbed

$P_2 = 6 \times 3 = 18W$ absorbed

$P_{9A} = 16 \times (-9) = -144W$ or $144W$ supplied

$P_3 = 8 \times 6 = 48W$ absorbed

In order to satisfy *Conservation of Power*,

$P_1 + P_2 + P_{9A} + P_{V_S} + P_3 = 0$

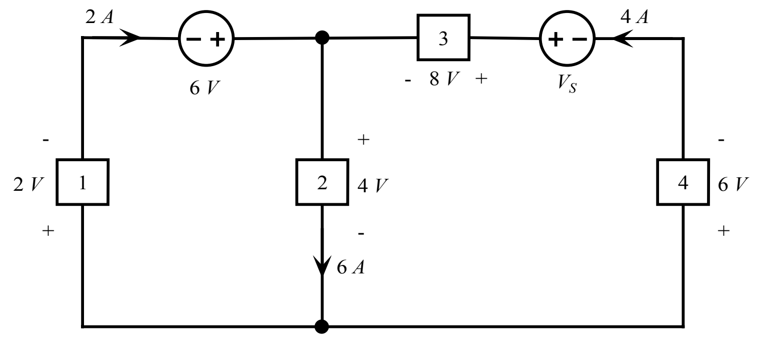
or $30 + 18 - 144 + P_{V_S} + 48 = 0$

Therefore, $P_{V_S} = 48W$.

$P_{V_S} = 48W$ absorbed

Problem 1.20

Find V_S in the network shown.



Suggested Solution

$P_1 = 2 \times 2 = 4W$ absorbed

$P_{6V} = 6 \times (-2) = -12W$ or $12W$ supplied

$P_2 = 4 \times 6 = 24W$ absorbed

$P_3 = 8 \times 4 = 32W$ absorbed

$P_{V_S} = V_S \times (-4) = -4V_S\, W$ or $4V_S\, W$ supplied

$P_4 = 6 \times 4 = 24W$ absorbed

In order to satisfy *Conservation of Power*,

$P_1 + P_{6V} + P_2 + P_3 + P_{V_S} + P_4 = 0$

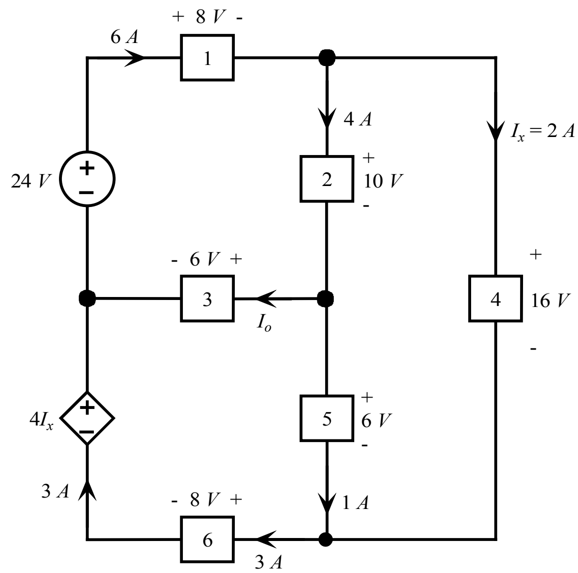
or $4 - 12 + 24 + 32 - 4V_S + 24 = 0$

Therefore, $4V_S = 72$ or $V_S = 18V$.

$V_S = 18V$

Problem 1.21

Find I_o in the network shown.



Suggested Solution

$P_{24V} = 24\times(-6) = -144W$ or $144W$ supplied

$P_{4I_x} = (4I_x)\times(-3) = 8\times(-3) = -24W$ or $24W$ supplied

$P_1 = 8\times6 = 48W$ absorbed

$P_3 = 6\times I_o = 6I_o\,W$ absorbed

$P_6 = 8\times3 = 24W$ absorbed

$P_2 = 10\times4 = 40W$ absorbed

$P_5 = 6\times1 = 6W$ absorbed

$P_4 = 16\times2 = 32W$ absorbed

In order to satisfy *Conservation of Power*,

$P_{24V} + P_{4I_x} + P_1 + P_3 + P_6 + P_2 + P_5 + P_4 = 0$

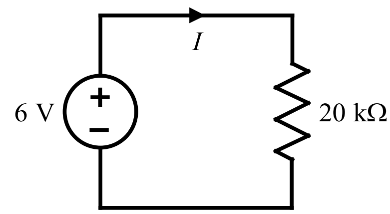
or $-144 - 24 + 48 + 6I_o + 24 + 40 + 6 + 32 = 0$

Therefore, $6I_o = 18$ or $I_o = 3\,A$.

$I_o = 3\,A$

Problem 2.1

Find the current I and the power supplied by the source in the network shown.



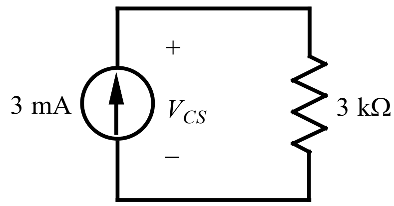
Suggested Solution

$$I = \frac{6}{20\times10^{3}} = 0.3 \text{ mA}$$

$$P = VI = (6)(0.3\times10^{-3}) = 1.8 \text{ mW}$$

Problem 2.2

In the circuit shown, find the voltage across the current source and the power absorbed by the resistor.



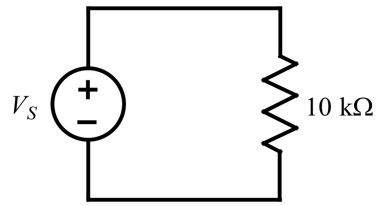
Suggested Solution

$V_{CS} = \left(3\times10^{-3}\right)\left(3\times10^{3}\right) = 9 \text{ V}$

$P = VI = \left(3\times10^{-3}\right)\left(9\right) = 27 \text{ mW}$

Problem 2.3

If the 10-kΩ resistor in the network shown absorbs 2.5 mW, find V_S.



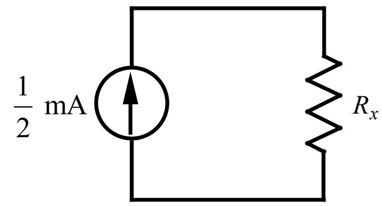
Suggested Solution

$$P = \frac{V_S^2}{10\ \text{k}\Omega}$$

or $V_S = \sqrt{P \times R} = \sqrt{\left(2.5\times10^{-3}\right)\left(10\times10^{3}\right)} = 5\ \text{V}$

Problem 2.4

In the network shown, the power absorbed by R_x is 5 mW. Find R_x.

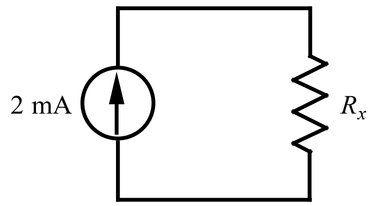


Suggested Solution

$$R_x = \frac{P}{I^2} = \frac{5\times10^{-3}}{\left(0.5\times10^{-3}\right)^2} = 20\ \text{k}\Omega$$

Problem 2.5

In the network shown, the power absorbed by R_x is 20 mW. Find R_x.

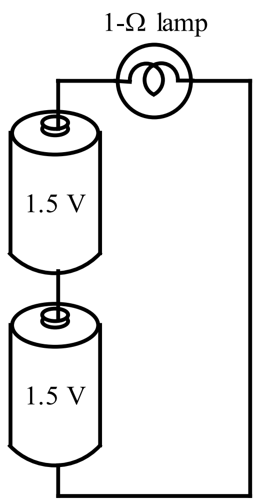


Suggested Solution

$$R = \frac{P}{I^2} = \frac{20\times10^{-3}}{\left(2\times10^{-3}\right)^2} = 5\ \text{k}\Omega$$

Problem 2.6

A model for a standard two D-cell flashlight is shown. Find the power dissipated in the lamp.



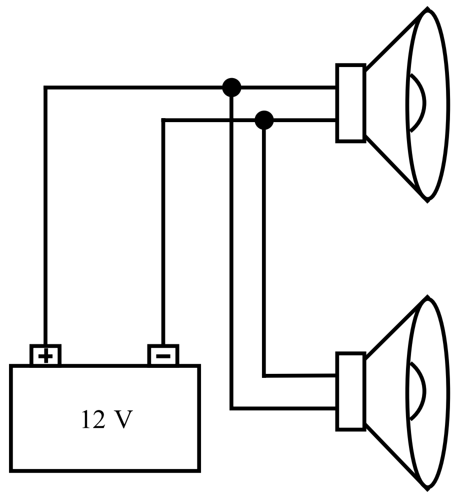
Suggested Solution

$V_{lamp} = 1.5 + 1.5 = 3\text{ V}$

$$P_{lamp} = \frac{V_{lamp}^2}{R_{lamp}} = \frac{(3)^2}{1} = 9\text{ W}$$

Problem 2.7

An automobile uses two halogen headlights connected as shown. Determine the power supplied by the battery if each headlight draws 3 A of current.

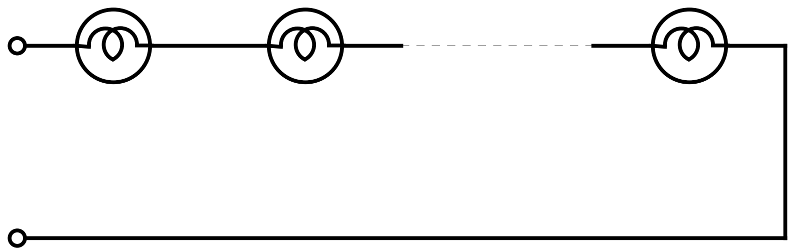


Suggested Solution

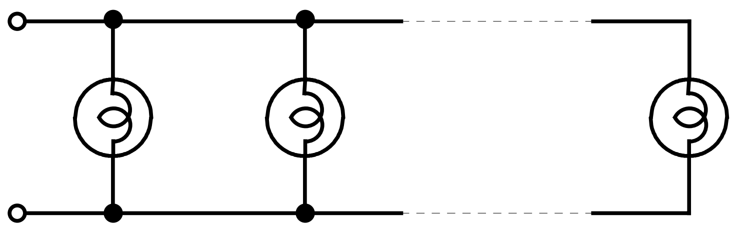
$P_S = VI = (12)(3+3) = 72 \text{ W}$

Problem 2.8

Many years ago a string of Christmas tree lights was manufactured in the form shown in (a). Today the lights are manufactured as shown in (b). Is there a good reason for this change?



(a)



(b)

Suggested Solution

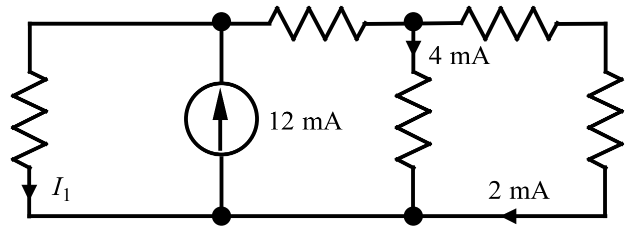
First of all, one must recognize the fact that a failed lamp becomes an open circuit. Then:

In (a), one faulty lamp would take out the whole string. There would be no way to identify the faulty lamp.

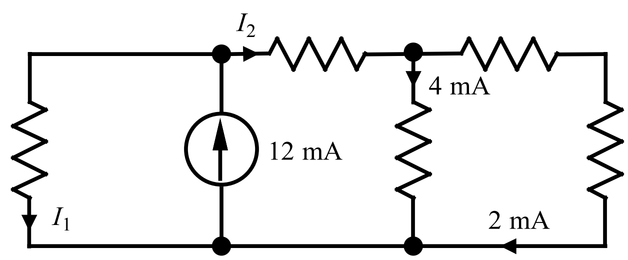
In (b), if one lamp fails, the others stay lit. It is easy to identify the faulty lamp.

Problem 2.9

Find I_1 in the network shown.



Suggested Solution



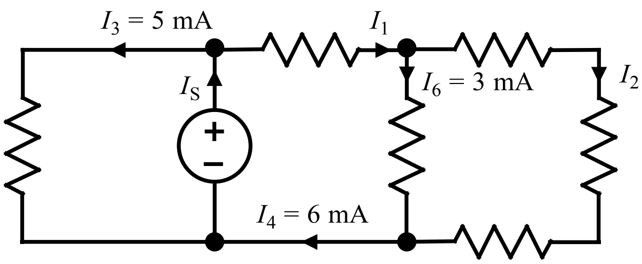
$I_2 = 0.004 + 0.002 = 0.006 \text{ A}$

$0.012 = I_1 + I_2$

$\Rightarrow \quad I_1 = 6 \text{ mA}$

Problem 2.10

Find I_1 and I_2 in the circuit shown.



Suggested Solution

By KCL, $I_1 = I_4 = 6$ mA

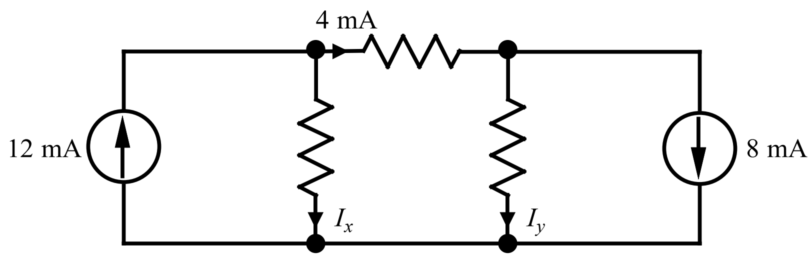
Also, $I_6 + I_2 = I_4$

or $0.003 + I_2 = 0.006$

$\Rightarrow \quad I_2 = 3$ mA

Problem 2.11

Find I_x and I_y in the network shown.



Suggested Solution

$$0.012 = 0.004 + I_x$$

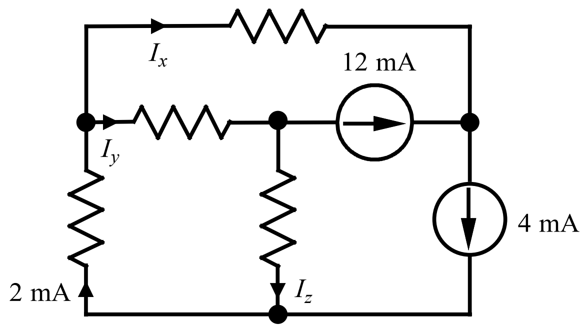
$$\Rightarrow \quad I_x = 8 \text{ mA}$$

$$0.004 = I_y + 0.008$$

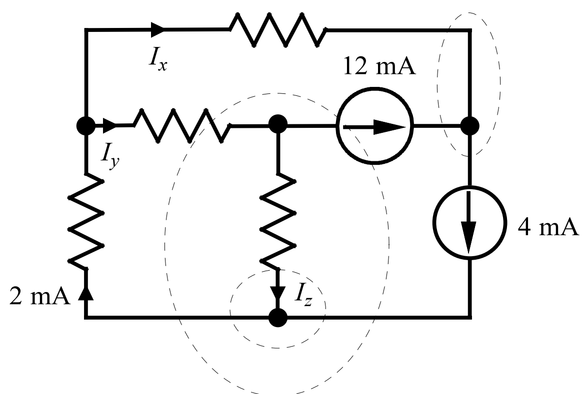
$$\Rightarrow \quad I_y = -4 \text{ mA}$$

Problem 2.12

Find I_x, I_y and I_z in the circuit shown.



Suggested Solution



$$I_x + 0.012 = 0.004$$

$$\Rightarrow \quad I_x = -8 \text{ mA}$$

$$I_y + 0.004 = 0.002 + 0.012$$

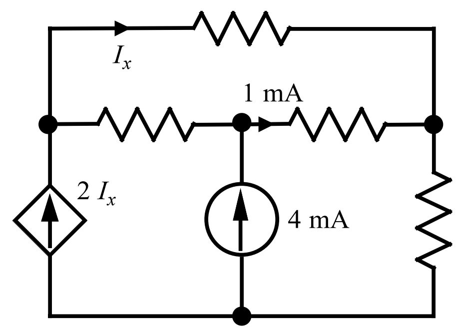
$$\Rightarrow \quad I_y = 10 \text{ mA}$$

$$I_z + 0.004 = 0.002$$

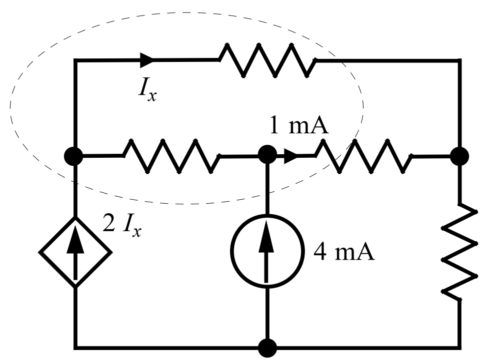
$$\Rightarrow \quad I_z = -2 \text{ mA}$$

Problem 2.13

Find I_x in the circuit shown.



Suggested Solution

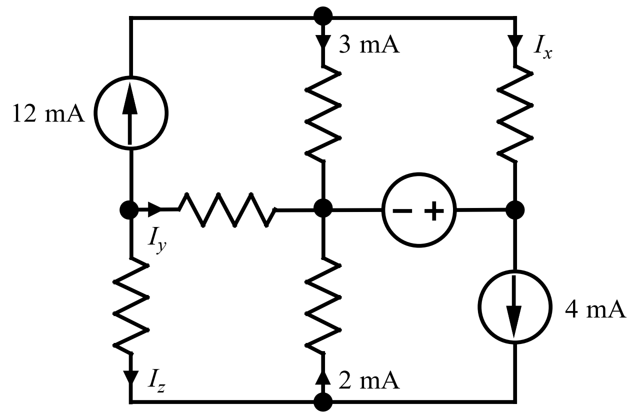


$$I_x + 0.001 = 0.004 + 2 I_x$$

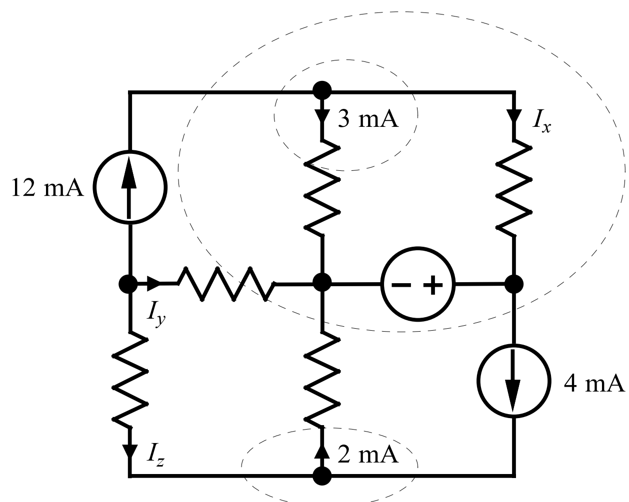
$$\Rightarrow \quad I_x = -3 \text{ mA}$$

Problem 2.14

Find I_x, I_y and I_z in the network shown.



Suggested Solution



$I_x + 0.003 = 0.012$

$\Rightarrow \quad I_x = 9 \text{ mA}$

$I_y + 0.012 + 0.002 = 0.004$

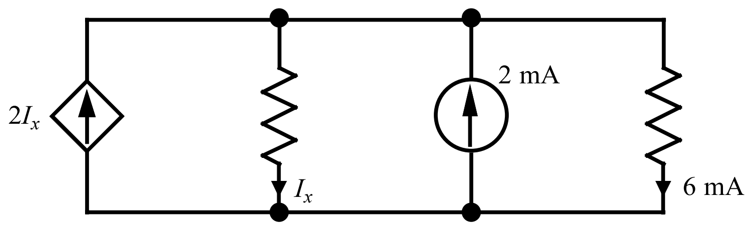
$\Rightarrow \quad I_y = -10 \text{ mA}$

$I_z + 0.004 = 0.002$

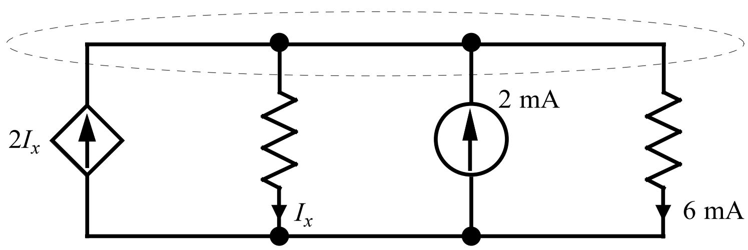
$\Rightarrow \quad I_z = -2 \text{ mA}$

Problem 2.15

Find I_x in the circuit shown.



Suggested Solution

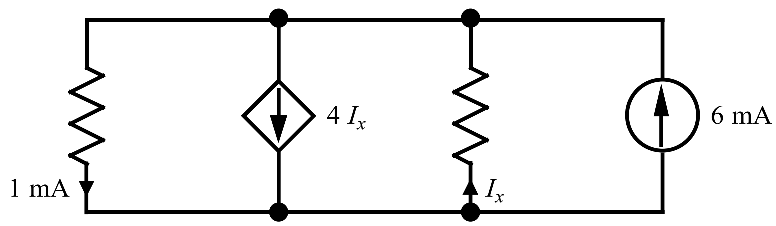


$$2I_x + 0.002 = I_x + 0.006$$

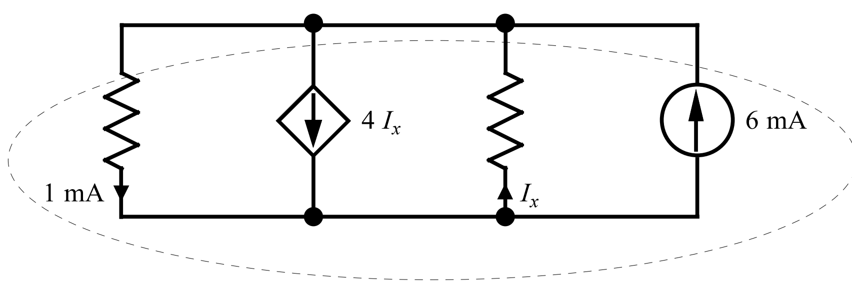
$$\Rightarrow \quad I_x = 4 \text{ mA}$$

Problem 2.16

Find I_x in the network shown.



Suggested Solution

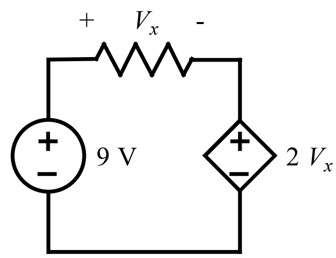


$$4I_x + 0.001 = I_x + 0.006$$

$$\Rightarrow \quad I_x = \frac{5}{3} \text{ mA}$$

Problem 2.17

Find V_x in the circuit shown.



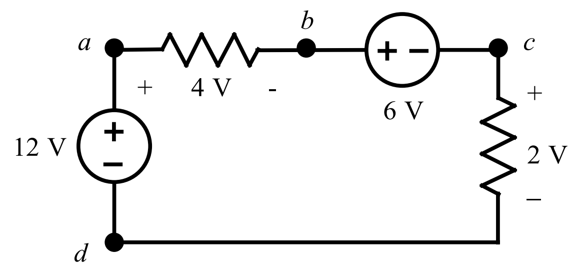
Suggested Solution

KVL: $-9+V_x+2V \ =0$

$\Rightarrow \quad V_x = 3\text{ V}$

Problem 2.18

Find V_{bd} in the circuit shown.



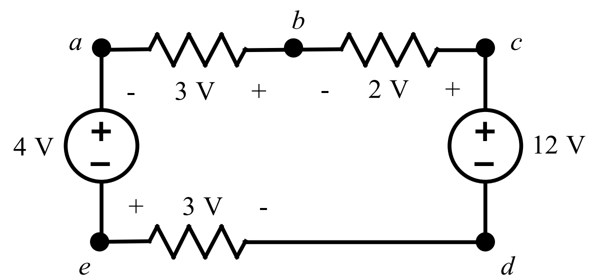
Suggested Solution

KVL: $-12+4+V_{bd}=0$

$\Rightarrow V_{bd}=8\text{ V}$

Problem 2.19

Find V_{ad} in the network shown.



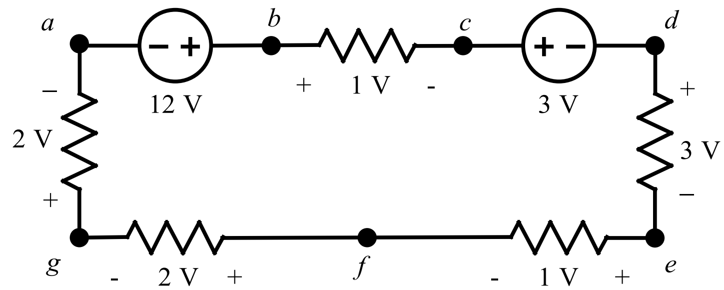
Suggested Solution

KVL: $-4+V_{ad}-3=0$

$\Rightarrow \; V_{ad}=7 \text{ V}$

Problem 2.20

Find V_{af} and V_{ec} in the circuit shown.



Suggested Solution

KVL: $2 + V_{af} + 2 = 0$

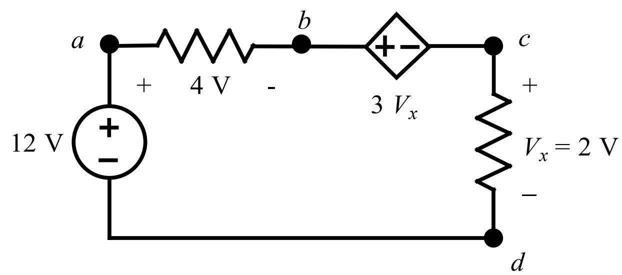
$\Rightarrow V_{af} = -4 \text{ V}$

KVL: $V_{ec} + 3 + 3 = 0$

$\Rightarrow V_{ec} = -6 \text{ V}$

Problem 2.21

Find V_{ac} in the circuit shown.



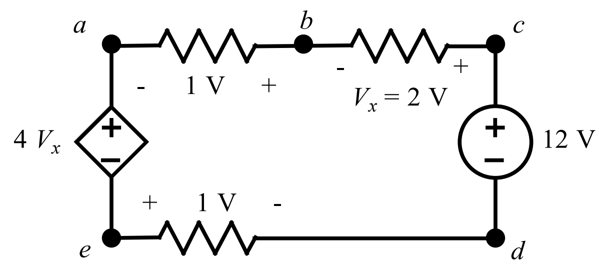
Suggested Solution

KVL: $V_{ac} + 2 - 12 = 0$

$\Rightarrow \; V_{ac} = 10 \text{ V}$

Problem 2.22

Find V_{ad} and V_{ce} in the circuit shown.



Suggested Solution

KVL: $V_{ad} - 12 + 2 + 1 = 0$

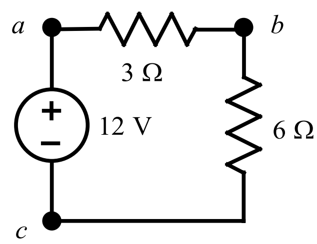
$\Rightarrow V_{ad} = 9\text{ V}$

KVL: $V_{ce} + 1 - 12 = 0$

$\Rightarrow V_{ce} = 11\text{ V}$

Problem 2.23

Find V_{ab} in the network shown.



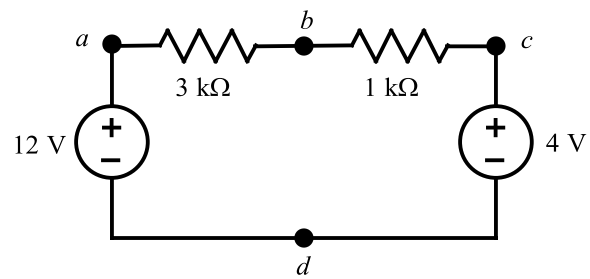
Suggested Solution

Using voltage division,

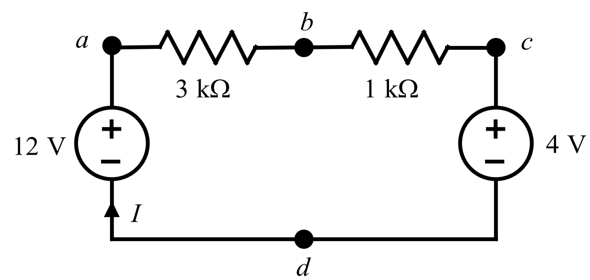
$$V_{ab} = \left(\frac{3}{3+6}\right)12 = 4 \text{ V}$$

Problem 2.24

Find V_{bd} in the network shown.



Suggested Solution



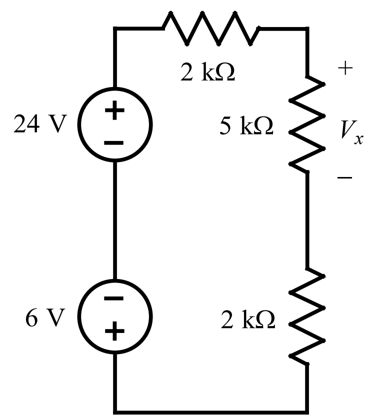
$$I = \frac{12-4}{3000+1000} = 2 \text{ mA}$$

KVL: $V_{bd} - 4 - 1000(0.002) = 0$

$\Rightarrow V_{bd} = 6 \text{ V}$

Problem 2.25

Find V_x in the circuit shown.



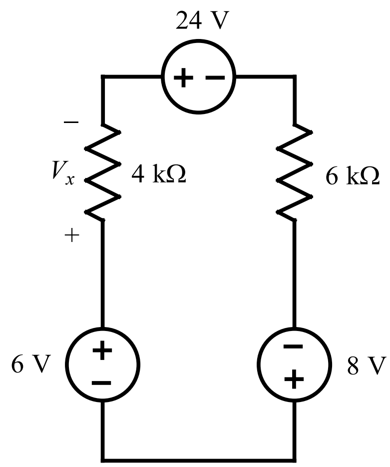
Suggested Solution

Using voltage division,

$$V_x = \left(\frac{5000}{2000 + 5000 + 2000} \right) (24 - 6) = 10 \text{ V}$$

Problem 2.26

Find V_x in the circuit shown.



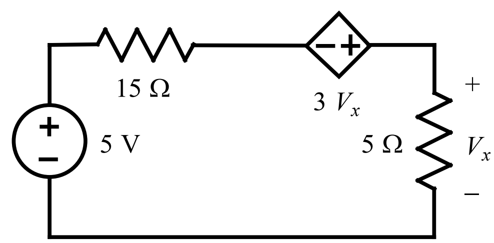
Suggested Solution

Using voltage division,

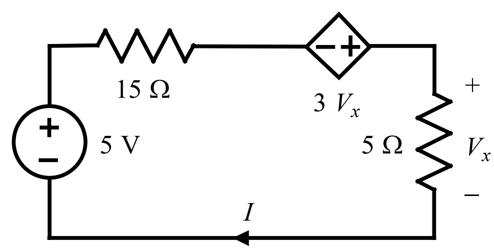
$$V_x = \left(\frac{4000}{4000+6000}\right)(6+8-24) = -4 \text{ V}$$

Problem 2.27

Find V_x in the network shown.



Suggested Solution



KVL: $-5+15I-3V_x+V_x=0$, where $V_x=5I$

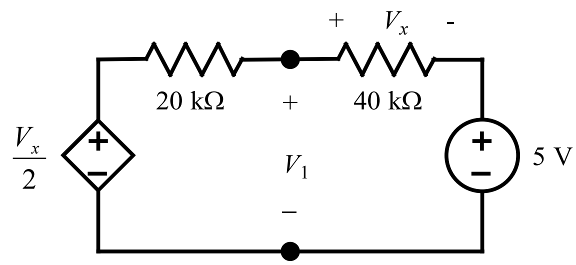
Therefore, $-5+15I-3(5I)+5I=0$

$\Rightarrow \quad I=1\text{ A}$

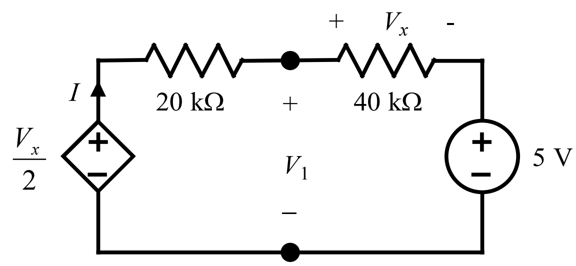
$\therefore \quad V_x=5\text{ V}$

Problem 2.28

Find V_1 in the network shown.



Suggested Solution



KVL: $-\frac{V_x}{2} + 20000\,I + 40000\,I + 5 = 0$, where $V_x = 40000\,I$

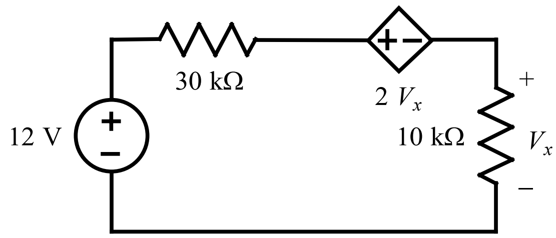
Therefore, $-\frac{40000\,I}{2} + 20000\,I + 40000\,I + 5 = 0$

$\Rightarrow \quad I = -0.125 \text{ mA}$

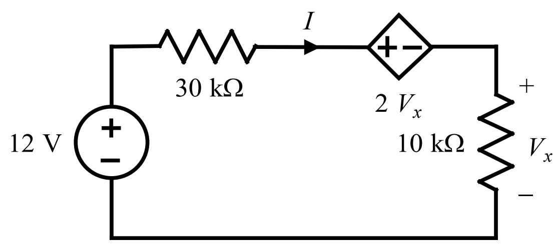
$V_1 = V_x + 5 = 40000\,I + 5 = -5 + 5 = 0 \text{ V}$

Problem 2.29

Find the power absorbed by the 30-kΩ resistor in the circuit shown.



Suggested Solution



KVL: $-12 + 30000\,I + 2V_x + V_x = 0$ where $V_x = 10000\,I$

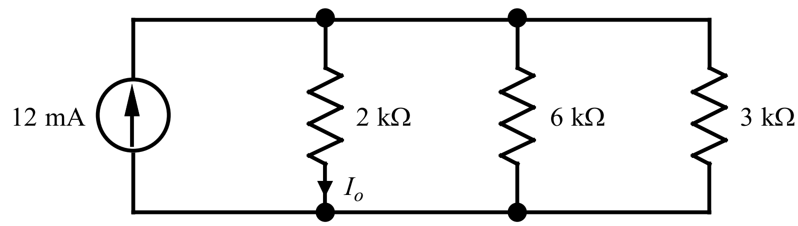
Therefore, $-12 + 30000\,I + 2(10000\,I) + 10000\,I = 0$

$\Rightarrow \quad I = 200\ \mu\text{A}$

$P_{30\,\text{k}\Omega} = I^2(30000)) = (200 \times 10^{-6})^2 (30000) = 1.2\ \text{mW}$

Problem 2.30

Find I_o in the network shown.

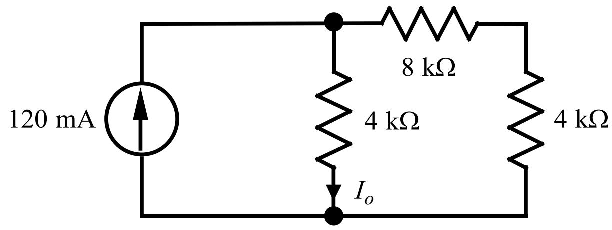


Suggested Solution

$$I_o = \left(\frac{\frac{1}{2000}}{\frac{1}{2000} + \frac{1}{6000} + \frac{1}{3000}} \right) (12 \text{ mA}) = 6 \text{ mA}$$

Problem 2.31

Find I_o in the circuit shown.



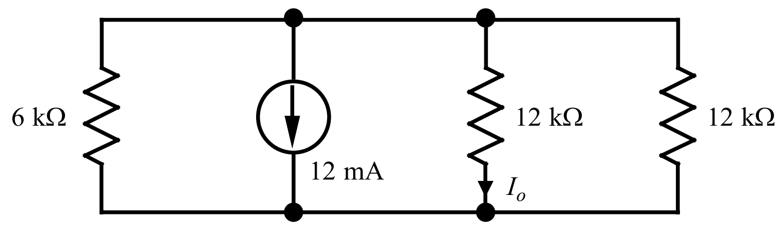
Suggested Solution

Using current division,

$$I_o = \left[\frac{\frac{1}{4000}}{\frac{1}{4000} + \frac{1}{8000 + 4000}} \right] (0.120) = 90 \text{ mA}$$

Problem 2.32

Find I_o in the network shown.



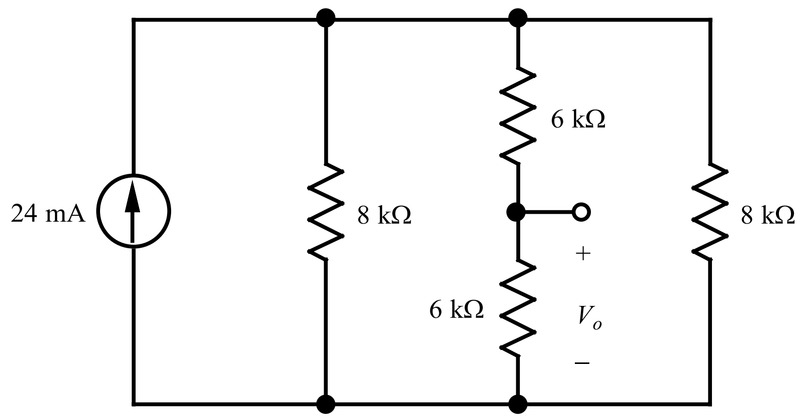
Suggested Solution

Using current division,

$$I_o = -\left(\frac{\frac{1}{12000}}{\frac{1}{6000}+\frac{1}{12000}+\frac{1}{12000}}\right)(0.012) = -3 \text{ mA}$$

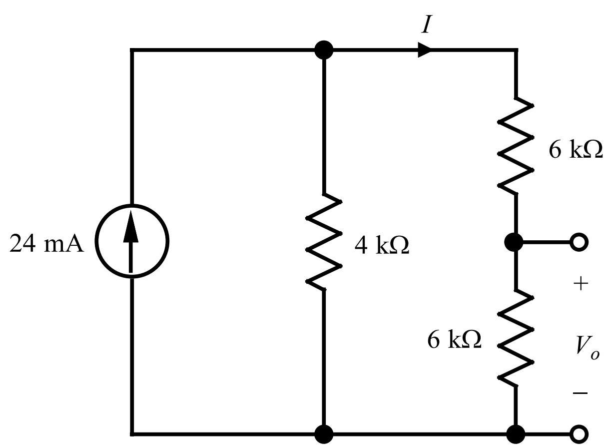
Problem 2.33

Find V_o in the circuit shown.



Suggested Solution

Combining the 8-kΩ resistors in parallel yields the following circuit.



Using current division,

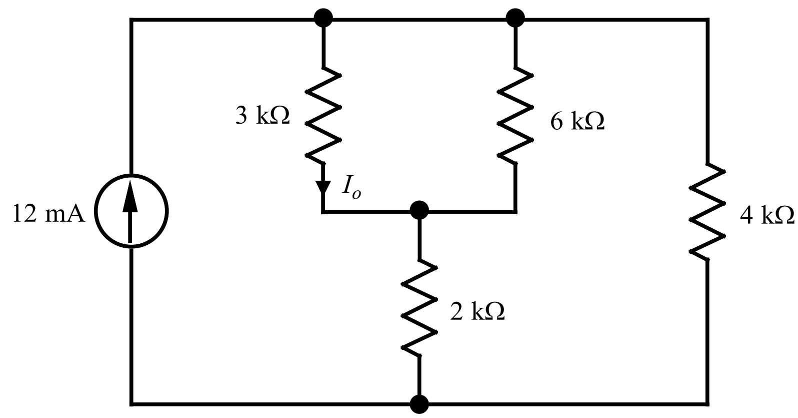
$$I = \left(\frac{\frac{1}{6000+6000}}{\frac{1}{4000} + \frac{1}{6000+6000}} \right)(0.024) = 6 \text{ mA}$$

Then,

$$V_o = 6000\,I = 36 \text{ V}$$

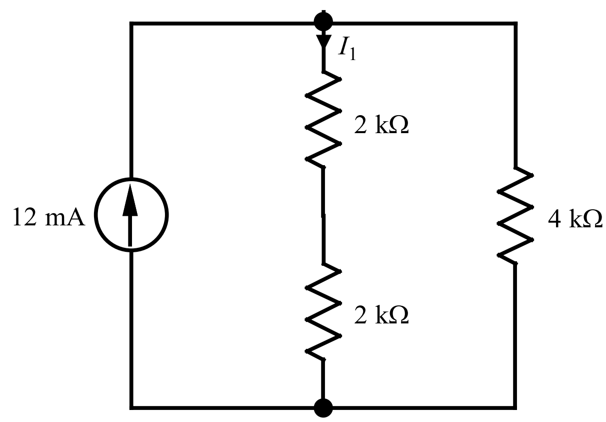
Problem 2.34

Find I_o in the network shown.



Suggested Solution

Combining the 3-kΩ and 6-kΩ resistors in parallel yields the following equivalent circuit.



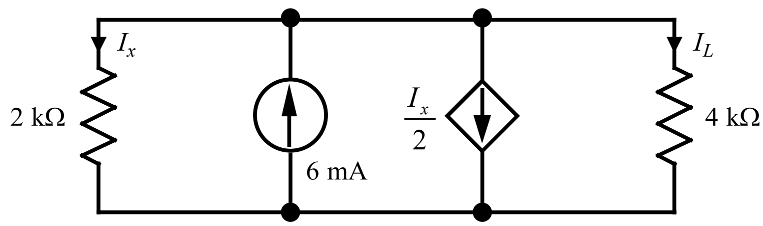
$$I_1 = \left[\frac{4000}{4000 + (2000 + 2000)}\right](0.012) = 6 \text{ mA}$$

Then,

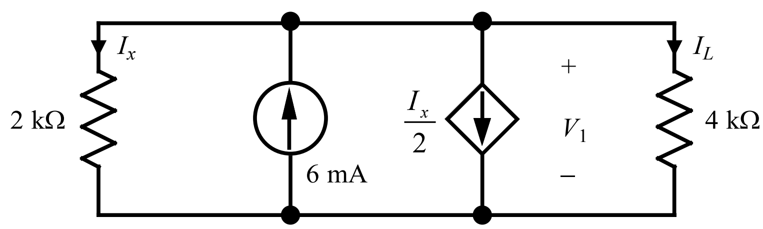
$$I_o = \left(\frac{6000}{6000 + 3000}\right) I_1 = 4 \text{ mA}$$

Problem 2.35

Determine I_L in the circuit shown.



Suggested Solution



KCL: $I_x - 0.006 + \frac{I_x}{2} + I_L = 0$, where $I_x = \frac{V_1}{2000}$ and $I_L = \frac{V_1}{4000}$

Therefore,

$$\frac{V_1}{2000} - 0.006 + \frac{V_1}{4000} + \frac{V_1}{4000} = 0$$

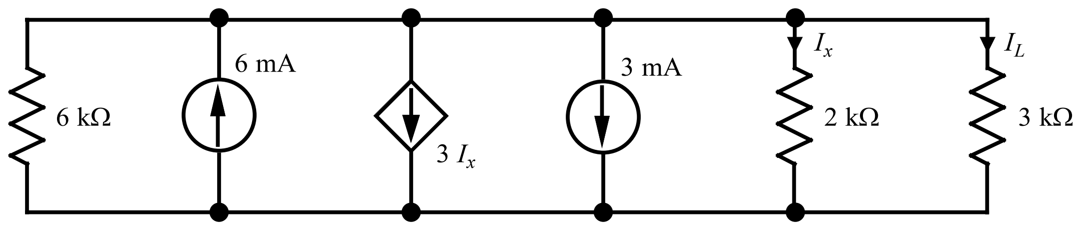
$$\Rightarrow \quad V_1 = 6 \text{ V}$$

Then,

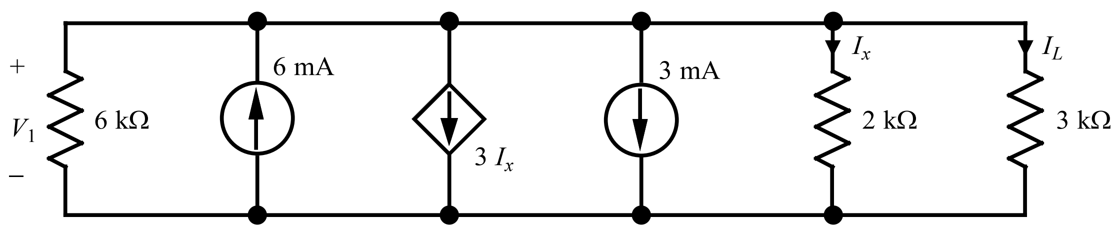
$$I_L = \frac{6}{4000} = 1.5 \text{ mA}$$

Problem 2.36

Determine I_L in the circuit shown.



Suggested Solution



KCL: $\frac{V_1}{6000} - 0.006 + 3I_x + 0.003 + I_x + I_L = 0$, where $I_x = \frac{V_1}{2000}$ and $I_L = \frac{V_1}{3000}$.

Therefore,

$$\frac{V_1}{6000} - 0.006 + 3\left(\frac{V_1}{2000}\right) + 0.003 + \frac{V_1}{2000} + \frac{V_1}{3000} = 0$$

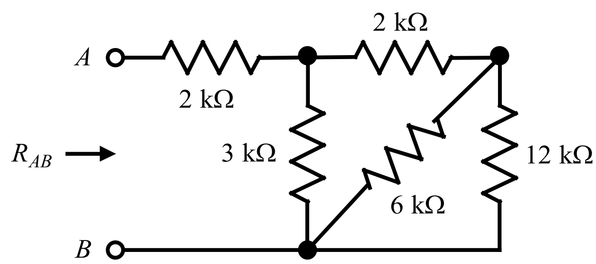
$$\Rightarrow \quad V_1 = \frac{6}{5} = 1.2 \text{ V}$$

and

$$I_L = \frac{1.2}{3000} = 0.4 \text{ mA}$$

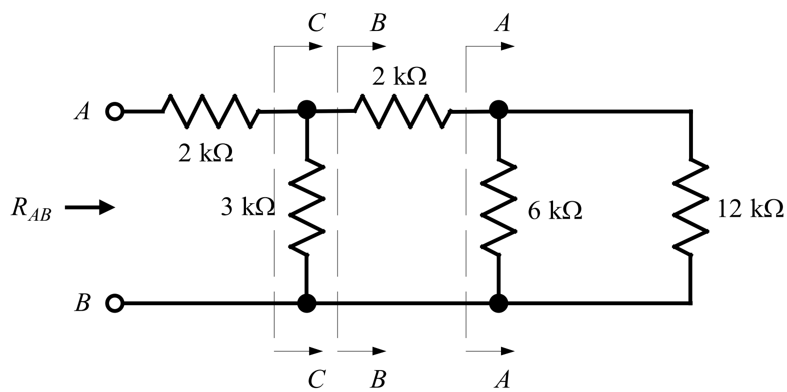
Problem 2.37

Find R_{AB} in the circuit shown.



Suggested Solution

The network can be redrawn as shown below.



Then,

At *A-A*: $6000 \| 12000 = 4 \text{ k}\Omega$

At *B-B*: $2000 + 4000 = 6 \text{ k}\Omega$

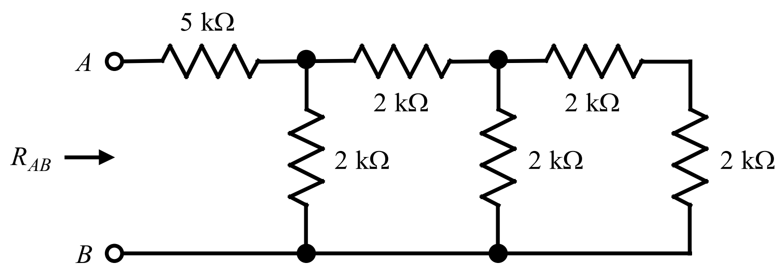
At *C-C*: $3000 \| 6000 = 2 \text{ k}\Omega$

and

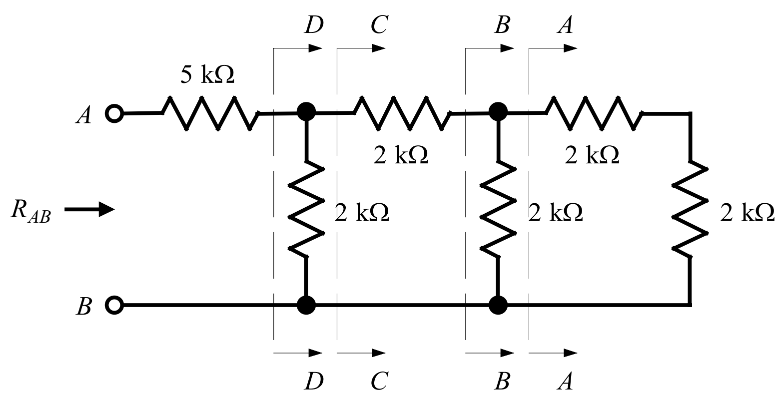
$R_{AB} = 2000 + 2000 = 4 \text{ k}\Omega$

Problem 2.38

Find R_{AB} in the circuit shown.



Suggested Solution



At *A*-*A*: $2000 + 2000 = 4\ \text{k}\Omega$

At *B*-*B*: $2000 \| 4000 = \frac{4}{3}\ \text{k}\Omega$

At *C*-*C*: $2000 + \frac{4000}{3} = \frac{10}{3}\ \text{k}\Omega$

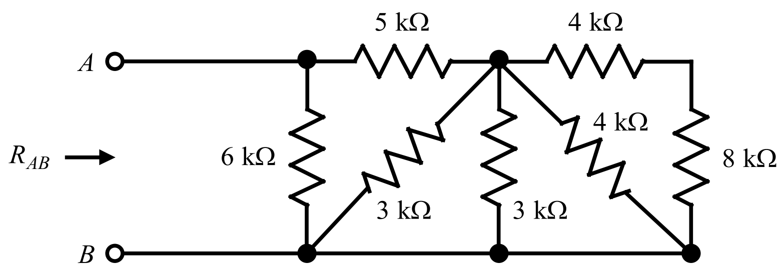
At *D*-*D*: $2000 \| \frac{10000}{3} = 1250\ \Omega$

Then,

$R_{AB} = 5000 + 1250 = 6250\ \Omega$

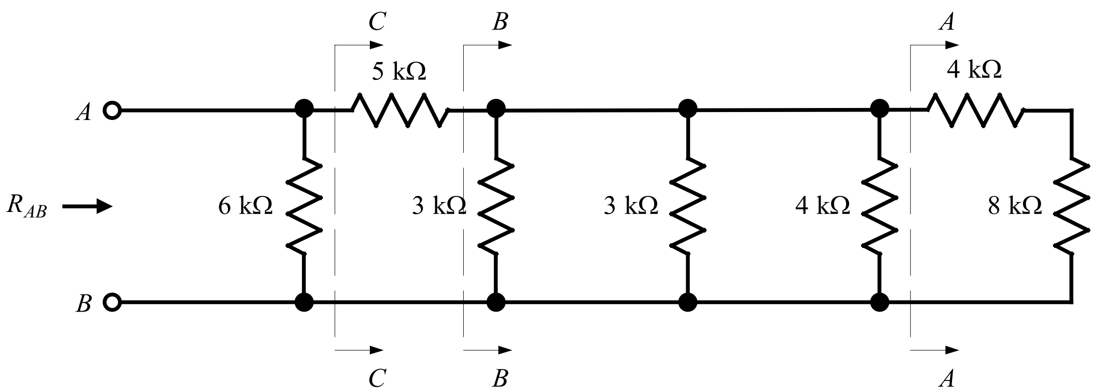
Problem 2.39

Find R_{AB} in the network shown.



Suggested Solution

The network can be redrawn as shown below.



At *A*-*A*: $4000 + 8000 = 12\ \text{k}\Omega$

At *B*-*B*: $3000 \| 3000 \| 4000 \| 12000 = 1\ \text{k}\Omega$

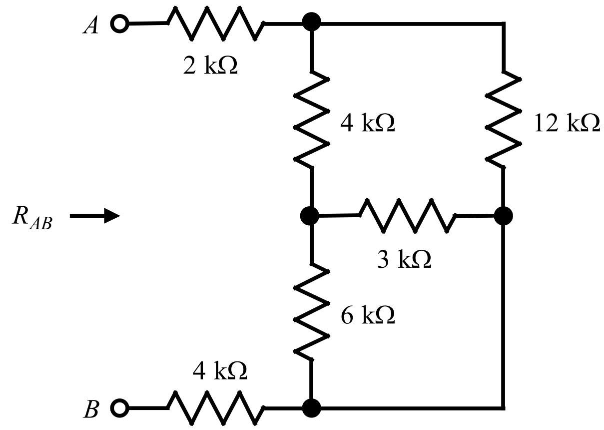
At *C*-*C*: $5000 + 1000 = 6\ \text{k}\Omega$

Therefore,

$R_{AB} = 6000 \| 6000 = 3\ \text{k}\Omega$

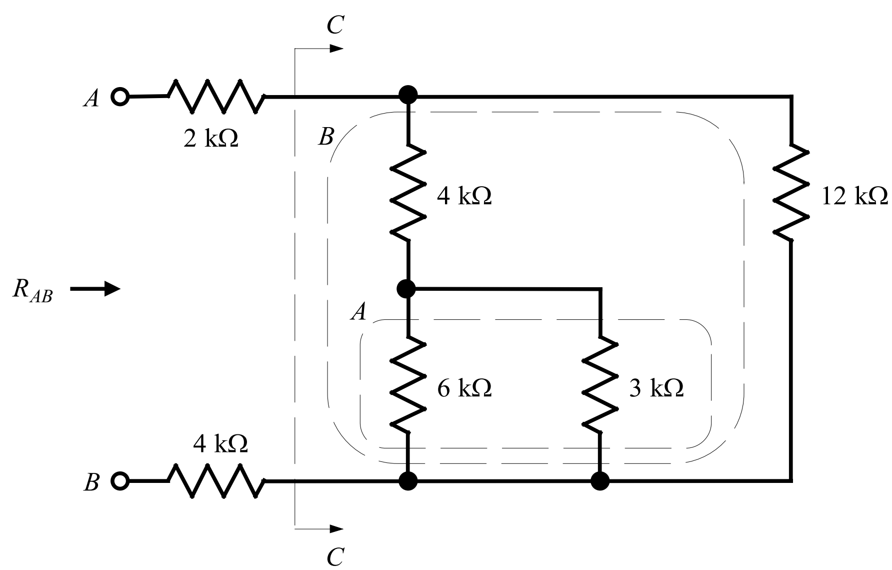
Problem 2.40

Find R_{AB} in the circuit shown.



Suggested Solution

The circuit can be redrawn as shown below.



At *A*: $6000 \| 3000 = 2\ \text{k}\Omega$

At *B*: $4000 + 2000 = 6\ \text{k}\Omega$

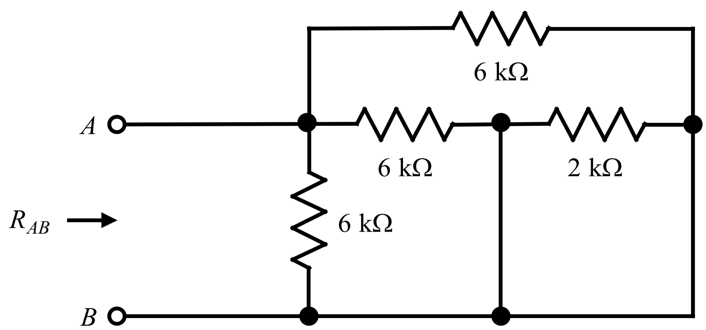
At *C*-*C*: $6000 \| 12000 = 4\ \text{k}\Omega$

Then,

$$R_{AB} = 2000 + 4000 + 4000 = 10\ \text{k}\Omega$$

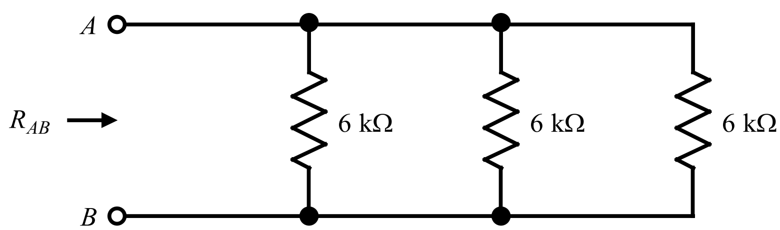
Problem 2.41

Find R_{AB} in the network shown.



Suggested Solution

The 2-kΩ resistor is shorted. Therefore, the network reduces to:

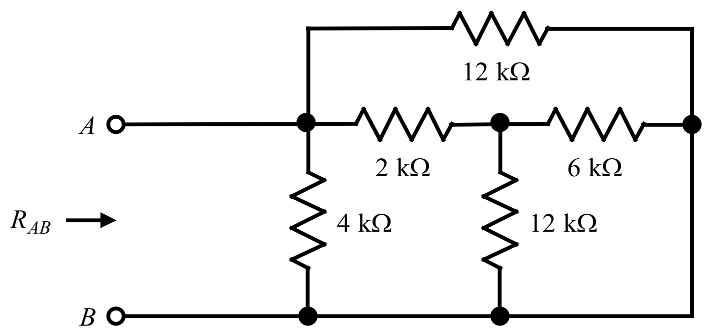


Then,

$$R_{AB} = 6000 \| 6000 \| 6000 = 2 \text{ k}\Omega$$

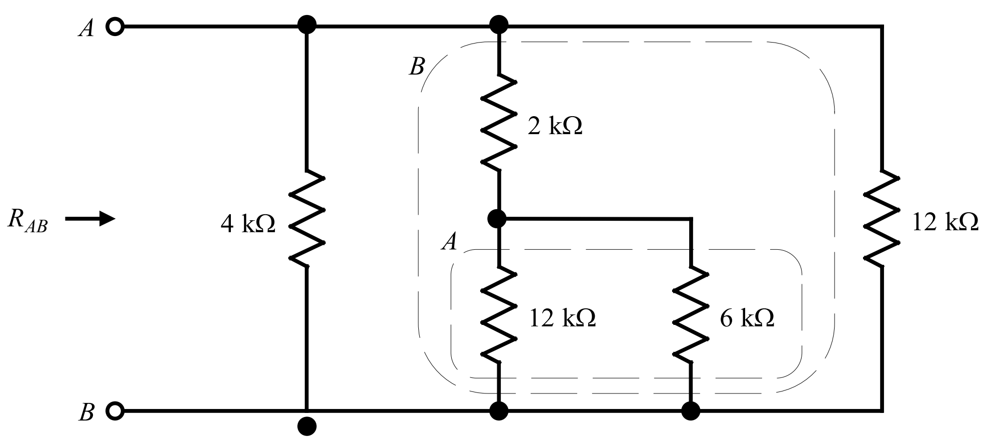
Problem 2.42

Find R_{AB} in the circuit shown.



Suggested Solution

The circuit maybe redrawn as follows:



At A: $12000 \| 6000 = 4 \text{ k}\Omega$

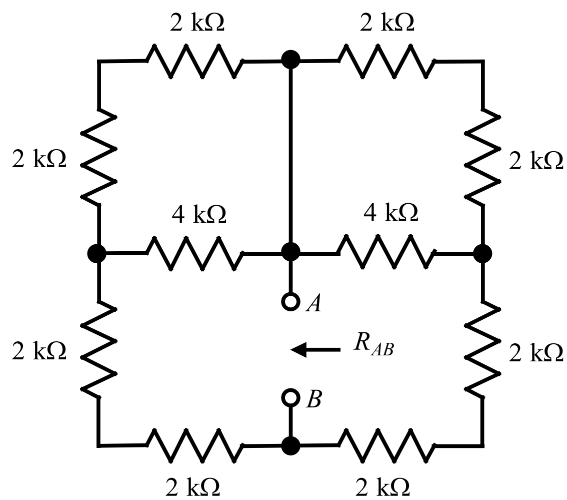
At B: $2000 + 4000 = 6 \text{ k}\Omega$

Then,

$$R_{AB} = 4000 \| 6000 \| 12000 = 2 \text{ k}\Omega$$

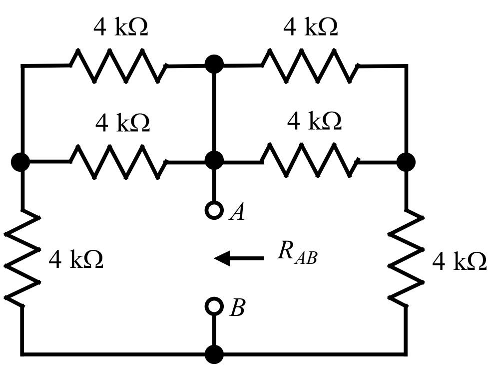
Problem 2.43

Find R_{AB} in the circuit shown.

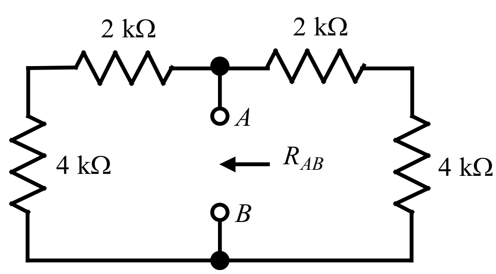


Suggested Solution

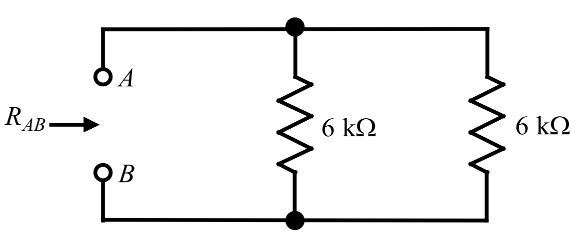
Combining each series pair of 2-kΩ resistors, the circuit can be redrawn as follows:



⇒



or



Then,

$R_{AB} = 6000 \| 6000 = 3 \text{ k}\Omega$

Problem 2.44

Find the range of resistance for the following resistors:

a) $1\ \text{k}\Omega$ with a tolerance of 5%.
b) $470\ \Omega$ with a tolerance of 2%.
c) $22\ \text{k}\Omega$ with a tolerance of 10%.

Suggested Solution

a) Minimum value = $(1-0.05)(1\ \text{k}\Omega) = 950\ \Omega$
   Maximum value = $(1+0.05)(1\ \text{k}\Omega) = 1050\ \Omega$

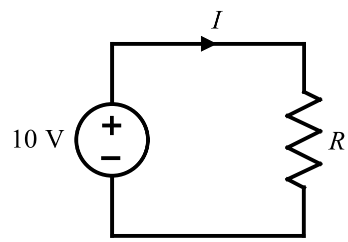
b) Minimum value = $(1-0.02)(470\ \Omega) = 460.6\ \Omega$
   Maximum value = $(1+0.02)(470\ \Omega) = 479.4\ \Omega$

c) Minimum value = $(1-0.1)(22\ \text{k}\Omega) = 19.8\ \text{k}\Omega$
   Maximum value = $(1+0.1)(22\ \text{k}\Omega) = 24.2\ \text{k}\Omega$

Problem 2.45

Given the network shown, find the possible range of values for the current and power dissipated by the following resistors:

a) $390\ \Omega$ with a tolerance of 1%.
b) $560\ \Omega$ with a tolerance of 2%.



Suggested Solution

Note that $I = \frac{10\text{ V}}{R}$ and $P = \frac{(10\text{ V})^2}{R} = \frac{100}{R}$.

a) Minimum resistor value = $(1-0.01)(390\ \Omega) = 386.1\ \Omega$
Maximum resistor value = $(1+0.01)(390\ \Omega) = 393.9\ \Omega$

Minimum current value = $\frac{10}{393.9} = 25.39$ mA

Maximum current value = $\frac{10}{386.1} = 25.90$ mA

Minimum power value = $\frac{100}{393.9} = 253.9$ mW

Maximum power value = $\frac{100}{386.1} = 259$ mW

b) Minimum resistor value = $(1-0.02)(560\ \Omega) = 548.8\ \Omega$
Maximum resistor value = $(1+0.02)(560\ \Omega) = 571.2\ \Omega$

Minimum current value = $\frac{10}{571.2} = 17.51$ mA

Maximum current value = $\frac{10}{548.8} = 18.22$ mA

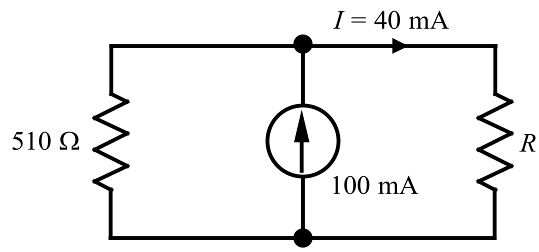
Minimum power value = $\frac{100}{571.2} = 175.1$ mW

Maximum power value = $\frac{100}{548.8} = 182.2$ mW

Problem 2.46

Given the circuit shown:

a) Find the required value of R .
b) Use Table 2.1 to select a standard 10% tolerance resistor for R .
c) Calculate the actual value of I .
d) Determine the percent error between the actual value of I and that shown in the circuit.
e) Determine the power rating for the resistor R .



Suggested Solution

a) From KCL, the current in the $510\text{-}\Omega$ resistor is $100\text{ mA} - 40\text{ mA} = 60\text{ mA}$. From Ohm's Law, the voltage across the circuit is $(510\ \Omega)(60\text{ mA}) = 30.6\text{ V}$. Therefore, the required value of R is $(30.6\text{ V})/(40\text{ mA}) = 765\ \Omega$.

b) From Table 2.1, the nearest 10% resistor value is $820\ \Omega$.

c) To find the actual value of I, first find the actual voltage across the circuit, which is $(100\text{ mA})(510\ \Omega \| 820\ \Omega) = 31.44\text{ V}$. Then, $I = \dfrac{31.44\text{ V}}{820\ \Omega} = 38.35\text{ mA}$.

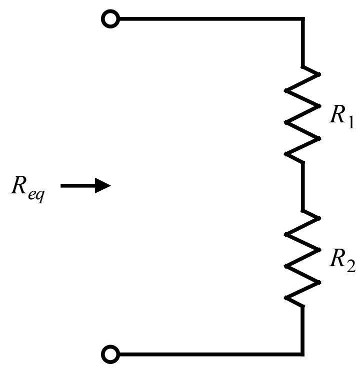
d) The percent error is $\dfrac{(38.35 - 40)}{40}(100) = -4.14\ \%$.

e) Since the power consumption in R is $\dfrac{(31.44\text{ V})^2}{820\ \Omega} = 1.206\text{ W}$, a 2-W resistor should be used.

Problem 2.47

The resistors R_1 and R_2 shown in the circuit are $1\ \Omega$ with a tolerance of 5% and $2\ \Omega$ with a tolerance of 10%, respectively.

a) What is the nominal value of the equivalent resistance?
b) Determine the positive and negative tolerance for the equivalent resistance.



Suggested Solution

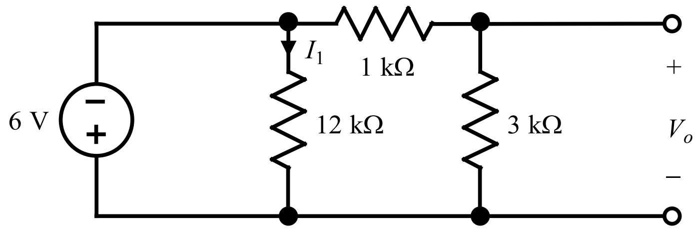
a) The nominal value is $R_{eq} = R_1 + R_2 = 3\ \Omega$
b) The minimum and maximum values of R_1 are $0.95\ \Omega$ and $1.05\ \Omega$, and the minimum and maximum values of R_2 are $1.8\ \Omega$ and $2.2\ \Omega$. Thus, the minimum and maximum values of R_{eq} are $0.95 + 1.8 = 2.75\ \Omega$ and $1.05 + 2.2 = 3.25\ \Omega$. The positive and negative tolerances are

$$\text{Positive Tolerance} = \frac{(3.25-3)}{3}(100) = 8.33\ \%$$

$$\text{Negative Tolerance} = \frac{(2.75-3)}{3}(100) = -8.33\ \%$$

Problem 2.48

Find I_1 and V_o in the circuit shown.



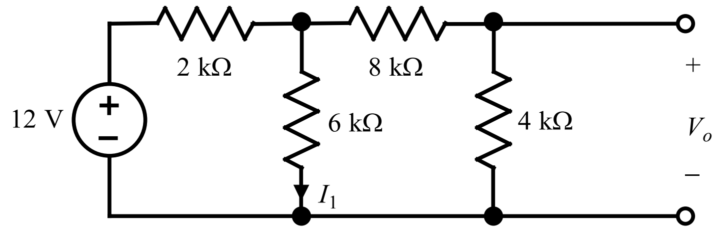
Suggested Solution

From Ohm's Law: $I_1 = \frac{-6\text{ V}}{12\text{ k}\Omega} = -0.5\text{ mA}$

By application of voltage division: $V_o = \left(\frac{3\text{ k}\Omega}{3\text{ k}\Omega + 1\text{ k}\Omega}\right)(-6\text{ V}) = -4.5\text{ V}$

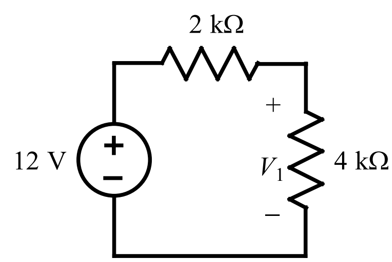
Problem 2.49

Find I_1 and V_o in the circuit shown.



Suggested Solution

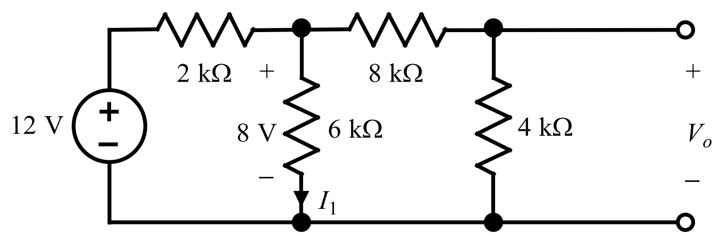
Combining resistors $\left[6\text{ k}\Omega \| (8\text{ k}\Omega + 4\text{ k}\Omega) = 4\text{ k}\Omega\right]$ reduces the network to the following:



Using voltage division, then

$$V_1 = \left(\frac{4000}{2000+4000}\right)(12\text{ V}) = 8\text{ V}.$$

Looking back at the original circuit,

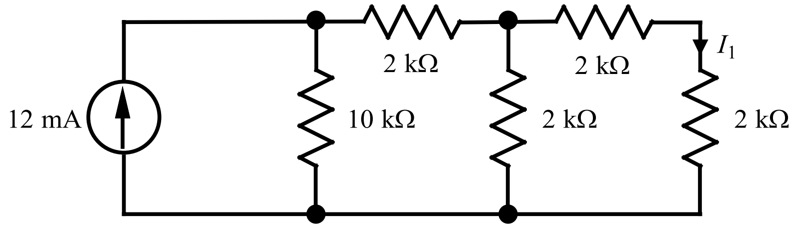


Ohm's Law: $I_1 = \dfrac{8\text{ V}}{6\text{ k}\Omega} = \dfrac{4}{3}\text{ mA}$

Voltage division: $V_o = \left(\dfrac{4000}{8000+4000}\right)(8) = \dfrac{8}{3}\text{ V}$

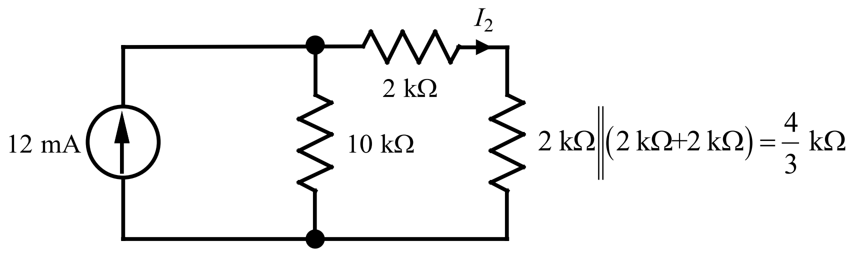
Problem 2.50

Find I_1 in the circuit shown.



Suggested Solution

The circuit can be simplified as follows:

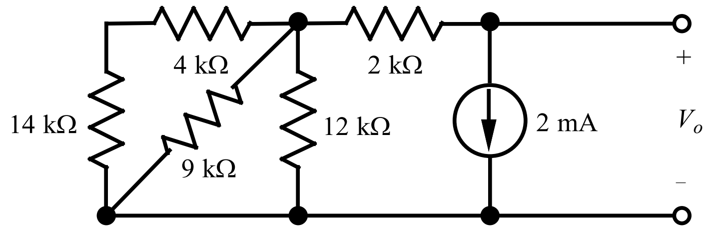


Then, $I_2 = \left[\dfrac{10000}{10000 + \left(2000 + \dfrac{4000}{3}\right)}\right](0.012) = 9\text{ mA}$

and $I_1 = \left[\dfrac{2000}{2000 + (2000 + 2000)}\right](9\text{ mA}) = 3\text{ mA}$

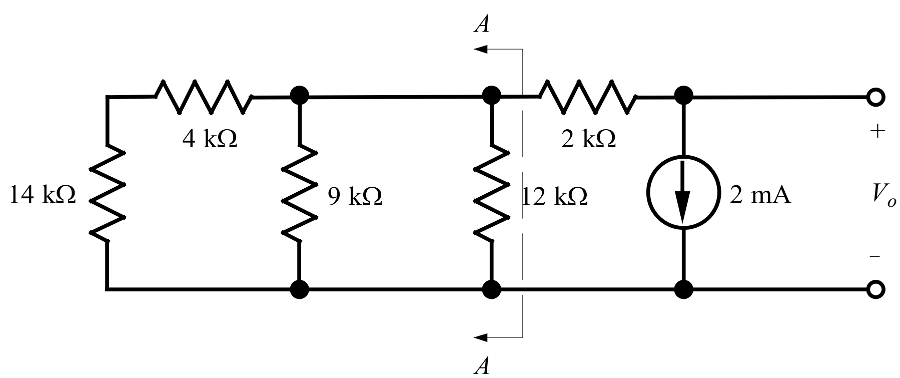
Problem 2.51

Determine V_o in the network shown.

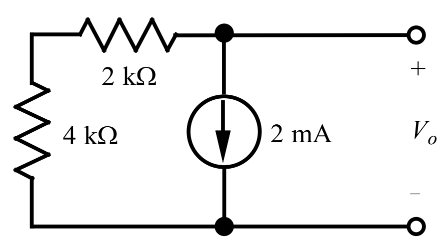


Suggested Solution

The network can be redrawn as:



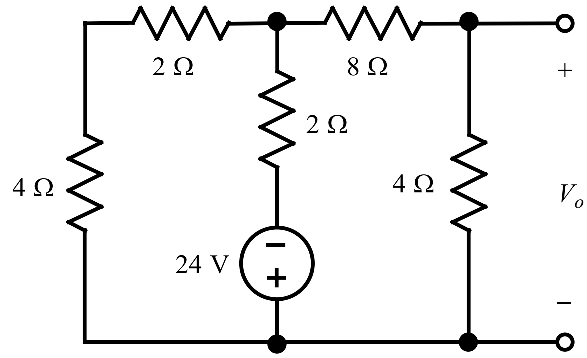
The equivalent resistance to the left of *A*-*A* is $(14\text{ k}\Omega + 4\text{ k}\Omega)\|9\text{ k}\Omega\|12\text{ k}\Omega = 4\text{ k}\Omega$.



Then, $V_o = -(2\text{ k}\Omega + 4\text{ k}\Omega)(2\text{ mA}) = -12\text{ V}$.

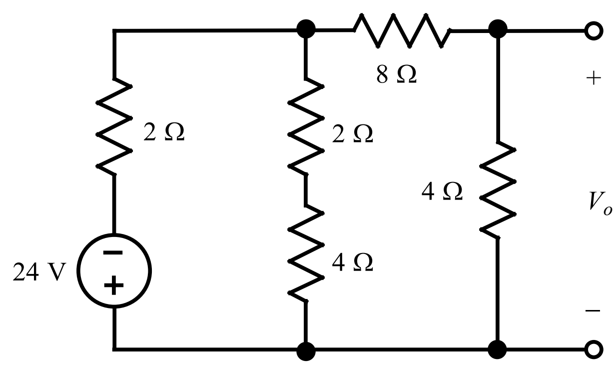
Problem 2.52

Find V_o in the network shown.

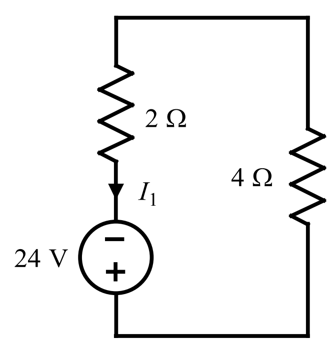


Suggested Solution

The network can be redrawn as:

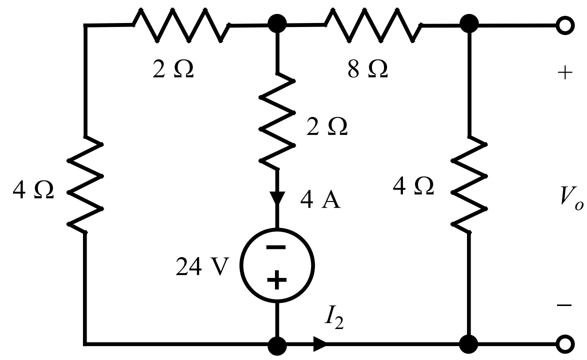


Combining the four resistors on the right-hand side $\left[(4\ \Omega + 2\ \Omega)\|(8\ \Omega + 4\ \Omega) = 4\ \Omega\right]$ yields:



and $I_1 = \dfrac{24\ \text{V}}{(2\ \Omega + 4\ \Omega)} = 4\ \text{A}$.

Then, reconsidering the original circuit,



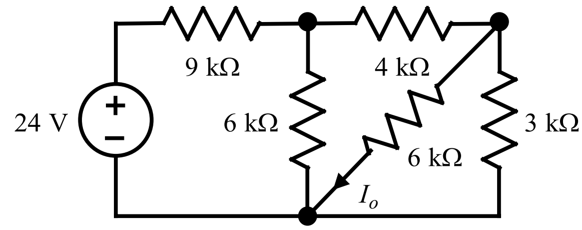
$$I_2 = \left[\frac{(4+2)}{(4+2)+(8+4)}\right](4\ \text{A}) = \frac{4}{3}\ \text{A}$$

and

$$V_o = -(4\ \text{A})\left(\frac{4}{3}\ \text{A}\right) = -\frac{16}{3}\ \text{V}$$

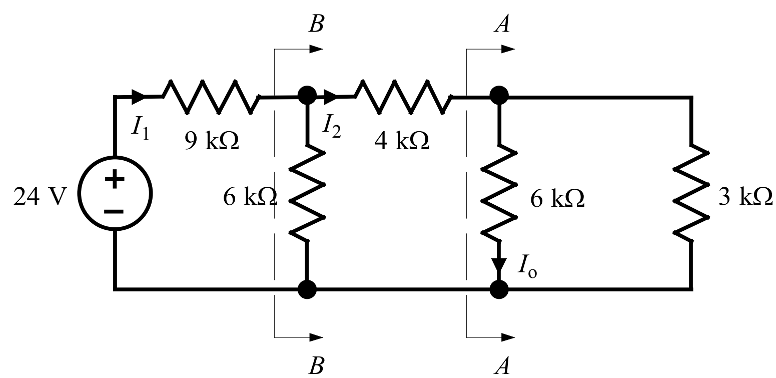
Problem 2.53

Find I_o in the circuit shown.



Suggested Solution

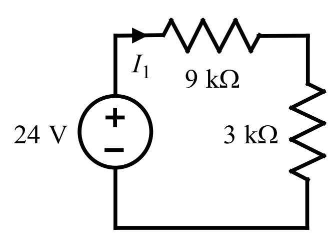
The circuit can be redrawn as:



At *A-A*: $6000\|3000 = 2\text{ k}\Omega$

At *B-B*: $6000\|(4000+2000) = 3\text{ k}\Omega$

The circuit simplifies to:

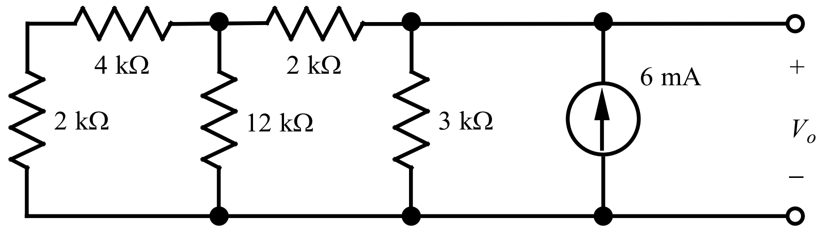


Then, $I_1 = \dfrac{24}{9000+3000} = 2\text{ mA} \Rightarrow I_2 = \left[\dfrac{6000}{6000+(4000+2000)}\right]I_1 = 1\text{ mA}$

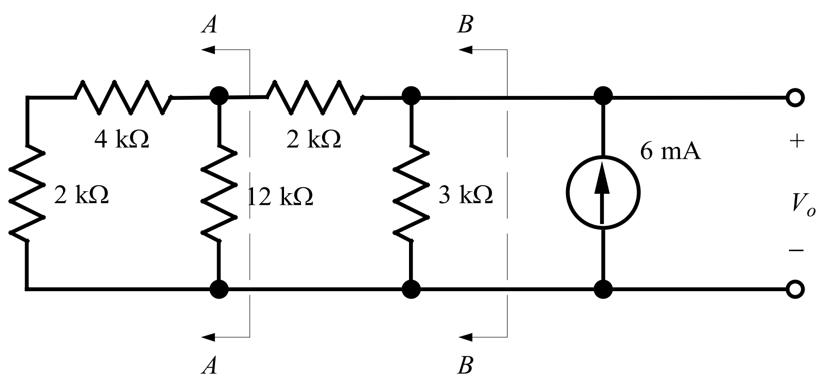
and $I_o = \left(\dfrac{3000}{3000+6000}\right)I_2 = \dfrac{1}{3}\text{ mA}$.

Problem 2.54

Find V_o in the circuit shown.



Suggested Solution



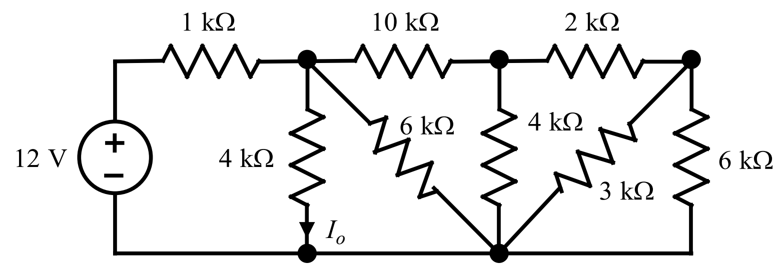
At *A-A*: $(2000+4000)\|12000 = 4\text{ k}\Omega$

At *B-B*: $(4000+2000)\|3000 = 2\text{ k}\Omega$

Then, $V_o = (2\text{ k}\Omega)(6\text{ mA}) = 12\text{ V}$.

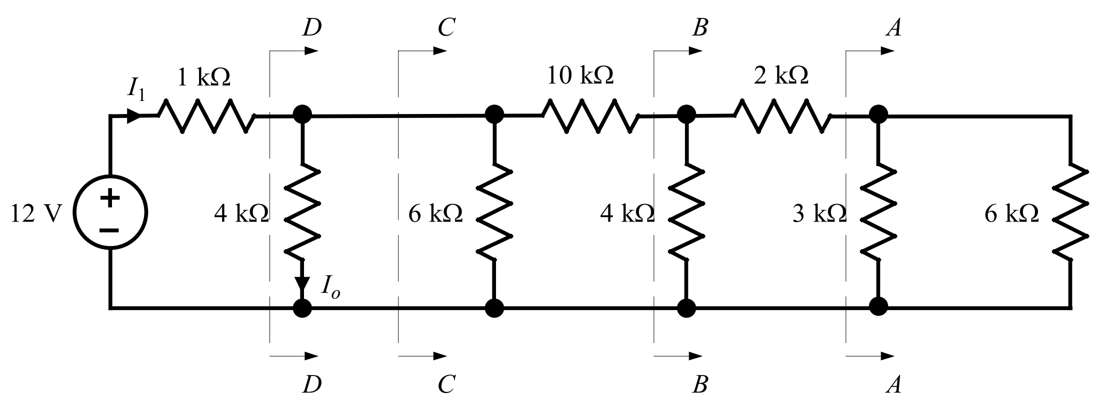
Problem 2.55

Find I_o in the network shown.



Suggested Solution

Redrawing the network:



At *A*-*A*: $3000 \| 6000 = 2\ \text{k}\Omega$

At *B*-*B*: $4000 \| (2000 + 2000) = 2\ \text{k}\Omega$

At *C*-*C*: $6000 \| (10000 + 2000) = 4\ \text{k}\Omega$

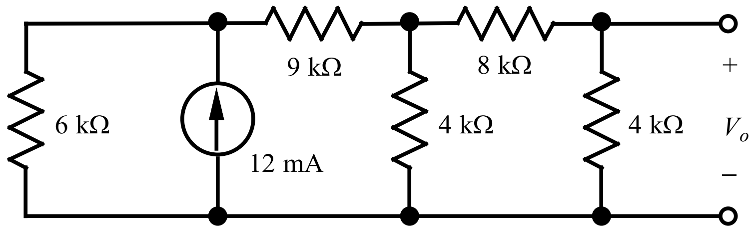
At *D*-*D*: $4000 \| 4000 = 2\ \text{k}\Omega$

$$I_1 = \frac{12}{1000 + 2000} = 4\ \text{mA}$$

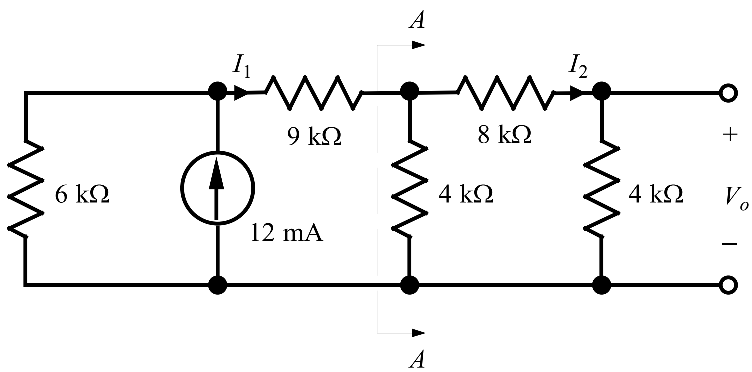
Using current division, $I_o = \left(\frac{4000}{4000 + 4000} \right) I_1 = 2\ \text{mA}$

Problem 2.56

Find V_o in the network shown.



Suggested Solution



At *A*-*A*: $4000 \| (8000+4000) = 3 \text{ k}\Omega$

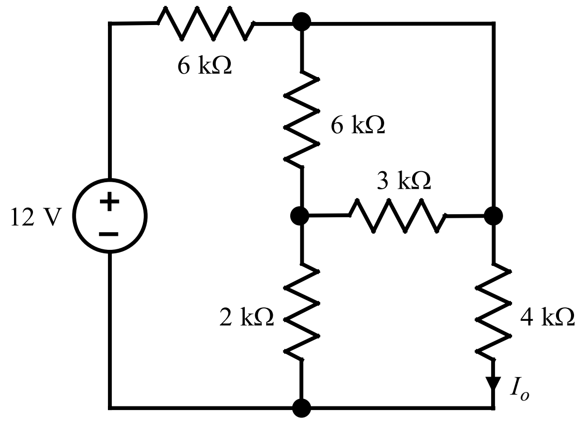
Using current division, $I_1 = \left[\dfrac{6000}{6000+(9000+3000)}\right]0.012 = 4 \text{ mA}$

Again using current division, $I_2 = \left[\dfrac{4000}{4000+(8000+4000)}\right]I_1 = 1 \text{ mA}$

Then, $V_o = 4000\,I_2 = 4 \text{ V}$.

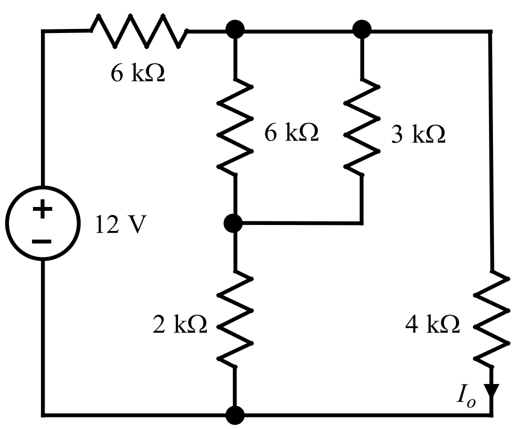
Problem 2.57

Find I_o in the circuit shown.

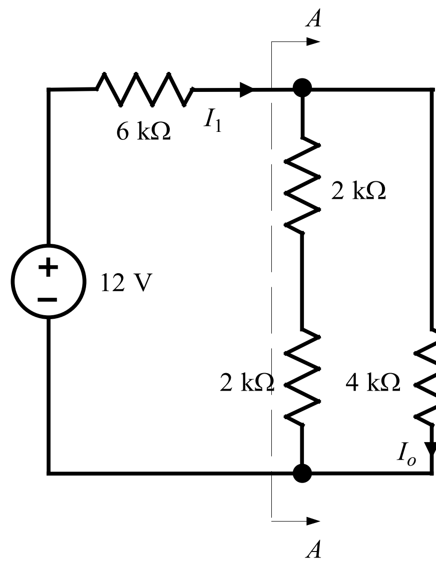


Suggested Solution

The circuit can be redrawn as:



⇒



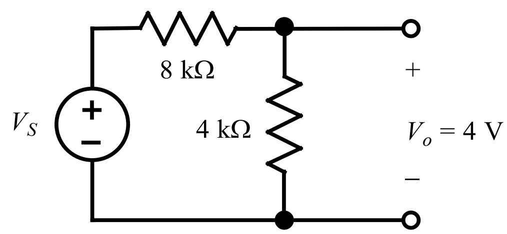
At *A*-*A*: $(2000+2000)\|4000 = 2\text{ k}\Omega$

$$I_1 = \frac{12}{6000+2000} = 1.5\text{ mA}$$

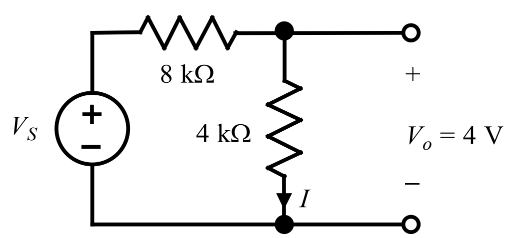
Using current division, $I_o = \left[\frac{(2000+2000)}{(2000+2000)+4000}\right] I_1 = 0.75\text{ mA}$

Problem 2.58

If $V_o = 4\text{ V}$ in the network shown, find V_S.



Suggested Solution

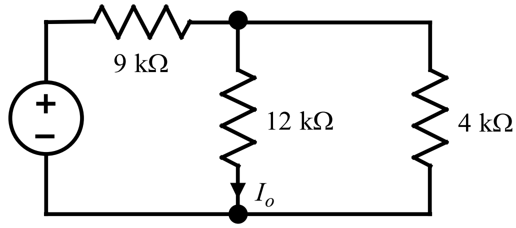


$$I = \frac{V_o}{4000} = \frac{4}{4000} = 1\text{ mA}$$

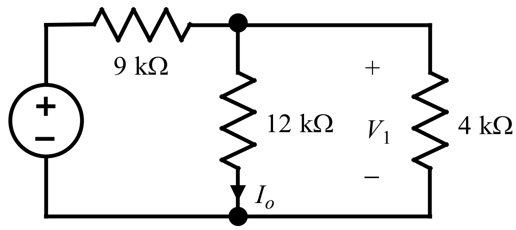
Then, $V_S = 8000\,I + V_o = 8 + 4 = 12\text{ V}$.

Problem 2.59

If the power absorbed by the 4-kΩ resistor in the network shown is 36 mW, find I_o.



Suggested Solution

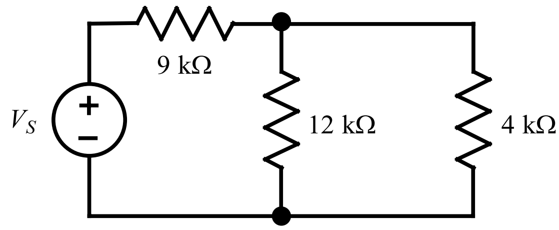


$$P_{4\text{ k}\Omega} = \frac{V_1^2}{4\text{ k}\Omega} = 36\text{ mW} \quad \Rightarrow \quad V_1 = 12\text{ V}$$

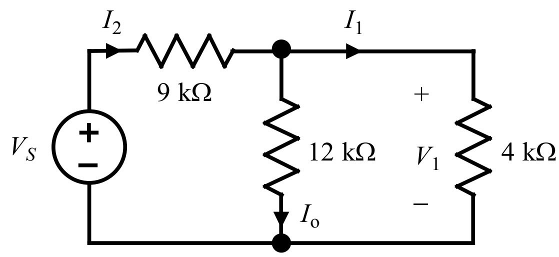
$$I_o = \frac{12\text{ V}}{12\text{ k}\Omega} = 1\text{ mA}$$

Problem 2.60

If the power absorbed by the 4-kΩ resistor in the circuit shown is 36 mW, find V_S.



Suggested Solution



$$P_{4\text{ k}\Omega} = 36\text{ mW} = \frac{V_1^2}{4000} \quad \Rightarrow \quad V_1 = 12\text{ V}$$

$$I_o = \frac{V_1}{12\text{ k}\Omega} = 1\text{ mA}$$

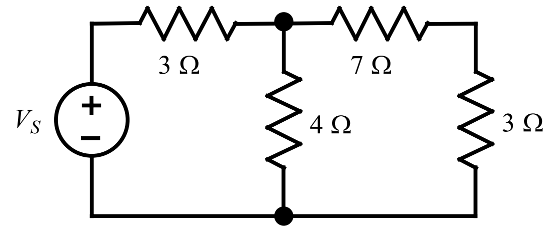
$$I_1 = \frac{V_1}{4\text{ k}\Omega} = 3\text{ mA}$$

$$I_2 = I_o + I_1 = 1\text{ mA} + 3\text{ mA} = 4\text{ mA}$$

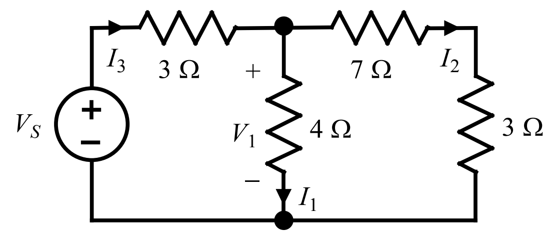
$$V_S = (9\text{ k}\Omega)I_2 + V_1 = 48\text{ V}$$

Problem 2.61

In the network shown, the power absorbed by the 4-Ω resistor is 100 W. Find V_S.



Suggested Solution



$$P_{4\,\Omega} = I_1^2 (4\ \Omega) = 100\ \text{W} \quad \Rightarrow \quad I_1 = 5\ \text{A}$$

$$V_1 = 4 I_1 = 20\ \text{V}$$

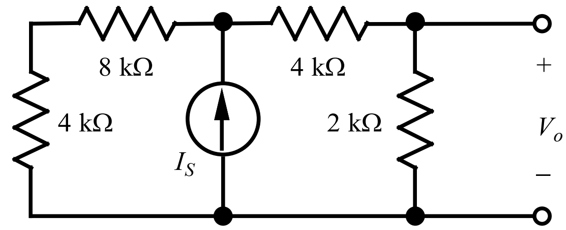
$$I_2 = \frac{V_1}{7\ \Omega + 3\ \Omega} = 2\ \text{A}$$

$$I_3 = I_1 + I_2 = 7\ \text{A}$$

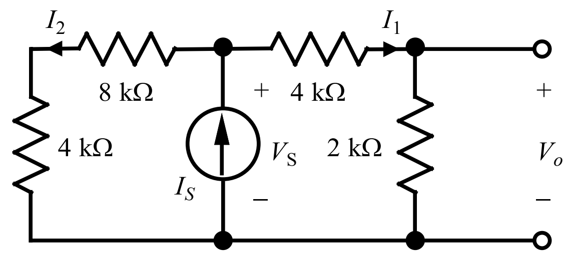
$$V_S = 3 I_S + V_1 = 21 + 20 = 41\ \text{V}$$

Problem 2.62

In the network shown, $V_o = 6\text{ V}$. Find I_S.



Suggested Solution



$$I_1 = \frac{V_o}{2\text{ k}\Omega} = \frac{6}{2000} = 3\text{ mA}$$

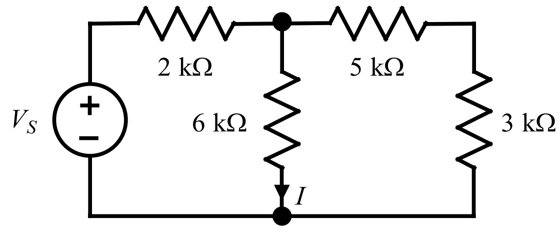
$$V_S = (4\text{ k}\Omega)I_1 + V_o = 12 + 6 = 18\text{ V}$$

$$I_2 = \frac{V_S}{8\text{ k}\Omega + 4\text{ k}\Omega} = \frac{18}{12000} = 1.5\text{ mA}$$

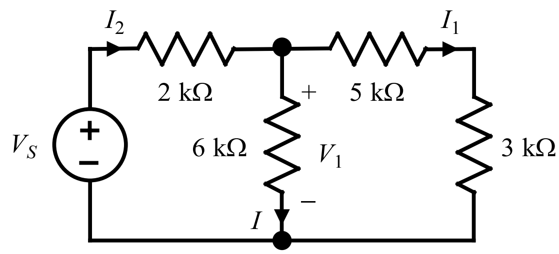
$$I_S = I_1 + I_2 = 3\text{ mA} + 1.5\text{ mA} = 4.5\text{ mA}$$

Problem 2.63

In the circuit shown, $I = 4\text{ mA}$. Find V_S.



Suggested Solution



$$V_1 = (6\text{ k}\Omega)I = 24\text{ V}$$

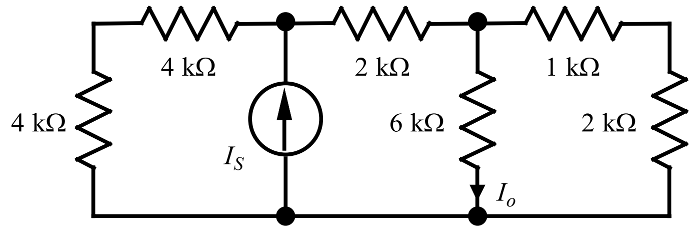
$$I_1 = \frac{V_1}{5\text{ k}\Omega + 3\text{ k}\Omega} = \frac{24}{8000} = 3\text{ mA}$$

$$I_2 = I + I_1 = 0.004 + 0.003 = 7\text{ mA}$$

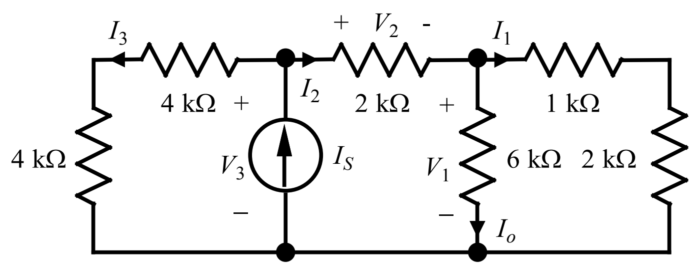
$$V_S = (2\text{ k}\Omega)I_2 + V_1 = 14 + 24 = 38\text{ V}$$

Problem 2.64

In the circuit shown, $I_o = 2$ mA . Find I_S .



Suggested Solution



$$V_1 = (6\text{ k}\Omega)I_o = 12\text{ V}$$

$$I_1 = \frac{V_1}{1\text{ k}\Omega + 2\text{ k}\Omega} = \frac{12}{3000} = 4\text{ mA}$$

$$I_2 = I_o + I_1 = 0.002 + 0.004 = 6\text{ mA}$$

$$V_2 = (2\text{ k}\Omega)I_2 = 12\text{ V}$$

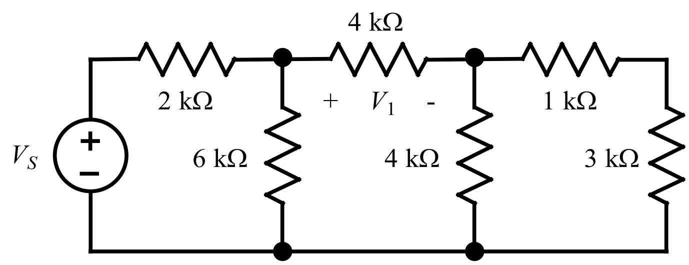
$$V_3 = V_2 + V_1 = 12 + 12 = 24\text{ V}$$

$$I_3 = \frac{V_3}{4\text{ k}\Omega + 4\text{ k}\Omega} = \frac{24}{8000} = 3\text{ mA}$$

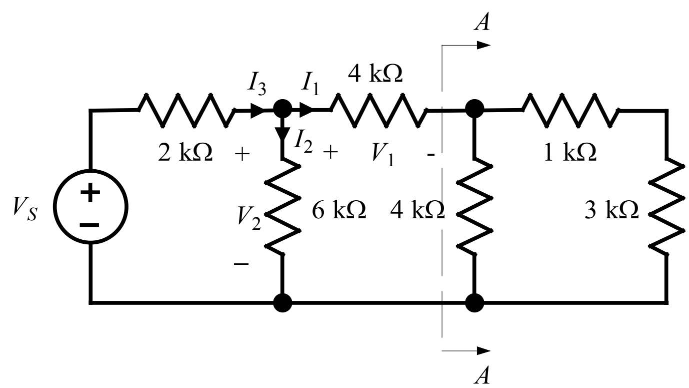
$$I_S = I_3 + I_2 = 0.003 + 0.006 = 9\text{ mA}$$

Problem 2.65

In the network shown, $V_1 = 12\text{ V}$. Find V_S.



Suggested Solution



At *A-A*: $4000 \| (1000 + 3000) = 2\text{ k}\Omega$

$$I_1 = \frac{V_1}{4\text{ k}\Omega} = \frac{12}{4000} = 3\text{ mA}$$

$$V_2 = V_1 + (2\text{ k}\Omega)I_1 = 12 + 6 = 18\text{ V}$$

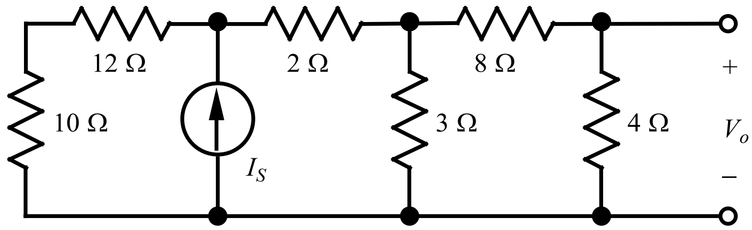
$$I_2 = \frac{V_2}{6\text{ k}\Omega} = \frac{18}{6000} = 3\text{ mA}$$

$$I_3 = I_1 + I_2 = 0.003 + 0.003 = 6\text{ mA}$$

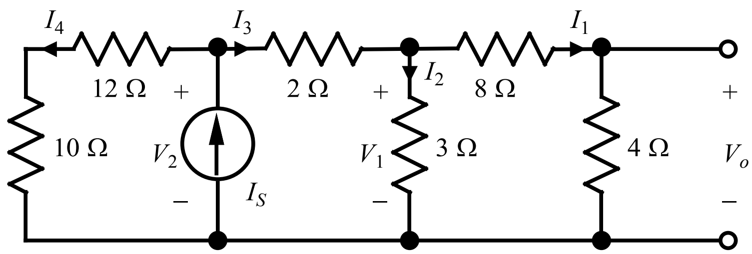
$$V_S = (2\text{ k}\Omega)I_3 + V_2 = 12 + 18 = 30\text{ V}$$

Problem 2.66

In the circuit shown, $V_o = 2\text{ V}$. Find I_S.



Suggested Solution



$$I_1 = \frac{V_o}{4\ \Omega} = \frac{2}{4} = 0.5\text{ A}$$

$$V_1 = (8\ \Omega)I_1 + V_o = 6\text{ V}$$

$$I_2 = \frac{V_1}{3\ \Omega} = 2\text{ A}$$

$$I_3 = I_2 + I_1 = 2.5\text{ A}$$

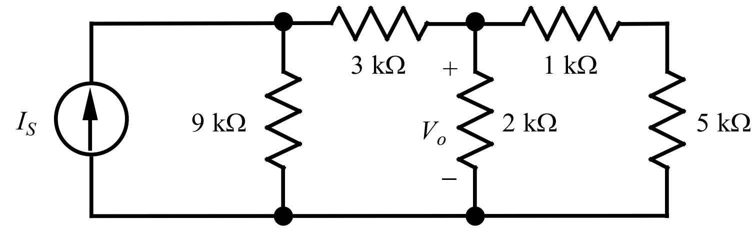
$$V_2 = (2\ \Omega)I_3 + V_1 = 11\text{ V}$$

$$I_4 = \frac{V_2}{10\ \Omega + 12\ \Omega} = 0.5\text{ A}$$

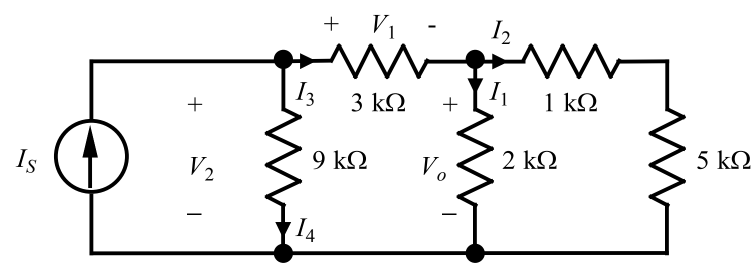
$$I_S = I_3 + I_4 = 2.5 + 0.5 = 3\text{ A}$$

Problem 2.67

In the network shown, $V_o = 6\text{ V}$. Find I_S.



Suggested Solution



$$I_1 = \frac{V_o}{2\text{ k}\Omega} = \frac{6}{2000} = 3\text{ mA}$$

$$I_2 = \frac{V_o}{1\text{ k}\Omega + 5\text{ k}\Omega} = \frac{6}{6000} = 1\text{ mA}$$

$$I_3 = I_1 + I_2 = 0.003 + 0.001 = 4\text{ mA}$$

$$V_1 = (3\text{ k}\Omega) I_3 = 12\text{ V}$$

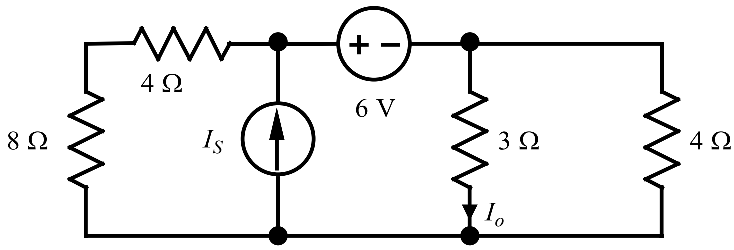
$$V_2 = V_1 + V_o = 12 + 6 = 18\text{ V}$$

$$I_4 = \frac{V_2}{9\text{ k}\Omega} = \frac{18}{9000} = 2\text{ mA}$$

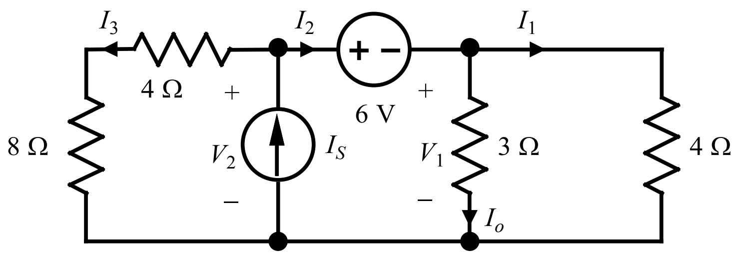
$$I_S = I_3 + I_4 = 0.004 + 0.003 = 6\text{ mA}$$

Problem 2.68

In the circuit shown, $I_o = 2\text{ A}$. Find I_S.



Suggested Solution



$$V_1 = (3\ \Omega)I_o = 6\text{ V}$$

$$I_1 = \frac{V_1}{4\ \Omega} = \frac{6}{4} = 1.5\text{ A}$$

$$I_2 = I_o + I_1 = 2 + 1.5 = 3.5\text{ A}$$

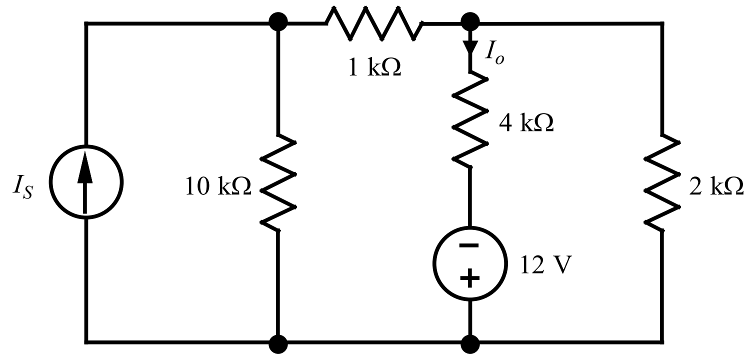
$$V_2 = 6\text{ V} + V_1 = 6 + 6 = 12\text{ V}$$

$$I_3 = \frac{V_2}{8\ \Omega + 4\ \Omega} = \frac{12}{12} = 1\text{ A}$$

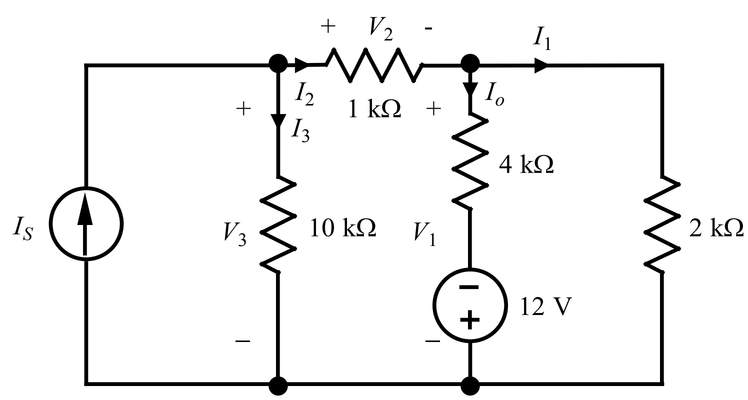
$$I_S = I_2 + I_3 = 3.5 + 1 = 4.5\text{ A}$$

Problem 2.69

If $I_o = 4$ mA in the circuit shown, find I_S.



Suggested Solution



$$V_1 = (4\text{ k}\Omega)I_o - 12\text{ V} = 16 - 12 = 4\text{ V}$$

$$I_1 = \frac{V_1}{2\text{ k}\Omega} = \frac{4}{2000} = 2\text{ mA}$$

$$I_2 = I_o + I_1 = 0.004 + 0.002 = 6\text{ mA}$$

$$V_2 = (1\text{ k}\Omega)I_2 = 6\text{ V}$$

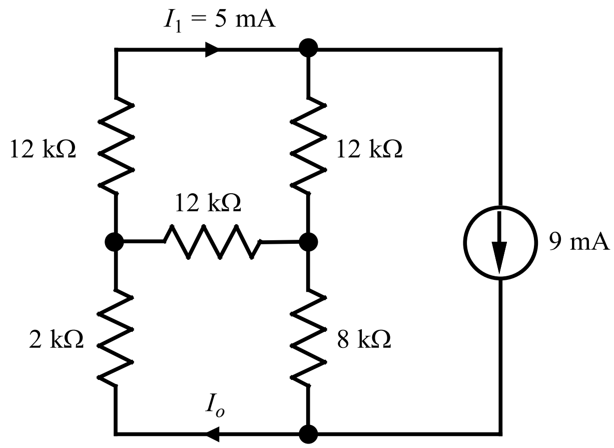
$$V_3 = V_2 + V_1 = 6 + 4 = 10\text{ V}$$

$$I_3 = \frac{V_3}{10\text{ k}\Omega} = \frac{10}{10000} = 1\text{ mA}$$

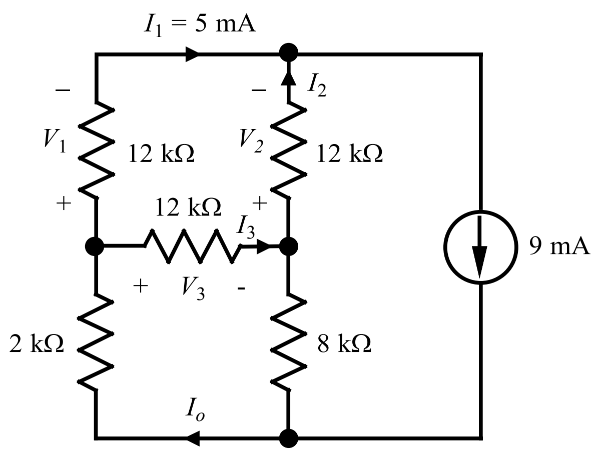
$$I_S = I_2 + I_3 = 0.006 + 0.001 = 7\text{ mA}$$

Problem 2.70

Find I_o in the circuit shown.



Suggested Solution



$I_1 + I_2 = 9 \text{ mA} \quad \Rightarrow \quad I_2 = 4 \text{ mA}$

$V_1 = (12 \text{ k}\Omega)I_1 = 60 \text{ V}$

$V_2 = (12 \text{ k}\Omega)I_2 = 48 \text{ V}$

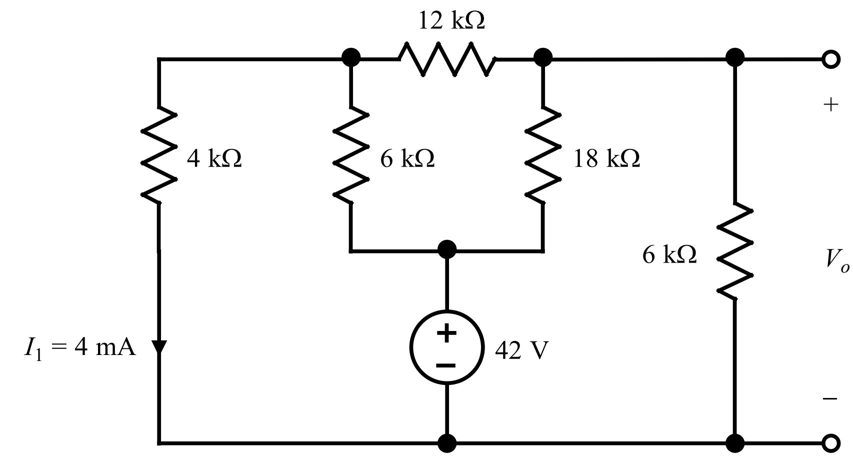
$V_3 = V_1 - V_2 = 60 - 48 = 12 \text{ V}$

$$I_3 = \frac{V_3}{12 \text{ k}\Omega} = \frac{12}{12000} = 1 \text{ mA}$$

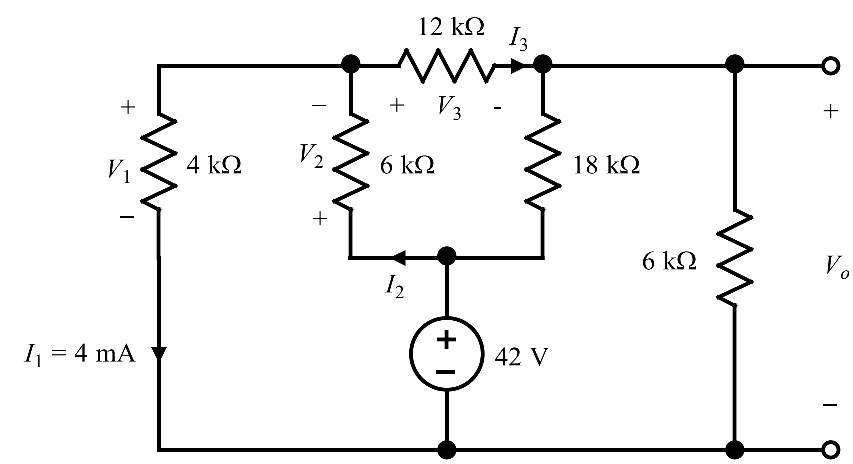
$I_o = I_1 + I_3 = 0.005 + 0.001 = 6 \text{ mA}$

Problem 2.71

Find V_o in the circuit shown.



Suggested Solution



$$V_1 = (4\text{ k}\Omega)I_1 = 16\text{ V}$$

$$V_2 = 42\text{ V} - V_1 = 42 - 16 = 26\text{ V}$$

$$I_2 = \frac{V_2}{6\text{ k}\Omega} = \frac{26}{6000} = \frac{13}{3}\text{ mA}$$

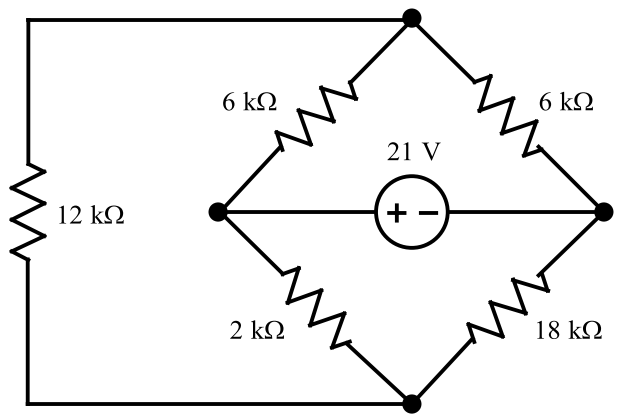
$$I_3 = I_2 - I_1 = \frac{13}{3}\text{ mA} - 4\text{ mA} = \frac{1}{3}\text{ mA}$$

$$V_3 = (12\text{ k}\Omega)I_3 = 4\text{ V}$$

$$V_o = V_1 - V_3 = 16 - 4 = 12\text{ V}$$

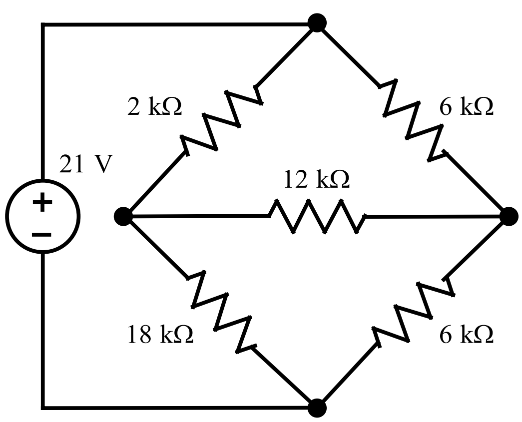
Problem 2.72

Find the power absorbed by the network shown.

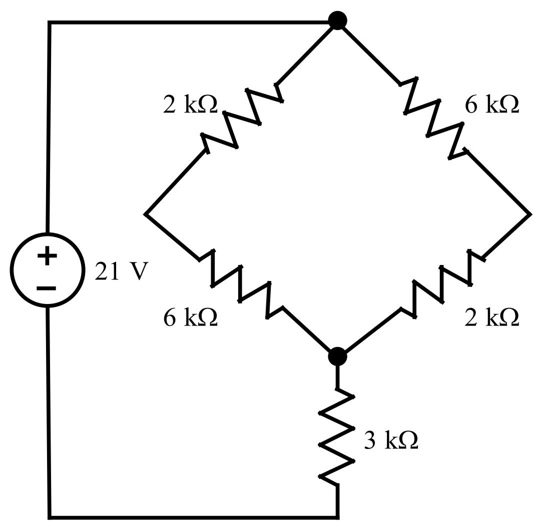


Suggested Solution

Redrawing the network:



$\Delta \rightarrow Y$
$\Rightarrow$



The equivalent resistance seen by the source is:

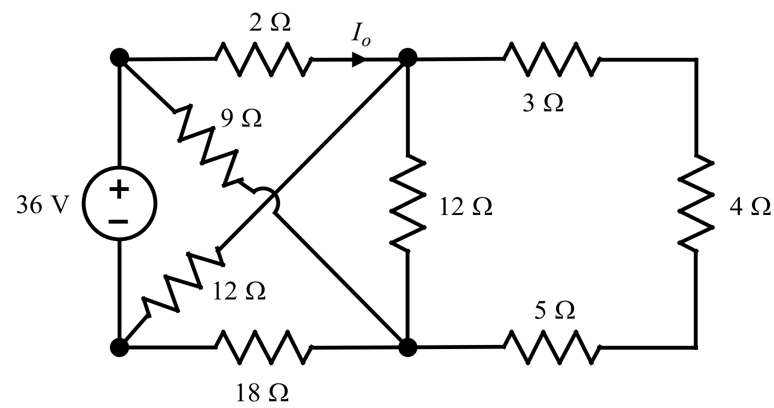
$$\begin{aligned} R_{eq} &= \left[\left(2\ \text{k}\Omega + 6\ \text{k}\Omega\right) \| \left(6\ \text{k}\Omega + 2\ \text{k}\Omega\right) \right] + 3\ \text{k}\Omega \\ &= 4\ \text{k}\Omega + 3\ \text{k}\Omega \\ &= 7\ \text{k}\Omega \end{aligned}$$

Then,

$$P = \frac{(21\ \text{V})^2}{R_{eq}} = \frac{441}{7000} = 63\ \text{mA}$$

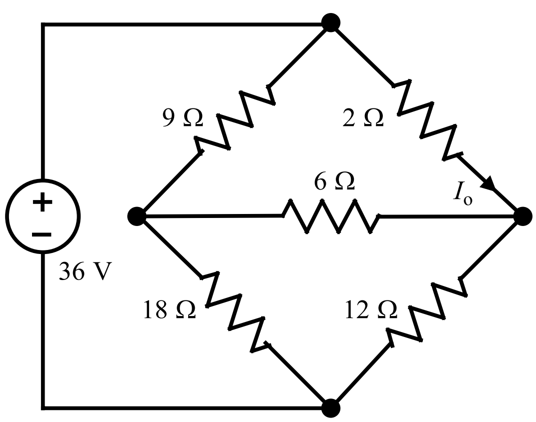
Problem 2.73

Find I_o in the circuit shown.



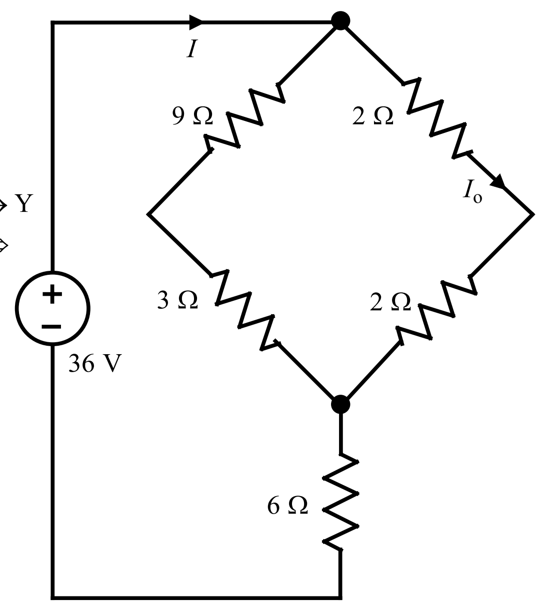
Suggested Solution

Note that the four right-most resistors can be combined as $(3\ \Omega + 4\ \Omega + 5\ \Omega) \| 12\ \Omega = 6\ \Omega$. Then the circuit can be redrawn as:



$\Delta \rightarrow Y$

$\Rightarrow$

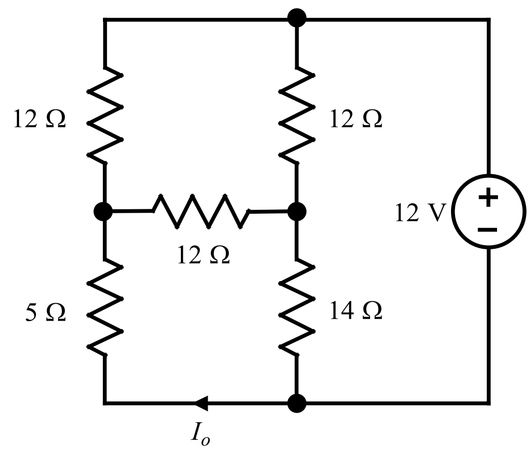


$$I = \frac{36\text{ V}}{\left[(9\ \Omega + 3\ \Omega) \| (2\ \Omega + 2\ \Omega)\right] + 6\ \Omega} = 4\text{ A}$$

$$I_o = \left[\frac{(9+3)}{(9+3)+(2+2)}\right] I = 3\text{ A}$$

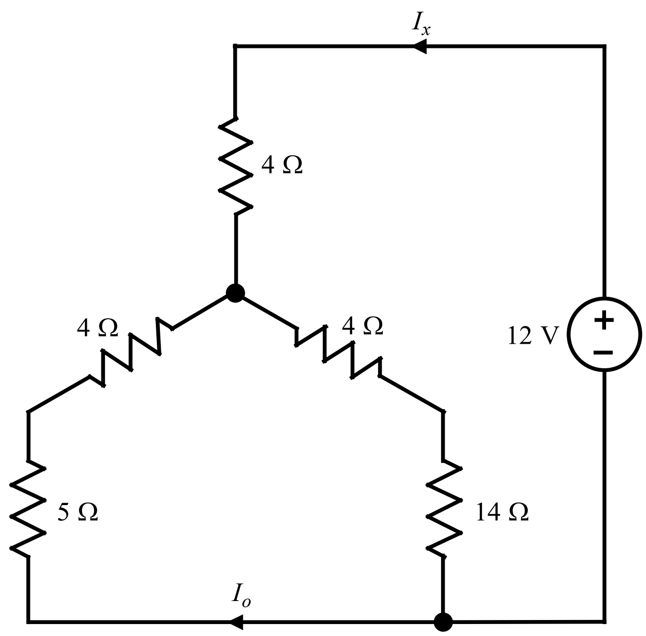
Problem 2.74

Find I_o in the circuit shown.



Suggested Solution

Applying the $\Delta \rightarrow Y$ transformation to the three 12-Ω resistors yields:

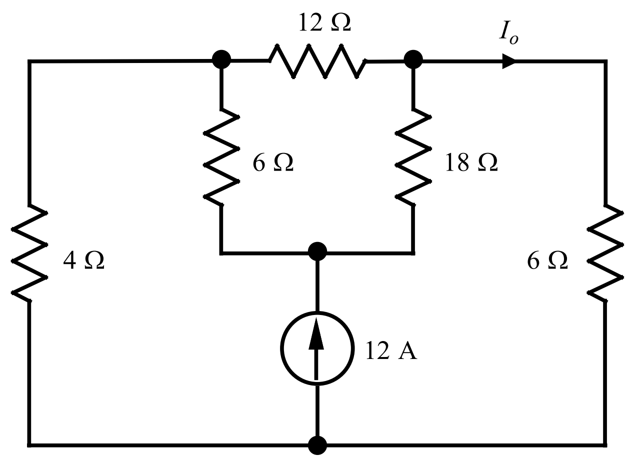


$$I_o = -\left[\frac{(4+14)}{(4+5)+(4+14)}\right] I_x \qquad \text{where } I_x = \frac{12\text{ V}}{4\ \Omega + \left[(4\ \Omega + 5\ \Omega) \| (4\ \Omega + 14\ \Omega)\right]} = \frac{12}{10} = 1.2\text{ A}$$

$$\therefore\ I_o = -\frac{18}{27} \bullet \frac{12}{10} = -\frac{4}{5} \approx -0.80\text{ A}$$

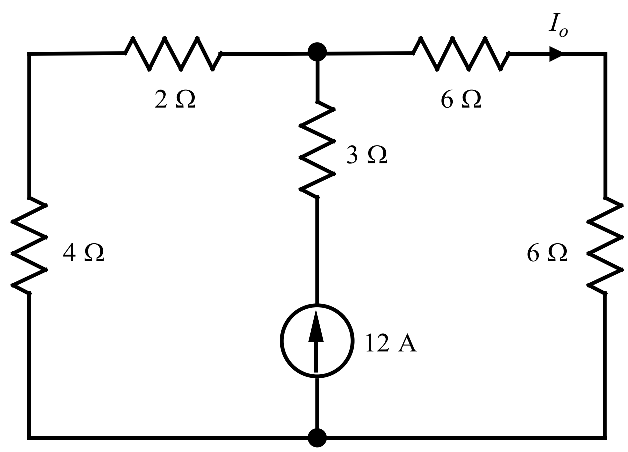
Problem 2.75

Find I_o in the circuit shown.



Suggested Solution

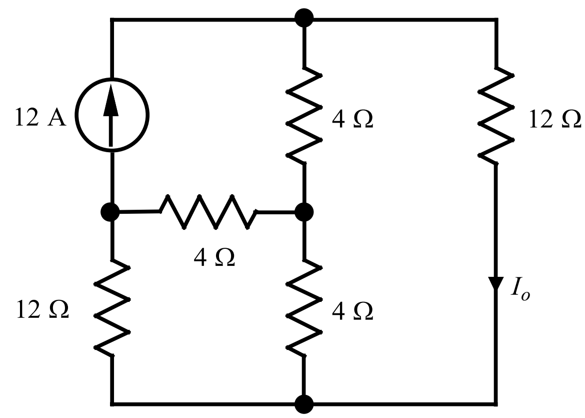
Converting the Δ to a Y yields the circuit:



$$I_o = \left[\frac{(2+4)}{(2+4)+(6+6)} \right] (12 \text{ A}) = 4 \text{ A}$$

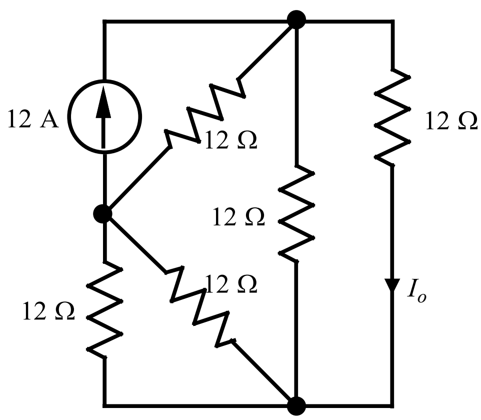
Problem 2.76

Find I_o in the circuit shown.

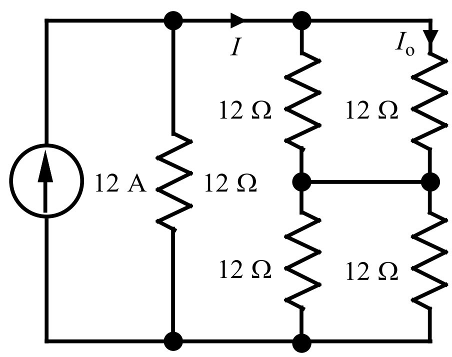


Suggested Solution

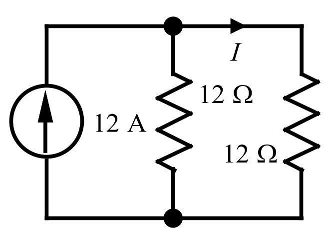
Applying the $Y \rightarrow \Delta$ transformation:



$\Rightarrow$



The circuit can be further simplified to:

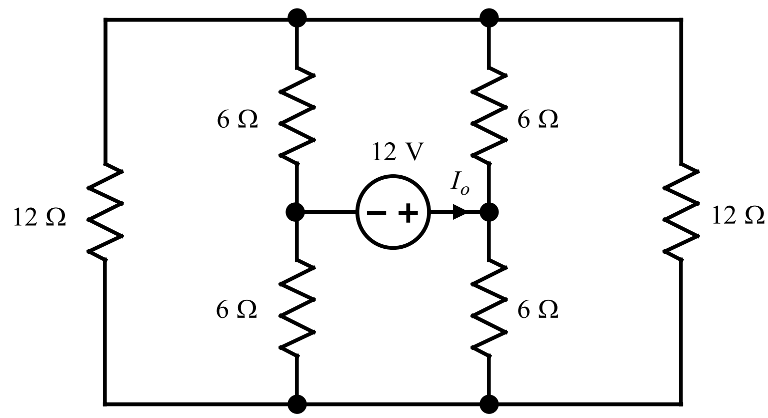


$$I = \frac{12\text{ A}}{2} = 6\text{ A}$$

$$I_o = \frac{I}{2} = 3\text{ A}$$

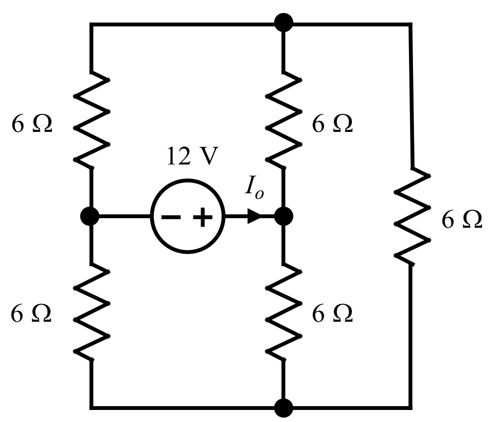
Problem 2.77

Find I_o in the circuit shown.

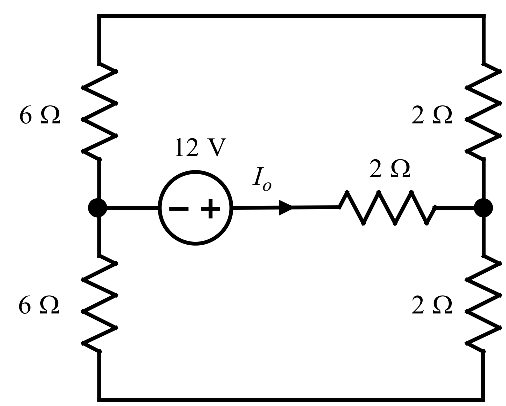


Suggested Solution

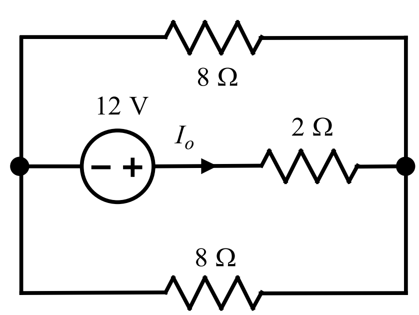
Combining the two 12-Ω resistors yields:



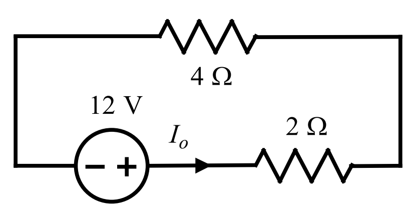
$\Delta \rightarrow Y$
$\Rightarrow$



The circuit can be further simplified to the following:



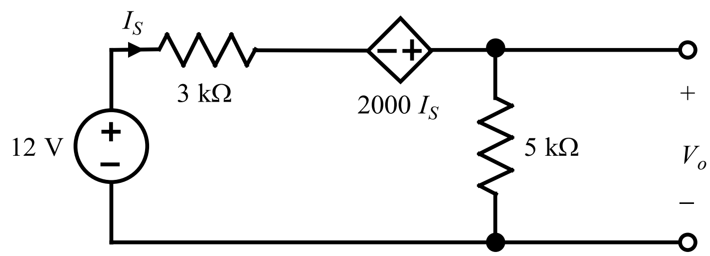
$\Rightarrow$



$$I_o = \frac{12 \text{ V}}{6\ \Omega} = 2 \text{ A}$$

Problem 2.78

Find V_o in the circuit shown.



Suggested Solution

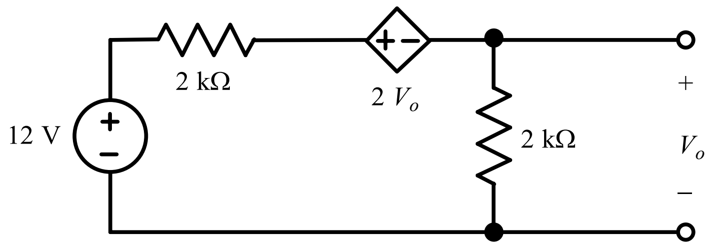
KVL: $-12 + 3000 I_S - 2000 I_S + 5000 I_S = 0$

$\Rightarrow \quad I_S = 2 \text{ mA}$

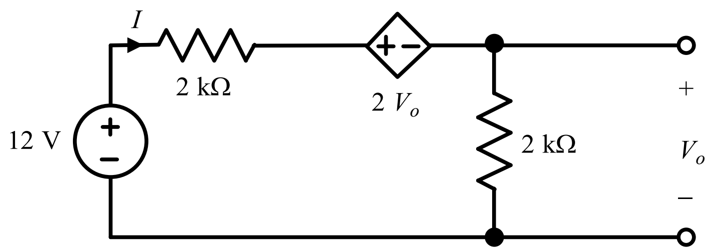
$V_o = 5000 I_S = 10 \text{ V}$

Problem 2.79

Find V_o in the network shown.



Suggested Solution



KVL: $-12 + 2000\,I + 2V_o + V_o = 0$ where $V_o = 2000\,I$

Therefore,

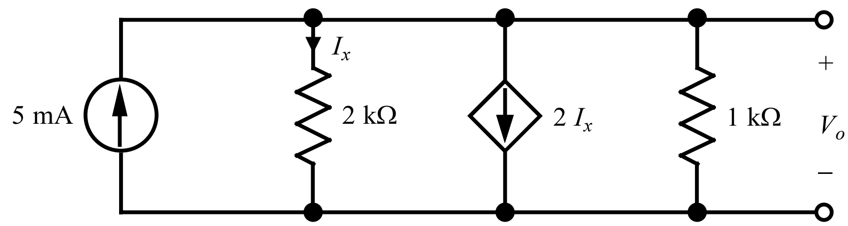
$-12 + 2000\,I + 4000\,I + 2000\,I = 0$

$\Rightarrow \quad I = 1.5 \text{ mA}$

$V_o = 2000\left(1.5 \times 10^{-3}\right) = 3 \text{ V}$

Problem 2.80

Find V_o in the network shown.



Suggested Solution

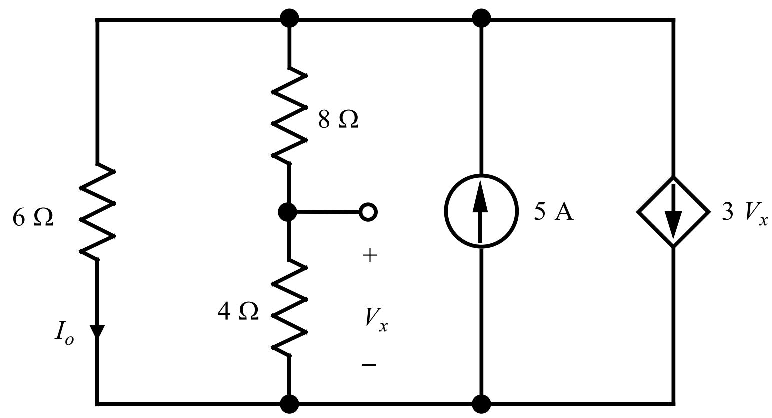
KCL: $-0.005+\frac{V_o}{2000}+2I_x+\frac{V_o}{1000}=0$ where $I_x=\frac{V_o}{2\text{ k}\Omega}$

Therefore,

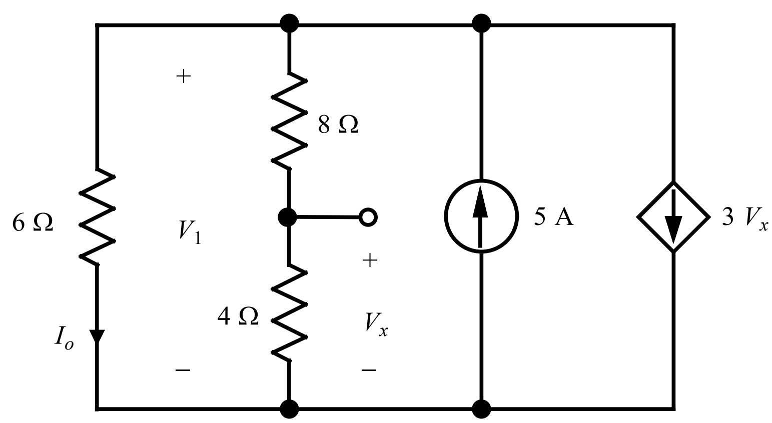
$$-0.005+\frac{V_o}{2000}+2\frac{V_o}{2000}+\frac{V_o}{1000}=0 \quad\Rightarrow\quad V_o=2\text{ V}$$

Problem 2.81

Find I_o in the network shown.



Suggested Solution



KCL: $\frac{V_1}{6}+\frac{V_1}{8+4}-5+3V_x=0$ where $V_x=\left(\frac{4}{8+4}\right)V_1=\frac{V_1}{3}$

Therefore,

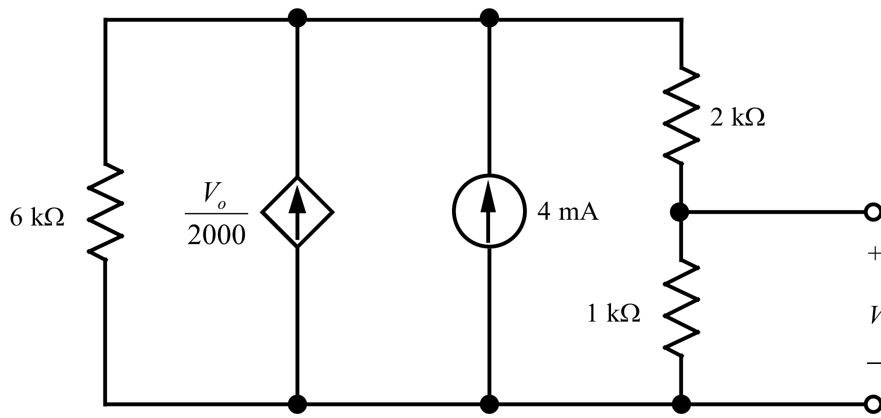
$$\frac{V_1}{6}+\frac{V_1}{12}-5+3\frac{V_1}{3}=0$$

$$\Rightarrow \quad V_1=4\text{ V}$$

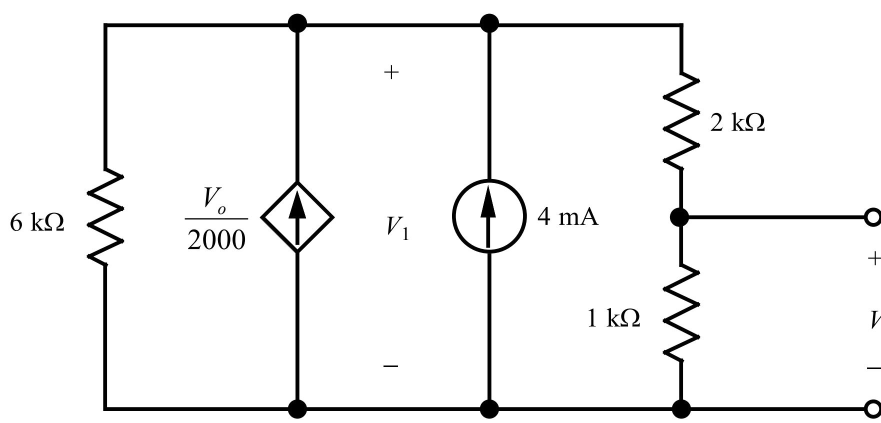
$$I_o=\frac{V_1}{6\ \Omega}=\frac{4}{6}=\frac{2}{3}\text{ A}$$

Problem 2.82

Find V_o in the circuit shown.



Suggested Solution



KCL: $\frac{V_1}{6000} - \frac{V_o}{2000} - 0.004 + \frac{V_1}{2000+1000} = 0$ where $V_o = \left(\frac{1000}{2000+1000}\right)V_1 = \frac{V_1}{3}$

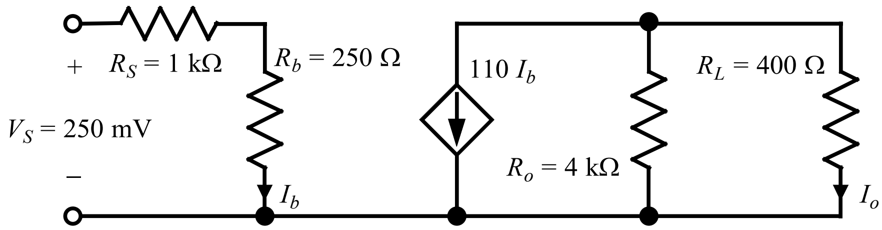
Therefore,

$$\frac{3V_o}{6000} - \frac{V_o}{2000} - 0.004 + \frac{3V_o}{3000} = 0$$

$$\Rightarrow \quad V_o = 4 \text{ V}$$

Problem 2.83

A single-stage transistor amplifier is modeled as shown. Find the current in the load R_L .



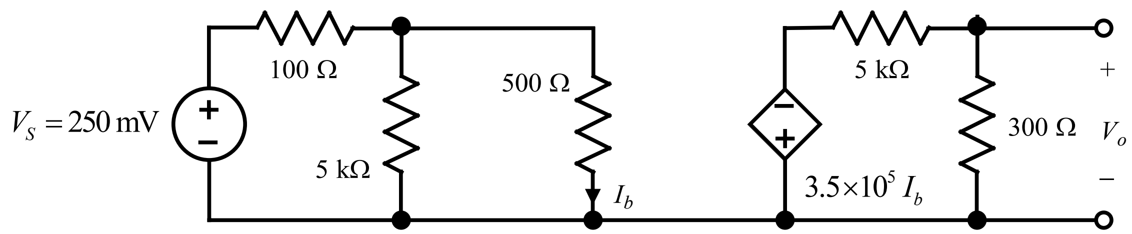
Suggested Solution

$$I_b = \frac{V_S}{R_S + R_b} = \frac{0.25}{1000 + 250} = 0.2 \text{ mA}$$

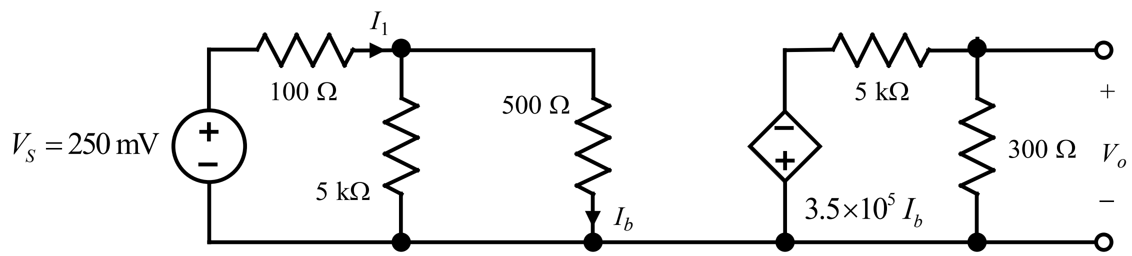
$$I_o = -\left(\frac{4000}{4000 + 400}\right)\left(110\, I_b\right) = -100\, I_b = -20 \text{ mA}$$

Problem 2.84

A typical transistor amplifier is shown. Find the amplifier gain G (i.e., the ratio of the output voltage to the input voltage).



Suggested Solution



$$I_1 = \frac{V_S}{100\ \Omega + \left(5\ \text{k}\Omega \| 500\ \Omega\right)} = \frac{0.25}{554.5} = 451\ \mu\text{A}$$

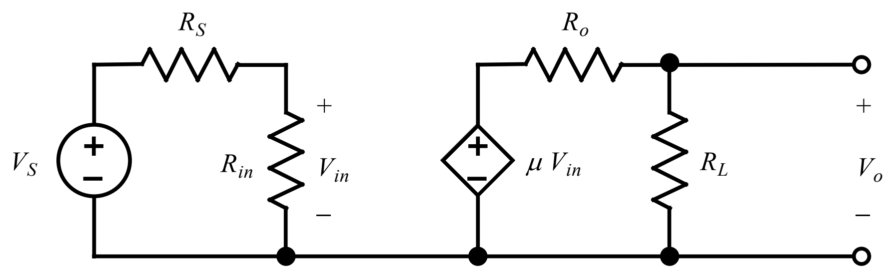
$$I_b = \left(\frac{5000}{5000+500}\right) I_1 = 409.8\ \mu\text{A}$$

$$V_o = \left(\frac{300}{300+5000}\right)\left[\left(-3.5\times10^5\right) I_b\right] = -8.12\ \text{V}$$

$$G \triangleq \frac{V_o}{V_S} = \frac{-8.12}{0.250} \approx -32.5$$

Problem 2.85

For the network shown, choose the values of R_{in} and R_o such that V_o is maximized. What is the resulting ratio, V_o/V_S ?



Suggested Solution

$$V_{in} = \left(\frac{R_{in}}{R_S + R_{in}}\right) V_S$$

$$V_o = \left(\frac{R_L}{R_o + R_L}\right)(\mu V_{in}) = \mu \left(\frac{R_{in}}{R_S + R_{in}}\right)\left(\frac{R_L}{R_o + R_L}\right) V_S$$

Therefore,

$$\frac{V_o}{V_S} = \mu \left(\frac{R_{in}}{R_S + R_{in}}\right)\left(\frac{R_L}{R_o + R_L}\right)$$

Clearly, to maximize $\frac{V_o}{V_S}$, we must maximize R_{in}. This is because $\frac{R_{in}}{R_S + R_{in}}$ is always less than 1, but gets closer and closer to 1 as R_{in} is made larger and larger.

On the other hand, to maximize $\frac{V_o}{V_S}$, we must minimize R_o. This is because $\frac{R_L}{R_o + R_L}$ is always less than 1, but gets closer and closer to 1 as R_o is made smaller and smaller.

In the limit,

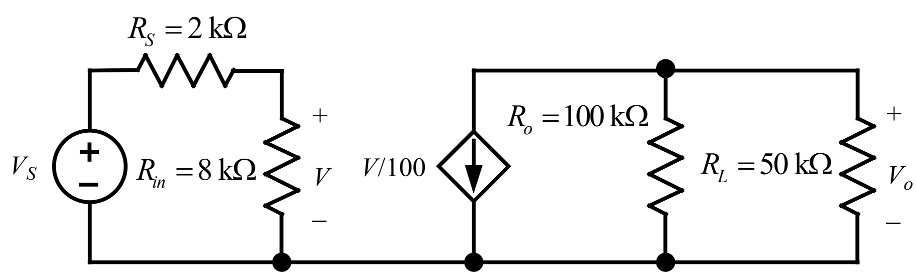
$$\lim_{R_{in}\to\infty}\left(\lim_{R_o\to 0}\frac{V_o}{V_S}\right) = \mu\left(\lim_{R_{in}\to\infty}\frac{R_{in}}{R_S + R_{in}}\right)\left(\lim_{R_o\to 0}\frac{R_L}{R_o + R_L}\right) = \mu \times 1 \times 1 = \mu$$

Problem 2.86

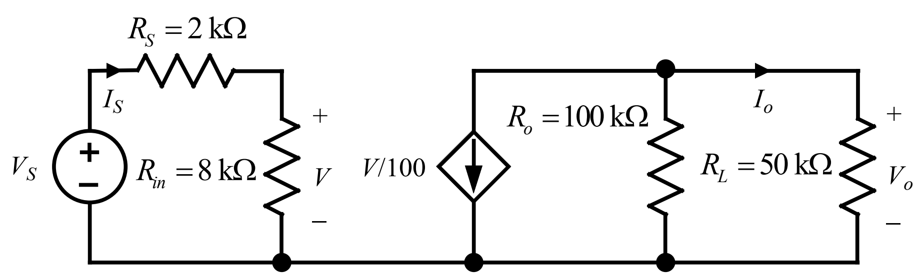
In many amplifier applications we are concerned not only with voltage gain, but also with power gain.

$$\text{Power gain} = A_p = (\text{power delivered to the load})/(\text{power delivered by the input})$$

Find the power gain for the circuit shown, where $R_L = 50\ \text{k}\Omega$.



Suggested Solution



$$V = \left(\frac{8000}{8000+2000}\right)V_S = 0.8V_S$$

$$I_o = -\left(\frac{100,000}{100,000+50,000}\right)\left(\frac{V}{100}\right) = -5.33\times10^{-3}\,V_S$$

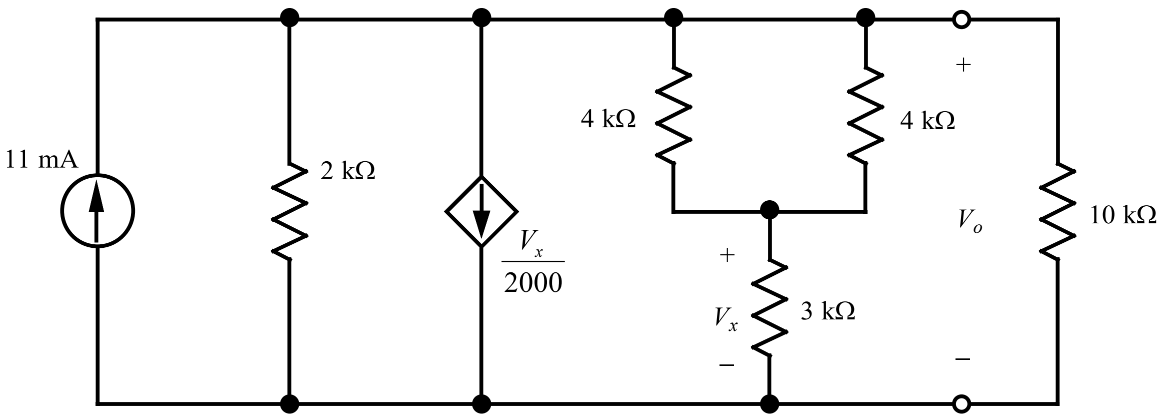
$$P_{load} = I_o^2\,R_L = \left(5.33\times10^{-3}\,V_S\right)^2\left(50,000\right)) = 1.422\,V_S^2$$

$$P_{in} = V_S\,I_S = V_S\left(\frac{V_S}{8000+2000}\right) = \frac{V_S^2}{10,000}$$

$$A_p = \frac{P_{load}}{P_{in}} = \frac{1.422\,V_S^2}{\left(\frac{V_S^2}{10,000}\right)} = 14.22\times10^3$$

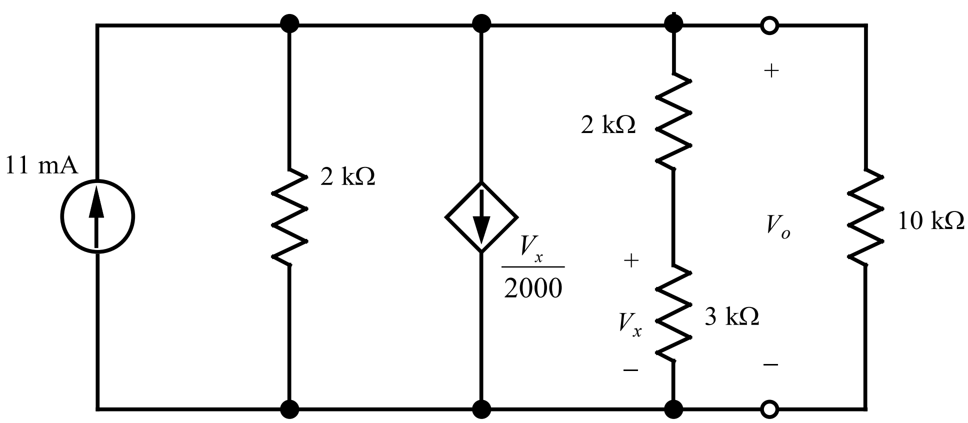
Problem 2.87

Find the power absorbed by the $10\text{-k}\Omega$ resistor in the circuit shown.



Suggested Solution

Combining the two $4\text{-k}\Omega$ resistors in parallel yields the following simplified circuit:



$$-0.011+\frac{V_o}{2\text{ k}\Omega}+\frac{V_x}{2000}+\frac{V_o}{2\text{ k}\Omega+3\text{ k}\Omega}+\frac{V_o}{10\text{ k}\Omega}=0 \text{ where } V_x=\left(\frac{3000}{2000+3000}\right)V_o=0.6V_o$$

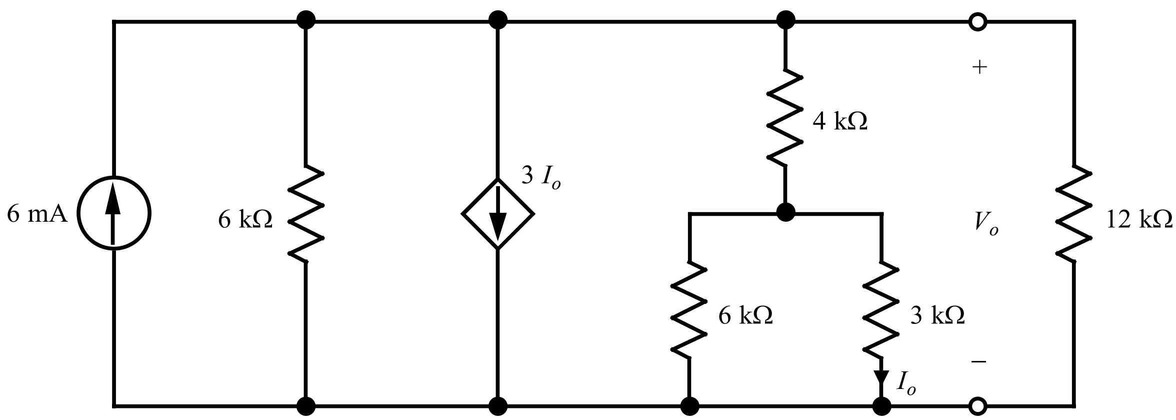
Therefore,

$$-0.011+\frac{V_o}{2000}+\frac{3V_o}{10,000}+\frac{V_o}{5000}+\frac{V_o}{10,000}=0 \quad \Rightarrow \quad V_o=10\text{ V}$$

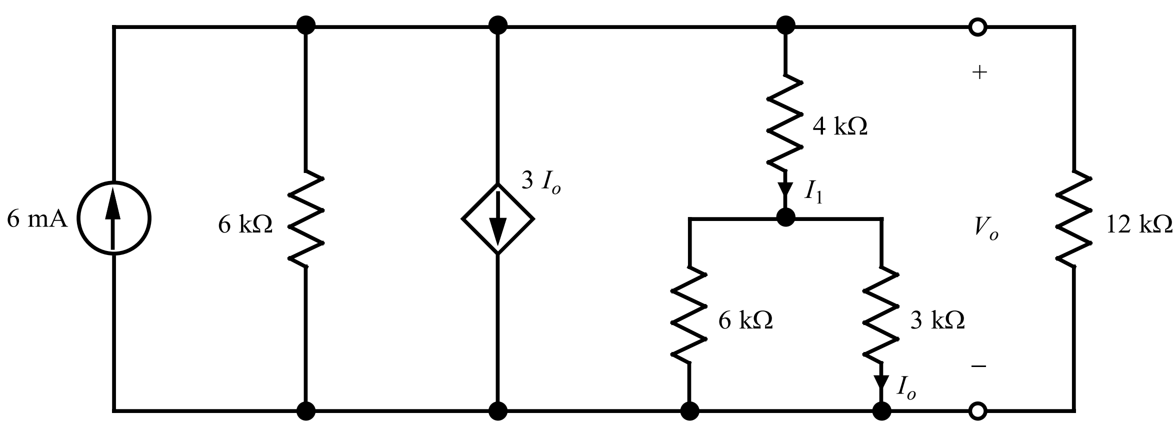
$$P_{10\text{ k}\Omega}=\frac{V_o^2}{10\text{ k}\Omega}=\frac{(10)^2}{10,000}=10\text{ mW}$$

Problem 2.88

Find the power absorbed by the 12-kΩ resistor in the circuit shown.



Suggested Solution



$$-0.006+\frac{V_o}{6\text{ k}\Omega}+3I_o+\frac{V_o}{4\text{ k}\Omega+\left(6\text{ k}\Omega\|3\text{ k}\Omega\right)}+\frac{V_o}{12\text{ k}\Omega}=0 \text{ where } I_o=\left(\frac{6000}{6000+3000}\right)I_1=\frac{2}{3}I_1$$

$$I_1=\frac{V_o}{4\text{ k}\Omega+\left(6\text{ k}\Omega\|3\text{ k}\Omega\right)}=\frac{V_o}{6000} \quad\Rightarrow\quad I_o=\frac{2}{3}\left(\frac{V_o}{6000}\right)=\frac{V_o}{9000}$$

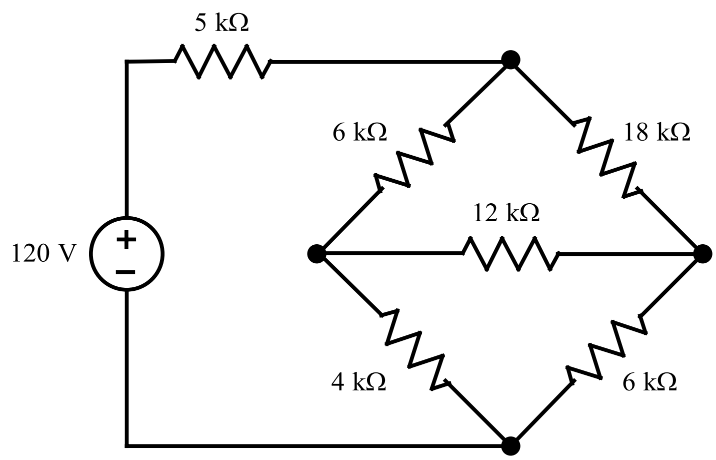
Then, substituting:

$$-0.006+\frac{V_o}{6000}+3\left(\frac{V_o}{9000}\right)+\frac{V_o}{6000}+\frac{V_o}{12,000}=0 \quad\Rightarrow\quad V_o=8\text{ V}$$

$$P_{12\text{ k}\Omega}=\frac{V_0^2}{12\text{ k}\Omega}=\frac{64}{12,000}=5.33\text{ mW}$$

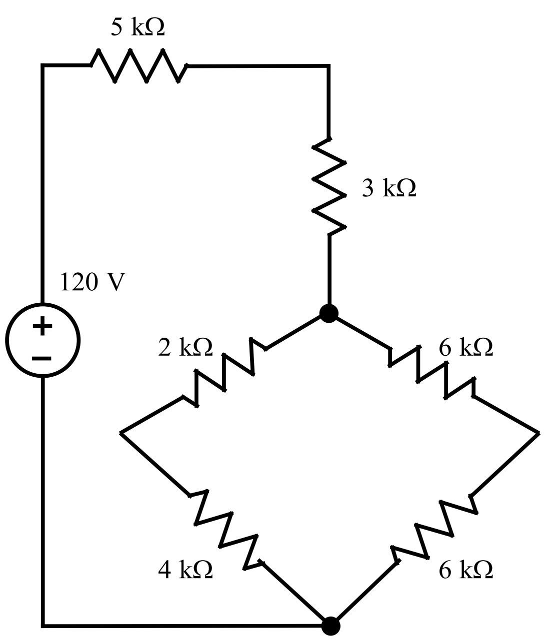
Problem 2FE-1

Find the power generated by the source in the network shown.

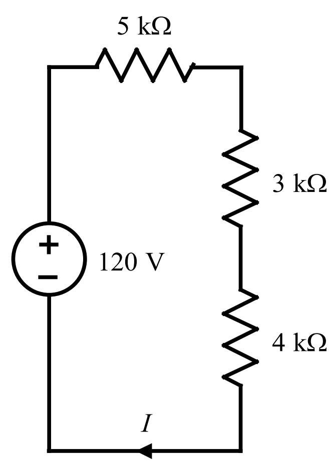


Suggested Solution

Applying the $\Delta \rightarrow Y$ transformation yields:



$\Rightarrow$

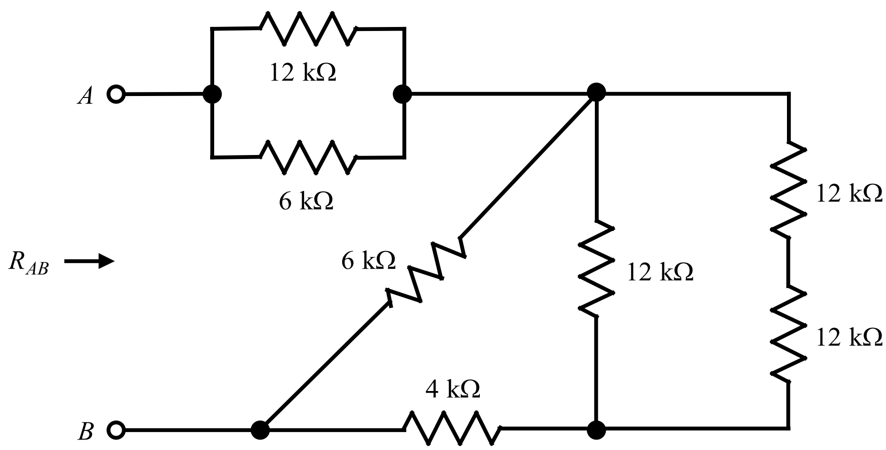


$$I = \frac{120 \text{ V}}{12 \text{ k}\Omega} = 10 \text{ mA}$$

$$P = (120 \text{ V}) I = 120 \times 0.010 = 1.2 \text{ W}$$

Problem 2FE-2

Find the equivalent resistance of the circuit shown at the terminals A-B.



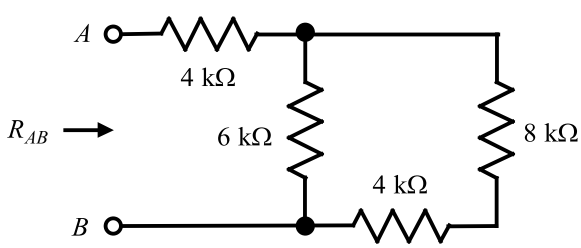
Suggested Solution

Combining resistors,

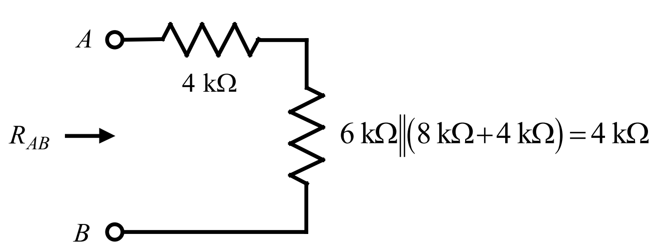
$$12\ \text{k}\Omega \| 6\ \text{k}\Omega = 4\ \text{k}\Omega$$

$$12\ \text{k}\Omega \| (12\ \text{k}\Omega + 12\ \text{k}\Omega) = 8\ \text{k}\Omega$$

The circuit is now reduced to:



and further to:

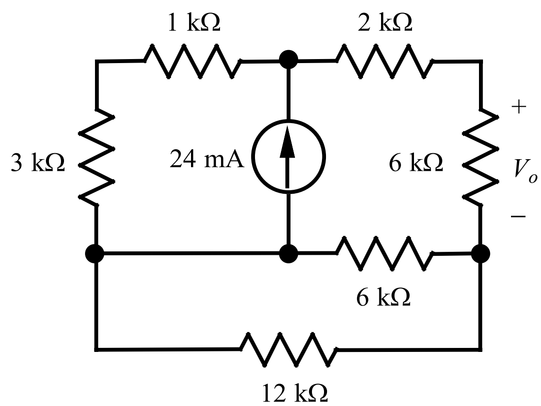


Therefore,

$$R_{AB} = 4\ \text{k}\Omega + 4\ \text{k}\Omega = 8\ \text{k}\Omega$$

Problem 2FE-3

Find the voltage V_o in the network shown.



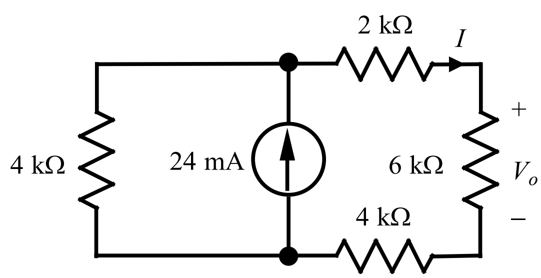
Suggested Solution

Combining resistors:

$1\ \text{k}\Omega + 3\ \text{k}\Omega = 4\ \text{k}\Omega$

$6\ \text{k}\Omega \| 12\ \text{k}\Omega = 4\ \text{k}\Omega$

The network is now reduced to:



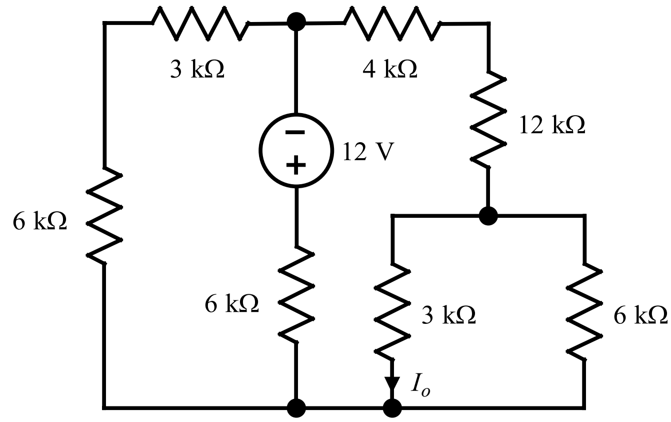
Using current division,

$$I = \left[\frac{4000}{4000 + (2000 + 6000 + 4000)}\right](24\ \text{mA}) = 6\ \text{mA}$$

$$V_o = (6\ \text{k}\Omega) I = 36\ \text{V}$$

Problem 2FE-4

Find the current I_o in the circuit shown.



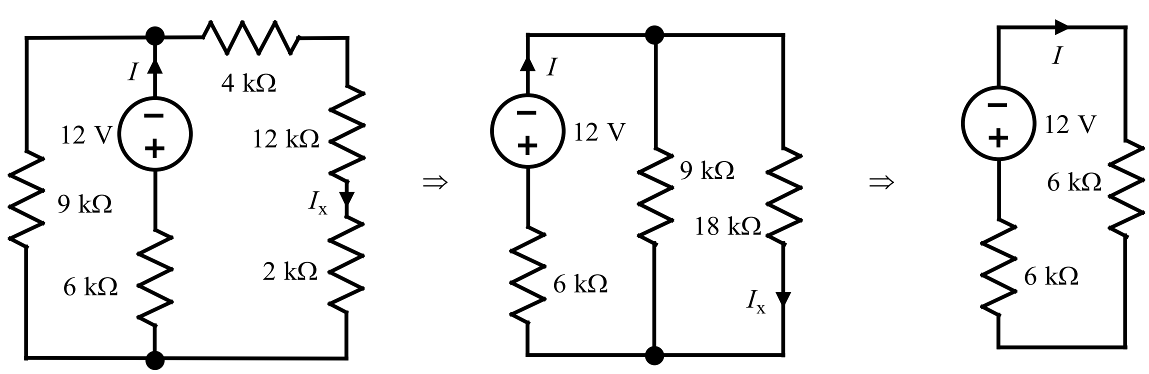
Suggested Solution

Combining resistors:

$$3\ \text{k}\Omega + 6\ \text{k}\Omega = 9\ \text{k}\Omega$$

$$3\ \text{k}\Omega \| 6\ \text{k}\Omega = 2\ \text{k}\Omega$$

Thus, the circuit simplifies to:



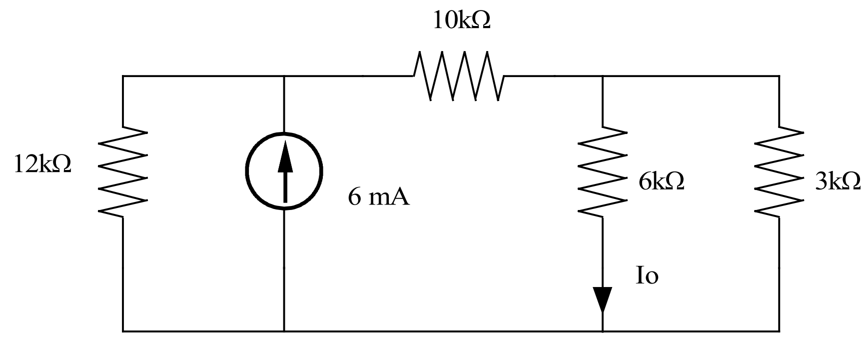
$$I = -\frac{12\text{V}}{6\ \text{k}\Omega + 6\ \text{k}\Omega} = -1\ \text{mA}$$

$$I_x = \left(\frac{9000}{9000 + 18{,}000}\right) I = -\frac{1}{3}\ \text{mA}$$

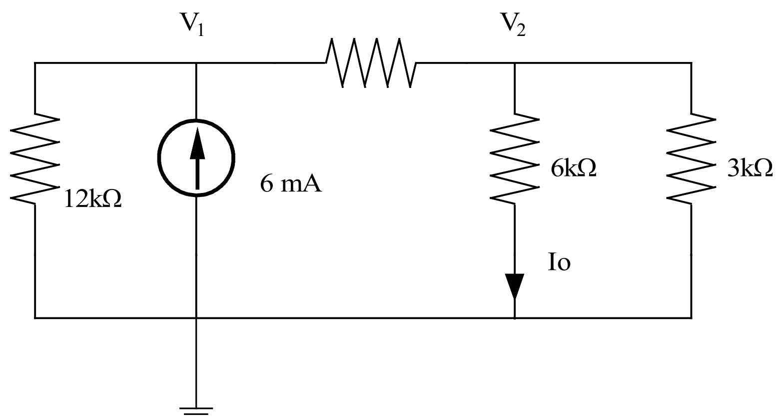
$$I_o = \left(\frac{6000}{3000 + 6000}\right) I_x = -\frac{2}{9}\ \text{mA}$$

Problem 3.1

Find I_0 in the circuit using nodal analysis.



Suggested Solution



$$\frac{V_1}{12k} + \frac{V_1 - V_2}{10k} = \frac{6}{k}$$

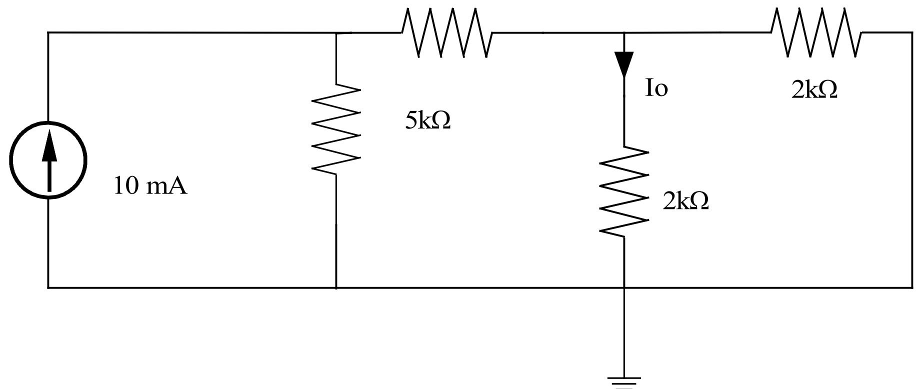
$$\frac{V_2 - V_1}{10k} + \frac{V_2}{6k} + \frac{V_2}{3k} = 0$$

$1mA$

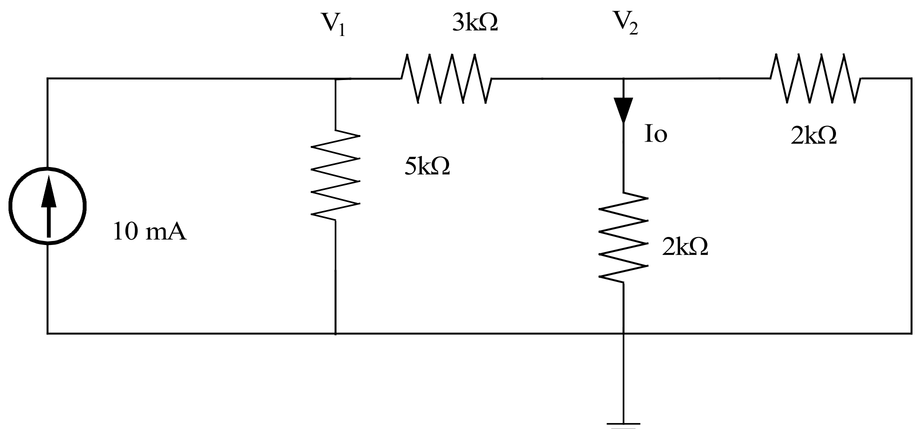
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{12k} + \frac{1}{10k} & \frac{-1}{10k} \\ \frac{-1}{10k} & \frac{1}{10k} + \frac{1}{6k} + \frac{1}{3k} \end{bmatrix}^{-1} \begin{bmatrix} \frac{6}{k} \\ 0 \end{bmatrix} = \begin{bmatrix} 36 \\ 6 \end{bmatrix} \therefore I_0 = \frac{6}{6} = 1mA$$

Problem 3.2

Find I0 in the circuit using nodal analysis.



Suggested Solution



$$\frac{V_1}{5k} + \frac{V_1 - V_2}{3k} = \frac{10}{k}$$

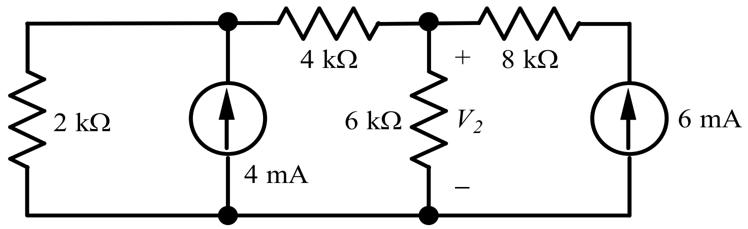
$$\frac{V_2 - V_1}{3k} + \frac{V_2}{2k} + \frac{V_2}{2k} = 0$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{5k} + \frac{1}{3k} & \frac{-1}{3k} \\ \frac{-1}{3k} & \frac{1}{3k} + \frac{1}{2k} + \frac{1}{2k} \end{bmatrix}^{-1} \begin{bmatrix} \frac{10}{k} \\ 0 \end{bmatrix} = \begin{bmatrix} 22.2 \\ 5.56 \end{bmatrix} \therefore I_0 = \frac{5.56}{2k} = 2.78mA$$

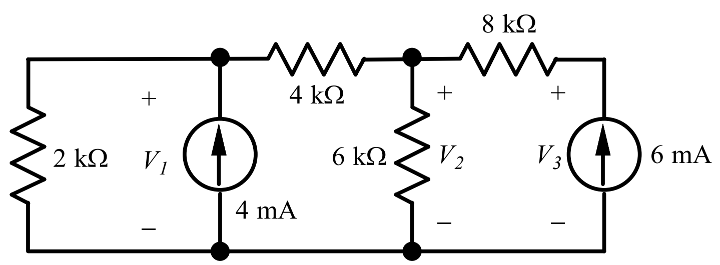
$2.78mA$

Problem 3.3

Find V_2 in the circuit shown using nodal analysis.



Suggested Solution



$$\frac{V_1}{2\text{ k}\Omega}+\frac{V_1-V_2}{4\text{ k}\Omega}=4\text{ mA}$$

$$\frac{V_2-V_1}{4\text{ k}\Omega}+\frac{V_2}{6\text{ k}\Omega}+\frac{V_2-V_3}{8\text{ k}\Omega}=0$$

$$\frac{V_3-V_2}{8\text{ k}\Omega}=6\text{ mA}$$

In matrix form:

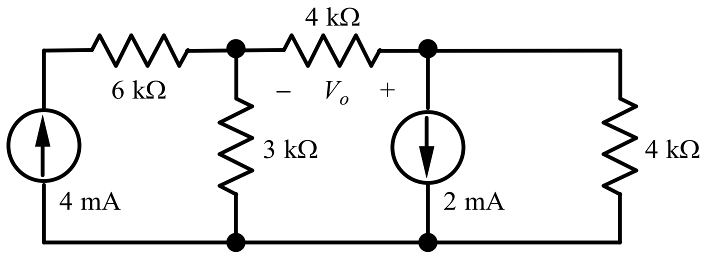
$$\begin{bmatrix} \frac{1}{2000}+\frac{1}{4000} & -\frac{1}{4000} & 0 \\ -\frac{1}{4000} & \frac{1}{4000}+\frac{1}{6000}+\frac{1}{8000} & -\frac{1}{8000} \\ 0 & -\frac{1}{8000} & \frac{1}{8000} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}=\begin{bmatrix} 0.004 \\ 0 \\ 0.006 \end{bmatrix} \quad \Rightarrow \quad V_2=22\text{ V}$$

Alternately, since we know the current through the $8\text{ k}\Omega$ resistor is 6 mA, we know that $V_3=V_2+48\text{ V}$. Therefore, we really need only 2 equations to solve this problem. Those are:

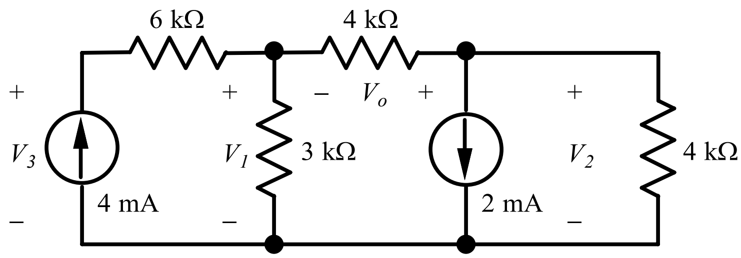
$$\frac{V_1}{2\text{ k}\Omega}+\frac{V_1-V_2}{4\text{ k}\Omega}=4\text{ mA}$$

Problem 3.4

Use nodal analysis to find V_o in the circuit shown.



Suggested Solution



$$\frac{V_3 - V_1}{6\text{ k}\Omega} = 4\text{ mA}$$

$$\frac{V_1 - V_3}{6\text{ k}\Omega} + \frac{V_1}{3\text{ k}\Omega} + \frac{V_1 - V_2}{4\text{ k}\Omega} = 0$$

$$\frac{V_2 - V_1}{4\text{ k}\Omega} + \frac{V_2}{4\text{ k}\Omega} = -2\text{ mA}$$

In matrix form:

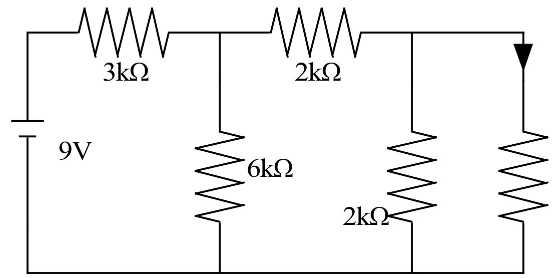
$$\begin{bmatrix} -\frac{1}{6000} & 0 & \frac{1}{6000} \\ \frac{1}{6000}+\frac{1}{3000}+\frac{1}{4000} & -\frac{1}{4000} & -\frac{1}{6000} \\ -\frac{1}{4000} & \frac{1}{4000}+\frac{1}{4000} & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0.004 \\ 0 \\ -0.002 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 6.5455 \\ -0.7273 \\ 30.5455 \end{bmatrix}$$

$$\therefore\ V_o = V_2 - V_1 = -0.73 - 6.55 = -7.28\text{ V}$$

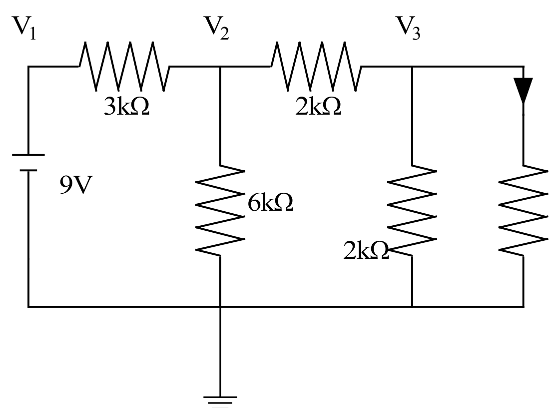
Alternately, since we know the current through the $6\text{ k}\Omega$ resistor is 4 mA, we know that $V_3 = V_1 + 24\text{ V}$. Therefore, we really need only 2 equations to solve this problem. Those are:

Problem 3.5

Find I_0 in the circuit using nodal analysis



Suggested Solution



$$V_1 = 9V$$

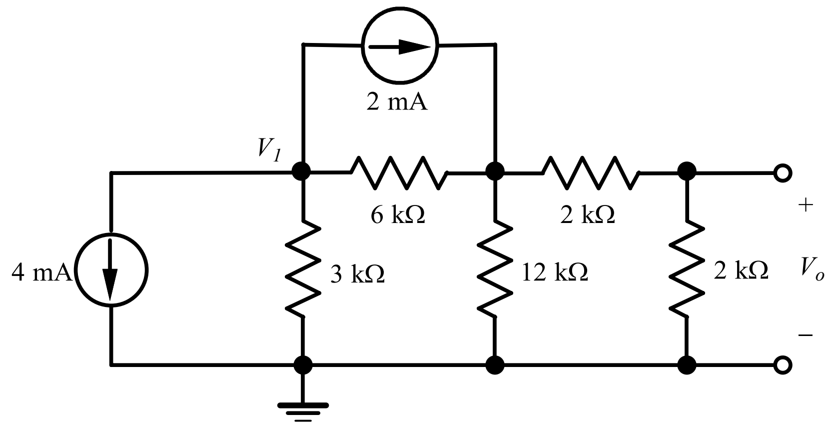
$$\frac{9 - V_2}{3k} = \frac{V_2}{6k} + \frac{V_2 - V_3}{2k}$$

$$\frac{V_3 - V_2}{2k} + \frac{V_3}{2k} + \frac{V_3}{2k} = 0 \Rightarrow V_3 = 1.2V, I_0 = \frac{1.2}{2k} = 0.6mA$$

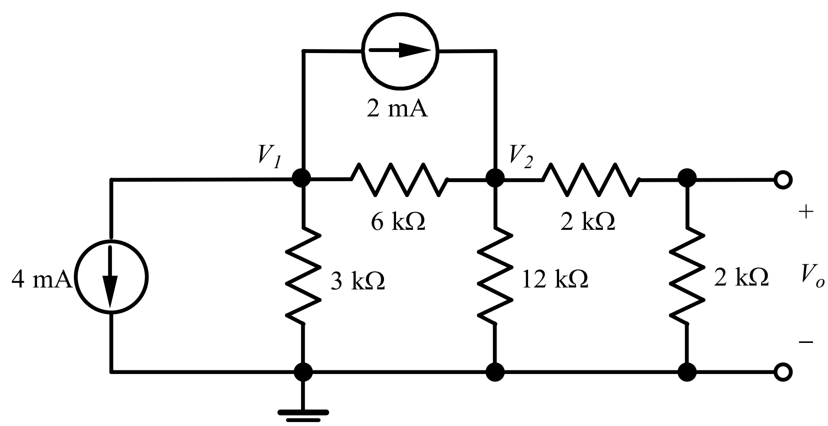
$$V_3 = 1.2V$$
$$I_0 = 0.6mA$$

Problem 3.6

Use nodal analysis to find both V_1 and V_o in the circuit shown.



Suggested Solution



$$\frac{V_1}{3\text{ k}\Omega}+\frac{V_1-V_2}{6\text{ k}\Omega}=-4\text{ mA}-2\text{ mA}$$

$$\frac{V_2-V_1}{6\text{ k}\Omega}+\frac{V_2}{12\text{ k}\Omega}+\frac{V_2}{2\text{ k}\Omega+2\text{ k}\Omega}=2\text{ mA}$$

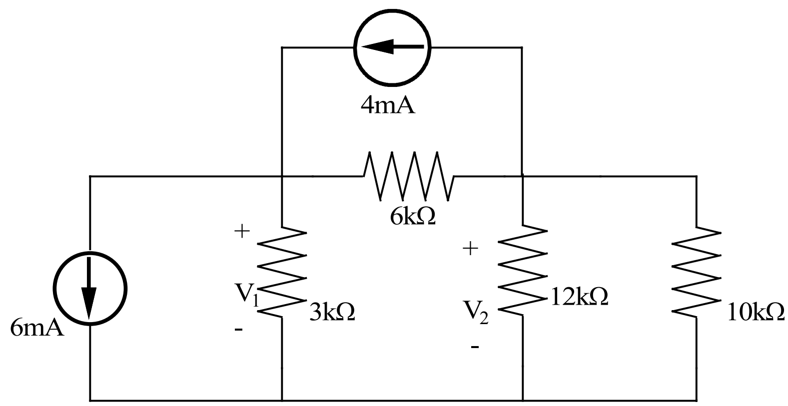
In matrix form:

$$\begin{bmatrix}\frac{1}{3000}+\frac{1}{6000} & -\frac{1}{6000}\\ -\frac{1}{6000} & \frac{1}{6000}+\frac{1}{12000}+\frac{1}{4000}\end{bmatrix}\begin{bmatrix}V_1\\V_2\end{bmatrix}=\begin{bmatrix}-0.006\\0.002\end{bmatrix}\quad\Rightarrow\quad\begin{bmatrix}V_1\\V_2\end{bmatrix}=\begin{bmatrix}-12\\0\end{bmatrix}$$

$$\therefore\ V_1=-12\text{ V}\ \text{ and }\ V_o=\frac{2000}{2000+2000}V_2=0\text{ V}$$

Problem 3.7

Find V1 and V2 in the circuit using nodal analysis Then solve the problem using Matlab and compare your answers



Suggested Solution

$$\frac{V_1}{6k} + \frac{V_1 - V_2}{4k} = \frac{4}{k} - \frac{6}{k}$$
$$\frac{V_2 - V_1}{4k} + \frac{V_2}{5k} + \frac{V_2}{10k} = \frac{-4}{k}$$

$$\Rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{6k} + \frac{1}{4k} & \frac{-1}{4k} \\ \frac{-1}{4k} & \frac{1}{4k} + \frac{1}{5k} + \frac{1}{10k} \end{bmatrix}^{-1} \begin{bmatrix} \frac{-2}{k} \\ \frac{-4}{k} \end{bmatrix}$$
$$V_1 = -12.6V; V_2 = -13V$$

$$Nodal - Equations$$
$$6m + V_1 / 6k + (V_1 - V_2) / 4k - 4m = 0$$
$$4m + (V_2 - V_1) / 4k + V_2 / 5k + V_2 / 10k = 0$$
$$or$$
$$-24 = 5V_1 - 3V_2$$
$$-80 = -5V_1 + 11V_2$$

In Matlab

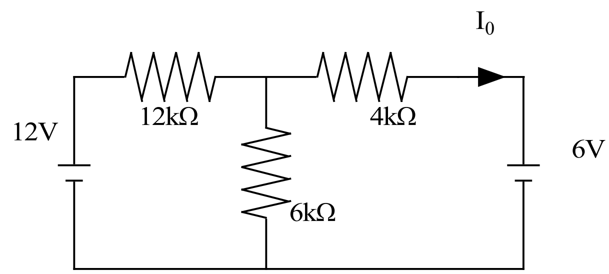
```
>> g=[5 -3;-5 11]

g =

     5    -3
    -5    11

>> i=[-24;-80]

i =

   -24
   -80

>> v=inv(g)*i

v =

  -12.6000
  -13.0000
```

$V_1 = -12.6V; V_2 = -13V$

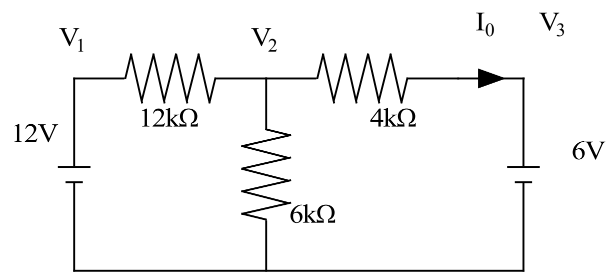
$V_1 = -12.6V; V_2 = -13V$

Problem 3.8

Find I_0 in the network using nodal analysis



Suggested Solution



$V_1 = 12V; V_3 = -6V$

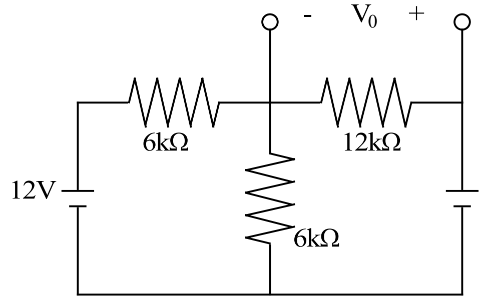
$$\frac{V_2 - 12}{12K} + \frac{V_2}{6k} + \frac{V_2 + 6}{4k} = 0 \Rightarrow V_2 = -1V$$

$$V_2 = -1V; I_0 = 1.25mA$$

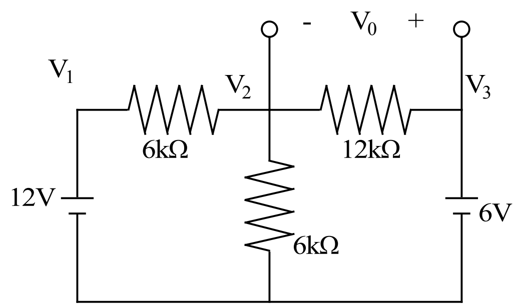
$$I_0 = \frac{V_2 - V_3}{4k} = \frac{-1+6}{4k} = 1.25mA$$

Problem 3.9

Find V_0 in the network using nodal analysis



Suggested Solution



$$V_1 = 12V; V_3 = 6V$$

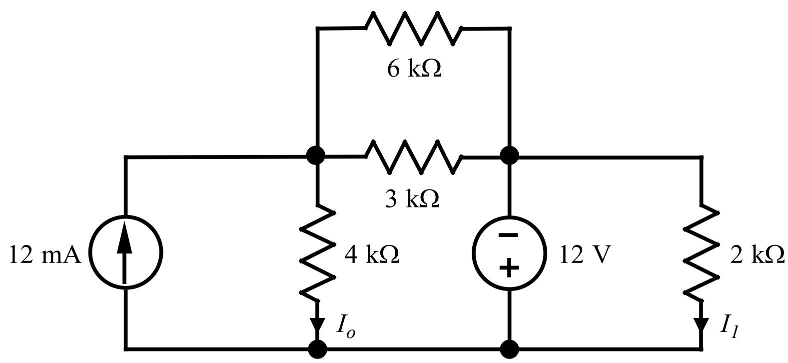
$$\frac{V_2 - 12}{6K} + \frac{V_2}{6k} + \frac{V_2 - 6}{12k} = 0 \Rightarrow V_2 = -6V$$

$$V_2 = 6v \therefore V_0 = V_2 - V_3 = 0V$$

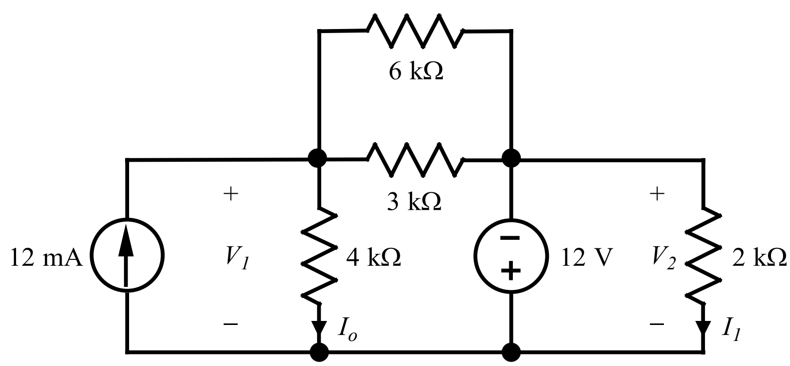
$$\Rightarrow V_2 = 6v \therefore V_0 = V_2 - V_3 = 0V$$

Problem 3.10

Use nodal analysis to find I_o and I_1 in the circuit shown.



Suggested Solution



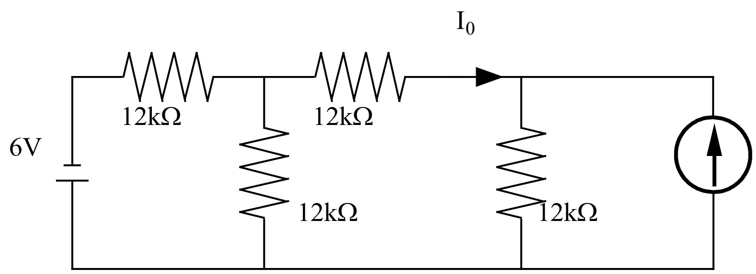
$$V_2 = -12 \text{ V} \quad \Rightarrow \quad I_1 = \frac{V_2}{2 \text{ k}\Omega} = \frac{-12}{2000} = -6 \text{ mA}$$

$$-12 \text{ mA} + \frac{V_1}{4 \text{ k}\Omega} + \frac{V_1 - (-12 \text{ V})}{6 \text{ k}\Omega} + \frac{V_1 - (-12 \text{ V})}{3 \text{ k}\Omega} = 0 \quad \Rightarrow \quad V_1 = 8 \text{ V} \quad \Rightarrow \quad I_o = \frac{V_1}{4 \text{ k}\Omega} = \frac{8}{4000} = 2 \text{ mA}$$

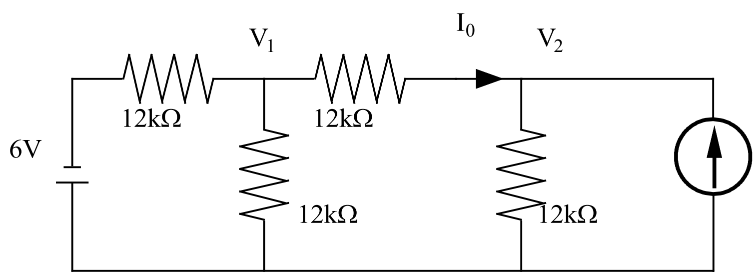
$$I_o = 2 \text{ mA}, \; I_1 = -6 \text{ mA}$$

Problem 3.11

Find I_0 in the circuit using nodal analysis



Suggested Solution



$$\frac{V_1+6}{12k}+\frac{V_1}{12k}+\frac{V_1-V_2}{12k}=0$$

$$\frac{V_2-V_1}{12k}+\frac{V_2}{12k}=\frac{2}{K}$$

$$\Rightarrow$$

$$\frac{3V_1}{12k}-\frac{V_2}{12k}=\frac{-1}{12k}$$

$$\frac{-V_1}{12k}+\frac{2V_2}{12k}=\frac{2}{12k}$$

$$3V_1-V_2=-6$$

$$-V_1+2V_2=24$$

$$\Rightarrow$$

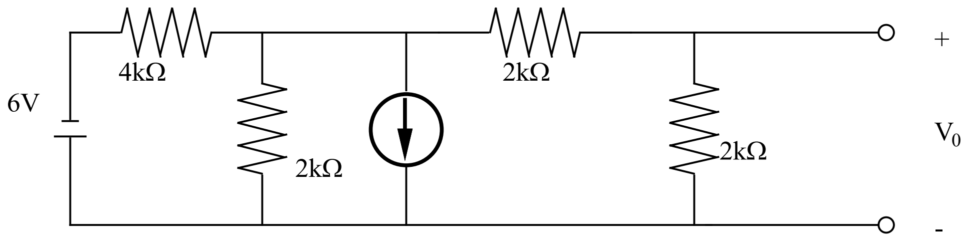
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}=\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -6 \\ 24 \end{bmatrix}=\begin{bmatrix} \frac{12}{5} \\ \frac{66}{5} \end{bmatrix}$$

$$I_0=\frac{\frac{12}{5}-\frac{66}{5}}{12k}=-0.9mA$$

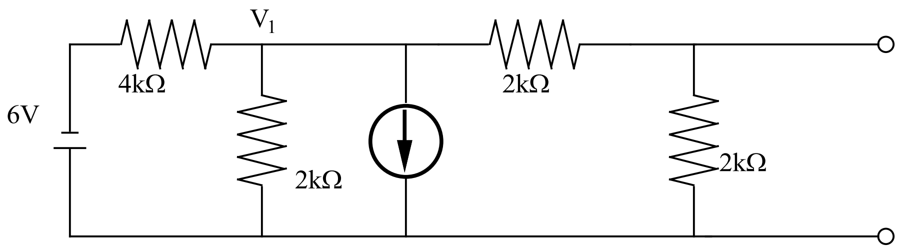
$$I_0=-0.9mA$$

Problem 3.12

Use nodal analysis to find V_0 in the network



Suggested Solution



$$\frac{V_1-(-6)}{4k}+\frac{V_1}{2k}+\frac{2}{k}+\frac{V_1}{2k+2k}=0$$

$\Rightarrow$

$$V_1=\frac{-7}{2}V;\therefore V_0=\frac{-7}{2}\left(\frac{2k}{2k+2k}\right)$$

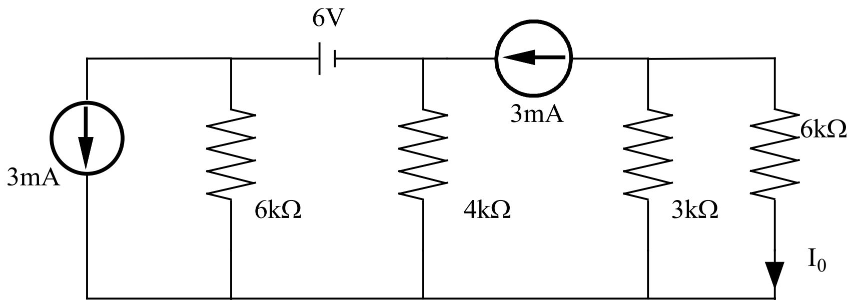
$$V_0=\frac{-7}{4}V$$

$$V_1=\frac{-7}{2}V$$

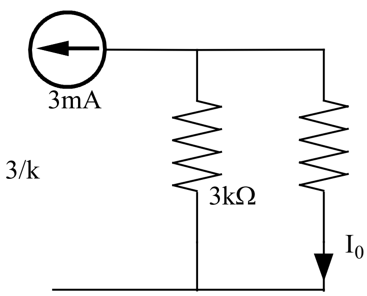
$$V_0=\frac{-7}{4}V$$

Problem 3.13

Find I_0 in the network



Suggested Solution

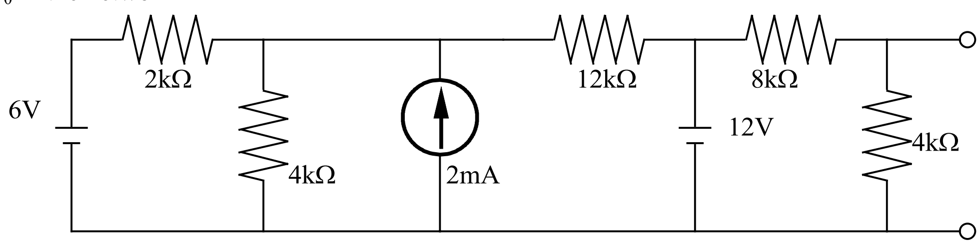


$$I_0 = \frac{-3}{k}\left(\frac{3k}{3k+6k}\right) = -1mA$$

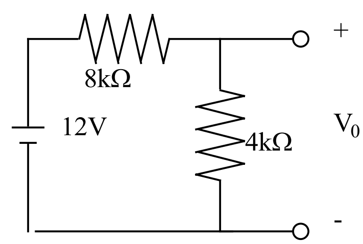
$$I_0 = -1mA$$

Problem 3.14

Find V_0 in the network



Suggested Solution

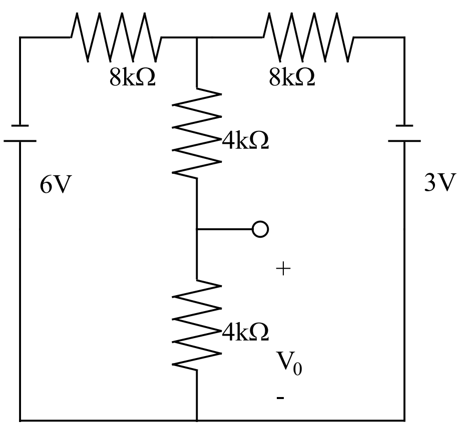


$$V_0 = 12\left(\frac{4k}{4k+8k}\right) = 4V$$

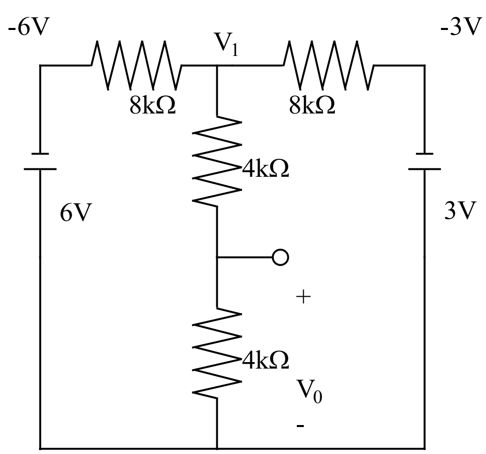
$$V_0 = 4V$$

Problem 3.15

Use nodal analysis to find V_0.



Suggested Solution



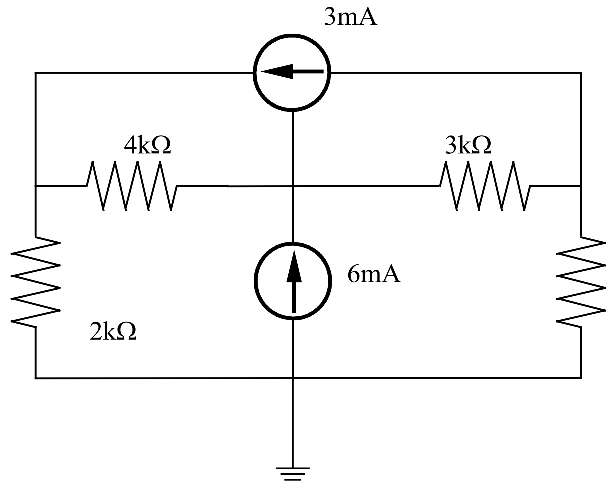
$$\frac{V_1+6}{6k}+\frac{V_1}{2k+2k}+\frac{V_1+3}{2k}=0 \Rightarrow V_1=\frac{-5}{2}V$$

$$V_0=\frac{-5}{2}\left(\frac{1k}{1k+2k}\right)=\frac{-5}{6}V$$

$$V_0=\frac{-5}{6}V$$

Problem 3.16

Write the node equations for the circuit in matrix form and find the node voltages using Matlab



Suggested Solution

$$\frac{V_1}{2k} + \frac{V_1 - V_2}{2k + 2k} = \frac{3}{k}$$

$$\frac{V_2 - V_1}{1k} + \frac{V_2 - V_3}{3k} = \frac{6}{k}$$

$$\frac{V_3 - V_2}{3k} + \frac{V_3}{4k} = \frac{-3}{k}$$

$$\Rightarrow$$

$$V_1(\frac{3}{2k}) - V_2(\frac{1}{k})0 = \frac{3}{k}$$

$$-V_1(\frac{1}{k}) + V_2(\frac{4}{3k}) - V_3(\frac{1}{3k}) = \frac{6}{k}$$

$$0 - V_2(\frac{1}{3k}) + V_3(\frac{7}{12k}) = \frac{-3}{k}$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \frac{3}{2k} & \frac{-1}{k} & 0 \\ \frac{-1}{k} & \frac{4}{3k} & \frac{-1}{3k} \\ 0 & \frac{-1}{3k} & \frac{7}{12k} \end{bmatrix} \begin{bmatrix} \frac{3}{k} \\ \frac{6}{k} \\ \frac{-3}{k} \end{bmatrix}$$

$$Nodal - Equations$$

$$V_1 / 2k(V_1 - V_2) / k - 3m = 0$$

$$(V_3 - V_2) / 3k + V_3 / 4k + 3m = 0$$

$$V_1 / 2k + V_3 / 4k - 6m = 0$$

$$or$$

$$6 = 3V_1 - 2V_2$$

$$-36 = -4V_1 + 7V_2$$

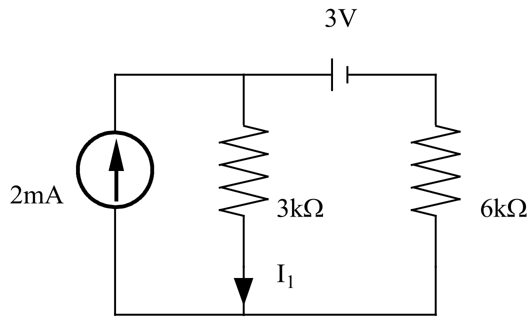
$$24 = 2V_1 + V_3$$

Using Matlab

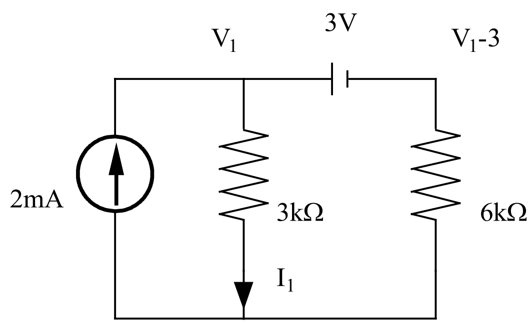
```
EDU» g=[3 -2 0;0 -4 7;2 0 1]

g =

     3    -2     0
     0    -4     7
     2     0     1

EDU» i=[6;-36;24]

i =

     6
   -36
    24

EDU» v=inv(g)*i

v =

   10.8000
   13.2000
    2.4000
```

Problem 3.17

Find I_1 in the network shown.



Suggested Solution

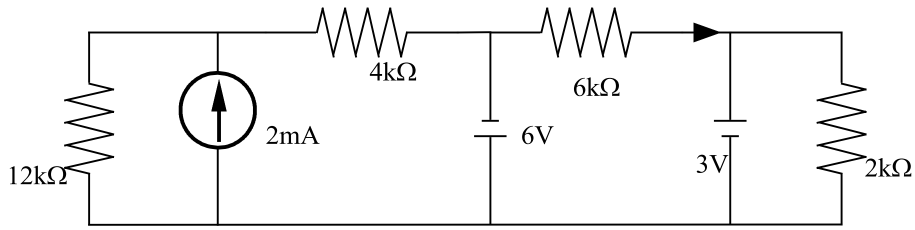


$$\frac{V_1}{3k} + \frac{V_1 - 3}{6k} = \frac{2}{k} \Rightarrow V_1 = 5V; I_1 = 1.67mA$$

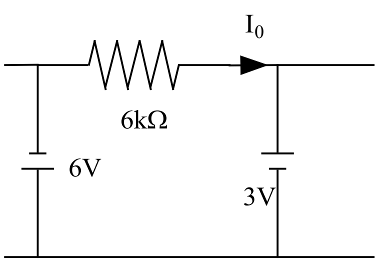
$I_1 = 1.67mA$

Problem 3.18

Find I_0 in the network shown.



Suggested Solution

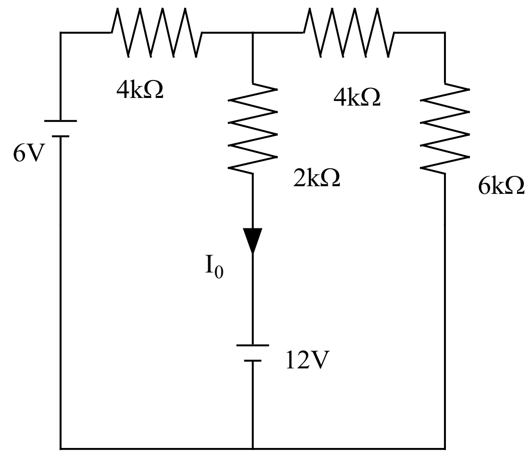


$$I_0 = \frac{-6-3}{6k} = -1.5mA$$

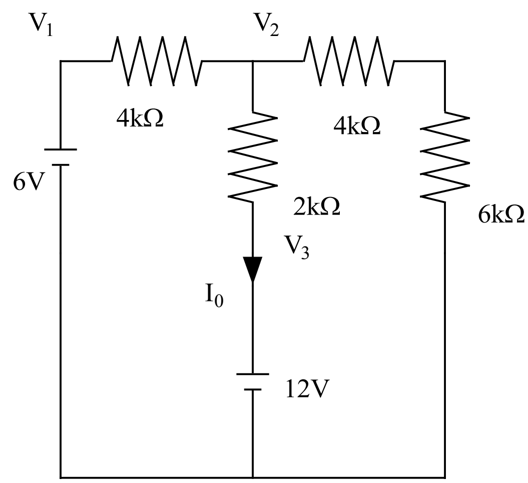
$$I_0 = -1.5mA$$

Problem 3.19

Find I_0 in the circuit shown.



Suggested Solution



$$V_1 = 6V, V_3 = 12V$$

$$\frac{V_2 - V_1}{3k} + \frac{V_2 - V_3}{2k} + \frac{V_2}{10k} = 0$$

$$\Rightarrow$$

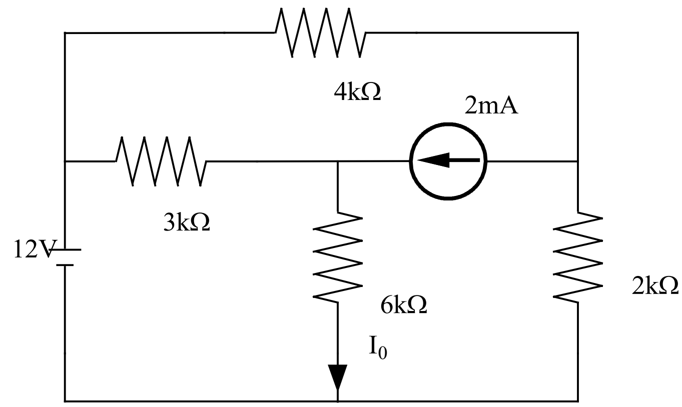
$$V_2 = \frac{150}{17}V, V_3 = 12 = \frac{204}{17}V$$

$$I_0 = (\frac{150}{17} - \frac{204}{17}) / 2k = \frac{-27}{17}mA = -1.59mA$$

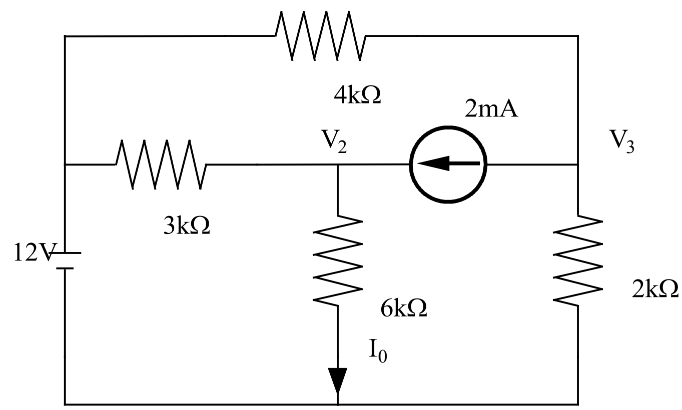
$$I_0 = -1.59mA$$

Problem 3.20

Find I_0 in the network using nodal analysis



Suggested Solution

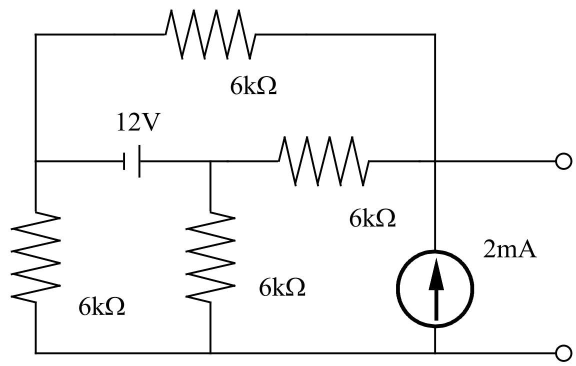


$$\frac{V_2 - 12}{3k} + \frac{V_2}{6k} = \frac{2}{k} \therefore V_2 = 12V and I_0 = 2mA$$

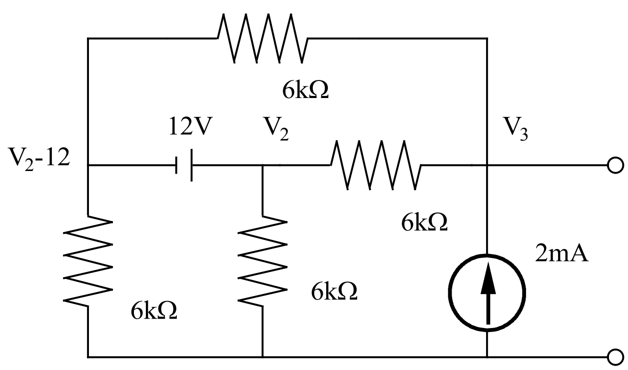
$$I_0 = 2mA$$

Problem 3.21

Use nodal analysis to find V_0 in the network shown. Then solve this problem using Matlab and compare your results.



Suggested Solution



$$\frac{V_2-12}{1k}+\frac{V_2-12-V_3}{1k}+\frac{V_2}{1k}+\frac{V_2-V_3}{1k}=0$$

$$\frac{V_3-V_2}{1k}+\frac{V_3-(V_2-12)}{1k}=\frac{2}{k}$$

$$\frac{4}{k}V_2-\frac{2}{k}V_3=\frac{24}{k};\frac{-2V_2}{k}+\frac{2V_3}{k}=\frac{-10}{k}$$

$$\Rightarrow$$

$$4V_2-2V_3=24;-2V_2+2V_3=-10$$

$$\Rightarrow$$

$$2V_2=14\Rightarrow V_2=7VandV_3=2V$$

$$Nodal-Equations$$

$$V_2-V_1=12$$

$$V_1/1k+V_2/1K=2m$$

$$(V_0-V_2)/1k+(V_0-V_1)/1k=2m$$

$$or$$

$$12=-V_1+V_2$$

$$2=V_1+V_2$$

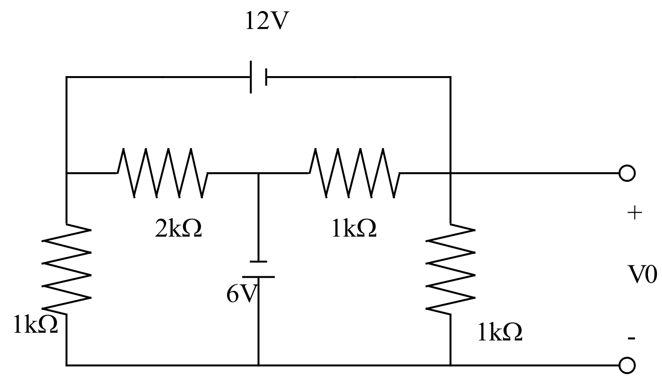
$$2=-V_1-V_2+2V_0$$

```
EDU» g=[-1 1 0;1 1 0;-1 -1 2]

g =

    -1     1     0
     1     1     0
    -1    -1     2

EDU» i=[12;2;2]

i =

    12
     2
     2

EDU» v=inv(g)*i

v =

    -5
     7
     2
```

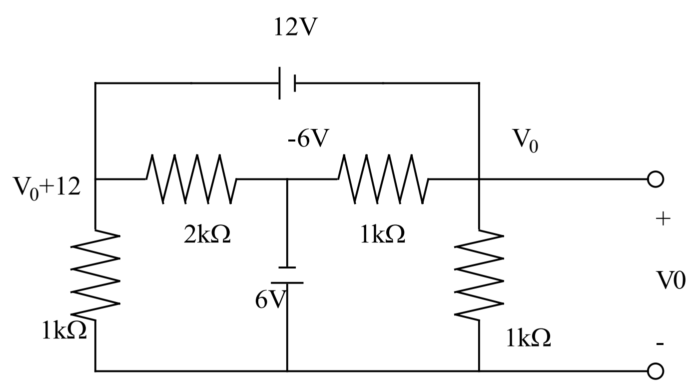
$$V_0 = 2V$$

Problem 3.22

Find V_0 in the circuit shown using Nodal Analysis



Suggested Solution



$$\frac{V_0+12}{1k}+\frac{V_0+12+6}{2k}+\frac{V_0+6}{1k}+\frac{V_0}{1k}=0$$

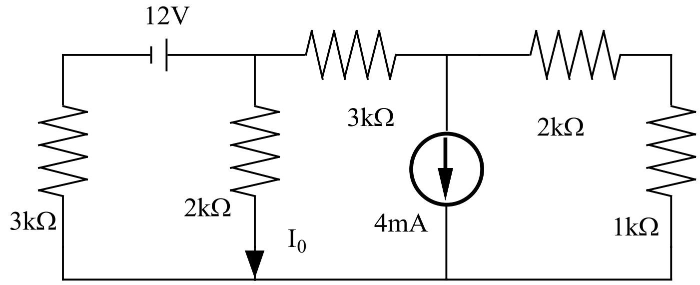
$$V_0\left(\frac{7}{2k}\right)=\frac{-27}{1k}$$

$$\therefore V_0=-7.17V$$

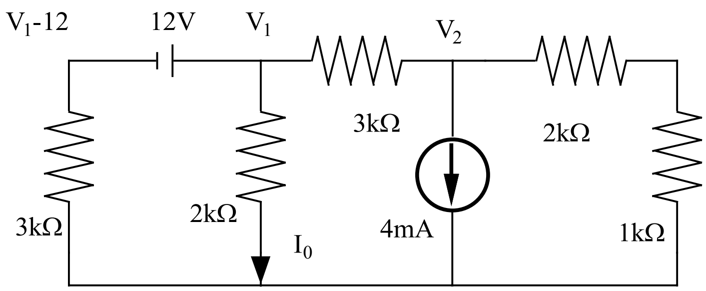
$$V_0=-7.17V$$

Problem 3.23

Use nodal analysis to find I_0 in the network shown.



Suggested Solution



$$\frac{V_1-12}{3k}+\frac{V_1}{2k}+\frac{V_1-V_2}{3k}=0$$

$$\frac{V_2-V_1}{3k}+\frac{4}{k}+\frac{V_2}{3k}=0$$

$\Rightarrow$

$$\frac{7V_1}{6k}-\frac{V_2}{3k}=\frac{4}{k}$$

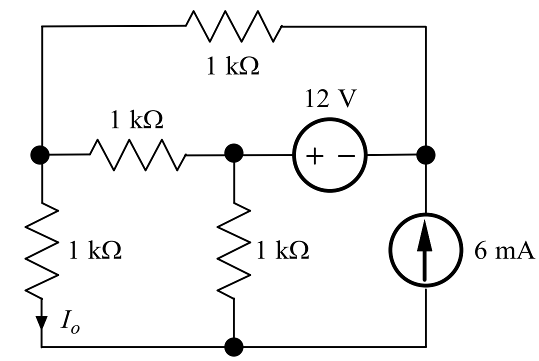
$$\frac{-1V_1}{3k}+\frac{2V_2}{3k}=\frac{-4}{k}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}=\begin{bmatrix} \frac{7}{6k} & \frac{-1}{3k} \\ \frac{-1}{3k} & \frac{2}{3k} \end{bmatrix}^{-1}\begin{bmatrix} \frac{4}{k} \\ \frac{-4}{k} \end{bmatrix}\Rightarrow V_1 = 2V and I_0 = \frac{2}{2k}=1mA$$

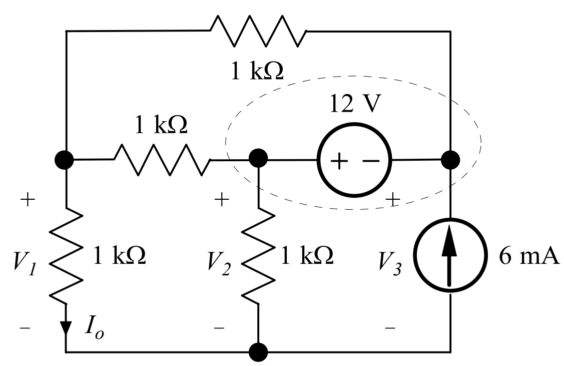
$$I_0 = 1mA$$

Problem 3.24

Find I_o in the network shown.



Suggested Solution



$V_2 = V_3 + 12 \text{ V} \qquad \Rightarrow \qquad V_2 - V_3 = 12$

$$\frac{V_1}{1\text{ k}\Omega} + \frac{V_1 - V_2}{1\text{ k}\Omega} + \frac{V_1 - V_3}{1\text{ k}\Omega} = 0 \qquad \Rightarrow \qquad 3V_1 - V_2 - V_3 = 0$$

$$\frac{V_2 - V_1}{1\text{ k}\Omega} + \frac{V_2}{1\text{ k}\Omega} + \frac{V_3 - V_1}{1\text{ k}\Omega} - 6\text{ mA} = 0 \qquad \Rightarrow \qquad -2V_1 + 2V_2 + V_3 = 6$$

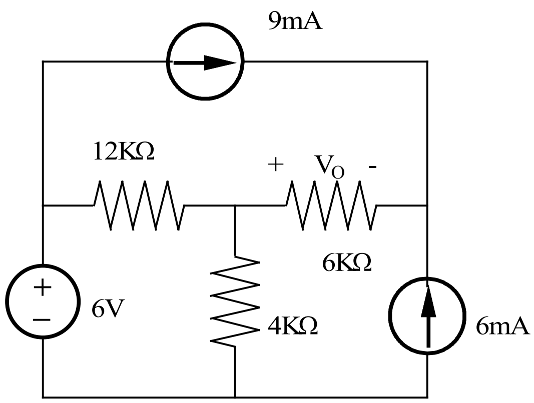
In matrix form:

$$\begin{bmatrix} 0 & 1 & -1 \\ 3 & -1 & -1 \\ -2 & 2 & 1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 6 \end{bmatrix} \qquad \Rightarrow \qquad V_1 = 0\text{ V}$$

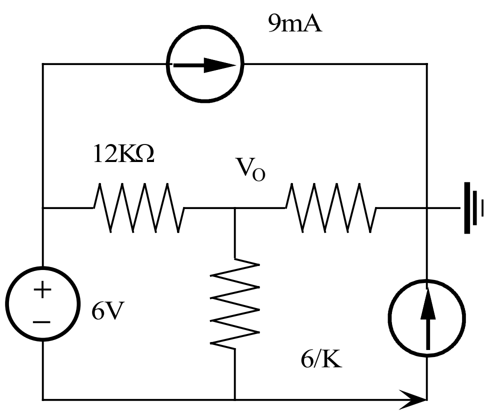
$$\therefore \quad I_o = \frac{V_1}{1\text{ k}\Omega} = 0\text{ A}$$

Problem 3.25

Use nodal analysis to find V_0 in the circuit shown.



Suggested Solution



Note the position of the reference node

KCL

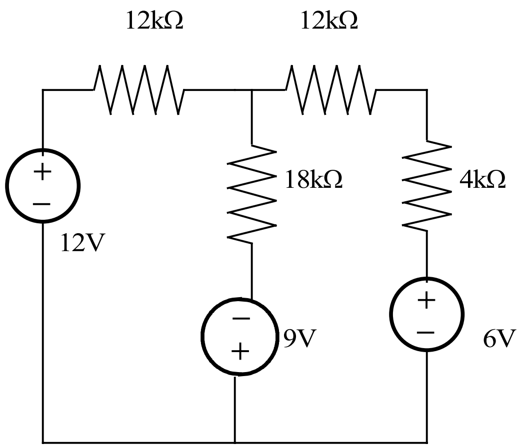
$$\frac{9}{K}+\frac{6}{K}+\frac{V_0}{6K}=0$$

$$V_0=-90V$$

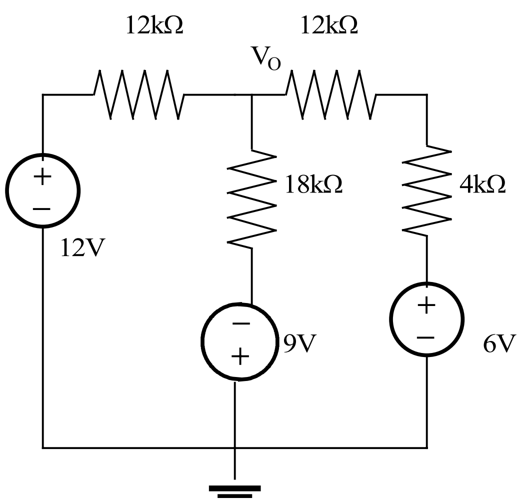
$$V_0=-90V$$

Problem 3.26

Find the V_O in the circuit shown using nodal analysis.



Suggested Solution



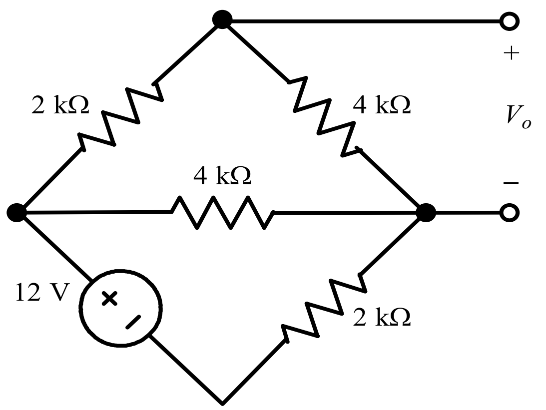
$$\frac{V_0 - 12}{12K} + \frac{V_0 + 9}{18K} + \frac{V_0 + 6}{16K} = 0$$

$$V_0 = 0.621V$$

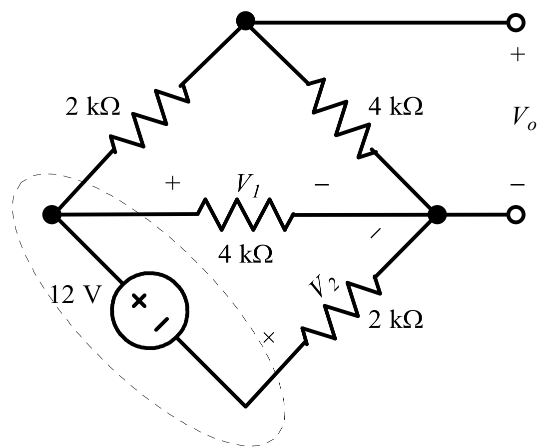
$$V_0\left(\frac{1}{12} + \frac{1}{18} + \frac{1}{16}\right) = 0.621V$$

Problem 3.27

Find V_o in the network shown using nodal analysis.



Suggested Solution



$V_1 - V_2 = 12 \text{ V}$

$$\frac{V_o - V_1}{2\text{ k}\Omega} + \frac{V_o}{4\text{ k}\Omega} = 0 \qquad \Rightarrow \qquad 3V_o - 2V_1 = 0$$

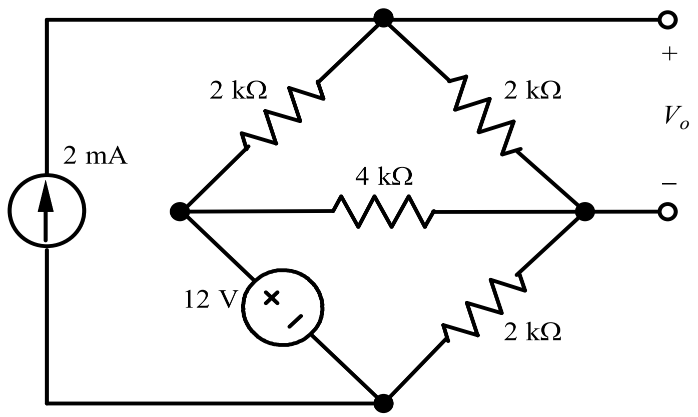
$$\frac{V_1 - V_o}{2\text{ k}\Omega} + \frac{V_1}{4\text{ k}\Omega} + \frac{V_2}{2\text{ k}\Omega} = 0 \qquad \Rightarrow \qquad -2V_o + 3V_1 + 2V_2 = 0$$

In matrix form:

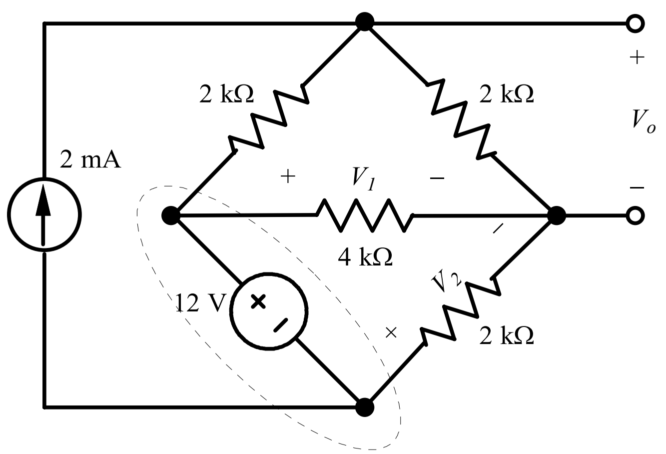
$$\begin{bmatrix} 0 & 1 & -1 \\ 3 & -2 & 0 \\ -2 & 3 & 2 \end{bmatrix} \begin{bmatrix} V_o \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix} \qquad \Rightarrow \qquad V_o = 4.36 \text{ V}$$

Problem 3.28

Find V_o in the network shown using nodal analysis.



Suggested Solution



$V_1 - V_2 = 12\text{ V}$

$$\frac{V_o - V_1}{2\text{ k}\Omega} + \frac{V_o}{2\text{ k}\Omega} - 2\text{ mA} = 0 \qquad \Rightarrow \qquad 2V_o - V_1 = 4$$

$$\frac{V_1 - V_o}{2\text{ k}\Omega} + \frac{V_1}{4\text{ k}\Omega} + \frac{V_2}{2\text{ k}\Omega} + 2\text{ mA} = 0 \qquad \Rightarrow \qquad -2V_o + 3V_1 + 2V_2 = -8$$

In matrix form:

$$\begin{bmatrix} 0 & 1 & -1 \\ 2 & -1 & 0 \\ -2 & 3 & 2 \end{bmatrix} \begin{bmatrix} V_o \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 4 \\ -8 \end{bmatrix} \qquad \Rightarrow \qquad V_o = 4.5\text{ V}$$

Problem 3.29

Suggested Solution

$$\frac{V_1-6}{4K}+\frac{2}{K}+\frac{V_1-V_2}{4K}=0 \Rightarrow V_1(\frac{1}{4K}+\frac{1}{4K})-V_2(\frac{1}{4K})=\frac{6}{4K}-\frac{2}{K}$$

$$\frac{V_2-V_1}{4K}-\frac{4}{K}+\frac{V_2}{2K+2K}=0 \Rightarrow -V_1(\frac{1}{4K})+V_2(\frac{1}{8K}+\frac{1}{4K})=\frac{4}{K}$$

$$V_2=10V$$

$$V_0=10(\frac{2K}{2K+2K})=5V$$

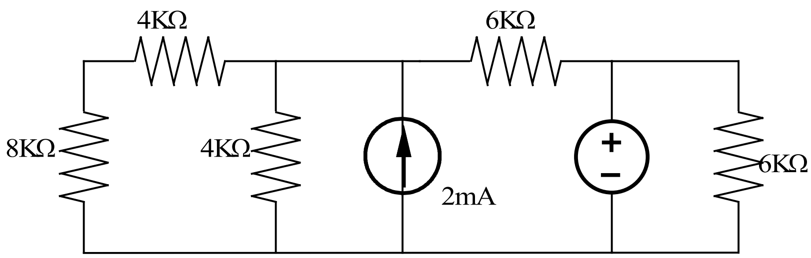
$$V_0=5V$$

$$V_2=10V$$

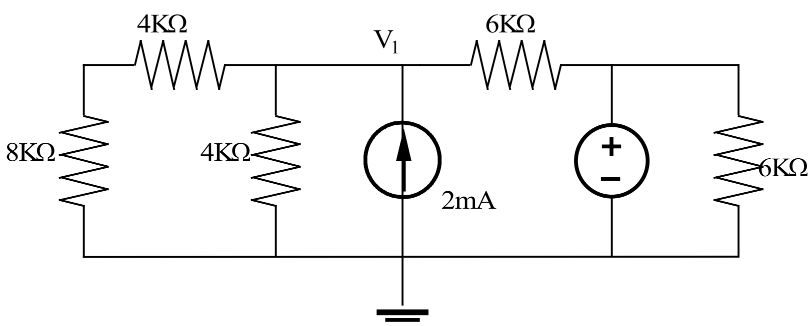
$$V_0=5V$$

Problem 3.30

Find Io in the circuit shown using nodal analysis.



Suggested Solution



$$\frac{V_1}{4K+8K}+\frac{V_1}{4K}-\frac{2}{K}+\frac{V_1-6}{6K}=0 \Rightarrow V_1=6V$$

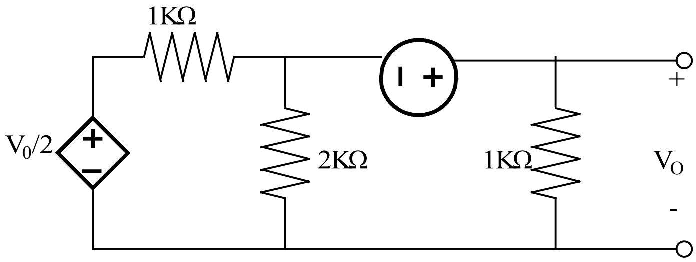
$$I_L=\frac{6}{4K}=1.5mA$$

$V_1 = 6V$

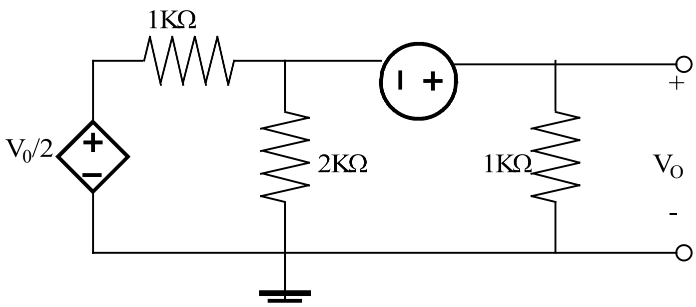
$I_L = 1.5mA$

Problem 3.31

Find Vo in the circuit shown using nodal analysis.



Suggested Solution



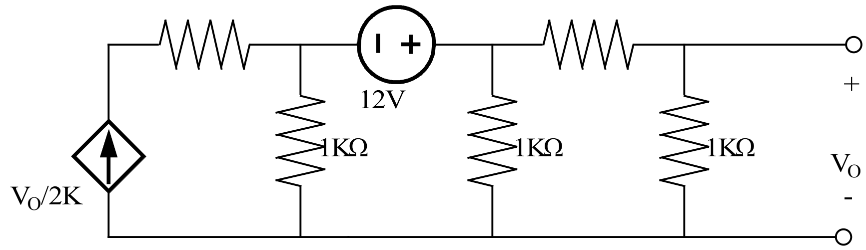
$$\frac{V_0 - 12 - \frac{V_0}{2}}{1K} + \frac{V_0 - 12}{2K} + \frac{V_0}{1K} = 0$$

$$V_0 = 9V$$

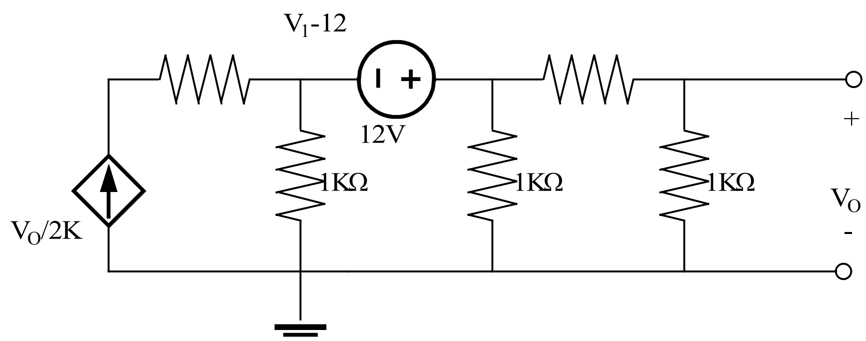
$$V_0 = 9V$$

Problem 3.32

Use nodal analysis to find Vo in the network shown.



Suggested Solution



$$\frac{V_1 - 12}{1K} - \frac{V_0}{2K} + \frac{V_1}{1K} + \frac{V_1}{1K + 1K} = 0$$

$$V_0 = \frac{1K}{1K + 1K}, V_1 = \frac{V_1}{2}$$

$$V_0 = 2.667V$$

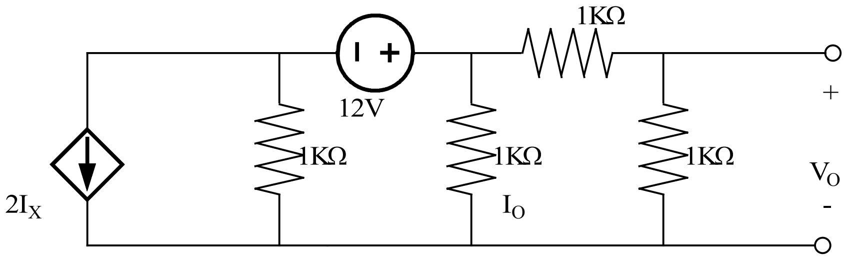
$$V_1(\frac{5}{2K}) - \frac{V_0}{2K} = \frac{12}{K}$$

$$2V_0(\frac{5}{2K} - \frac{V_0}{2K}) = \frac{12}{K}$$

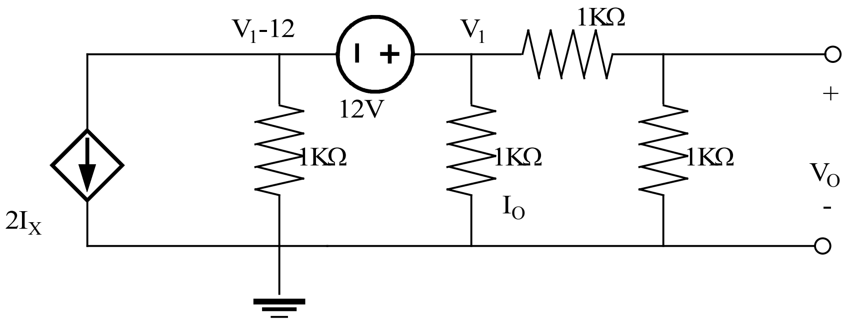
$$V_0 = 2.667V$$

Problem 3.33

Find the Vo in the circuit shown using nodal analysis. Then solve the problem using matlab and compare your results.



Suggested Solution



$$2I_X + \frac{V_1 - 12}{1K} + \frac{V_1}{1K} + \frac{V_1}{2K} = 0$$

Where

$$I_X = \frac{V_1}{1K}$$

$$\frac{2V_1}{1K} + \frac{V_1}{1K} - \frac{12}{K} + \frac{V_1}{1K} + \frac{V_1}{2K} = 0 \Rightarrow V_1 = 24/9V$$

$$V_0 = 1/2V_1 = \frac{4}{3}V$$

$$V_0 = 1/2V_1 = \frac{4}{3}V$$

$Nodal\_Equations$

$$2I_X + V_1/1K + V_2/1K + V_0/1K = 0$$
$$(V_2 - V_0)/1K = V_0/1K$$
$$V_2 - V_1 = 12$$
$$I_X = V_2/1K$$

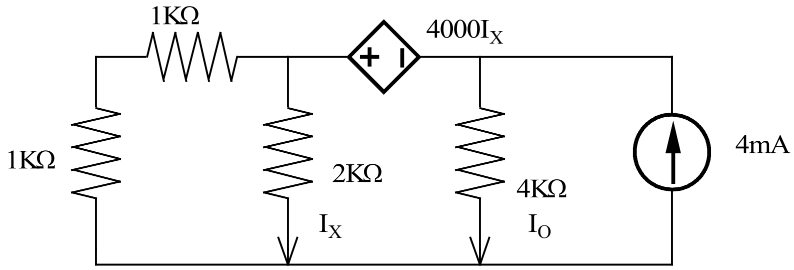
OR

$$V_1 + 3V_2 + V0 = 0$$
$$V_2 - 2V_0 = 0$$
$$-V_1 + V_2 = 12$$

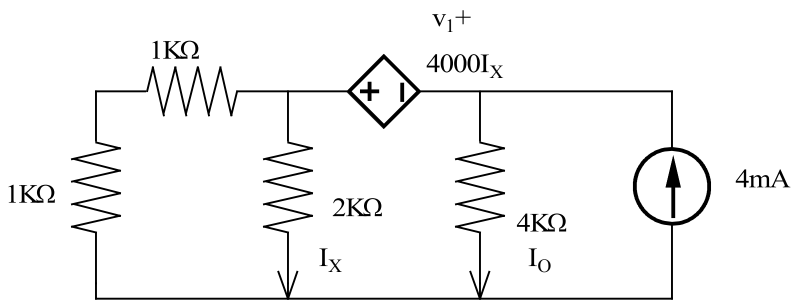
```
EDU» G=[1 3 1;0 1 -2;-1 1 0]

G =

     1     3     1
     0     1    -2
    -1     1     0

EDU» I=[0;0;12]

I =

     0
     0
    12

EDU» V=inv(G)*I

V =

   -9.3333
    2.6667
    1.3333
```

Problem 3.34

Find Io in the network shown.



Suggested Solution



$$\frac{V_1 + 4KI_X}{2K} + \frac{V_1 + 4KI_X}{2K} + \frac{V_1}{4K} = \frac{4}{K}$$

$$\frac{V_1 + 4KI_X}{2K} = I_X \Rightarrow I_X = \frac{-V_1}{2K}$$

$$\frac{5}{4K}V_1 - \frac{4V_1}{2K} = \frac{4}{K}$$

$$V_1 = \frac{-16}{3}V$$

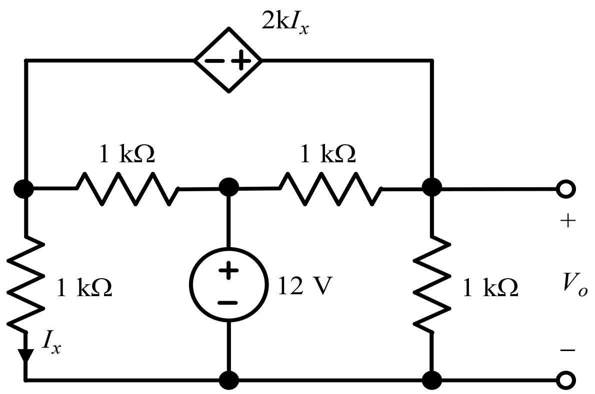
$$I_0 = \frac{-4}{3}mA$$

$$V_1 = \frac{-16}{3}V$$

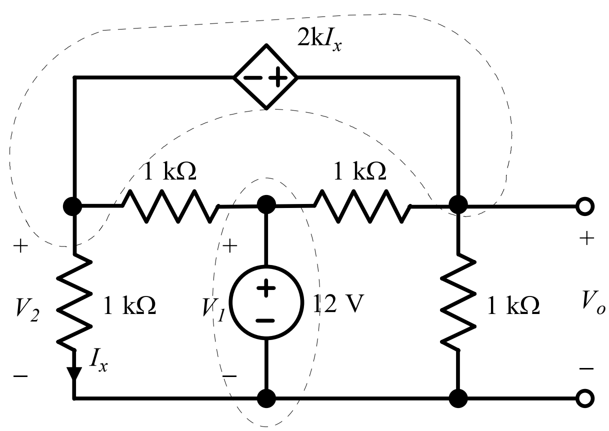
$$I_0 = \frac{-4}{3}mA$$

Problem 3.35

Find V_o in the circuit shown using nodal analysis.



Suggested Solution



$$V_o - V_2 = 2\text{k}I_x \qquad \Rightarrow \qquad V_o - V_2 - 2\text{k}I_x = 0$$

$$V_1 = 12 \text{ V}$$

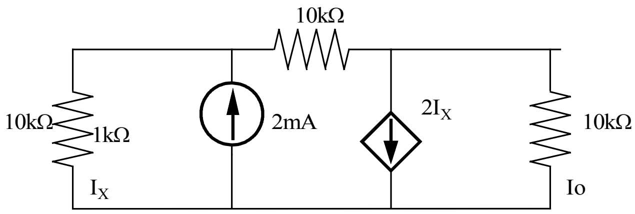
$$\frac{V_2 - V_1}{1 \text{ k}\Omega} + \frac{V_2}{1 \text{ k}\Omega} + \frac{V_o - V_1}{1 \text{ k}\Omega} + \frac{V_o}{1 \text{ k}\Omega} = 0 \qquad \Rightarrow \qquad V_o - V_1 + V_2 = 0$$

$$I_x = \frac{V_2}{1 \text{ k}\Omega} \qquad \Rightarrow \qquad V_2 - 1000I_x = 0$$

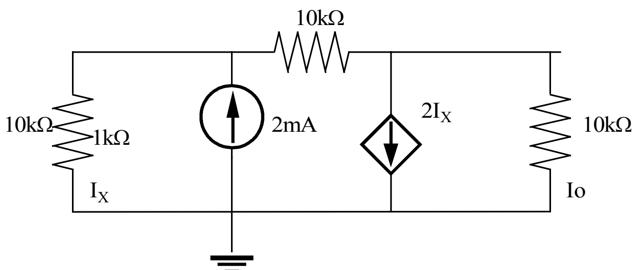
In matrix form:

Problem 3.36

Find Io in the circuit shown using nodal analysis.



Suggested Solution



$$\frac{V_1}{10K} - \frac{2}{K} + \frac{V_1 - V_2}{10K} = 0$$

$$\frac{V_2 - V_1}{10K} + 2I_X + \frac{V_2}{10K} = 0$$

$$V_2 = -4V$$

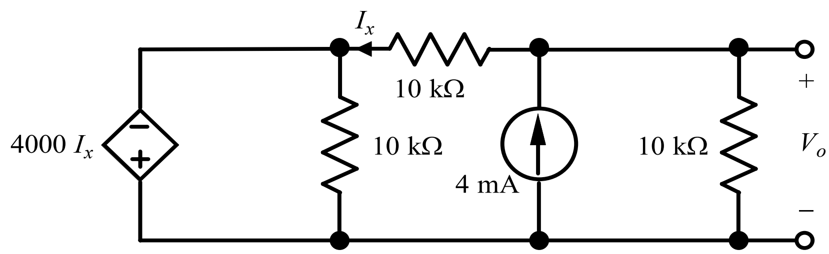
$$I_0 = -0.4mA$$

$$V_2 = -4V$$

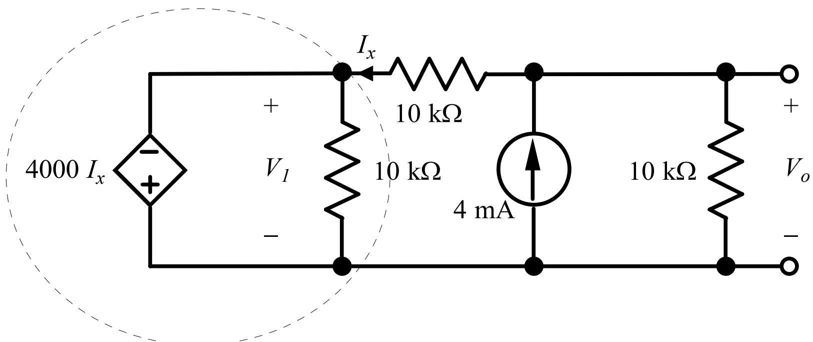
$$I_0 = -0.4mA$$

Problem 3.37

Find V_o in the network shown using nodal analysis.



Suggested Solution



$$V_1 = -4000 I_x \qquad \Rightarrow \qquad V_1 + 4000 I_x = 0$$

$$\frac{V_o - V_1}{10\text{ k}\Omega} - 4\text{ mA} + \frac{V_o}{10\text{ k}\Omega} = 0 \qquad \Rightarrow \qquad 2V_o - V_1 = 40$$

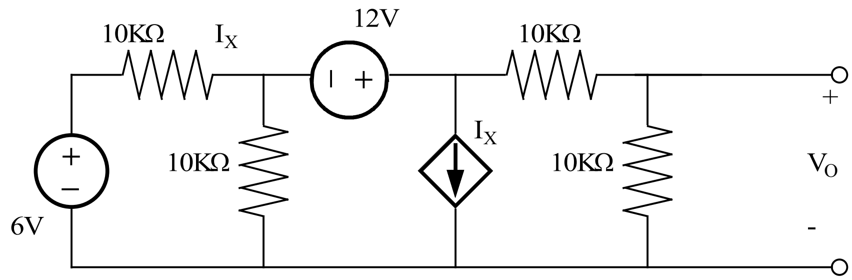
$$I_x = \frac{V_o - V_1}{10\text{ k}\Omega} \qquad \Rightarrow \qquad V_o - V_1 - 10000 I_x = 0$$

In matrix form:

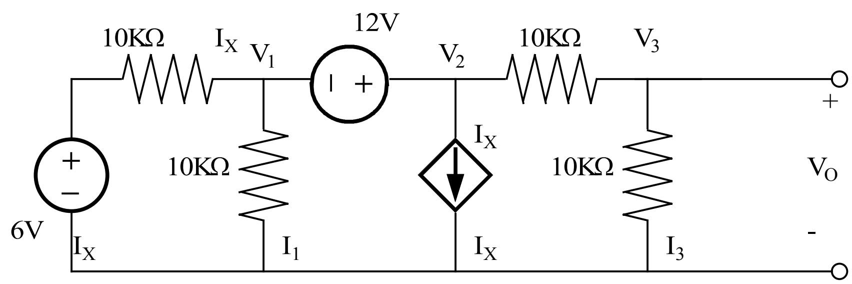
$$\begin{bmatrix} 0 & 1 & 4000 \\ 2 & -1 & 0 \\ 1 & -1 & -10000 \end{bmatrix} \begin{bmatrix} V_o \\ V_1 \\ I_x \end{bmatrix} = \begin{bmatrix} 0 \\ 40 \\ 0 \end{bmatrix} \qquad \Rightarrow \qquad V_o = 15\text{ V}$$

Problem 3.38

Find Vo in the network shown. In addition, determine all branch currents and check KCL at every node.



Suggested Solution



$$\frac{V_1-6}{10K}+\frac{V_1}{10K}+I_X+\frac{V_2}{20K}=0$$

$$I_X=\frac{6-V_1}{10K}$$

$$V_0=V_2/2$$

$$V_1=-4V, V_2=8V, V_0=4V$$

$$I_3=\frac{V_0}{10K}=0.4mA$$

$$I_X=\frac{6-V_1}{10K}=1mA$$

$$I_2=I3+I_X=1.4mA$$

$$I_1=\frac{V1}{10K}=-0.4mA$$

$$I_3 = \frac{V_0}{10K} = 0.4mA$$

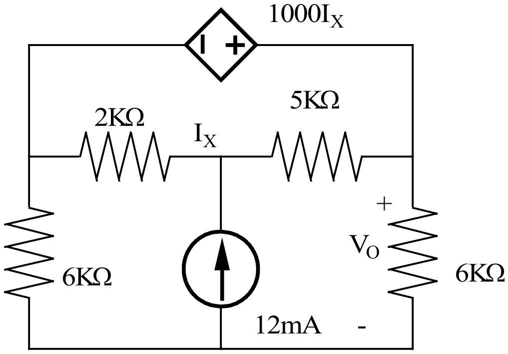
$$I_X = \frac{6 - V_1}{10K} = 1mA$$

$$I_2 = I3 + I_X = 1.4mA$$

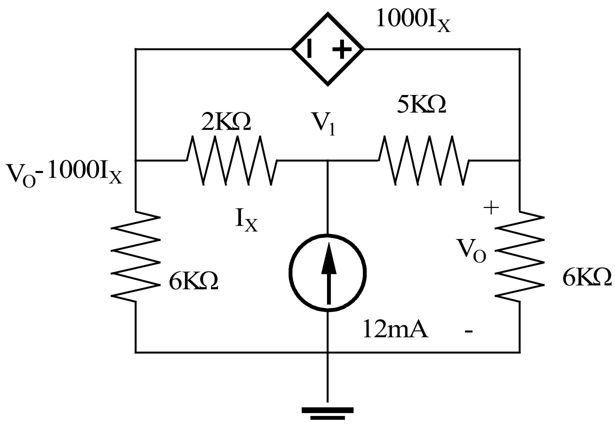
$$I_1 = \frac{V1}{10K} = -0.4mA$$

Problem 3.39

Find Vo in the circuit shown.



Suggested Solution



$$\frac{V_0 - KI_X}{6K} + \frac{V_0 - KI_X - V_1}{2K} + \frac{V_0 - V_1}{5K} + \frac{V_0}{6K} = 0$$

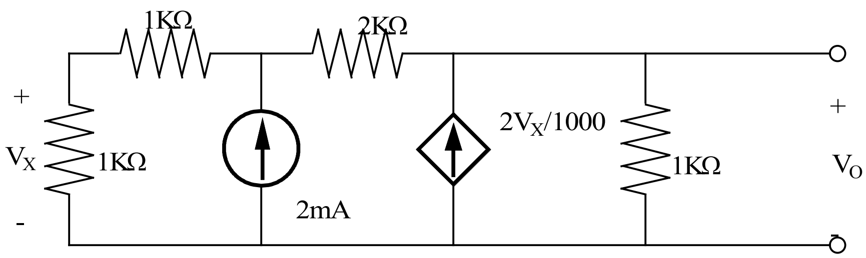
$$I_X = \frac{V_0 - KI_X - V_1}{2K}$$

$$V_0 = 32.25$$

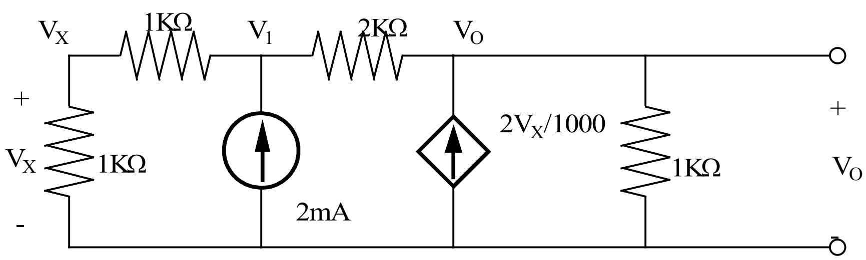
$$V_0 = 32.25$$

Problem 3.40

Find Vo in the circuit shown.



Suggested Solution



$$\frac{V_1}{2K} - \frac{2}{K} + \frac{V_1 - V_0}{2K} = 0$$

$$V_X = 1/2V_1$$

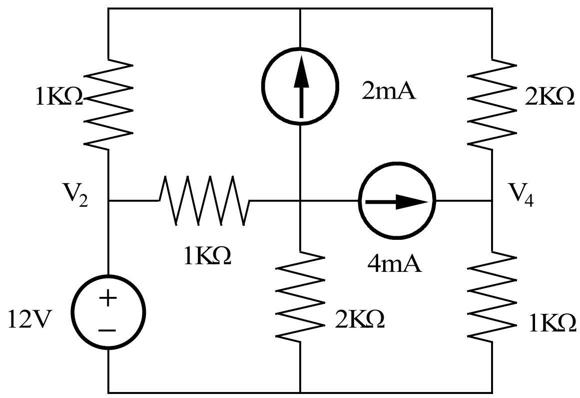
$$\frac{V_0 - V_1}{2K} - \frac{2V_X}{1K} + \frac{V_0}{1K} = 0$$

$$V_0 = 4V$$

$$V_0 = 4V$$

Problem 3.41

Use matlab to find the node voltages in the network shown.



Suggested Solution

$$Applying\_kcl$$

$$\frac{V_1-V_2}{1K}-\frac{2}{K}+\frac{V_1-V_4}{2K}=0$$

$$V_2=12$$

$$\frac{V_3-V_2}{1K}+\frac{2}{K}+\frac{4}{K}+\frac{V_3}{2K}=0$$

$$\frac{V_4-V_1}{2K}-\frac{4}{K}+\frac{V_4}{1K}=0$$

$$MATRIX\_FORM$$

$$\begin{bmatrix} 0.0015 & -0.001 & 0 & -0.0005 \\ 0 & 1 & 0 & 0 \\ 0 & -0.001 & 0.0015 & 0 \\ -0.0005 & 0 & 0 & 0.0015 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_X \end{bmatrix}=\begin{bmatrix} 0.002 \\ 12 \\ -0.006 \\ 0.004 \end{bmatrix}$$

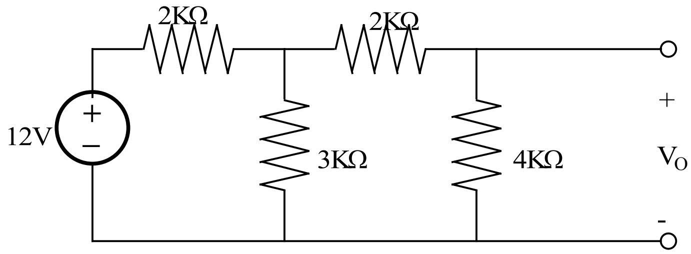
```
EDU» g=[0.0015 -0.001 0 -0.0005;0 1 0 0;0 -0.001 0.0015 0;-0.0005 0 0 0.0015]

g =

    0.0015   -0.0010         0   -0.0005
         0    1.0000         0         0
         0   -0.0010    0.0015         0
   -0.0005         0         0    0.0015

EDU» i=[0.002;12;-0.006;0.004]

i =
```

```
   0.0020
  12.0000
  -0.0060
   0.0040

EDU» v=inv(g)*i

v =

  11.5000
  12.0000
   4.0000
   6.5000
```

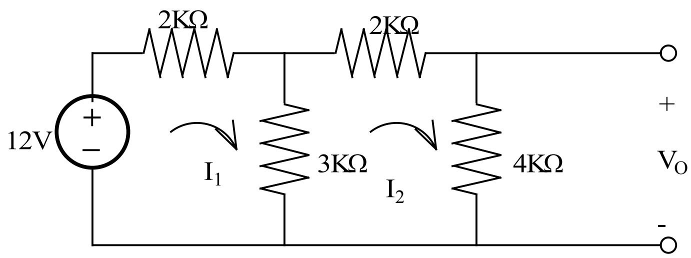
```
V =

  11.5000
  12.0000
   4.0000
   6.5000
```

Problem 3.42

Use mesh equations to find Vo in the circuit shown.



Suggested Solution



$$\begin{bmatrix} 5K & -3K \\ -3K & 4K \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 5K & -3K \\ -3K & 4K \end{bmatrix}^{-1} \begin{bmatrix} 1_2 \\ 0 \end{bmatrix}$$

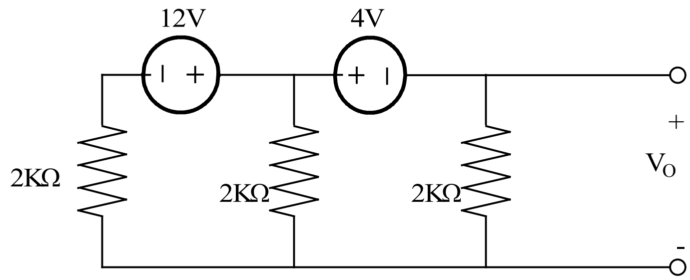
$$V_0 = 4V$$

$$I_2 = \frac{1}{K} A$$

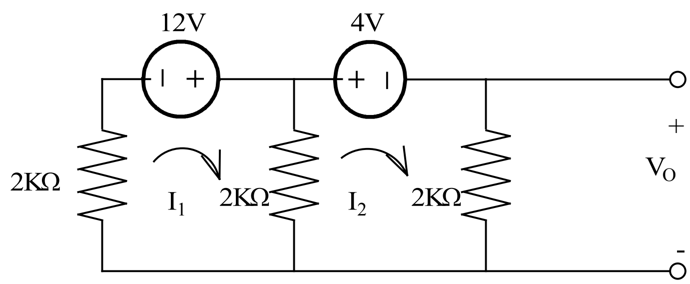
$$V_0 = 4V$$

Problem 3.43

Find Vo in the network shown using mesh equation.



Suggested Solution



$$\begin{bmatrix} 4K & -2K \\ -2K & 4K \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 4K & -2K \\ -2K & 4K \end{bmatrix}^{-1} \begin{bmatrix} I_2 \\ 0 \end{bmatrix}$$

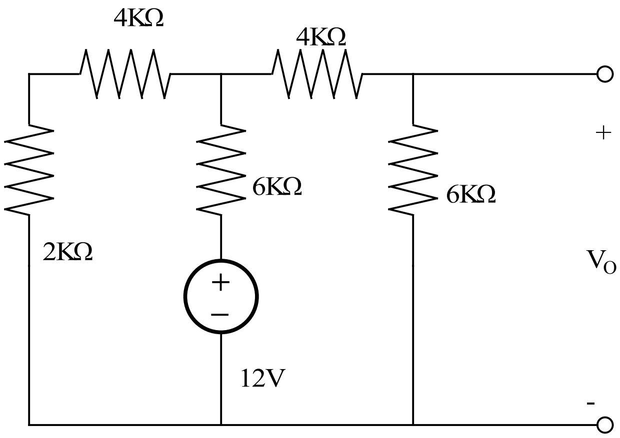
$$V_0 = \frac{4}{3} V$$

$$I_2 = \frac{2}{3K} A$$

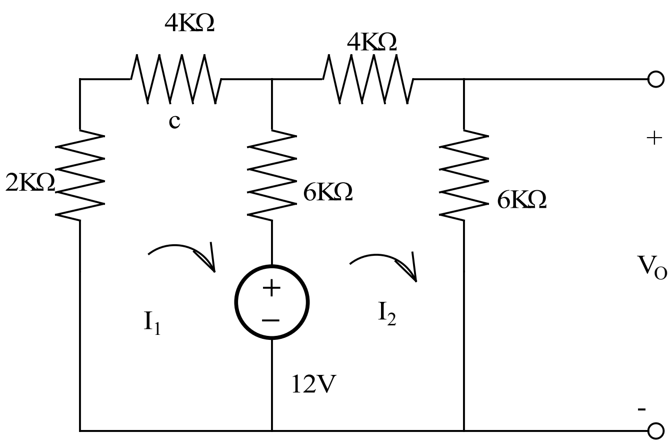
$$V_0 = \frac{4}{3} V$$

Problem 3.44

Use mesh analysis to find Vo in the circuit shown.



Suggested Solution



$$\begin{bmatrix} 12K & -6K \\ -6K & 12K \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -12 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12K & -6K \\ -6K & 12K \end{bmatrix}^{-1} \begin{bmatrix} -12 \\ 12 \end{bmatrix}$$

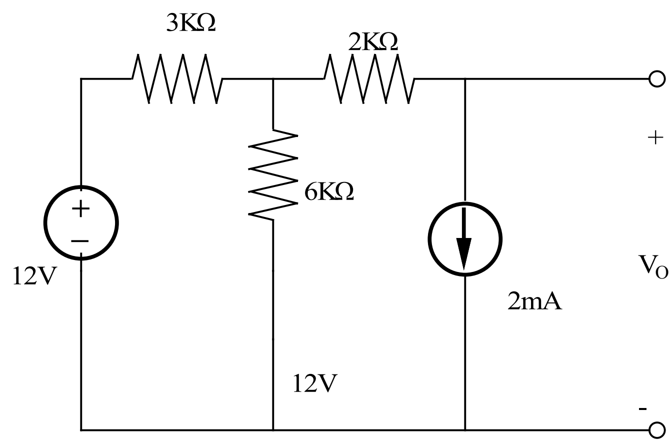
$$I_2 = \frac{2}{3} mA$$

$$V_0 = \frac{4}{3} V$$

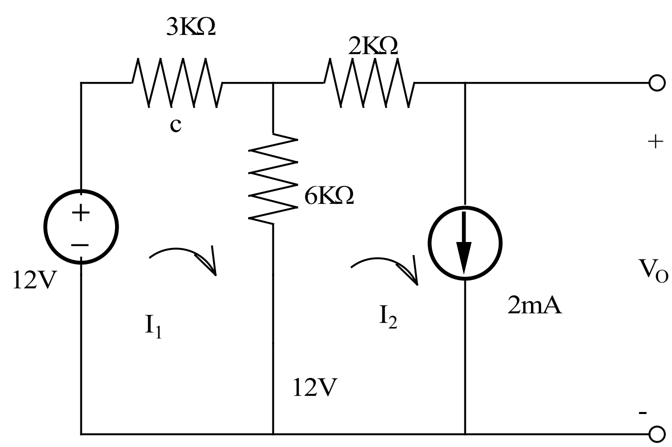
$$V_0 = \frac{4}{3} V$$

Problem 3.45

Use mesh analysis to find Vo in the network shown.



Suggested Solution



$$-12+3KI_1+K(I_1-I_2)=0$$

$$I_2=\frac{2}{K}$$

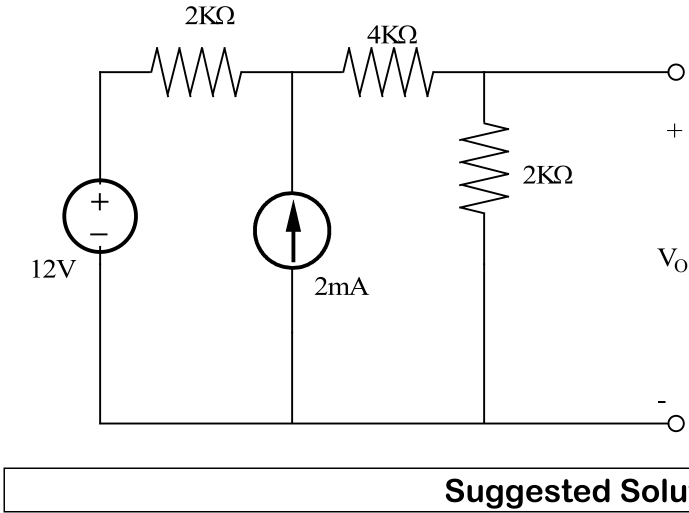
$$I_1=\frac{8}{3K}$$

$$-12+3K(\frac{8}{3K})+2K(\frac{2}{K})=V_0=0V$$

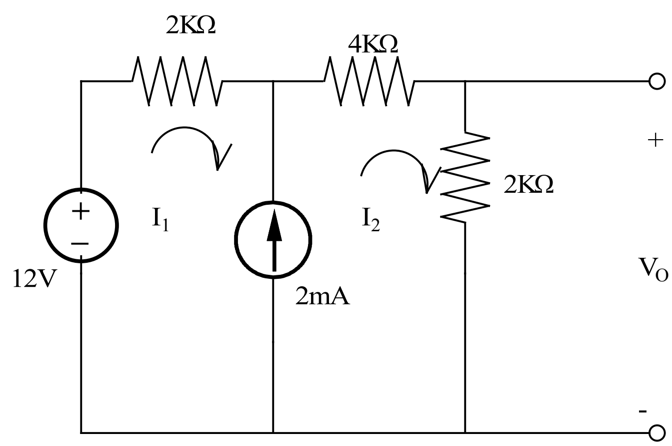
$$V_0=0V$$

Problem 3.46

Use loop analysis to find Vo in the circuit shown.



Suggested Solution



$$12 = 2KI_1 + 6KI_2$$

$$I_2 - I_1 = \frac{2}{K}$$

$$I_2 = \frac{2}{K}$$

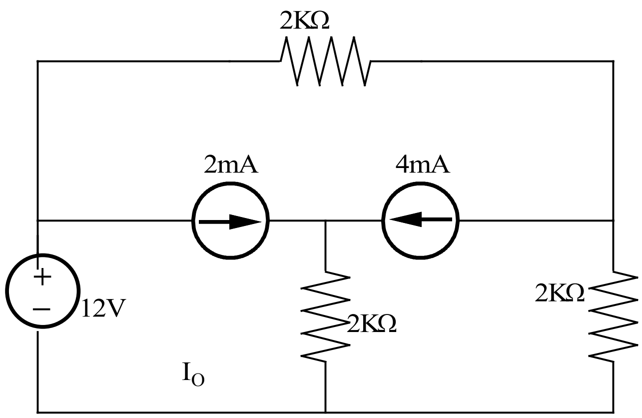
$$V_0 = (2K)(\frac{2}{K}) = 4V$$

$$V_0 = 4V$$

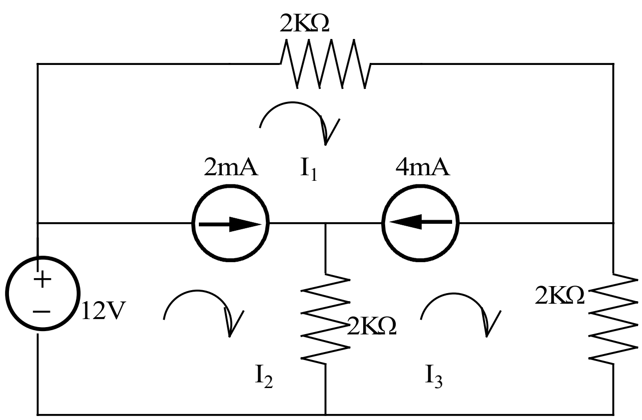
$$V_0 = 4V$$

Problem 3.47

Use loop analysis to find Io in the circuit shown.



Suggested Solution



$$12 = 2KI_2 + 2KI_3$$

$$I_1 - I_2 = \frac{2}{K}$$

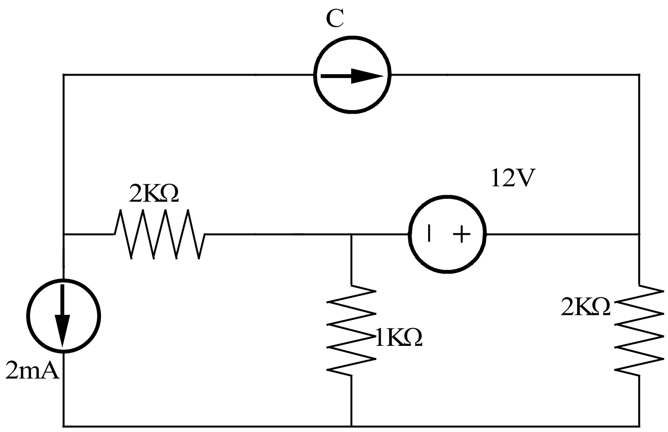
$$I_2 - I_3 = \frac{4}{K}$$

$$I_1 = I_0 = 7mA$$

$$I_1 = I_0 = 7mA$$

Problem 3.48

Use both nodal analysis and mesh analysis to find Io in the circuit shown.



Suggested Solution

$$\frac{2}{K}+\frac{4}{K}\frac{V_1-V_2}{2K}=0$$

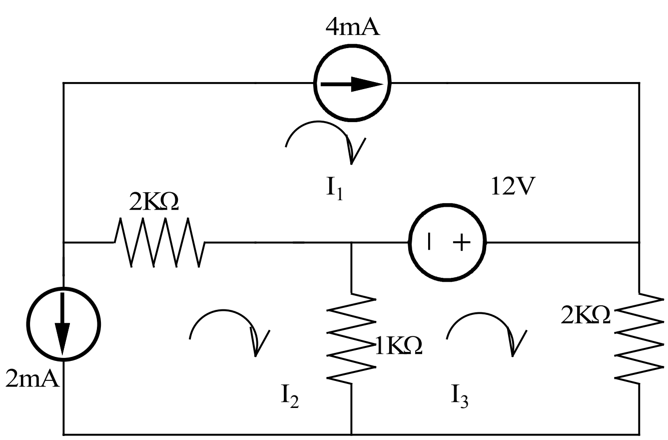
$$\frac{V_2-V_1}{2K}+\frac{V_2}{1K}+\frac{V_3}{2K}-\frac{4}{K}=0$$

Where

$$V_3-V_2=12$$

$$V_2=\frac{-16}{3}V$$

$$I_0=-5.33mA$$



$$I1=\frac{-2}{K}, I2=\frac{4}{K}$$

$$1K(I_1-I_3)-12+2KI_3=0$$

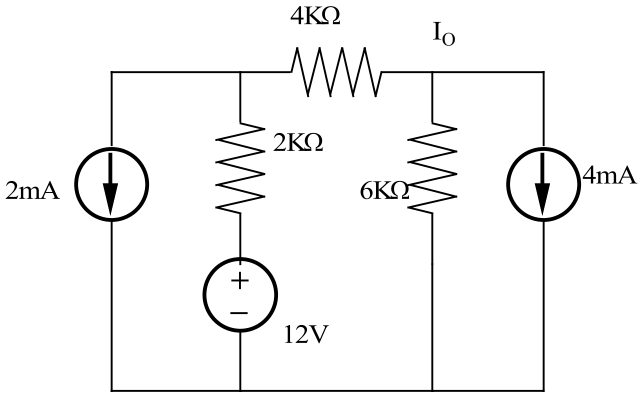
$$I_3=\frac{10}{3K}$$

$$I_0=I_1-I_3=-5.33mA$$

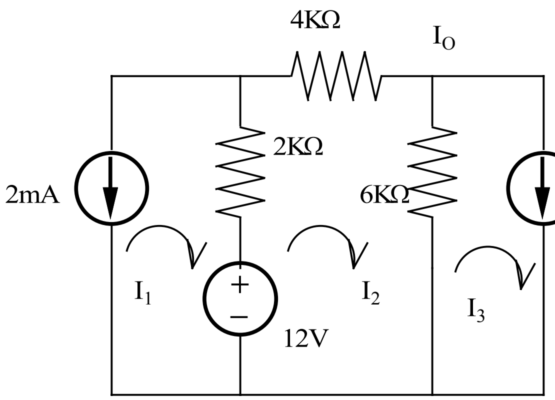
$$I_0=I_1-I_3=-5.33mA$$

Problem 3.49

Find Io in the network shown using mesh analysis.



Suggested Solution



$$I_1 = \frac{-2}{K}$$

$$I_3 = \frac{4}{K}$$

$$-12 + 2K(I_2 - I_1) + 4KI_2 + 6K(I_2 - I_3) = 0$$

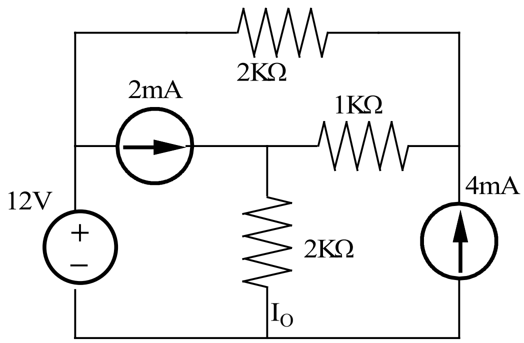
$$I_2 = \frac{16}{6} mA$$

$$I_0 = I_2 = 2.67mA$$

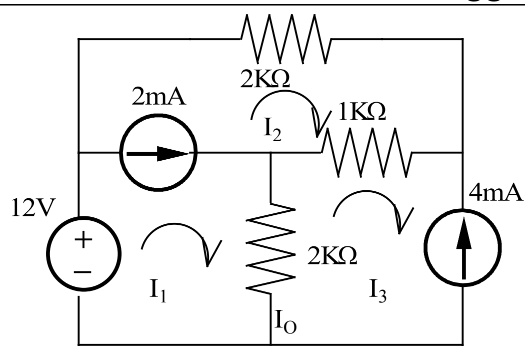
$$I_0 = I_2 = 2.67mA$$

Problem 3.50

Find Io in the network shown using mesh analysis.



Suggested Solution



$$-12 + 2K(I_1 - I_3) + 2KI_2 + 1K(I_2 - I_3) = 0$$

$$I_1 - I_2 = \frac{2}{K}$$

$$I_3 = \frac{-4}{K}$$

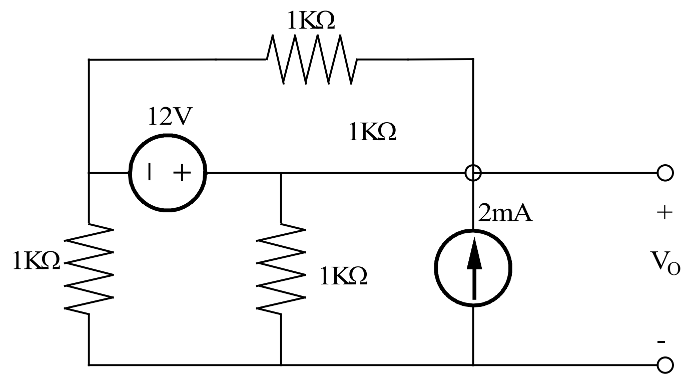
$$I_1 = \frac{6}{5K}A$$

$$I_0 = I_1 - I_3 = 5.2mA$$

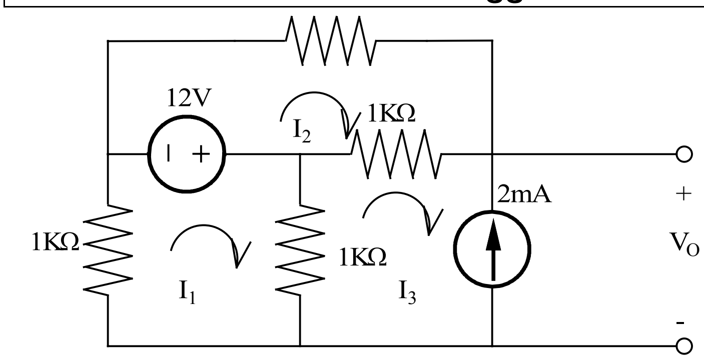
$$I_0 = I_1 - I_3 = 5.2mA$$

Problem 3.51

Find Vo in the circuit shown using mesh analysis.



Suggested Solution



$$2KI_1 - 1KI_3 = -12$$
$$2KI_2 - 1KI_3 = 12$$
$$I_3 = \frac{-2}{K}$$
$$\Rightarrow$$
$$2KI_1 + 2 = -12$$
$$2KI_2 + 2 = 12$$

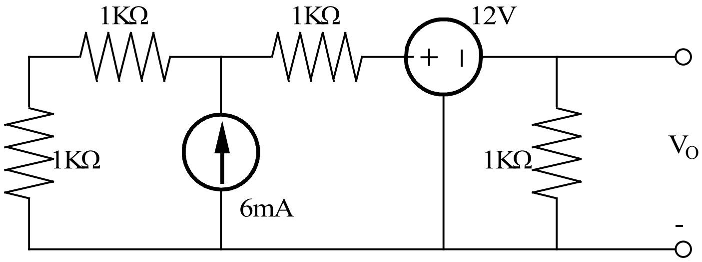
$$V_0 = 2V$$

$$I_1 = \frac{-7}{K}, I_2 = \frac{5}{K}$$

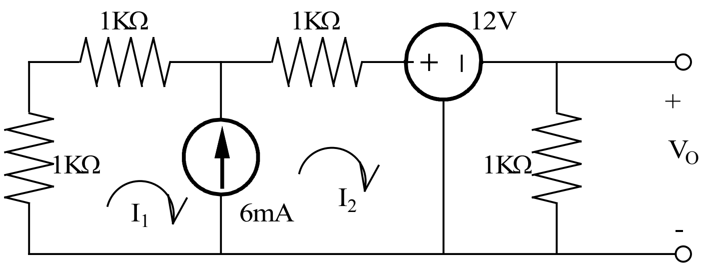
$$V_O = \frac{7}{K}(1K) - \frac{5}{K}(1K) = 2V$$
$$V_0 = 2V$$

Problem 3.52

Use loop analysis to find Vo in the network shown.



Suggested Solution



$$I_1 = \frac{-6}{K}$$

$$1K(I_1 + I_2) + 1K(I_1 + I_2) + 1KI_2 + 12 + 1KI_2 = 0$$

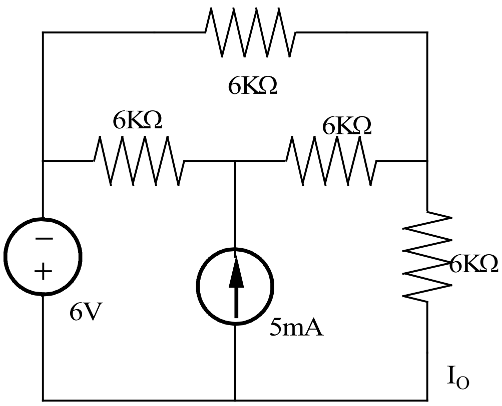
$$I_2 = 0$$

$$V_0 = 0$$

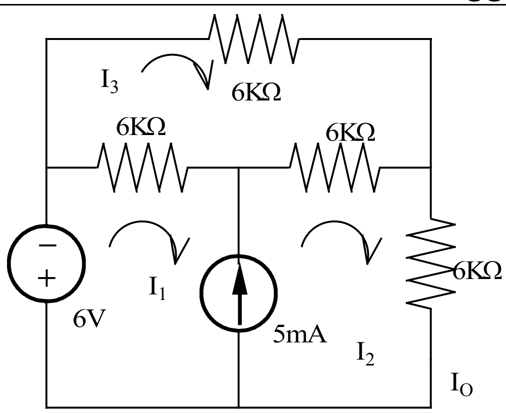
$$V_0 = 0$$

Problem 3.53

Find Io in the network shown using loop analysis. Then solve the problem using matlab and compare your answers.



Suggested Solution



$$6 + 6K(I_1 - I_3) + +6K(I_2 - I_3) + 6KI_2 = 0$$

$$6KI_3 + 6K(I_3 - I_2) + 6K(I_3 - I_1) = 0$$

$$I_2 - I_1 = \frac{5}{K}$$

$$\begin{bmatrix} 6K & 12K & -12K \\ -6K & -6K & 18K \\ -1K & 1K & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \\ 5 \end{bmatrix}$$

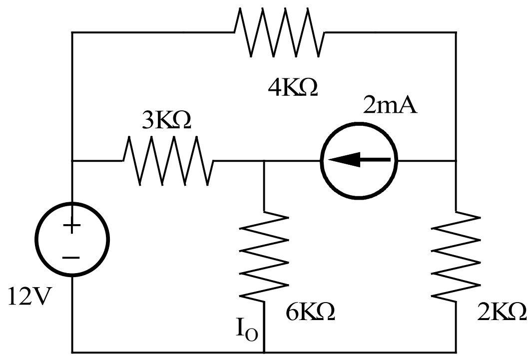
Using Matlab

```
EDU» Z=[6000 12000 -12000;-6000 -6000 18000;-1000 1000 0]

Z =

    6000      12000     -12000
   -6000      -6000      18000
   -1000       1000          0
```

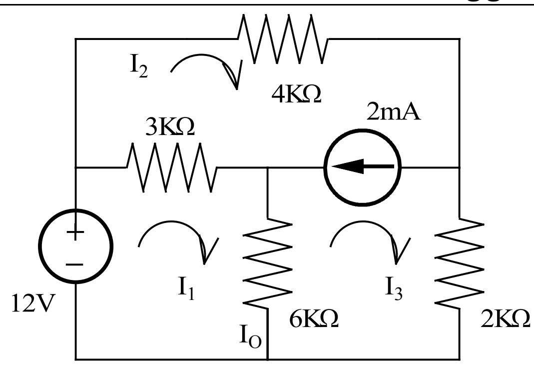
```
EDU» V=[-6;0;5]

V =

    -6
     0
     5


EDU» I=inv(Z)*V



I =

   -0.0046
    0.0004
   -0.0014
```

Problem 3.54

Use loop analysis to find Io in the circuit shown.



Suggested Solution



$$-12 + 3K(I_1 - I_2) + 6KI_2 = 0$$

$$I_2 = \frac{2}{K}$$

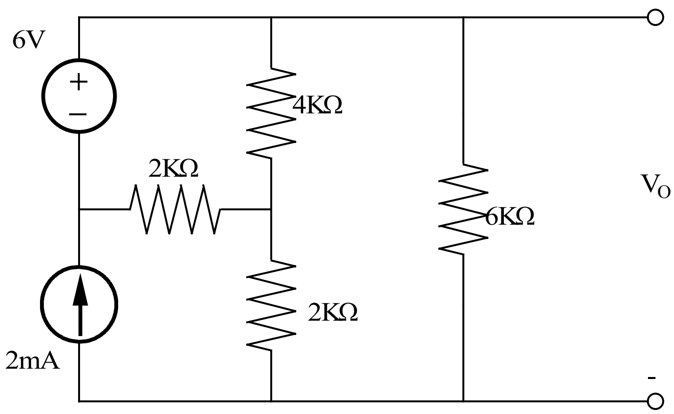
$$9KI_1 = 18$$

$$I_1 = I_0 = 2mA$$

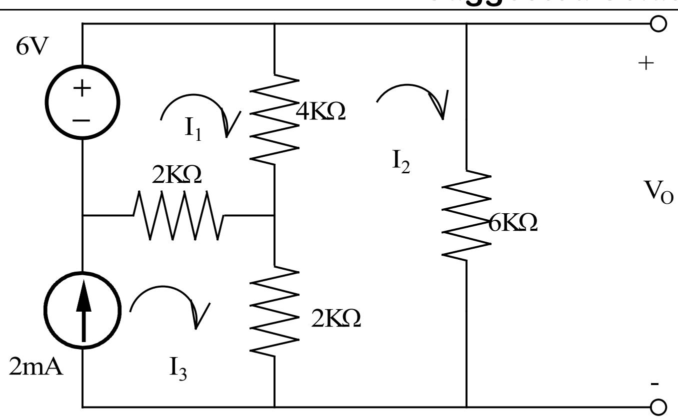
$$I_1 = I_0 = 2mA$$

Problem 3.55

Find Vo in the network shown using both mesh and nodal analysis.



Suggested Solution



$$I_3 = \frac{2}{K}$$

$$6 = 6KI_1 - 4KI_2 - 2KI_3$$

$$0 = -4KI_1 + 12KI_2 - 2KI_3$$

$$V_0 = 6.86V$$

$$I_2 = 1.14mA$$

$$V_0 = 6KI_2 = 6.86V$$

$$\frac{V_1 - V_2}{2K} + \frac{V_3 - V_2}{4K} = \frac{V_2}{2K} \Rightarrow 2V_1 + V_3 - 5V_2 = 0$$

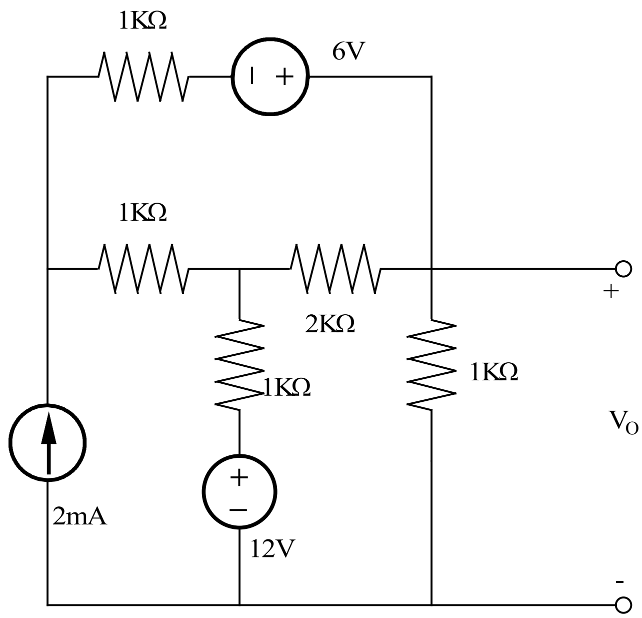
$$\frac{V_2}{2K} + \frac{V_3}{6K} = \frac{2}{K} \Rightarrow 3V_2 + V_3 = 12$$

$$V_3 - V_1 = 6$$

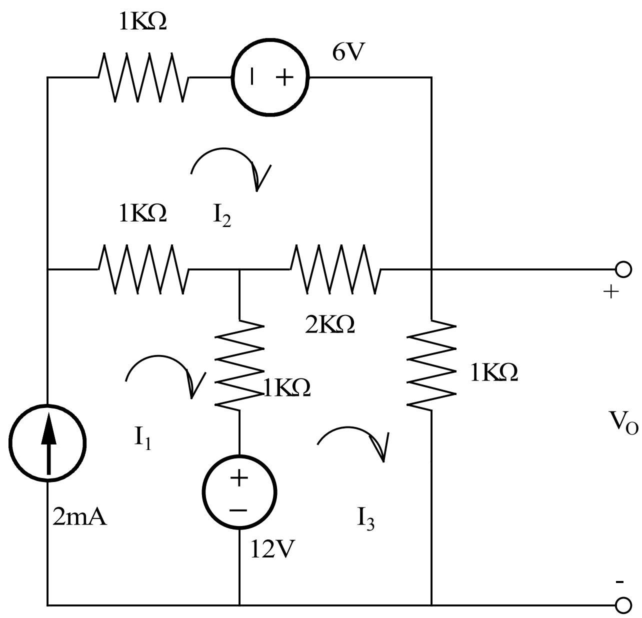
$$V_3 = V_0 = 6.86V$$

Problem 3.56

Use loop analysis to find Vo in the circuit shown.



Suggested Solution



$$I_1 = \frac{2}{K}$$

$$1KI_2 - 6 + 2K(I_2 - I_3) + 1K(I_2 - I_1) = 0$$

$$-12 + 1K(I_3 - I_1) + 2K(I_3 - I_2) + 1K(I_3) = 0$$

$$\begin{bmatrix} 4K & -2K \\ -2K & 4K \end{bmatrix} \begin{bmatrix} I2 \\ I3 \end{bmatrix} = \begin{bmatrix} 8 \\ 14 \end{bmatrix}$$

$$6KI_3 = 36$$

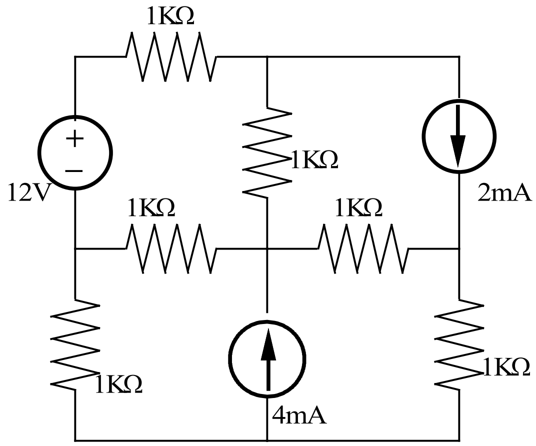
$$I_3 = 6mA$$

$$V_0 = 6V$$

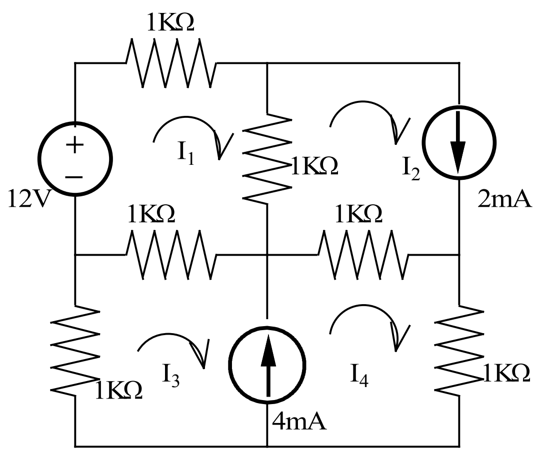
$$I_3 = 6mA$$
$$V_0 = 6V$$

Problem 3.57

Use loop analysis to find Io in the network shown.



Suggested Solution



$$-12+1K(I_1-I_2)+1K(I_1-I_3-I_4)+1K(I_1)=0$$

$$I_2=\frac{2}{K}$$

$$I_3=\frac{-4}{K}$$

$$1K(I_3+I_4)+1K(I_3+I_4-I_1)+1K(I_4-I_2)+1KI_4=0$$

$$3KI_1-1KI_4=10$$

$$-1KI_1+4KI_4=10$$

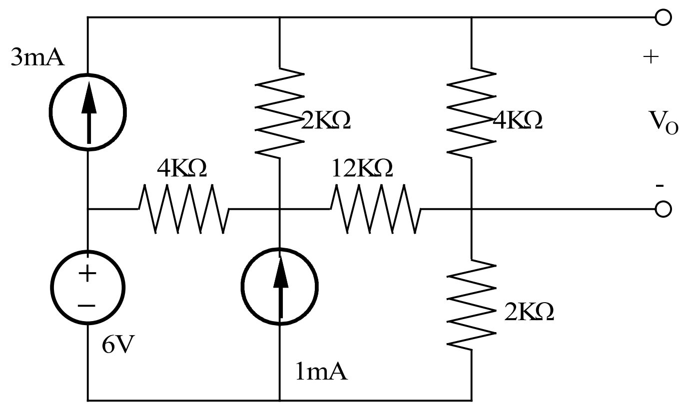
$$I_0=I_4-I_2=1.64mA$$

$$I_4=\frac{40}{11K}A$$

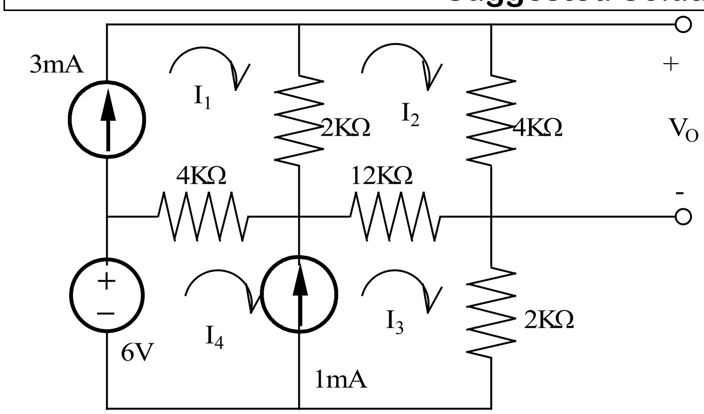
$$I_0=I_4-I_2=1.64mA$$

Problem 3.58

Use loop analysis to find Vo in the network shown.



Suggested Solution



$$I_1 = \frac{3}{K}$$

$$I_3 - I_4 = \frac{1}{K}$$

$$-6 + 4K(I_4 - I_1) + 12K(I_3 - I_2) + 2KI_3$$

$$0 = -2KI_1 - 12KI_3 + 18KI_2$$

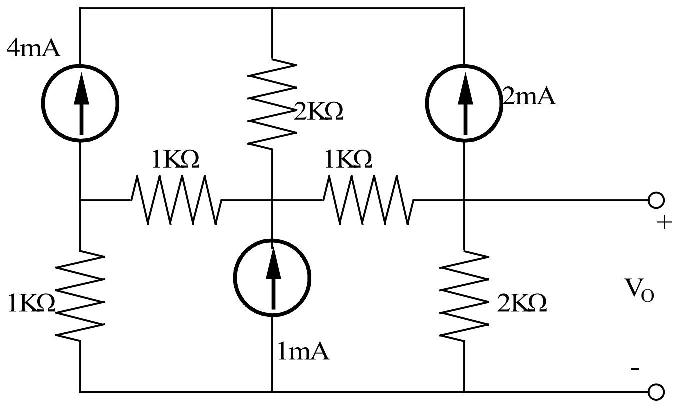
$$I_2 = \frac{31}{15K}A$$

$$V_0 = 4I_2 = 8.27V$$

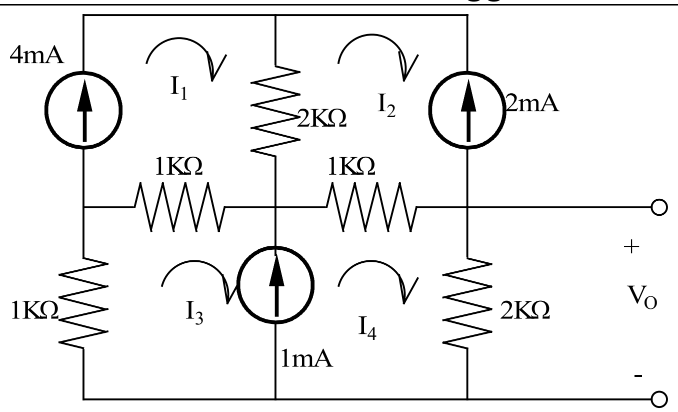
$$V_0 = 4I_2 = 8.27V$$

Problem 3.59

Find Vo in the network shown.



Suggested Solution



$$I_1 = \frac{4}{K}, I_2 = \frac{-2}{K}, I_3 = \frac{-1}{K}$$

$$1K(I_3 + I_4) + 1K(I_3 + I_4 - I_1) + 1K(I_4 - I_{2)} + 2KI_4 = 0$$

$$5KI_4 = 4$$

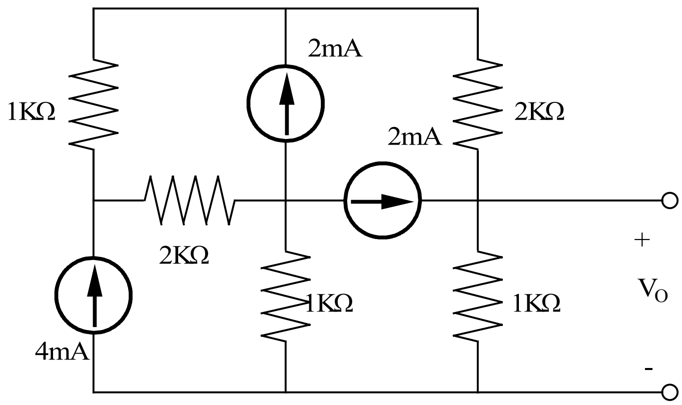
$$I_4 = \frac{4}{5K}$$

$$V_0 = \frac{8}{5}V$$

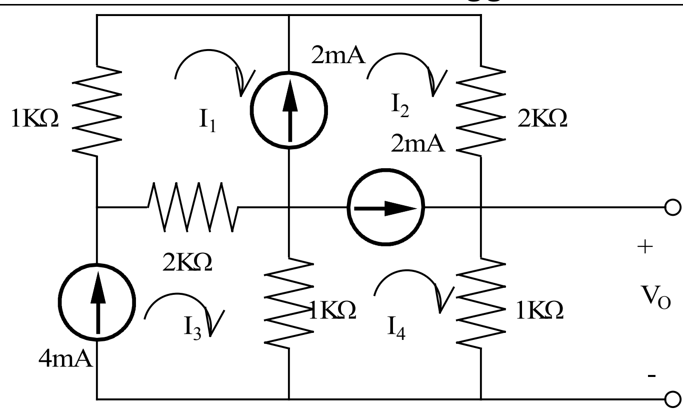
$$V_0 = \frac{8}{5}V$$

Problem 3.60

Find Vo in the circuit shown.



Suggested Solution



$$0 = 2KI_2 + 1KI_4 + 1K(I_4 - I_3) + 2K(I_1 - I_3) + 1KI_1 = 0$$

$$I_3 = \frac{4}{K}, I_4 - I_2 = \frac{2}{K}, I_2 - I_1 = \frac{2}{K}$$

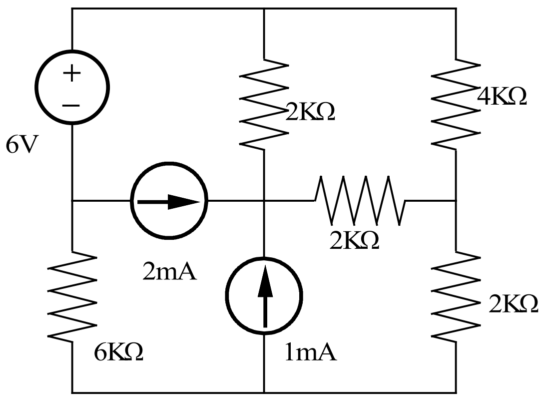
$$I_4 = \frac{4}{K}$$

$$V_0 = 1KI_4 = 4V$$

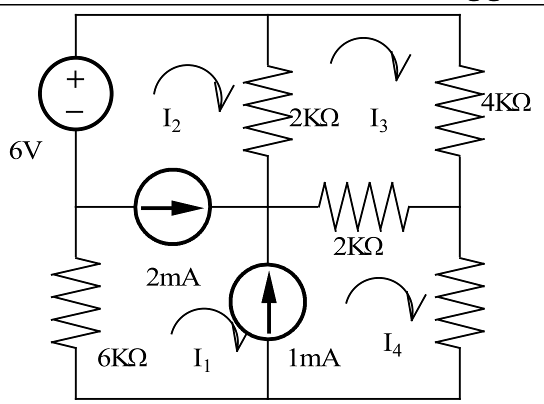
$$V_0 = 1KI_4 = 4V$$

Problem 3.61

Find Io in the circuit shown.



Suggested Solution



$$6 = 4KI_3 + 2KI_4 + 6KI_1$$

$$0 = -2KI_2 - 2KI_4 + 8KI_3$$

$$I_1 - I_2 = \frac{2}{K}, I_4 - I_1 = \frac{1}{K}$$

$$I_2 = -1.5mA$$

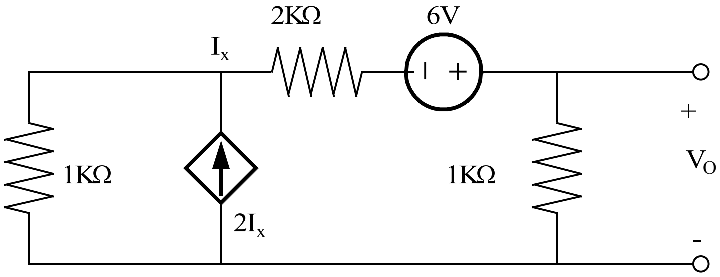
$$I_3 = 0$$

$$I_0 = I_2 - I_3 = -1.5mA$$

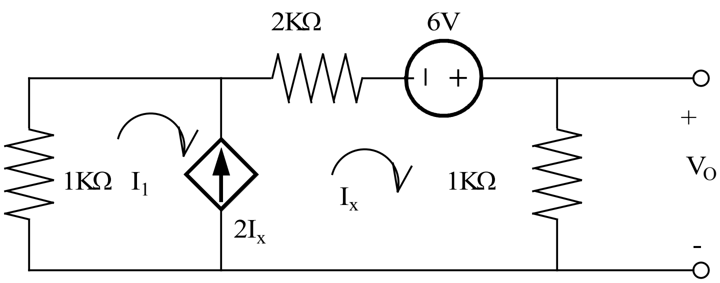
$$I_0 = I_2 - I_3 = -1.5mA$$

Problem 3.62

Use loop analysis to find Vo in the network shown.



Suggested Solution



$$I_1 = -2I_X$$

$$1K(I_1 + I_X) + 2KI_X - 6 + 1KI_X = 0$$

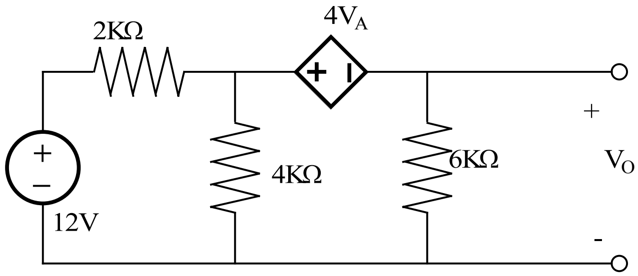
$$I_X = \frac{3}{K}A$$

$$V_0 = 3V$$

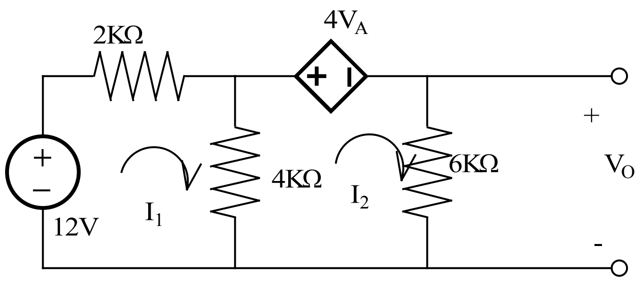
$$V_0 = 3V$$

Problem 3.63

Use mesh analysis to find Vo in the circuit shown.



Suggested Solution



$12 = 6KI_1 - 4KI_2$

$0 = -4KI_1 + 10KI_2 + 4V_A$

$V_A = 2KI_1$

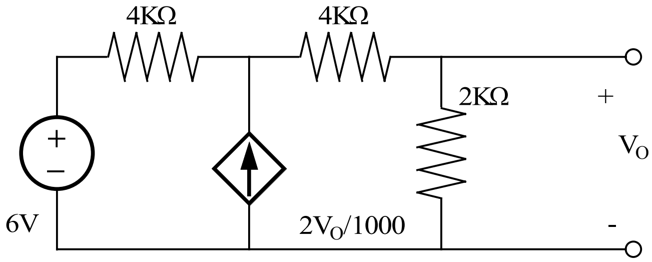
$I_2 = -0.63mA$

$V_O = 6KI2 = -3.79V$

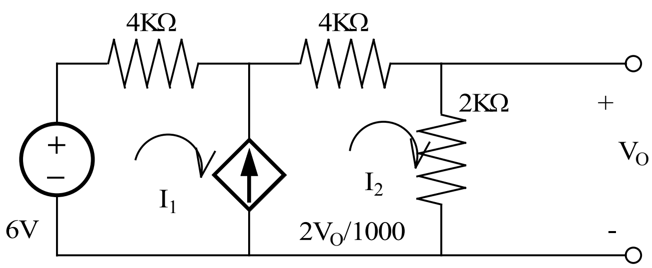
$V_O = 6KI2 = -3.79V$

Problem 3.64

Use loop analysis to find Vo in the circuit shown.



Suggested Solution



$$I_2 - I_1 = \frac{2V_0}{K}, V_0 = 2KI_2$$

$$6 = 4KI_1 + 6KI_2$$

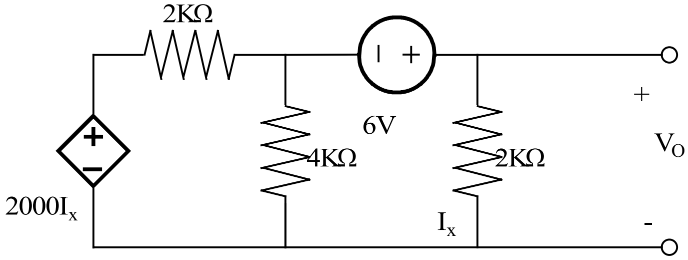
$$I_2 = -1mA$$

$$V_0 = 2KI_2 = -2V$$

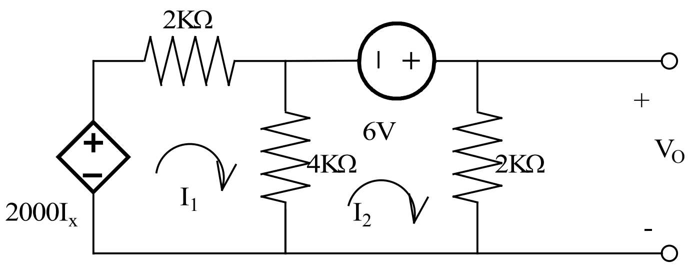
$$V_0 = 2KI_2 = -2V$$

Problem 3.65

Find Vo in the circuit shown using mesh analysis.



Suggested Solution



$I_2 = I_X$

$2KI_X = 6KI_1 - 4KI_2$

$6 = -4KI_1 + 6KI_2$

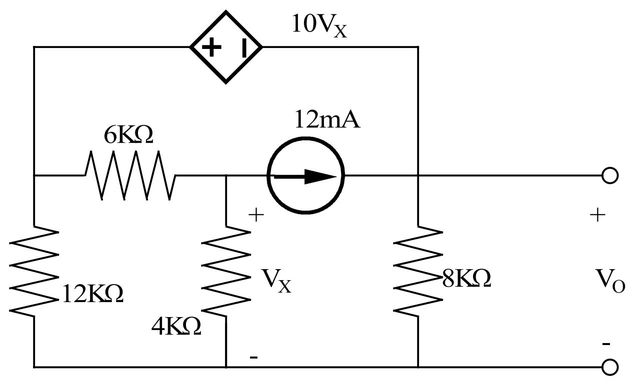
$I_2 = 3mA$

$V_0 = 2KI_2 = 6V$

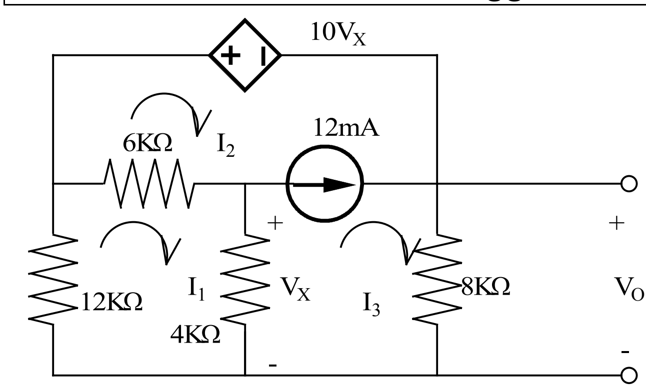
$V_0 = 2KI_2 = 6V$

Problem 3.66

Use both nodal analysis and mesh analysis to find Vo in the circuit shown.



Suggested Solution



$$22KI_1 - 6KI_2 - 4KI_3 = 0$$
$$12KI_1 + 10V_X + 8KI_3 = 0$$
$$V_X = 4K(I_1 - I_3)$$
$$I_3 - I_2 = \frac{12}{K}$$
$$I_3 = -20.35mA$$
$$V_0 = 8KI_3 = -162.78V$$

$Nodal\_Analysis$

$$\frac{V_2 - V_1}{6K} + \frac{V_2}{4K} + \frac{12}{K} = 0$$
$$\frac{V_1}{12K} + \frac{V_2}{4K} + \frac{V_3}{8K} = 0$$
$$V_1 - V_3 = 10VX$$
$$V_X = V_2$$

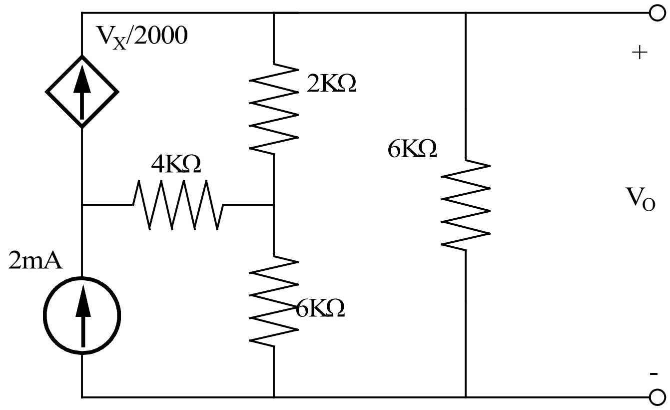
$$V_3 = V_0 = -162.78V$$

$$V_3 = V_0 = -162.78V$$

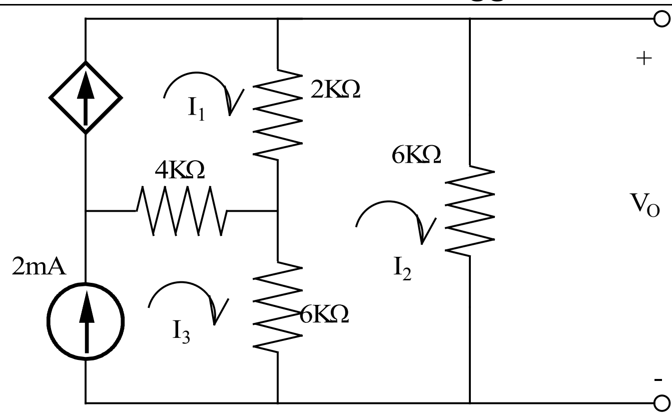
Problem 3.67

Using mesh analysis finds Vo in the circuit shown

.



Suggested Solution



$$I_1 = \frac{V_X}{2K}, R_3 = \frac{2}{K}, V_X = (I_3 - I_2)6K$$

$$(I_2 - I_3)6K + (I_2 - I_1)2K + 6KI_2 = 0$$

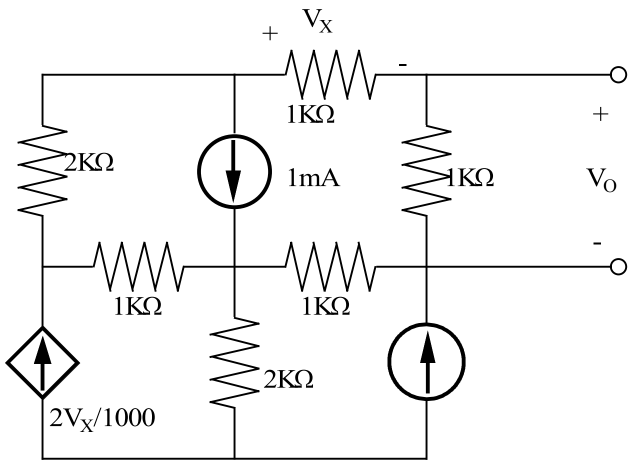
$$I_2 = \frac{6}{5K}A$$

$$V_0 = 6KI_2 = \frac{36}{5}V$$

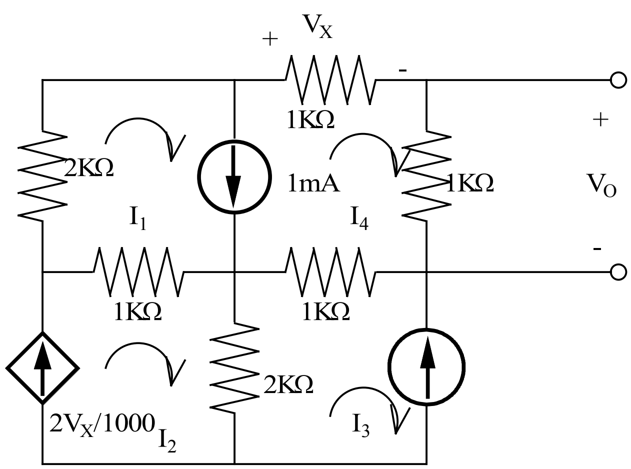
$$V_0 = 6KI_2 = \frac{36}{5}V$$

Problem 3.68

Find Vo in the network shown.



Suggested Solution



$$I_1 = \frac{1}{2K}, I_2 = \frac{2}{K}, V_X = -1KI_4, I_3 = \frac{-4}{K}$$

$$2K(I_1 + I_4) + 1KI_4 + 1KI_4 + 1K(I_4 - I_3) + 1K(I_4 + I_1 - I_2) = 0$$

$$I_2 = -2I_4$$

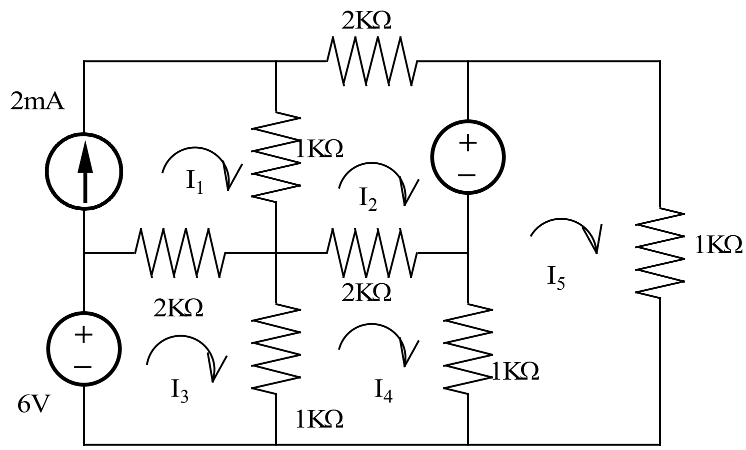
$$I_4 = \frac{-7}{8K}$$

$$V_0 = \frac{-7}{8}V$$

$$V_0 = \frac{-7}{8}V$$

Problem 3.69

Use matlab to find the mesh currents in the network shown.



Suggested Solution

$$I_1 = \frac{2}{K}$$

$$-I_1(1K) + I_2(5K) - I_4(2K) = -12$$

$$-I_1(2K) - I_4(1K) + I_3(3K) = 6$$

$$-I_3(1K) - I_2(2K) + I_4(4K) - I_5(1K) = 0$$

$$-I_4(1K) + I_5(2K) = 12$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1000 & 5000 & 0 & -2000 & 0 \\ -2000 & 0 & 3000 & -1000 & 0 \\ 0 & -2000 & -1000 & 4000 & -1000 \\ 0 & 0 & 0 & -1000 & 2000 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 0.002 \\ -12 \\ 6 \\ 0 \\ 12 \end{bmatrix}$$

```
EDU» r=[1 0 0 0 0;-1000 5000 0 -2000 0;-2000 0 3000 -1000 0;0 -2000 -1000 4000 -1000;0 0 0 -1000
2000]
```

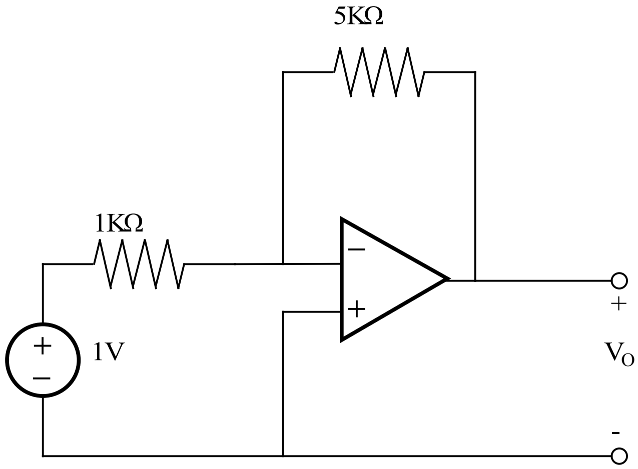
Using Matlab

```
r =

       1        0        0        0        0
   -1000     5000        0    -2000        0
   -2000        0     3000    -1000        0
       0    -2000    -1000     4000    -1000
       0        0        0    -1000     2000

EDU» v=[0.002;-12;6;0;12]
```

```
v =

    0.0020
  -12.0000
    6.0000
         0
   12.0000

EDU» i=inv(r)*v

i =

    0.0020
   -0.0011
    0.0041
    0.0023
    0.0071
```

Problem 3.70

Find Vo in the circuit shown.



Suggested Solution

KCL at the intersecting input

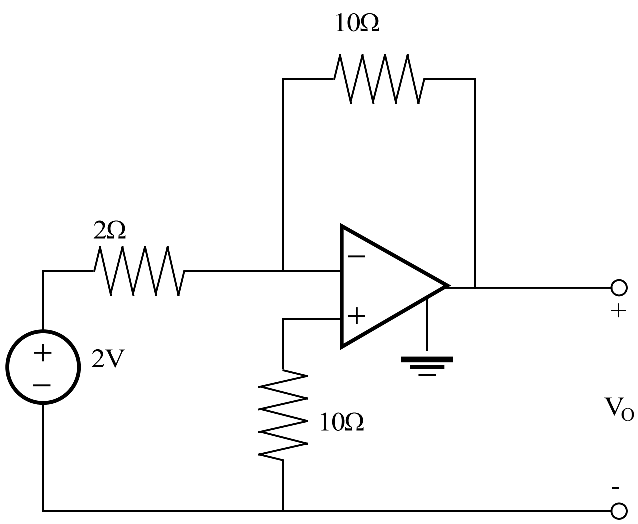
$$\frac{1\text{-}0}{0.1\text{K}} = \frac{0 - V0}{5K}$$

$$V0 == -5V$$

$$V0 == -5V$$

Problem 3.71

Find Vo in the network shown and explain what effect R1 has on the output.



Suggested Solution

No current in R_1. It has no effect. KCL at the inverting input.

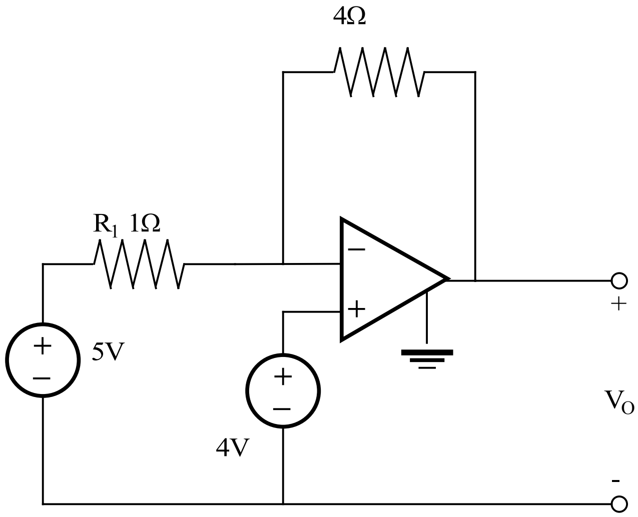
$$\frac{2\text{-}0}{2} = \frac{0 - V_0}{10}$$

$$V_0 = -10V$$

$$V_0 = -10V$$

Problem 3.72

Find Vo in the network shown.



Suggested Solution

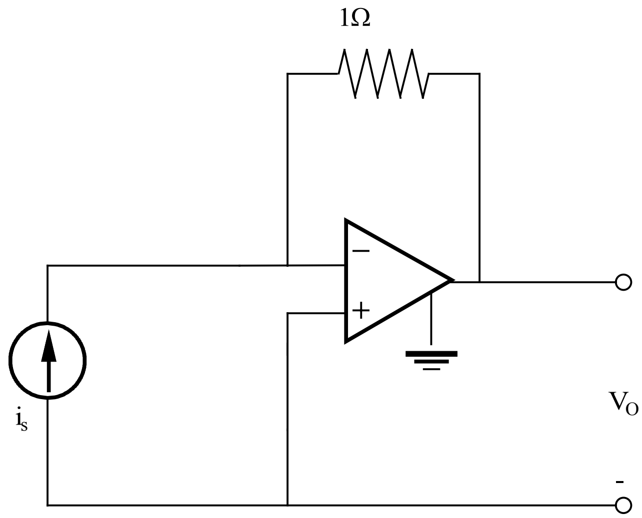
KCL at the inverting input is

$$\frac{5-4}{1} = \frac{4-V_0}{4}$$

$$V_0 = 0V$$

Problem 3.73

The network shown is a current-to-voltage converter or transresistance amplifier. Find Vo/Is for this network.



Suggested Solution

KCL at the inverting input is

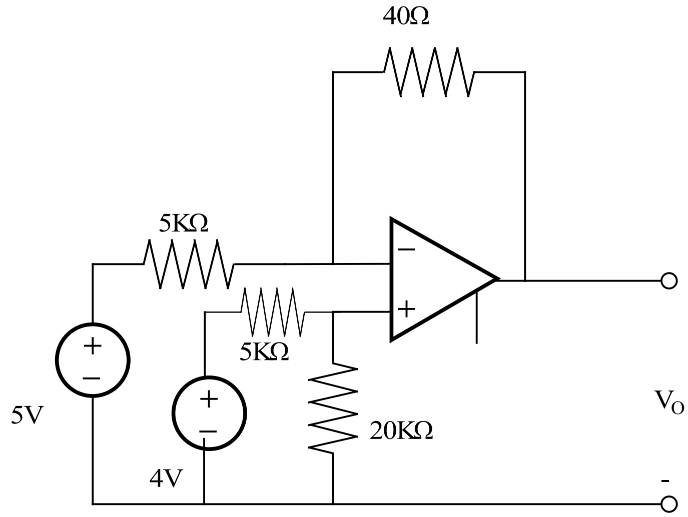
$$\frac{5-4}{1}=\frac{0-V_0}{1}$$

$$\frac{V_0}{i_s}\bigg|_0=-1$$

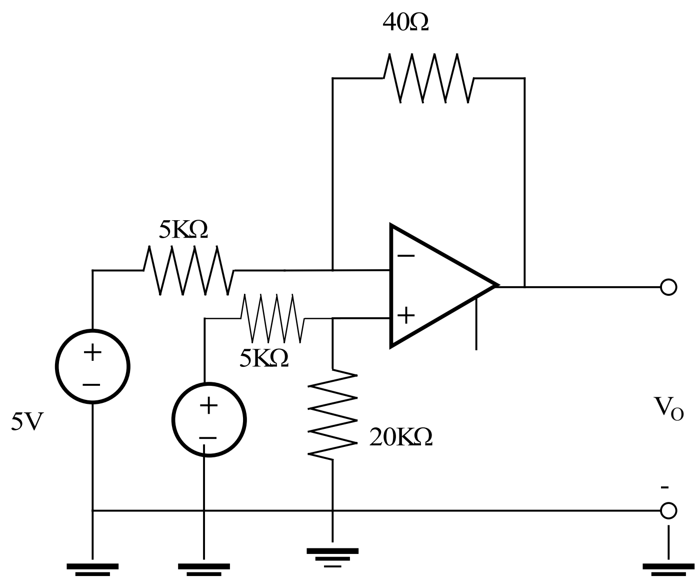
$$\frac{V_0}{i_s}\bigg|_0=-1$$

Problem 3.74

Find Vo in the circuit shown.



Suggested Solution



Applying Voltage Division

$$V_X = 4\left(\frac{20}{20+5}\right) = 3.2V$$

KCL at the inverting input

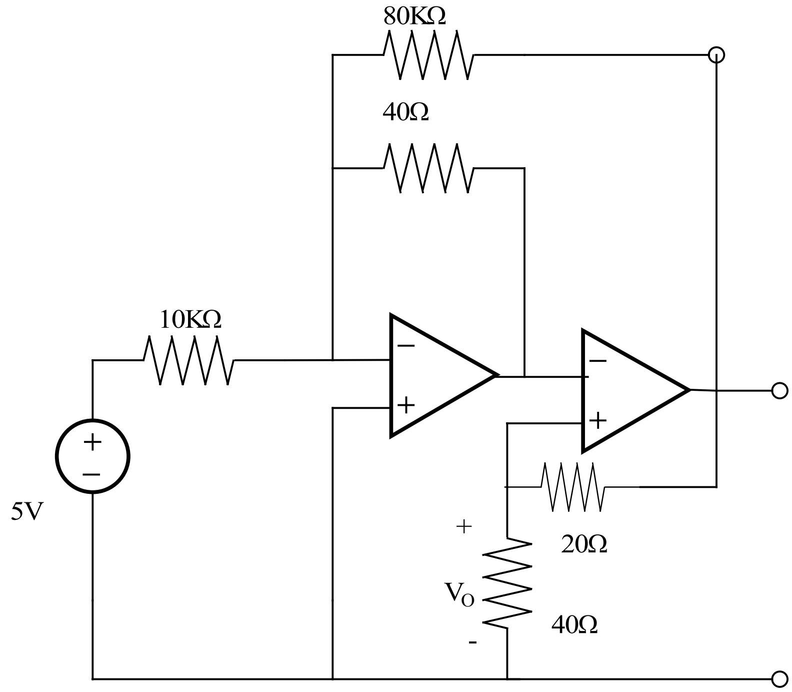
$$\frac{5 - V_X}{5K} = \frac{V_X - V_0}{40K}$$

$$V_0 = -11.2V$$

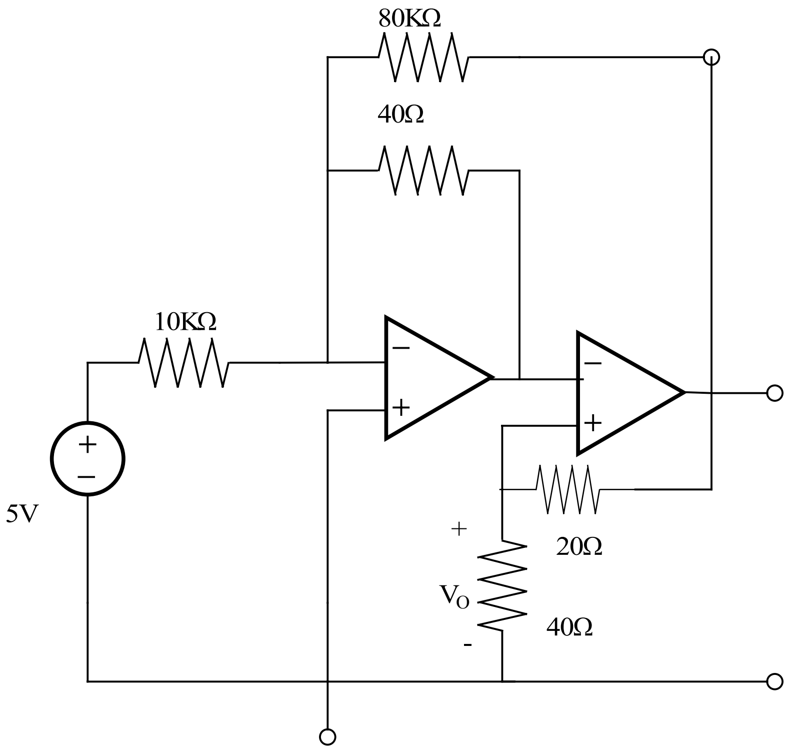
$$V_0 = -11.2V$$

Problem 3.75

Find Vo in the circuit shown.



Suggested Solution



KCL at the inverting input

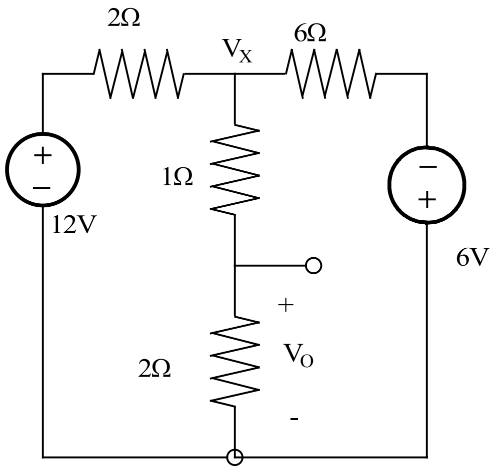
$$\frac{9-6}{10K}+\frac{10-6}{20K}+\frac{-12-6}{30K}=\frac{6}{40K}+\frac{6-V_0}{100K}$$
$$V_0 = 31V$$

$$V_X = 0, V_Y = V_0$$
$$Node-Z$$
$$V_Z = \left(\frac{40K+20}{40K}\right)V_0 = \frac{3}{2}V_0$$
$$Node-X$$
$$\frac{5}{10K}+\frac{3V_0}{160K}+\frac{V_0}{40K}=0$$
$$V_0 = -11.4V$$

$$V_0 = -11.4V$$

Problem 3FE-1

Find Vo in the circuit shown.



Suggested Solution

KCLat node Vx

$$\frac{V_X - 12}{2} + \frac{V_X}{3} + \frac{V_X + 6}{6} = 0$$

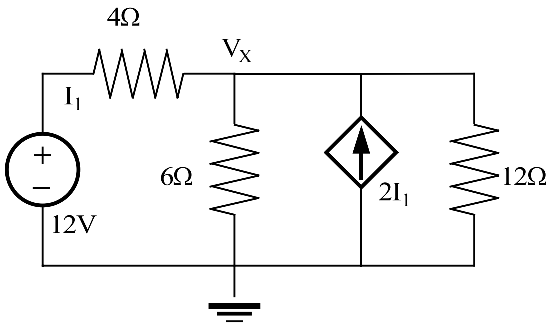
$$V_X = 5V$$

$$V_0 = V_X\left(\frac{2}{2+1}\right) = \frac{10}{3}V$$

$$V_0 = V_X\left(\frac{2}{2+1}\right) = \frac{10}{3}V$$

Problem 3FE-2

Determine the power dissipated in the 6-ohm resistor in the network shown.



Suggested Solution

KCL at node Vx

$$\frac{V_X - 12}{4} + \frac{V_X}{6} - 2\left(\frac{12 - V_X}{4}\right) + \frac{V_X}{12} = 0$$

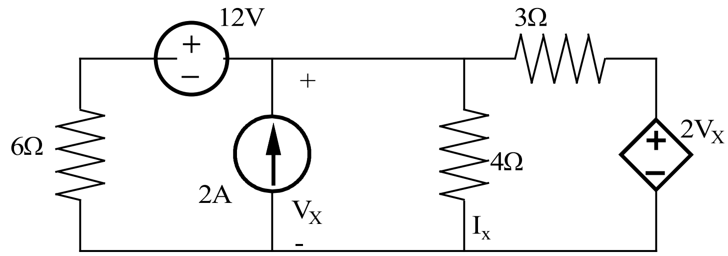
$$V_X = 9V$$

$$P_{6\Omega} = \frac{V^2}{R} = \frac{9^2}{6} = 13.5W$$

$$P_{6\Omega} = 13.5W$$

Problem 3FE-3

Find the current Ix in the 4-ohm resistor in the circuit shown.



Suggested Solution

KCL at

$$\frac{V_X - 12}{6} - 2 + \frac{V_X}{4} + \frac{V_X - 2V_X}{3} = 0$$

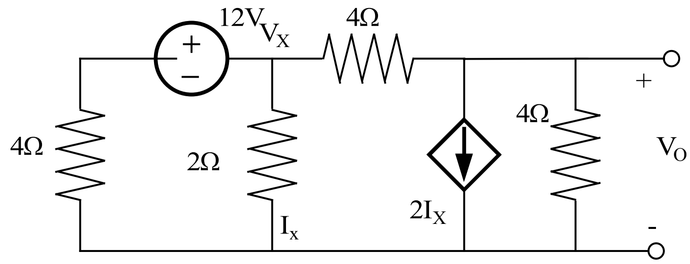
$V_X = 48V$

$I_X = 12A$

$I_X = 12A$

Problem 3FE-4

Determine the voltage Vo in the circuit shown.



Suggested Solution

KCL at Non Reference nodes

$$\frac{V_X - 12}{4} + \frac{V_X}{2} + \frac{V_X - V_0}{4} = 0$$

$$\frac{V_0 - V_X}{4} + 2I_X + \frac{V_0}{4} = 0$$

$$I_X = \frac{V_X}{2}$$

$$V_X - \frac{1}{4}V_0 = 3$$

$$\frac{3}{4}V_X + \frac{1}{2}V_0 = 0$$

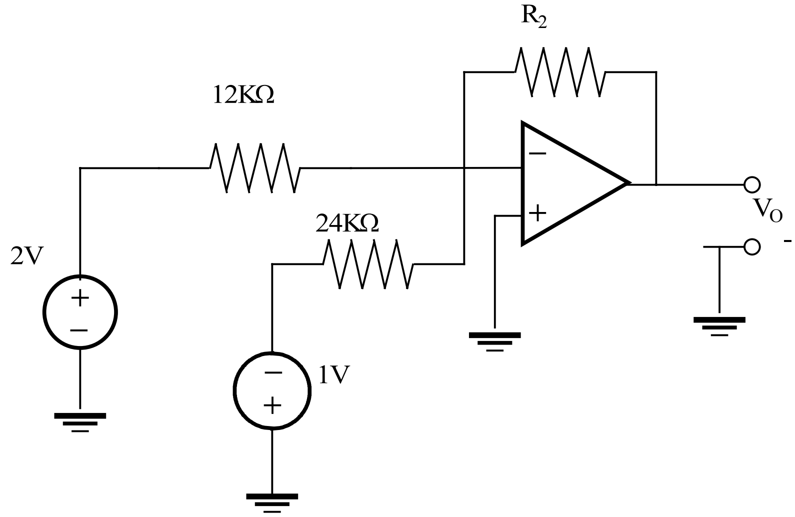
$$\begin{bmatrix} V_X \\ V_0 \end{bmatrix} = \begin{bmatrix} 1 & \frac{-1}{4} \\ \frac{3}{4} & \frac{1}{2} \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$V_0 = -3.27V$$

$$V_0 = -3.27V$$

Problem 3FE-5

Given the summing amplifier shown, select the values of R2 that will produce an output voltage of –3v.



Suggested Solution

$$V_0 = \frac{-R_2}{12K}(2) - \frac{R_2}{24K}(-1)$$

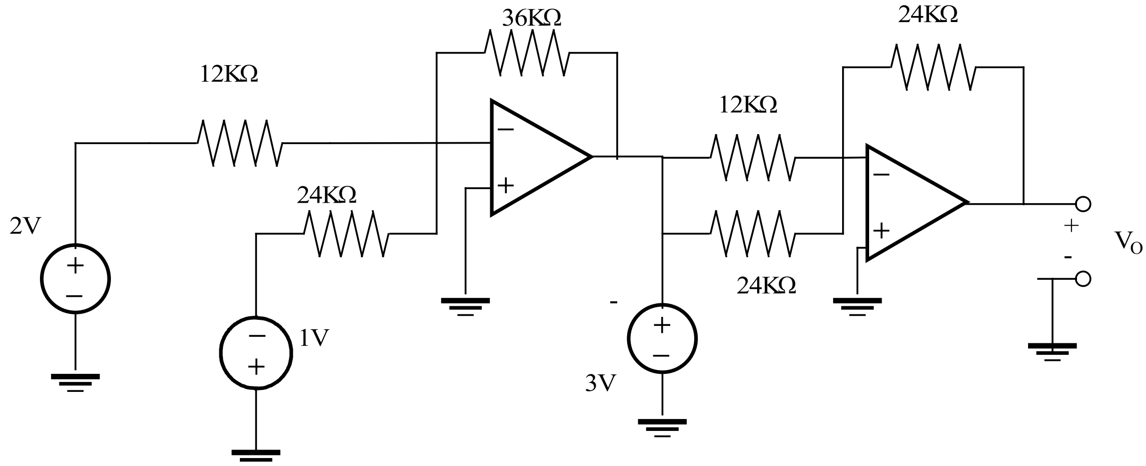
$$-3 = \frac{-2R_2}{12K} + \frac{R_2}{24K} = 0$$

$$R_2 = 24K\Omega$$

$$R_2 = 24K\Omega$$

Problem 3FE-6

Determine the output voltage Vo of the summing op-amp circuit shown.



Suggested Solution

At the output of 1st op-amp

$$V_1 = \frac{-36(2)}{12} - \frac{36(-1)}{24} = \frac{-9}{2} V$$

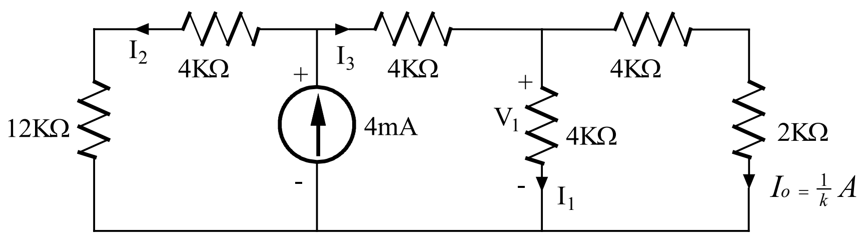
$$V_0 = -\left(\frac{24}{12}\right)\left(-\frac{9}{2}\right) - \left(\frac{24}{24}\right)(-3) = 6V$$

$$V_0 = 6V$$

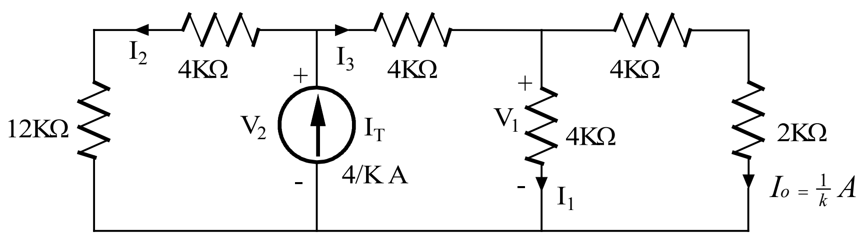
$$V_0 = 6V$$

Problem 4.1

Find I_o in the circuit shown using linearity and the assumption that I_o = 1mA.



Suggested Solution



If $I_o = \frac{1}{k} A$ Then $V_2 = \frac{1}{k}(4K + 2K) = 6V$, $I_1 = \frac{6}{4k} = \frac{3}{2} mA$

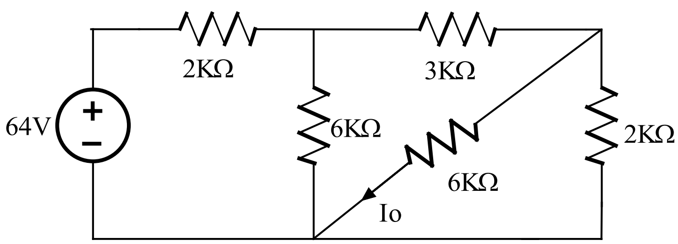
Then $I_2 = I_1 + I_o = \frac{5}{2} mA$ and $V_2 = V_1 + 4KI_2 = 16V$

Then $I_3 = \frac{V2}{4K + 12K} = 1mA \therefore I_T = I_2 + I_3 = \frac{7}{2} mA$

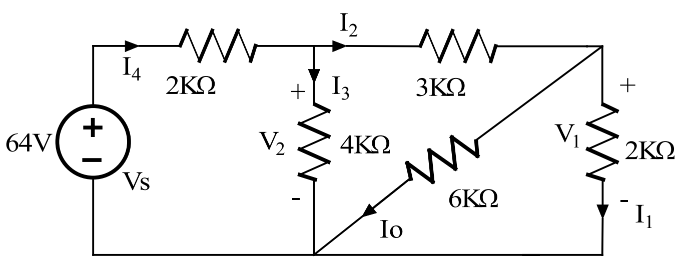
So $\frac{7/2mA}{1mA} = \frac{4mA}{x} \therefore x = \boxed{I_o = \frac{8}{7} mA}$

Problem 4.2

Find Io in the network shown using linearity and the assumption that Io=1mA.



Suggested Solution



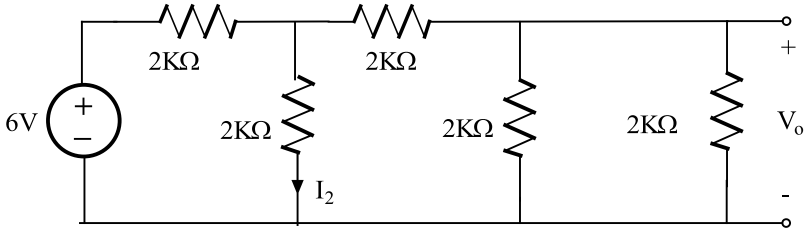
If $Io = 1mA,\ V_1 = 6V.\ I_1 = \frac{6}{2K} = 3mA.\ I_2 = Io + I_1 = 4mA$

$V_2 = V_1 + 3KI_2 = 18V,\ I_3 = \frac{V_2}{6K} = 3mA$ Then $I_4 = I_2 + I_3 = 7mA$

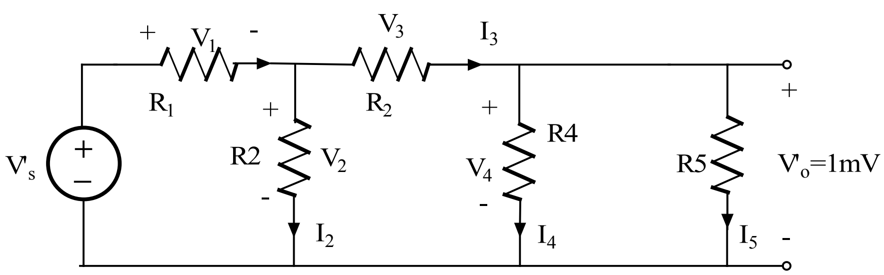
$Vs = V_2 + 2KI_4 = 32V \therefore \frac{32}{1mA} = \frac{64}{x} \therefore\ x = \boxed{I_o = 2mA}$

Problem 4.3

Find Vo in the network shown using linearity and the assumption that Vo=1mV



Suggested Solution



All $R = 2K\Omega, \quad Vs = 6V$

$I_5 = \frac{V'o}{R5} = \frac{1}{2}\mu A \qquad I_4 = \frac{V'o}{RT} = \frac{1}{2}\mu A \qquad I_3 = I_4 + I_5 = 1\mu A$

$V_3 = I_3 R_3 = 2mV \qquad V_2 = V_3 + V'o = 3mV \qquad I_2 = \frac{V2}{R2} = 1.5\mu A$

$I_1 = I_2 + I_3 = 2.5\mu A \qquad V_1 = I_1 R_1 = 5mV \qquad V's = V_1 + V_2 = 8mV$

$\frac{Vo}{Vs} = \frac{Vs}{V's} \Rightarrow \quad Vo = (\frac{6}{8m})(1m) = 0.75V \qquad \boxed{Vo = 0.75V}$

Problem 4.4

Find Vo in the circuit shown using linearity and the assumption that Vo=1V

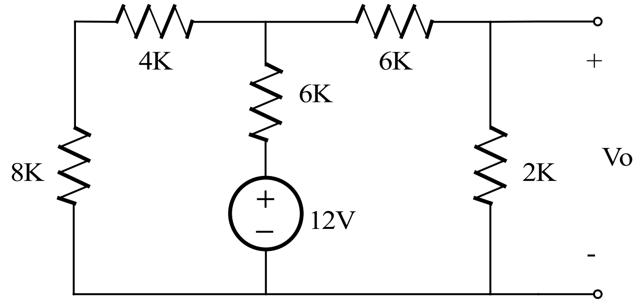
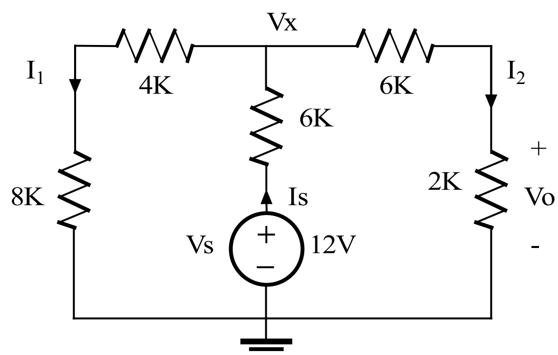


Figure P4.4

Suggested Solution



Assume $Vo = 1V$

$I_2 = \frac{Vo}{2K} = \frac{1}{2} mA \Rightarrow \quad Vx = I2(8K) = 4V$

$I_1 = \frac{Vx}{12K} = 0.333mA$

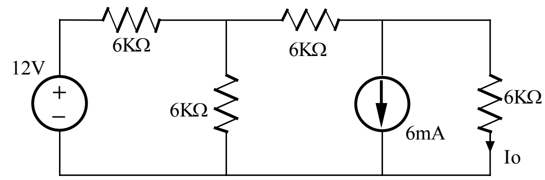
$Is = I_1 + I_2 = 0.8333mA$

$Vs = Is(6K) + Vx = 9V$

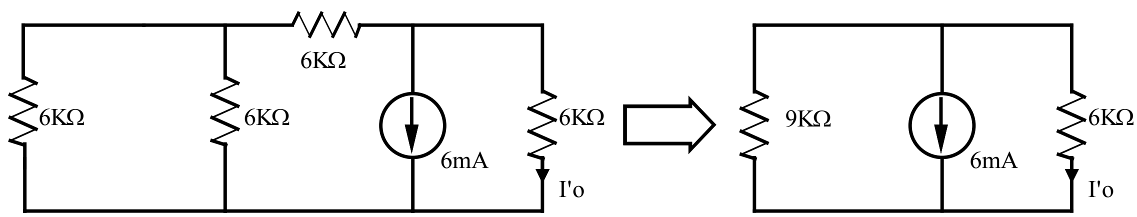
$\frac{9V}{1V} = \frac{12V}{Vo} \Rightarrow \quad \boxed{Vo = 1.33V}$

Problem 4.5

In the network shown find Io using superposition

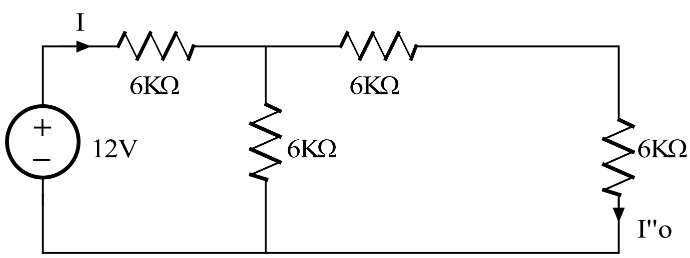


Suggested Solution



Zero the indep. voltage source

$$I'_o = -0.006\left(\frac{9K}{9K+6K}\right) = -\frac{18}{5}\,mA$$



Zero the indep. current souce

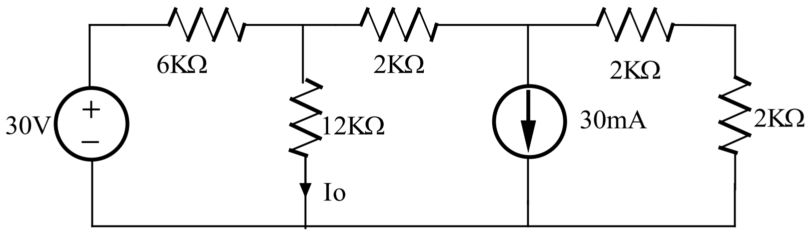
$$I = \frac{12}{6K+6K\|(6K+6K)} = \frac{6}{5}\,mA$$

$$I''_o = I\left(\frac{6K}{6K+12K}\right) = \frac{2}{5}\,mA$$

$$\boxed{I_o = I'_o + I''_o = \left(-\frac{18}{5}+\frac{2}{5}\right)mA = -\frac{16}{5}\,mA}$$

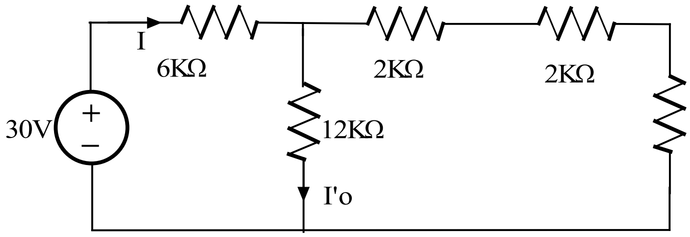
Problem 4.6

Find Io in the circuit shown using superposition



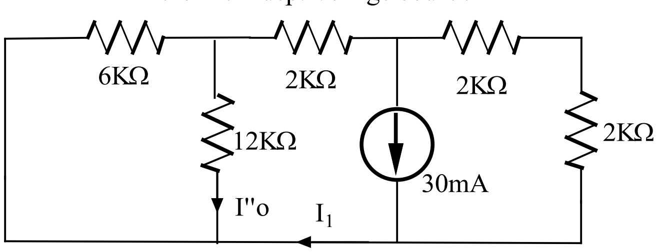
Suggested Solution

Zero the indep. current source



$I = \frac{30}{6K + 12K \| 6K} = 3mA, \qquad I'o = I\,\frac{6K}{18K} = 1mA$

Zero the indep. voltage source



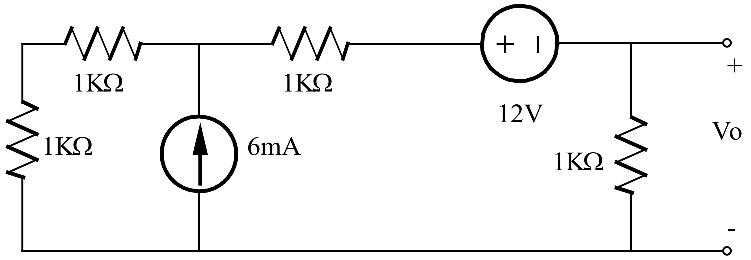
$I_1 = 0.03\left(\frac{4K}{4K + 2K + 6K \| 12K}\right) = 12mA$

$I"o = -0.012\left(\frac{6K}{18K}\right) = -4mA$

$\boxed{Io = I'o + I"o = 3mA}$

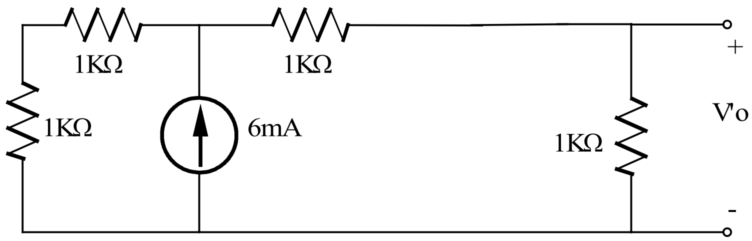
Problem 4.7

In the network shown find Vo using superposition



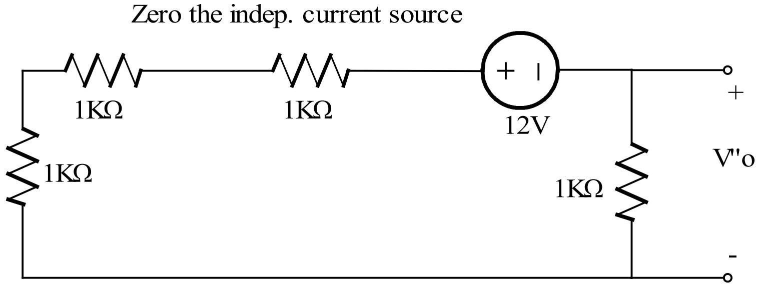
Suggested Solution

Zero the indep. voltage source



$V'o = 0.006(\frac{2K}{2K+2K})(1k) = 3V$

Zero the indep. current source

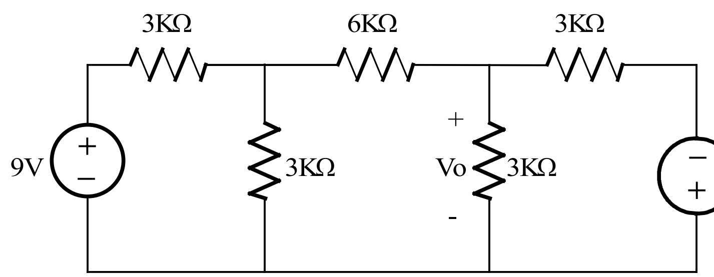


$V''o = \frac{-12}{4K}(1K) = -3V$

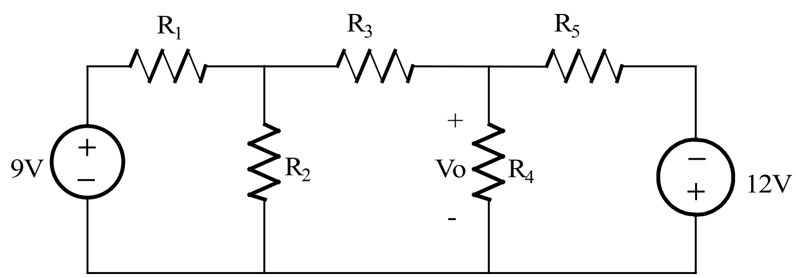
$\boxed{Vo = V'o + V''o = 0V}$

Problem 4.8

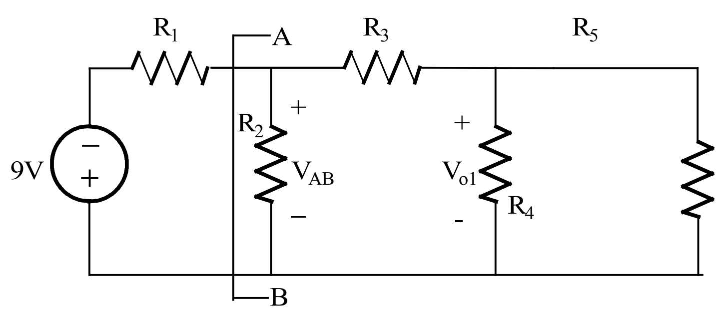
Find Vo in the network shown using superposition



Suggested Solution



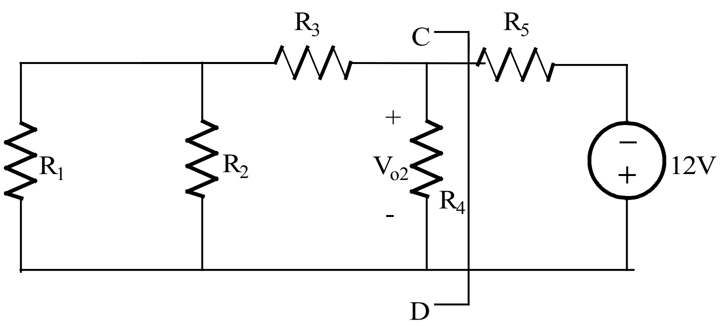
R3=6KΩ   All other R=3KΩ



$$R_{AB} = R_2 \,||\, [R_3 + (R_4 \,||\, R_5)] = 2.14K\Omega$$

$$V_{AB} = 9\left(\frac{R_{AB}}{R_{AB}+R_1}\right) = 3.75V$$

$$V_{01} = V_{AB}\left(\frac{R_4 || R_5}{(R_4 || R_5)+R_3}\right) = 0.75V$$



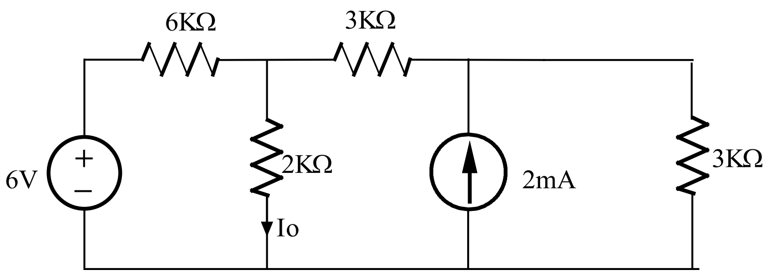
$$R_{CD} = R_4 \| [R_3 + (R_1 \| R_2)] = 2.14K\Omega$$

$$V_{02} = -12\left(\frac{R_{CD}}{R_{CD}+R_5}\right) = -5V$$

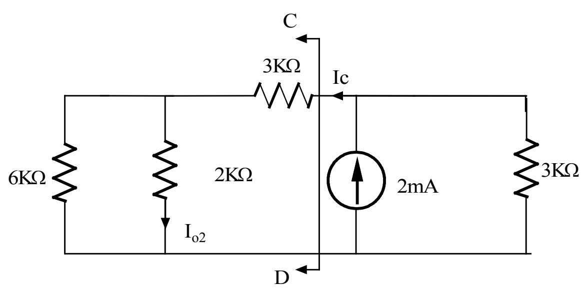
$$\boxed{Vo = V_{01} + V_{02} = -4.25V}$$

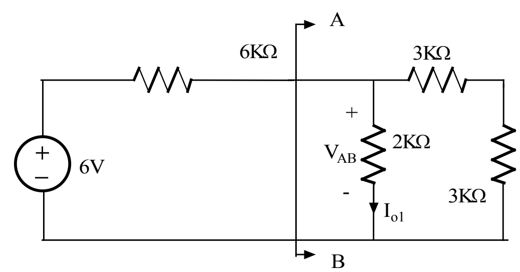
Problem 4.9

Find Io in the network shown using superposition.



Suggested Solution





$$R_{CD} = 3K + (6K \parallel 2K) = 4.5K\Omega$$

$$Ic = 2m\left(\frac{3K}{3K + R_{CD}}\right) = 0.8mA$$

$$I_{02} = Ic\left(\frac{6K}{6K + 2K}\right) = 0.6mA$$

$$R_{AB} = 2K \parallel (3K + 3K) = 1.5K\Omega$$

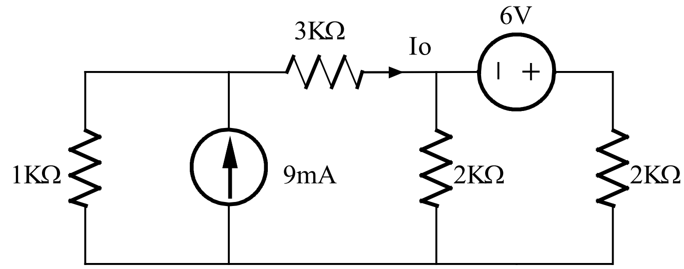
$$V_{AB} = 6\left(\frac{R_{AB}}{R_{AB} + 6K}\right) = 1.2V$$

$$I_{01} = V_{AB} / 2K = 0.6mA$$

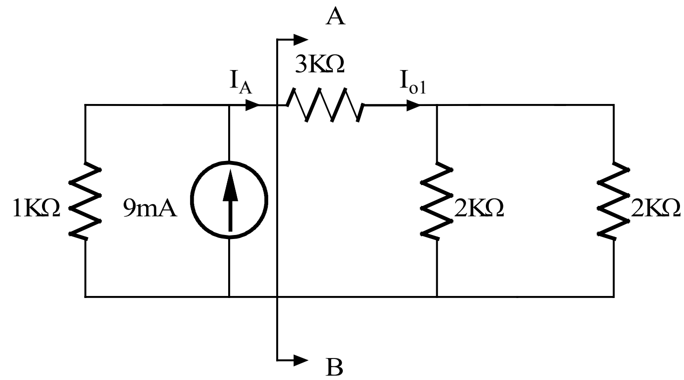
$$\boxed{Io = I_{01} + I_{02} = 1.2mA}$$

Problem 4.10

Find Io in the network shown using superposition

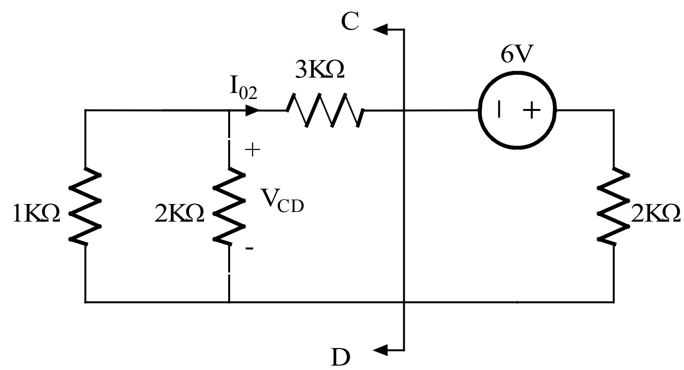


Suggested Solution



$$R_{AB} = 3K + (2K \parallel 2K) = 4K\Omega$$

$$I_A = I_{01} = 9m\left(\frac{1K}{1K + R_{AB}}\right) = 1.8mA$$



$$R_{CD} = (1K + 3K) \parallel 2K = 1.33K\Omega$$

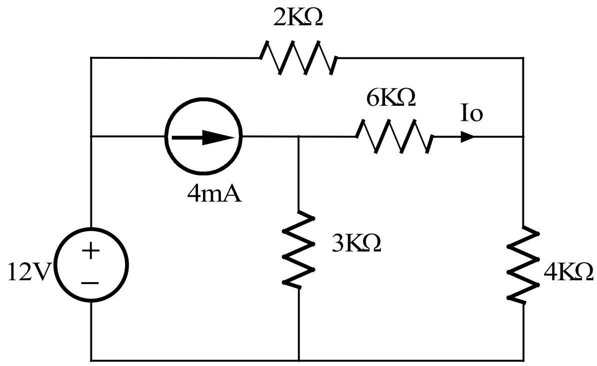
$$V_{CD} = -6\left[\frac{R_{CD}}{R_{CD} + 2K}\right] = -2.4V$$

$$I_{02} = -\frac{V_{CD}}{1K + 3K} = 0.6mA$$

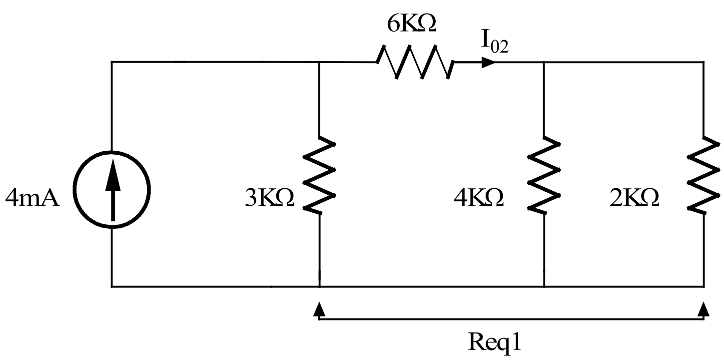
$$I_0 = I_{01} + I_{02} = 2.4mA, \quad \boxed{I0 = 2.4mA}$$

Problem 4.11

Find Io in the network shown using superposition.

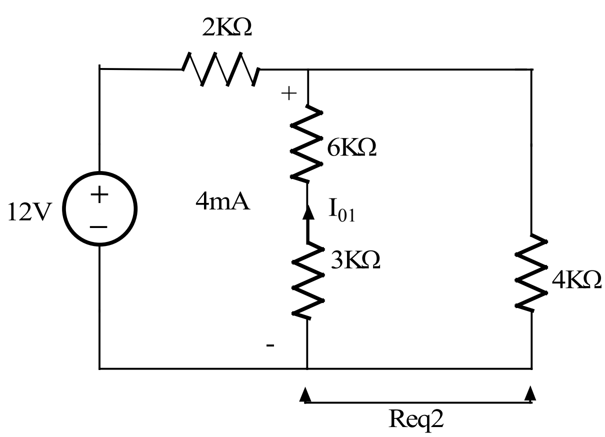


Suggested Solution



$$\mathrm{Re}q1 = 6K + (2K \,||\, 4K) = 7.33K$$

$$I_{02} = 4m[\tfrac{3K}{3K+\mathrm{Re}q1}] = 1.16mA$$



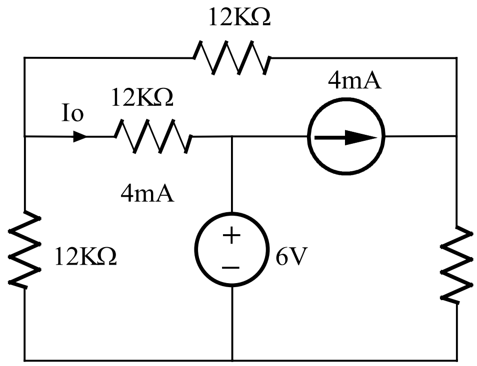
$$\mathrm{Re}q2 = 9K \,||\, 4K = 2.77k\Omega, \quad Vx = 12[\tfrac{\mathrm{Re}q2}{\mathrm{Re}q2+2K}] = 6.97V$$

$$I_{01} = -\tfrac{Vx}{9K} = -0.77mA$$

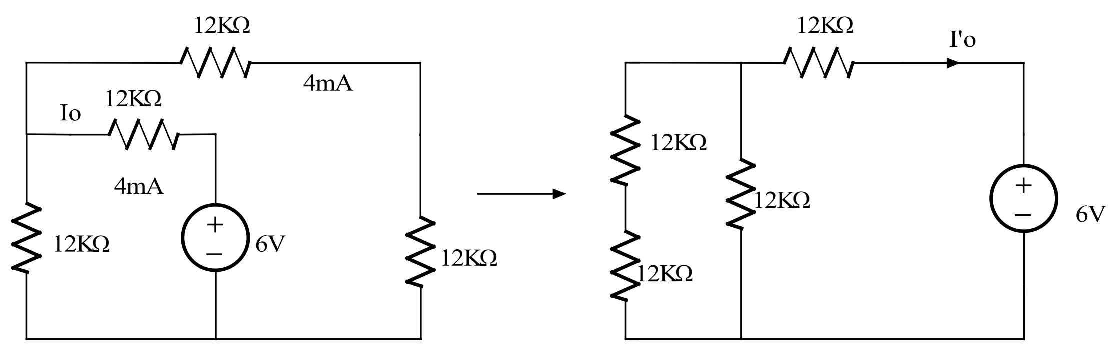
$$\boxed{I_0 = I_{01} + I_{02} = 0.39mA}$$

Problem 4.12

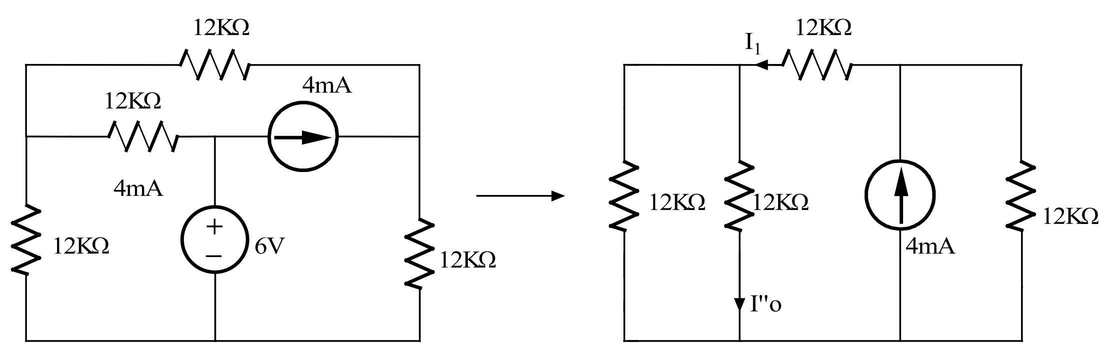
Find Io in the network shown using superposition



Suggested Solution



$$I'o = \frac{-6}{12K+12K\|(12K+12K)} = \frac{-3}{10}mA$$



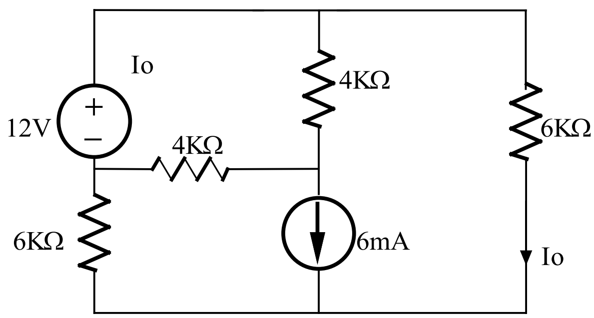
$$I_1 = 4m\left(\frac{12K}{12K+12K\|12K}\right) = 1.6mA$$

$$I''o = \frac{1}{2} I_1 = 0.8mA$$

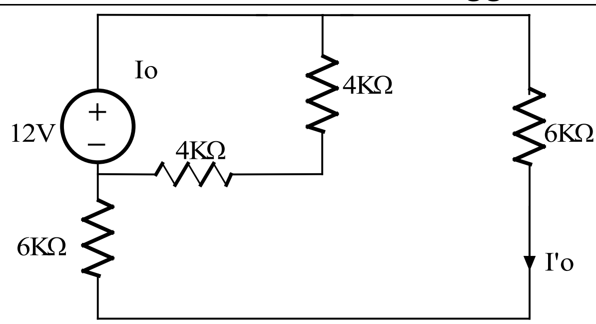
$$\boxed{Io = I'o + I''o = \frac{-0.003}{10} + \frac{0.008}{10} = \frac{1}{2}mA}$$

Problem 4.13

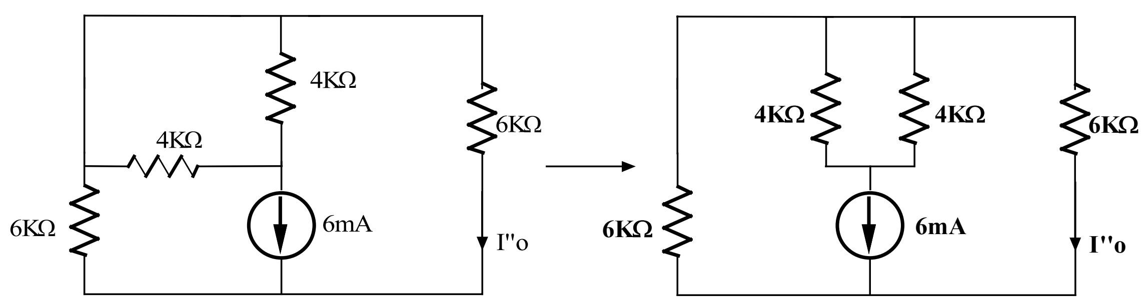
Find Io in the circuit shown using superposition



Suggested Solution



$I'o = \frac{0.012}{12} = 1mA$

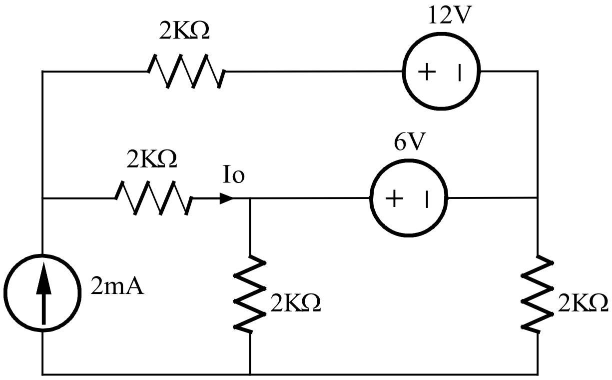


Current splits equally: $I''o = -3mA$

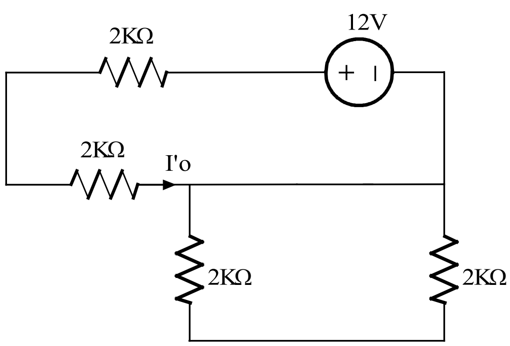
$\boxed{Io = I'o + I''o = -2mA}$

Problem 4.14

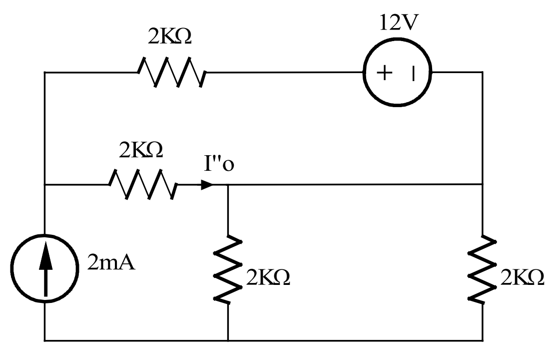
Use superposition to find Io in the circuit shown



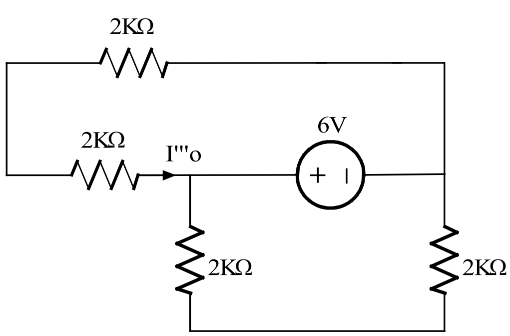
Suggested Solution



I'o = 12/4K = 3mA



Current Splits equally
I''o=1mA

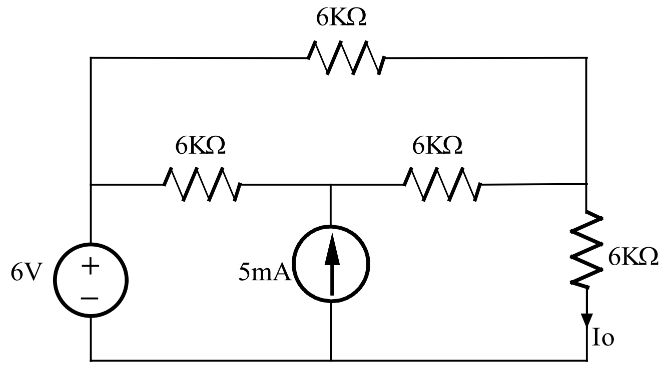


I'''o = -6/4K = -3/2 mA

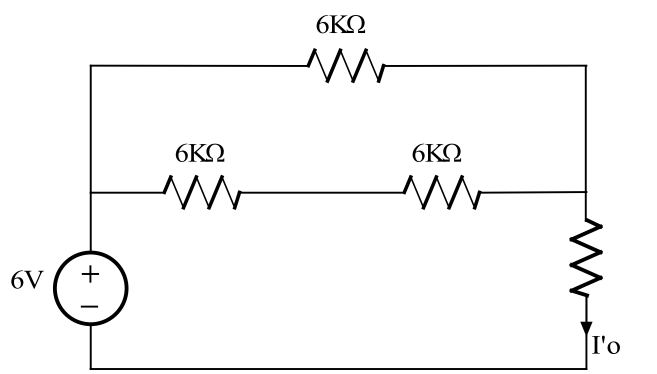
Then Io=I'o + I''o + I'''o = 3mA + 1mA -3/2 mA = 5/2 mA

Problem 4.15

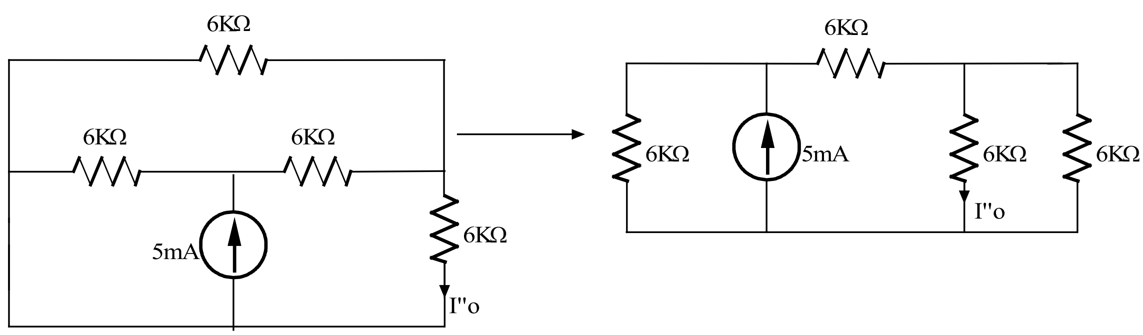
Find Io in the network shown using superposition



Suggested Solution



$$I'o = \frac{-6}{6K + 6K \| (6K + 6K)} = \frac{-6}{10K} A$$

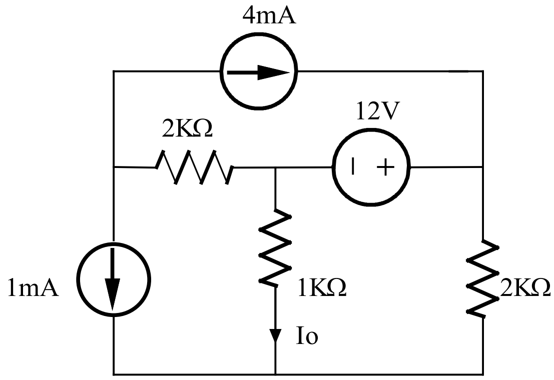


$$I''o = 5m\left(\frac{6K}{6K + 6K + 6K \| 6K}\right)\left(\frac{1}{2}\right) = 1mA$$

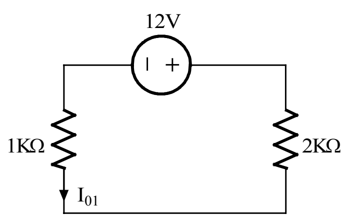
$$\boxed{Io = I'o + I''o = 1mA + \frac{3}{5}mA = \frac{2}{5}mA}$$

Problem 4.16

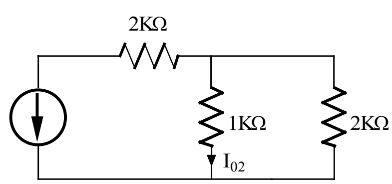
Find Io in the network shown using superposition.



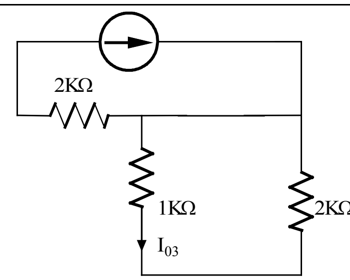
Suggested Solution



Io due to 12V source



Io due to 2mA source



Io due to 4mA source

I_{01}= -12/(1K+12K) = -4mA

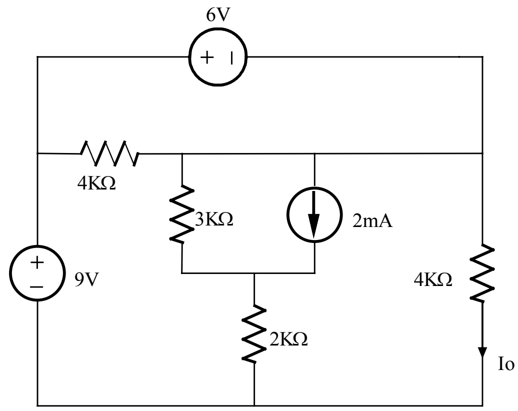
I_{02} = 2m[2K/(2K+3K)] = -1.33mA

I_{03}=0A

Io = $I_{01} + I_{02} + I_{03}$ = -5.33mA

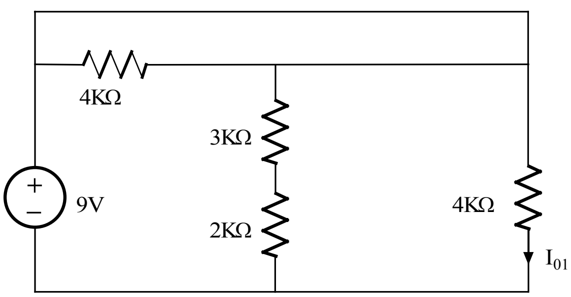
Problem 4.17

Find Io in the network shown using superposition



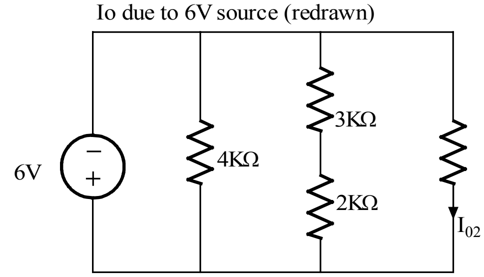
Suggested Solution

Io due to 9V source



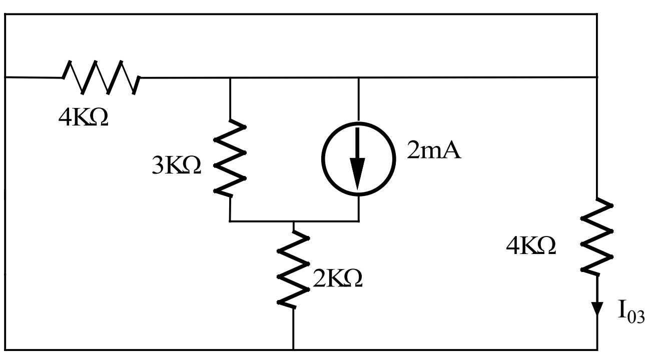
$I_{01} = \frac{9}{4K} = 2.25mA$

Io due to 6V source (redrawn)



$I_{02} = \frac{-6}{4K} = -1.5mA$

Io due to 2mA source



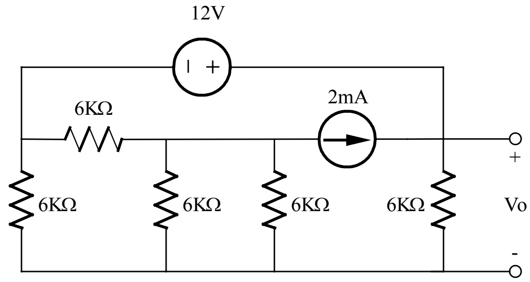
$(4K)I_{03} = 0 \rightarrow I_{03} = 0$

$Io = I_{01} + I_{02} + I_{03}$

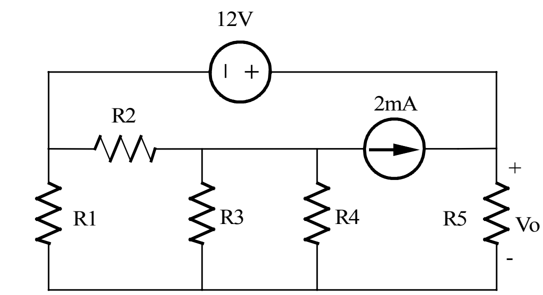
$\boxed{Io = 0.75mA}$

Problem 4.18

Find Vo in the network shown using superposition

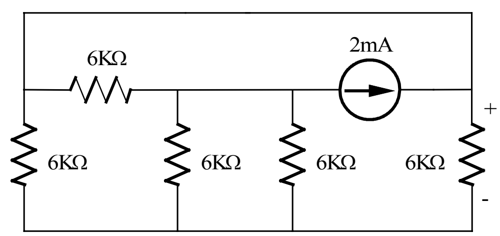


Suggested Solution

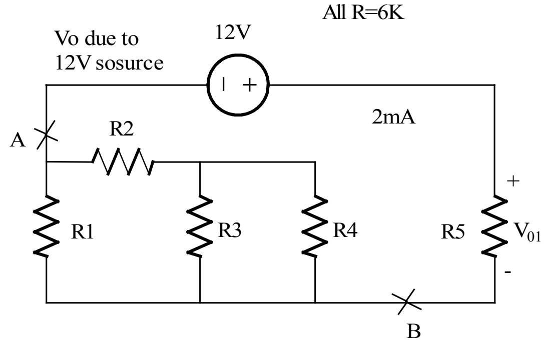


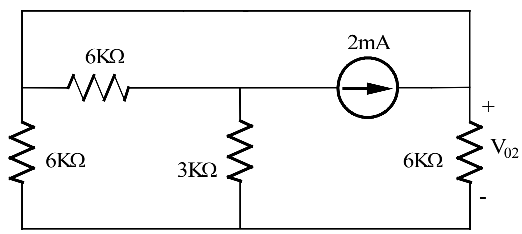
All R=6K

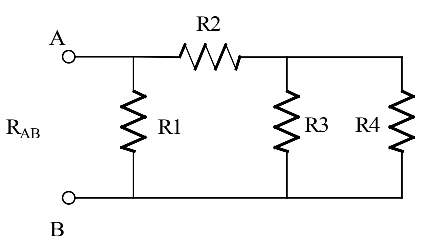
Vo due to 2mA source

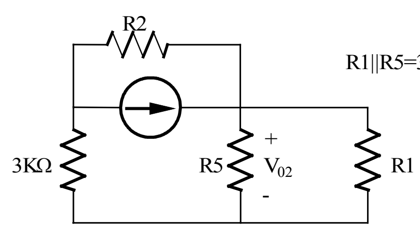


6K||6K=3K

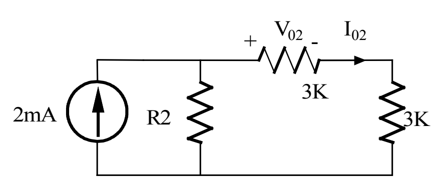








R1||R5=3K



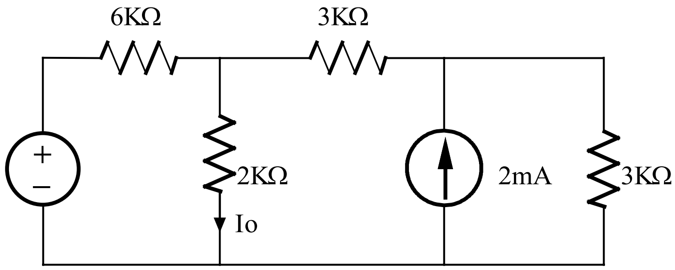
$RAB = R1||[R2+(R3||R4)]$
$RAB = 3.6K$ ohms
$V_{01} = 12[R5/(R5+RAB)]$
$V_{01} = 7.5V$

$I_{02} = 2m[R2/(R2+3K+3K)] = 1mA$
$V_{02} = I_{02}(3K)=3V$
$Vo = V_{01}+V_{02}=10.5V$

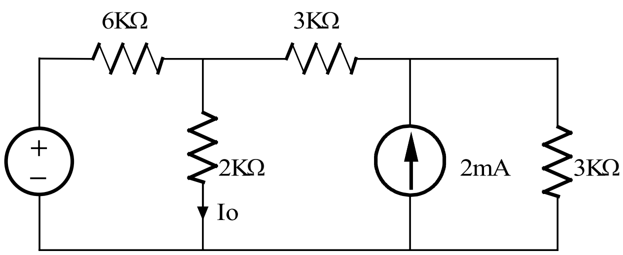
Vo=10.5

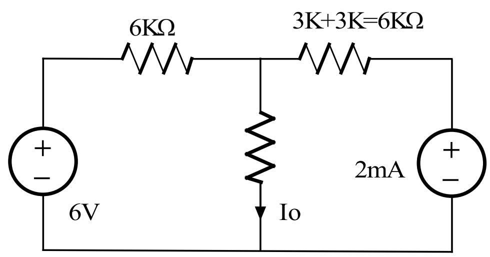
Problem 4.19

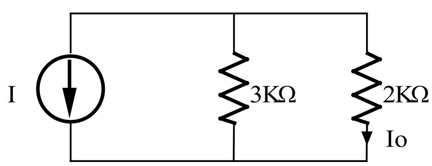
Use source transformation to find Io in the circuit shown



Suggested Solution





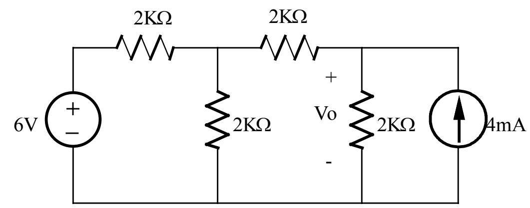


I=6/6K + 6/6K = 2mA
3K = 6K || 6K

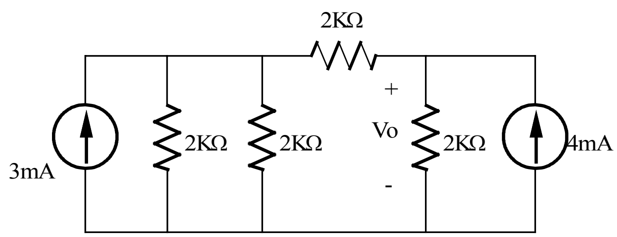
Io = I(3K/(2K+3K)) = 1.2mA

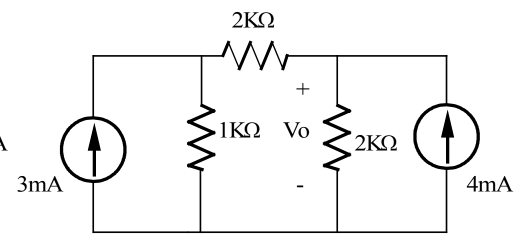
Problem 4.20

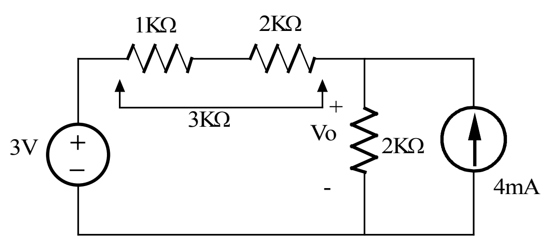
Find Vo in the network shown using source transformation

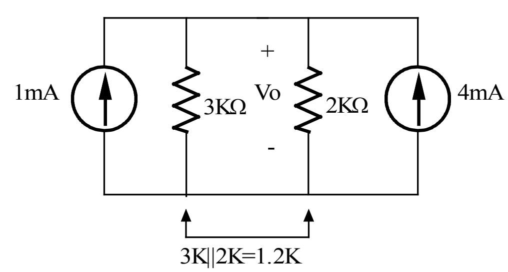


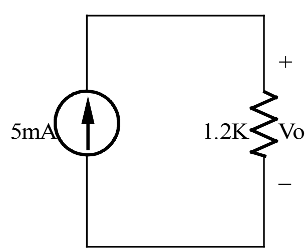
Suggested Solution









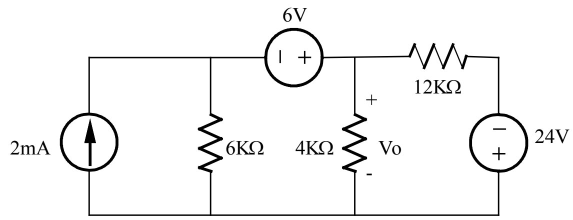


Vo = (5m)(1.2K) = 6V

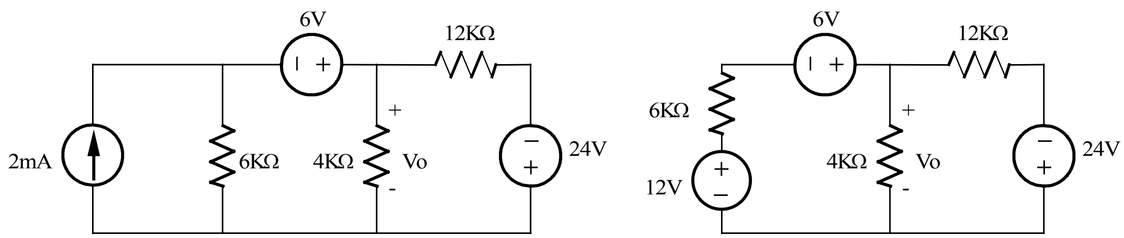
Vo = 6V

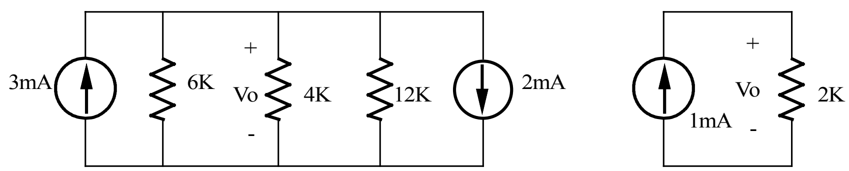
Problem 4.21

Use source transformation to find Vo in the network shown.



Suggested Solution

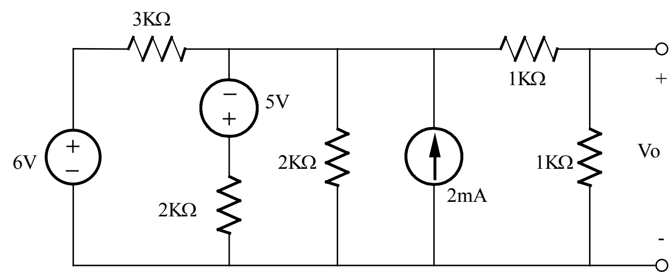




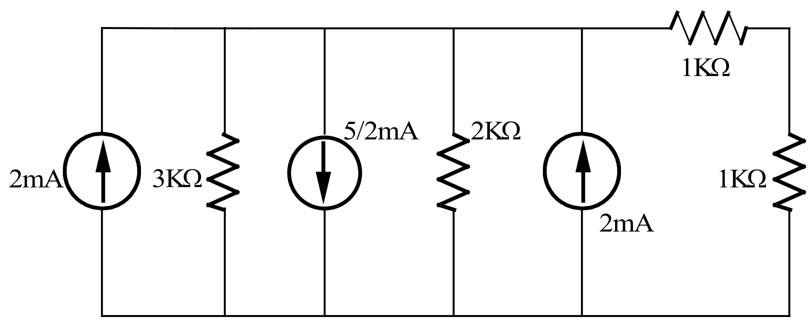
Vo = 2V

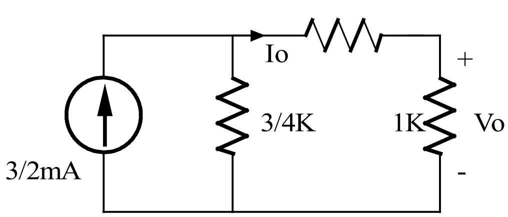
Problem 4.22

Find Vo in the network shown using source transformation



Suggested Solution



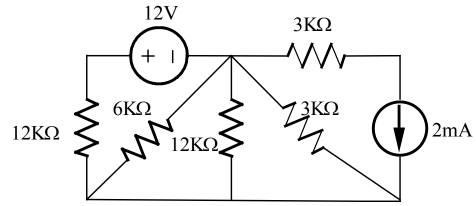


$$Io = \frac{3}{2K}\left(\frac{3/4K}{3/4K+2K}\right) = 0.41mA$$

$$\boxed{Vo = 1K(Io) = 0.41V}$$

Problem 4.23

Find Io in the circuit shown using source transformation



Suggested Solution

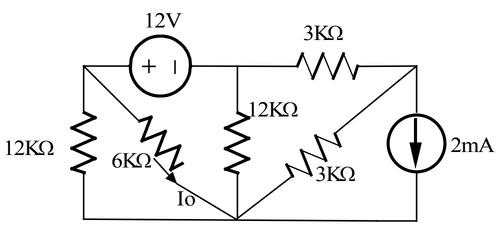
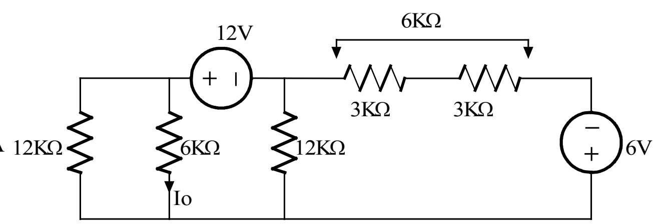
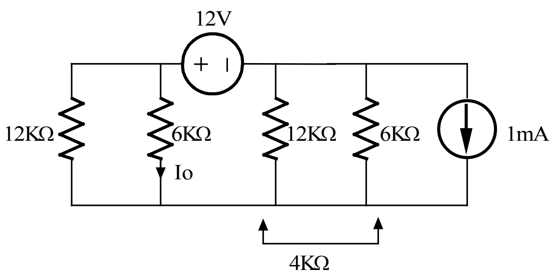
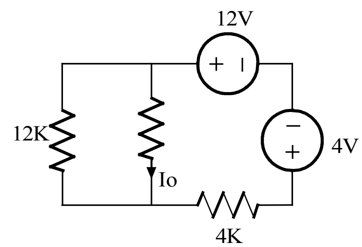
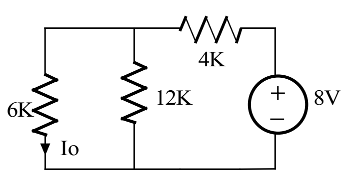


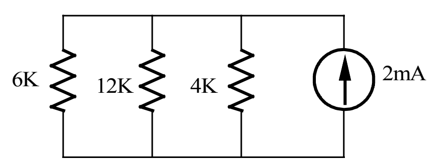
Figure P4.23

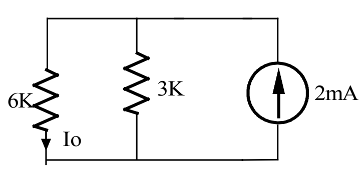










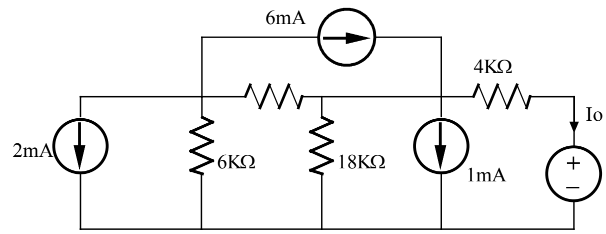


Io = 2m(3K/(3K+6K)) = 0.67mA

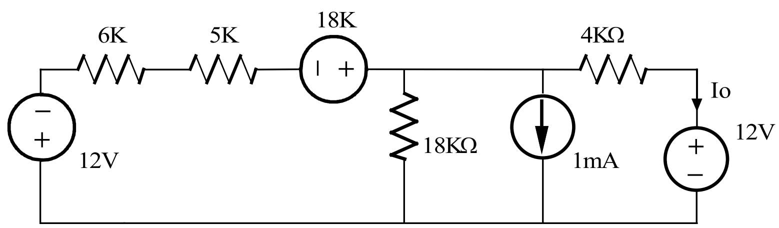
Io = 0.67mA

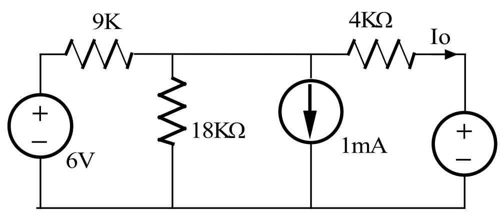
Problem 4.24

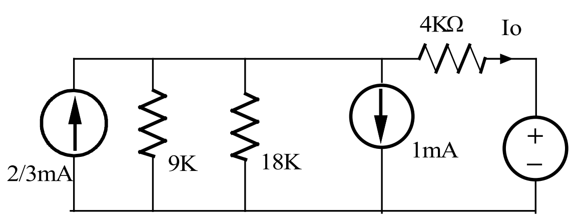
Find Io in the network shown using source transformation.

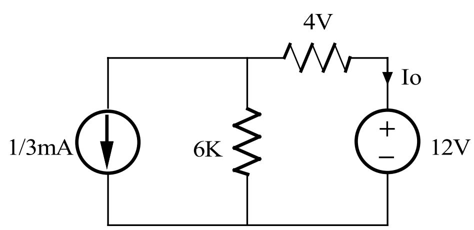


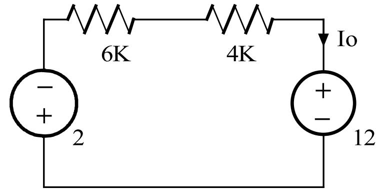
Suggested Solution







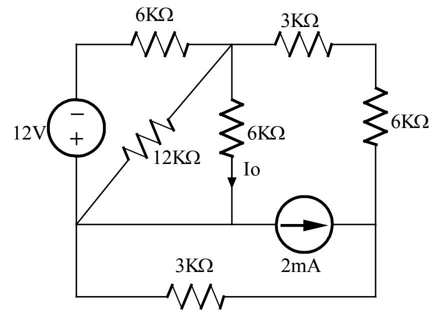




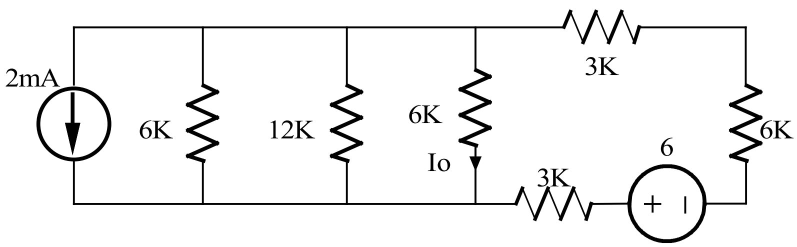
Io = -14/10K=-7/5mA

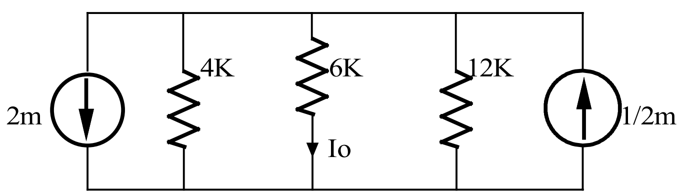
Problem 4.25

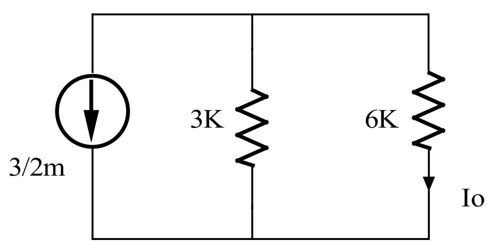
Use source transformation to find Io in the circuit shown.



Suggested Solution



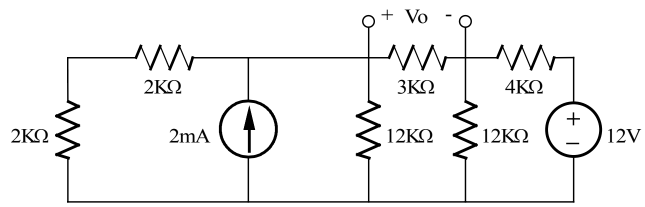




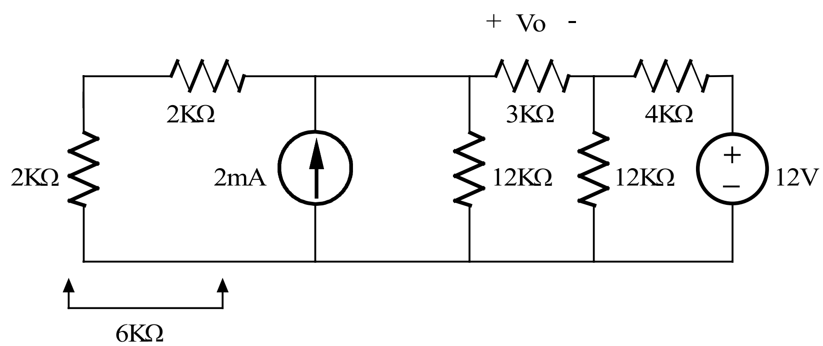
Io = -3/2m(3K/9K) = -1mA

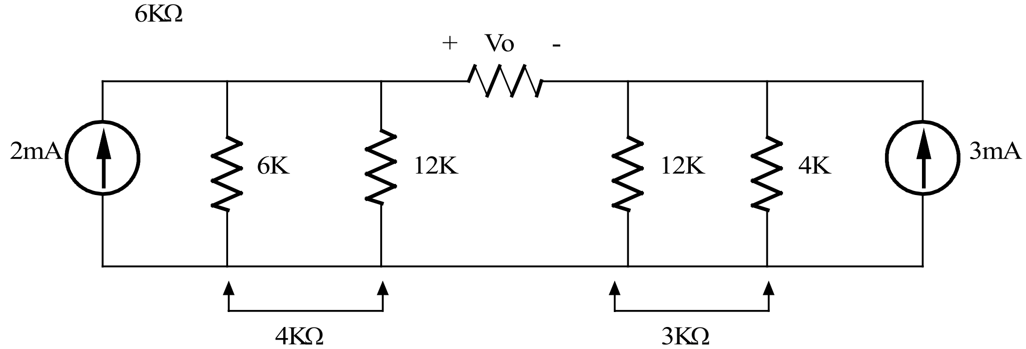
Problem 4.26

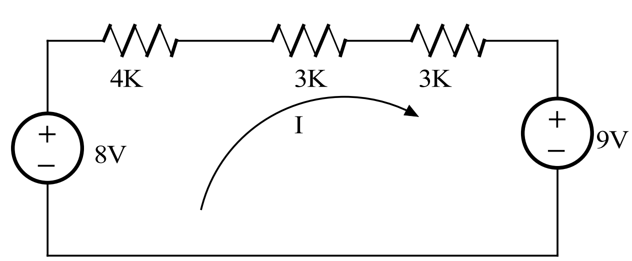
Find Vo in the network shown using source transformation.



Suggested Solution







$$-8 + I\,(4K + 3K + 3K) + 9 = 0$$

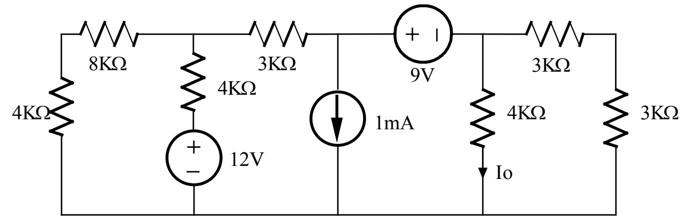
$$I = -\,0.1mA$$

$$Vo = I\,(3K) = -\,0.3V$$

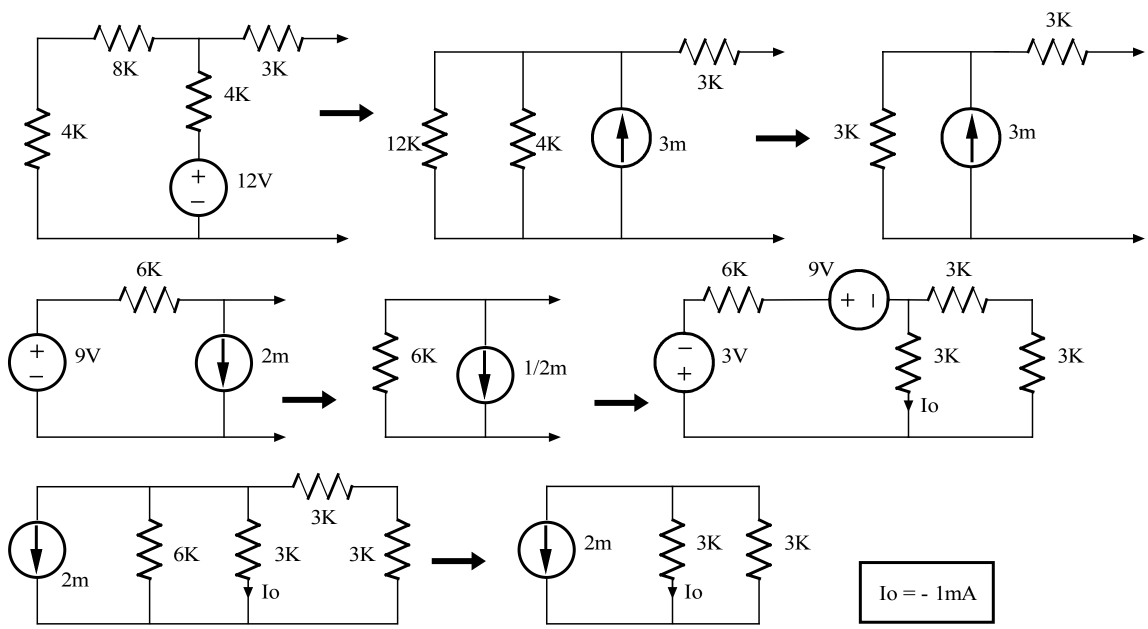
Vo = -0.3V

Problem 4.27

Find Vo in the circuit shown using source transformation

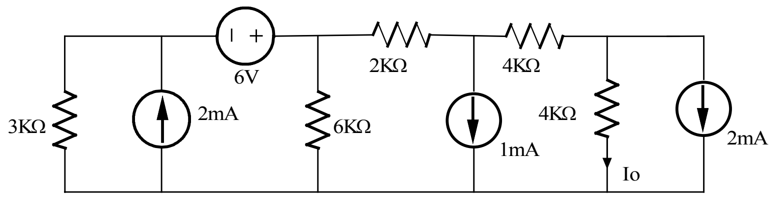


Suggested Solution

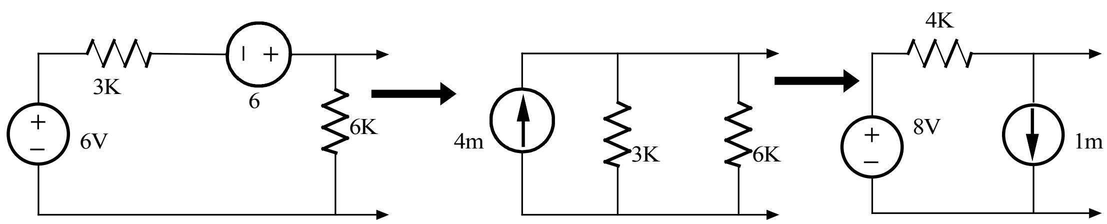


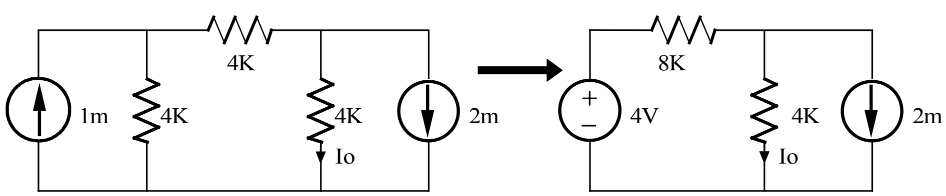
Problem 4.28

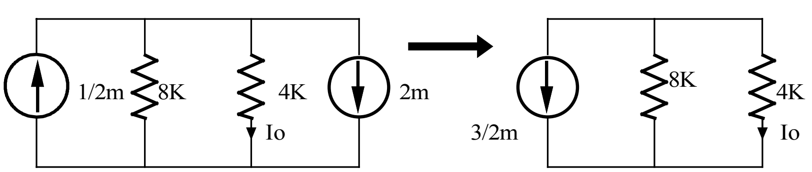
Find Io in the network shown using source transformation.



Suggested Solution





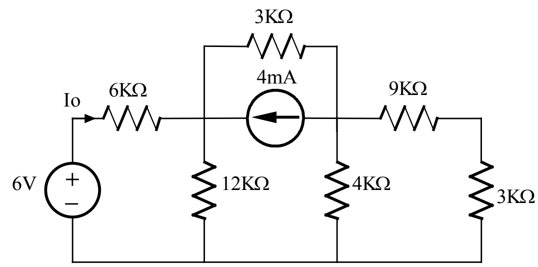


Io = -3/2m(8K/12K)

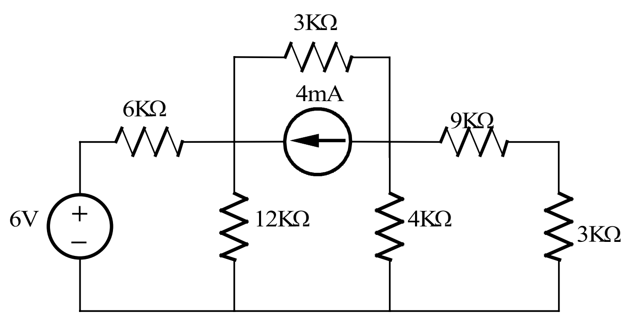
Io = -1mA

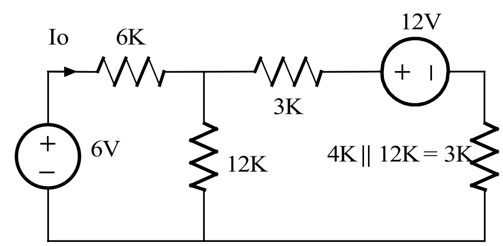
Problem 4.29

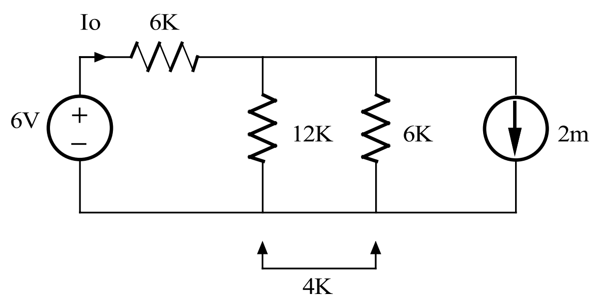
Find Io in the network shown using source transformation.

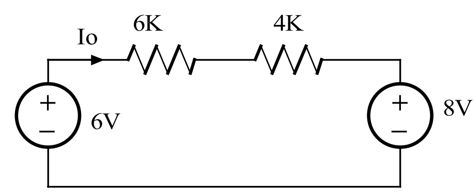


Suggested Solution







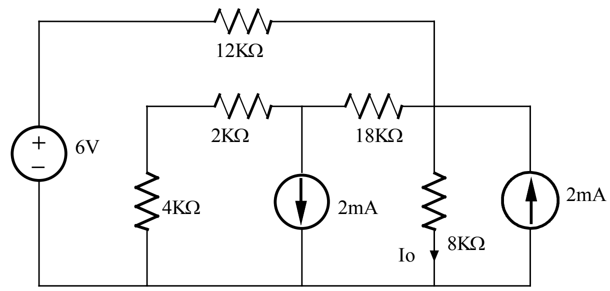


- 6 + Io (6K + 4K) + 8 = 0

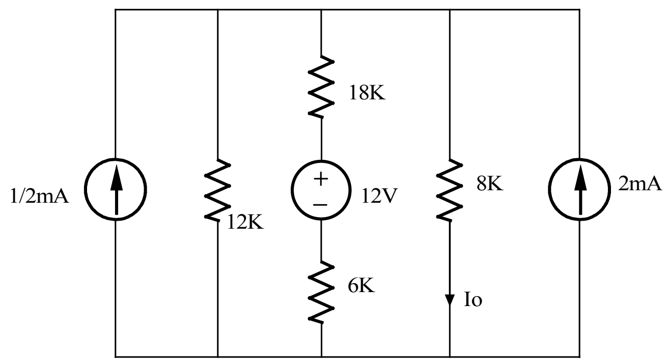
Io = - 0.2 mA

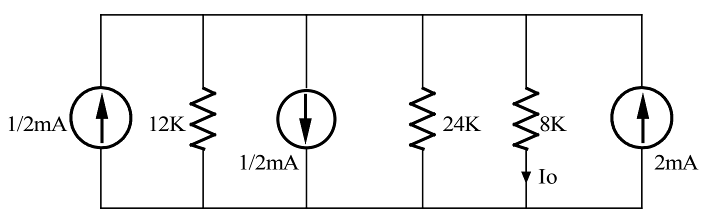
Problem 4.30

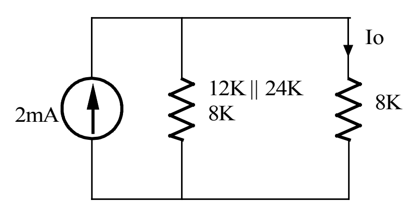
Find Io in the network shown using source transformation.



Suggested Solution





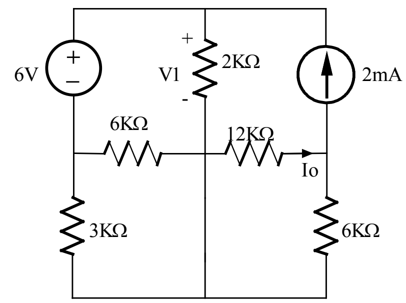


Io = 2m [8K / (8K + 8K)]

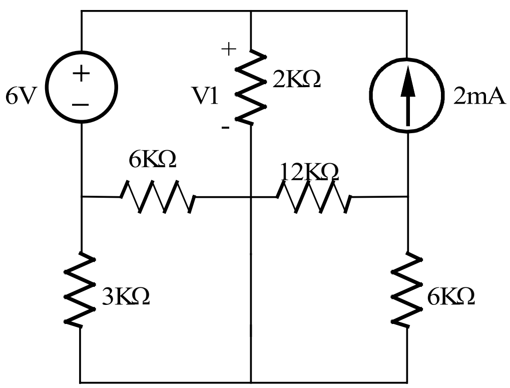
Io = 1 mA

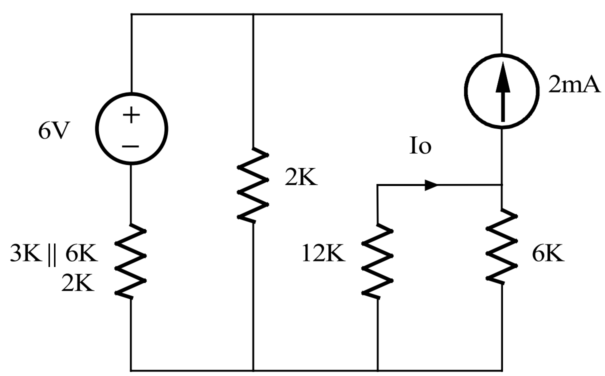
Problem 4.31

Find Io in the network shown using source transformation.



Suggested Solution



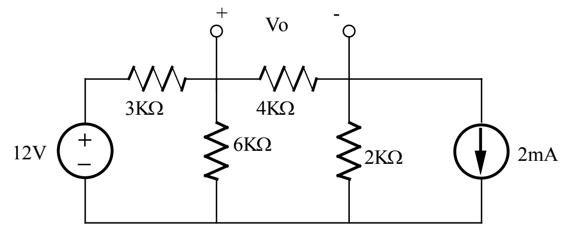


Io = 2m (6 / (6 + 12) = 0.67mA

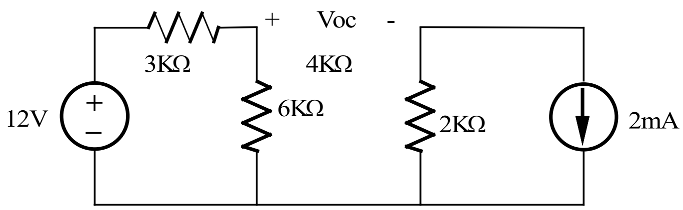
Io = 0.67 mA

Problem 4.32

Use Thevenin's Theorem to find Vo in the network shown.

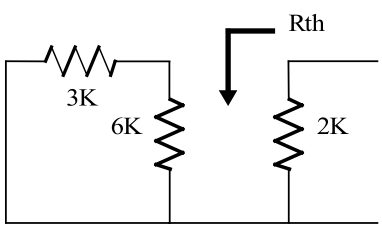


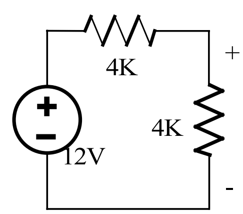
Suggested Solution



Voc = 12 (6K / 9K) + 2m 2K = 12V

Rth = 3K || 6K + 12K = 4K

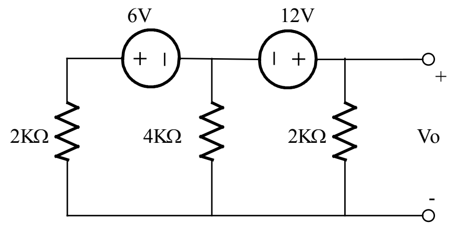




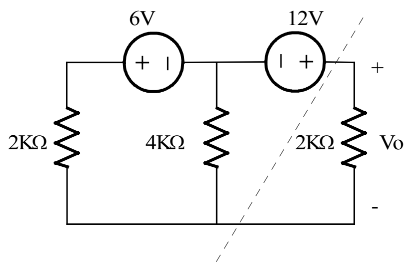
Vo = 6V

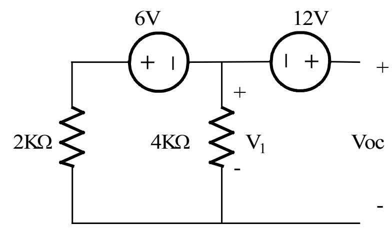
Problem 4.33

Use Thevenin's Theorem to find Vo in the network shown.



Suggested Solution



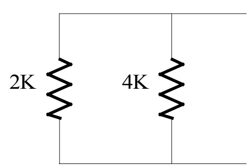


Find Voc

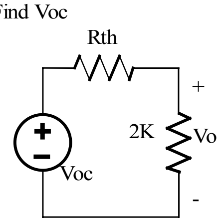
$V_1 = -6\ [\ 4K / (4K + 2K\ )]$

$V_1 = -4V$

$Voc = 12 + V_1$



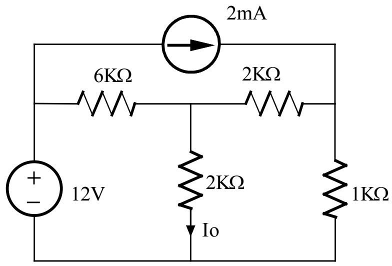
Rth = 2K || 4K = 1.33KΩ



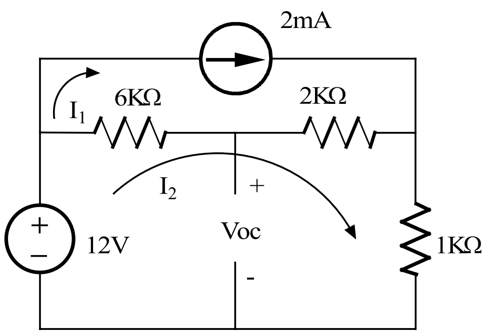
Vo = 4.8V

Problem 4.34

Use Thevenin's Theorem to find Io in the network shown.

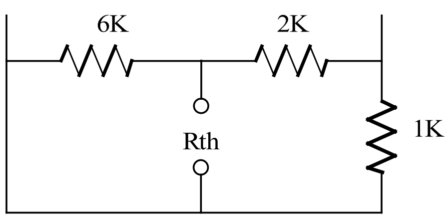


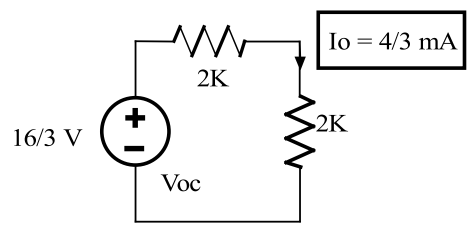
Suggested Solution



$I_1 = 2mA$
$-1_2 + 6K(I_2 - 2m) + 2K(I_2 - 2m) + 1K I_2 = 0$
$I2 = 28 / 9$ mA

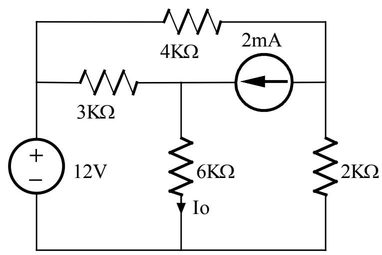
Then $Voc = 12 - 6K(I2 - 2m) = 16 / 3V$



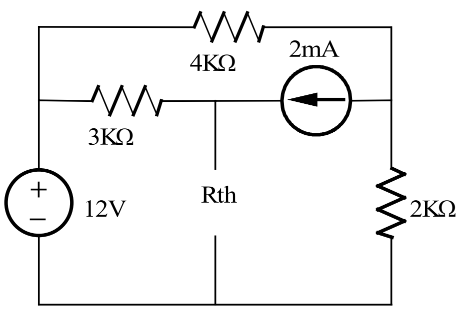


Problem 4.35

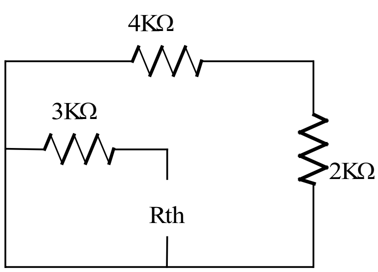
Find Io in the network shown using Thevenin's Theorem.



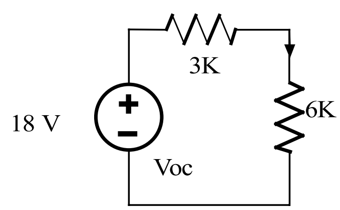
Suggested Solution



Voc = 12 + 3K (2m) = 18V



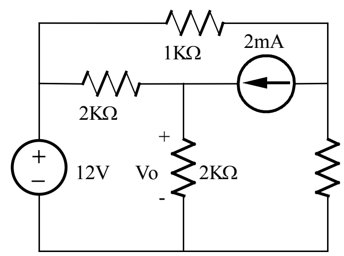
Rth = 3KΩ



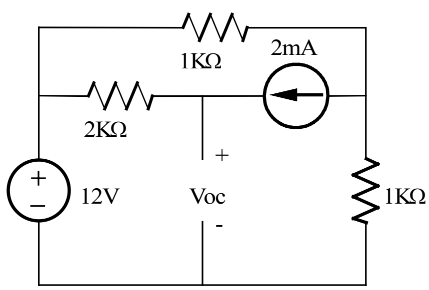
Io = 2 mA

Problem 4.36

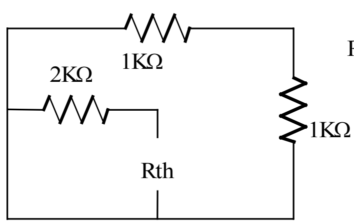
Find Vo in the circuit shown using Thevenin's Theorem.



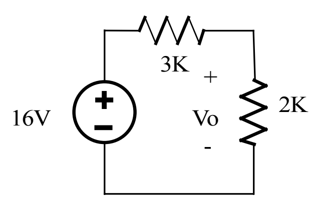
Suggested Solution



Voc = 12 + 2K (2m) = 16V



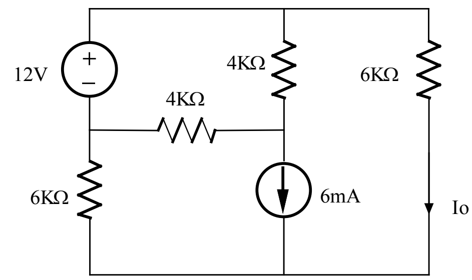
Rth = 2KΩ



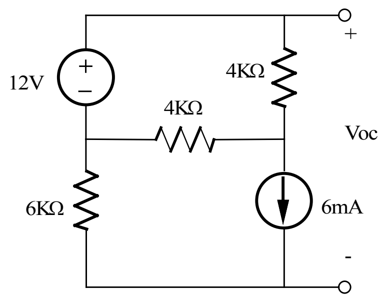
Vo = 16 (2K / (2K + 2K)) = 8V

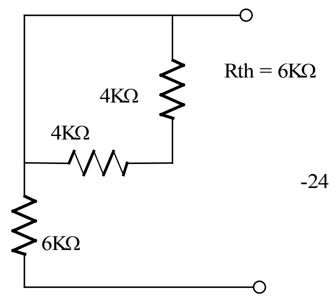
Problem 4.37

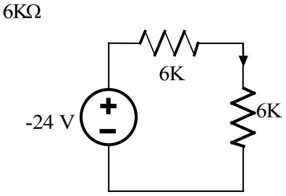
Find Io in the circuit shown using Thevenin's Theorem.



Suggested Solution





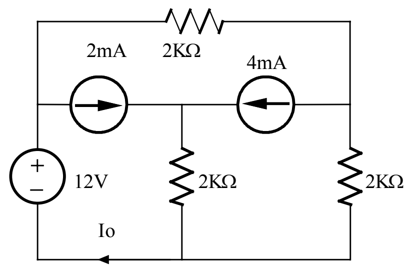


Io = -2 mA

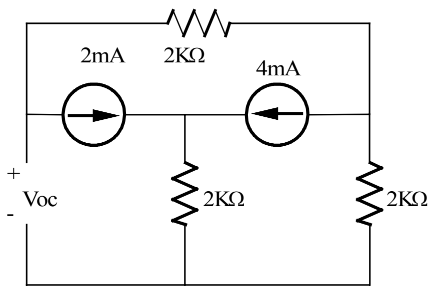
Voc = 12 - 6K (6m) = -24V

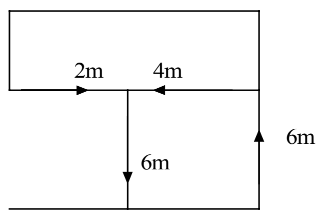
Problem 4.38

Find Io in the network shown using Thevenin's Theorem.

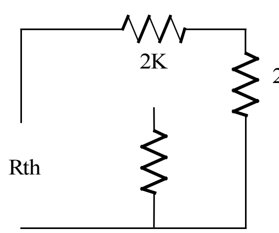


Suggested Solution

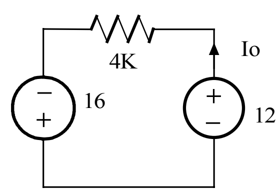




Voc = - 2K (2m) - 6m (2K) = - 16V



Rth = 4KΩ

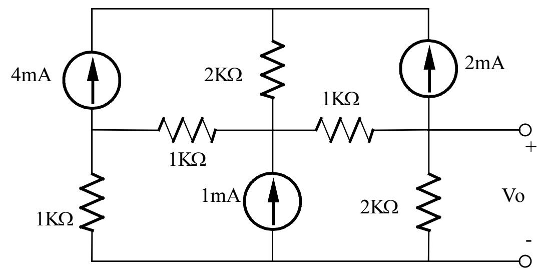


Io = (16 + 12) / ($K)

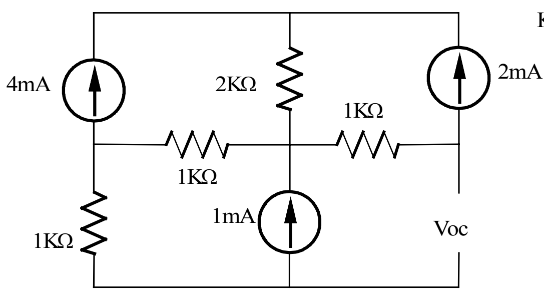
Io = 7mA

Problem 4.39

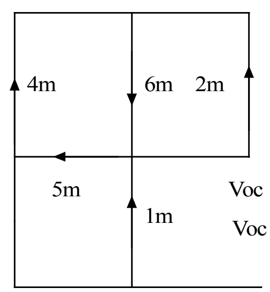
Find Vo in the circuit shown using Thevenin's Theorem.



Suggested Solution

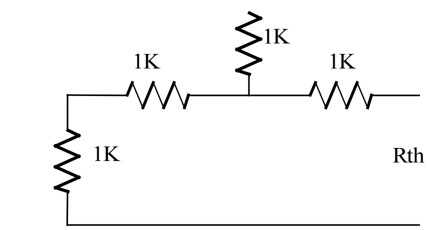


KCL

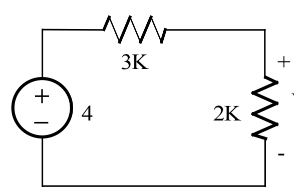


Voc = 1K(-2m)+1K(5m)+1K(1m)

Voc = 4V



Rth = 3KΩ

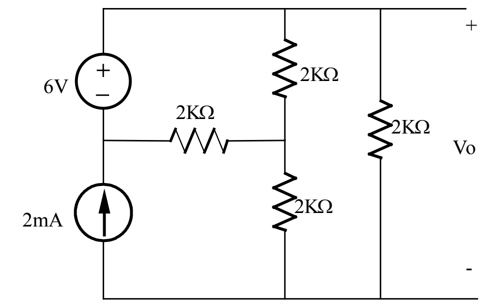


Vo = 4 / (3K + 2K) *2K

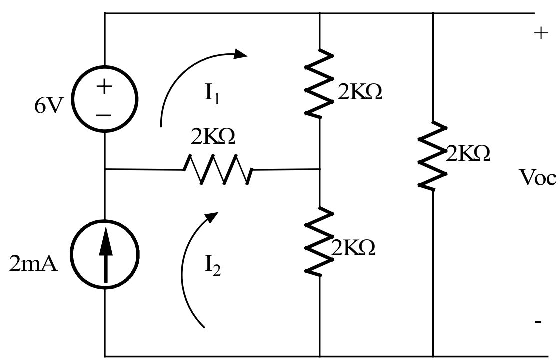
Vo = 8/5 V

Problem 4.40

Find Vo in the circuit shown using Thevenin's Theorem.



Suggested Solution

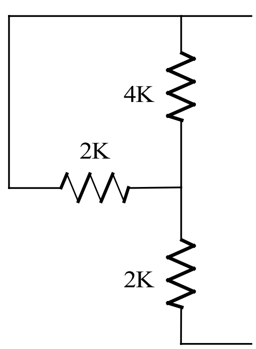


$I_2 = 2m$

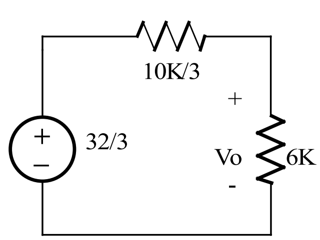
$-6 + 4K\,I_1 + 2K(I_1 - 2m) = 0$

$I_1 = 10/6K = 5/3\ mA$

$Voc = 4K\,I_1 + 2K\,I_2 = 4K\,(5/3m) + 2K\,2m = 32/3V$



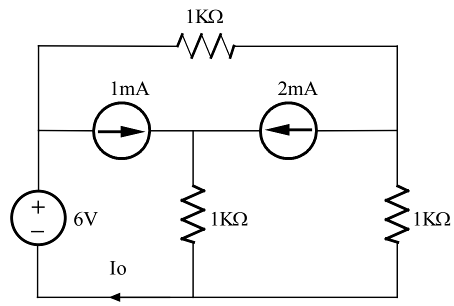
$Rth = 2K + 4K \,||\, 2K = 10K/3$



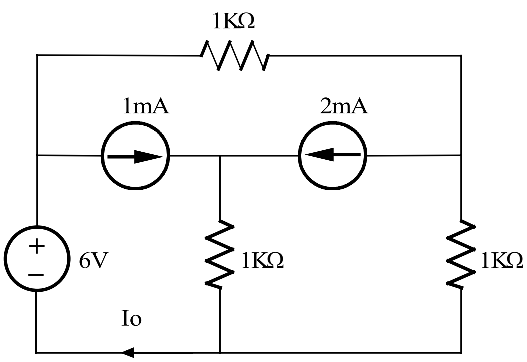
$Vo = 32/3\ [6K/(6K + 10/3\ K)] = 48/7\ V$

Problem 4.41

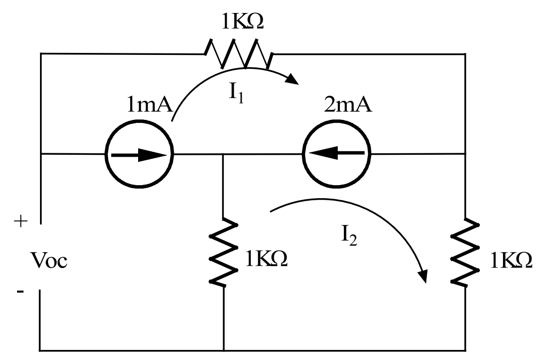
Find Io in the network shown using Thevenin's Theorem.



Suggested Solution



Find Voc

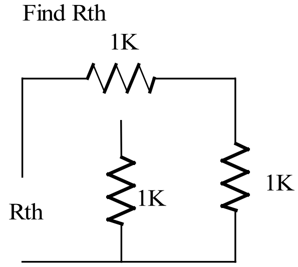


$I_1 = -1$ mA

$I_1 - I_2 = 2$mA , $I_2 = -3$ mA

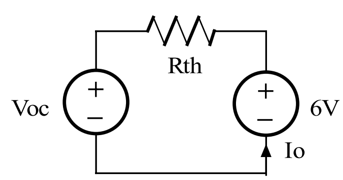
$Voc = (1K)I_1 + (1K)I_2 = -4V$

Find Rth



Rth = 1K + 1K

Rth = 2KΩ



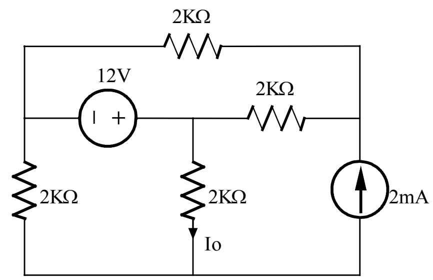
$-6 + Io\,(Rth) + Voc = 0$

$Io = (6 - Voc) / Rth$

Io = 5mA

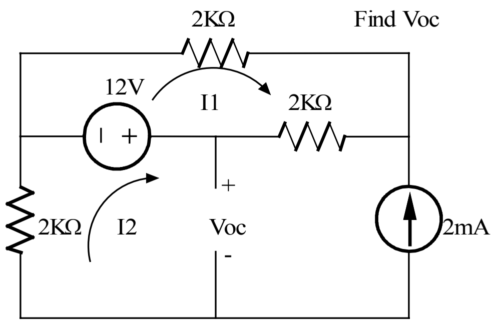
Problem 4.42

Find Io in the network shown using Thevenin's Theorem.

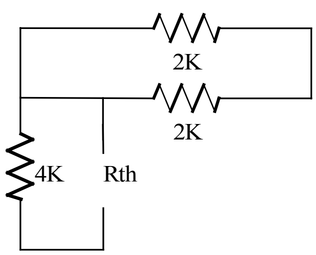


Suggested Solution

Find Voc

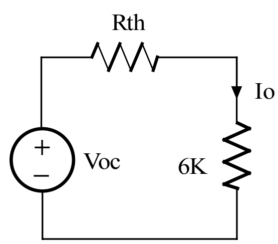


$I_2 = -2mA$
$Voc = 12 - (4K)\, I_2$
$Voc = 20V$



Both 2K Ω resistors are shorted!

Rth = 4KΩ

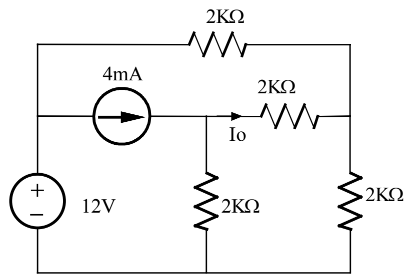


Io = Voc / (Rth + 6K)

Io = 2mA

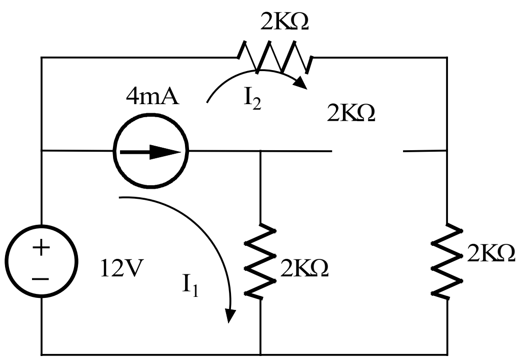
Problem 4.43

Find Io in the network shown using Thevenin's Theorem.



Suggested Solution

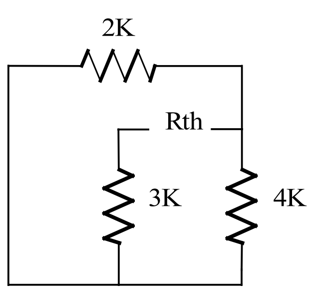
Find Voc



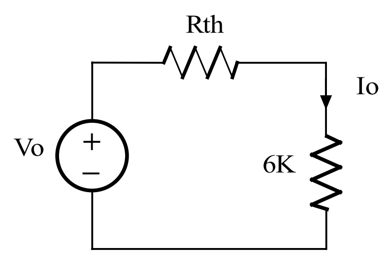
$I_1 - I_2 = 4mA$
$12 = (2K)\ I_2 + 4K\ (I_2)$
so, $I_2 = 2mA$, $I_1 = 6mA$

$Voc = 3K(I_1 - I_2) - 4K\ (I_2)$
$Voc = 4V$

Find Rth



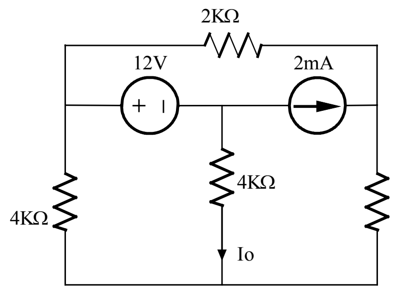
$Rth = 3K + (2K \mid\mid 4K)$
$Rth = 4.33K\Omega$



$Io = Voc/(Rth + 6K)$

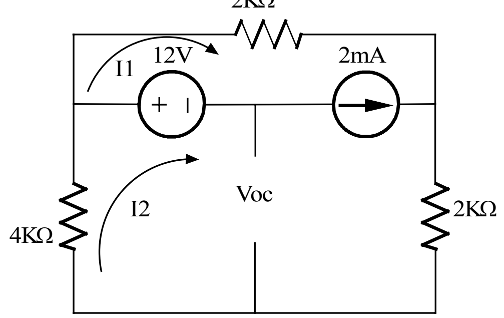
$Io = 0.39mA$

Problem 4.44

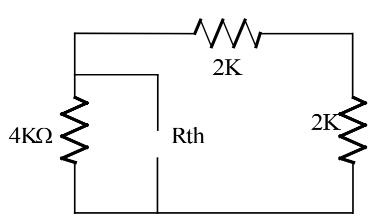


Suggested Solution

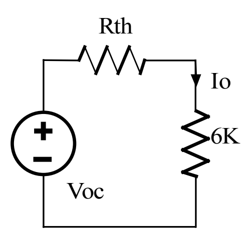
Find Voc



I2 - I1 = 2mA
(4K)I2 + (2K)I1 + (2K)I2 = 0
yields: I2 = 0.5mA
Voc = 12 -(4K)I2 = 10V



Rth = 4K || (2K + 2K)
Rth = 2K

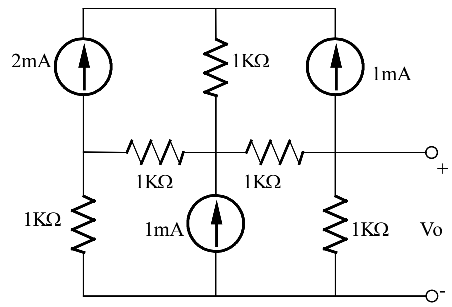


Io = Voc / (Rth + 6K)

Io = 1.25 mA

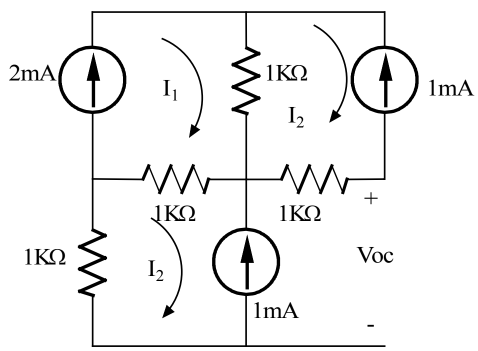
Problem 4.45

Find Vo in the network shown using Thevenin's Theorem.



Suggested Solution

Find Voc

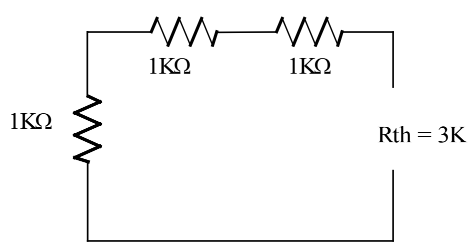


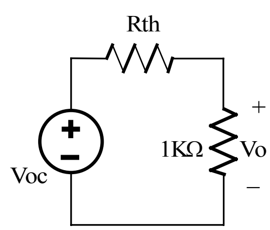
$I_1 = 2mA,\ I_2 = -1mA,\ I_3 = 1mA$

$(1K)I_3 + 1K(I_3 - I_1) - (1K)I_2 + Voc = 0$

$Voc = 3V$

Find Rth



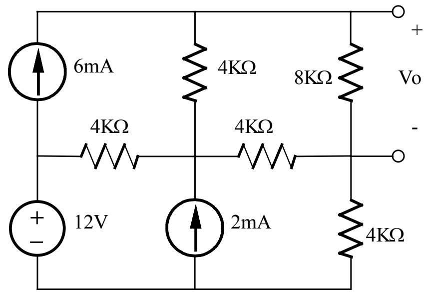


$Vo = Voc\ [\ 1K / (1K + Rth)]$

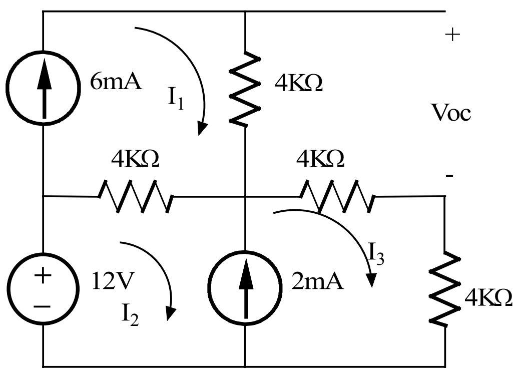
Vo = 0.75 V

Problem 4.46

Find Vo in the network shown using Thevenin's Theorem.



Suggested Solution



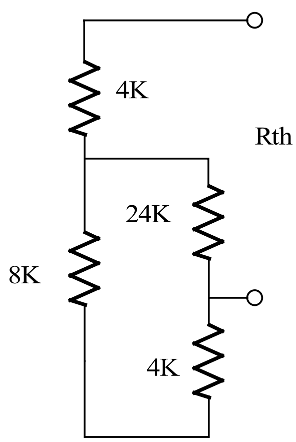
$I_1 = 6mA$

I_3 - I2 = 2mA

$-1_2 + 8K(I_2 - I_1) + (28K)\ I_3 = 0$

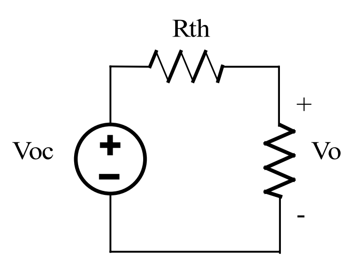
$Voc = (4K)I_1 + (24K)\ I_3$

Yields Voc = 74.67V



Rth = 4K + [(8K + 4K) || 24K]

Rth = 12KΩ

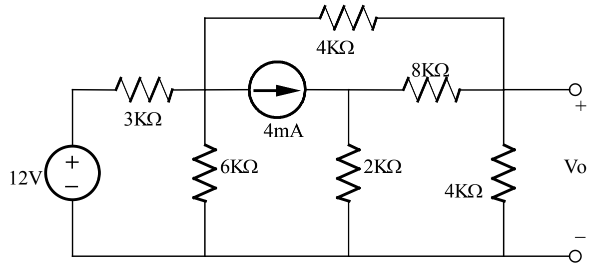


Vo = Voc[8K / (8K + Rth)]

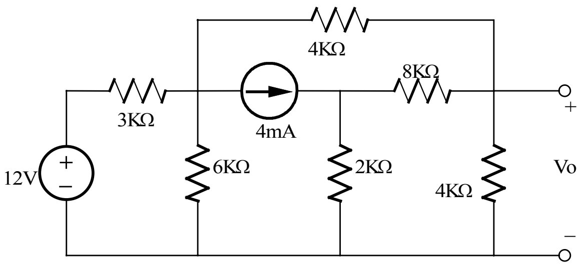
Vo = 29.87V

Problem 4.47

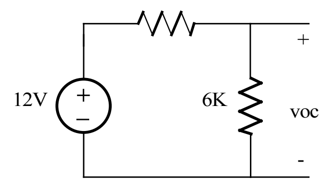
Use a combination of Thevenin's Theorem and superposition to find Vo in the circuit shown.



Suggested Solution



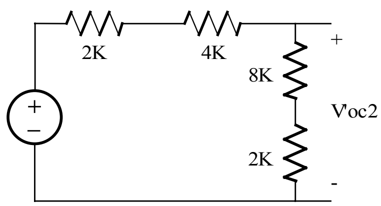
Find Voc1 and Rth1



Voc1 = 12[6/(6+3)]
Voc1 = 8V

Rth1 = 3K || 6K = 2K

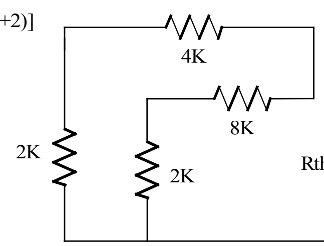
New circuit: Find Voc2 due to 8V



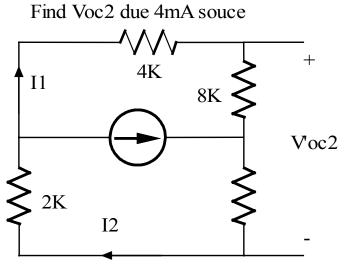
Voc2 = 8[(8+2)/(8+2+4+2)]

Voc2 = 5V

Find Rth2



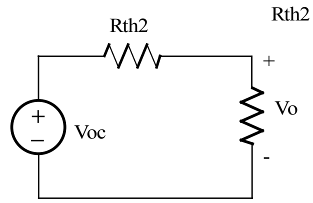
Find Voc2 due 4mA souce



I1 = -4m(8+2)/(2+2+8+4)

I1 = - 1mA

Rth2 = 6K||10K = 3.75K



Voc = Voc2 + V'oc2
Voc = 3V

I2 = 4m [(4 + 8) / (4 + 8 + 2 + 20] = 3mA

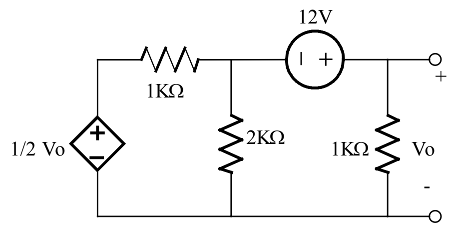
V'oc2 = 8K I1 + 2K I2 = - 2V

Vo = Voc[4K/(Rth + 4K)]

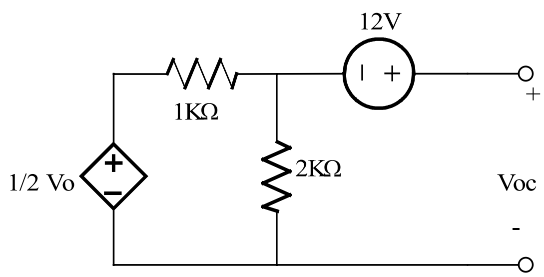
Vo = 1.55V

Problem 4.48

In the network shown find Vo using Thevenin's Theorem.



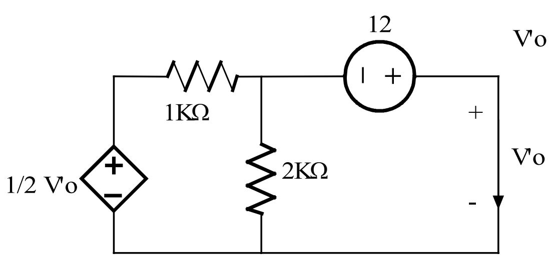
Suggested Solution

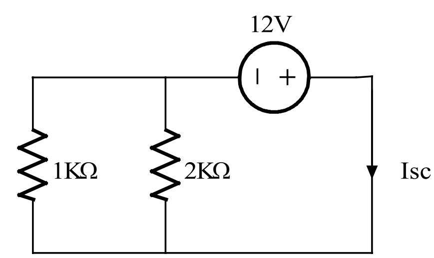


Voc = 12 + Voc/2 [2K / (1K 5K)] = 12 + Voc/3

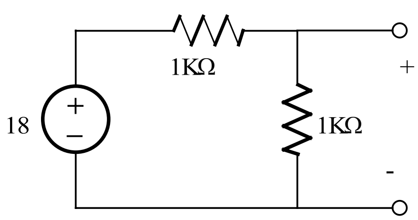
Voc = 18V

V'o = 0V





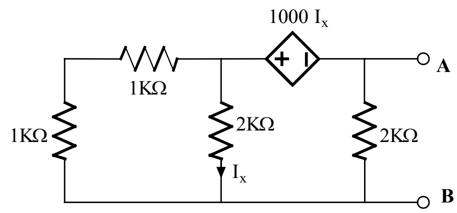
Isc = 12 / (1K || 2K) = 18mA , Rth = Voc/Isc = 1KΩ



Vo = 9V

Problem 4.49

Find the Thevenin equivalent of the network shown at the terminals A-B.



Suggested Solution

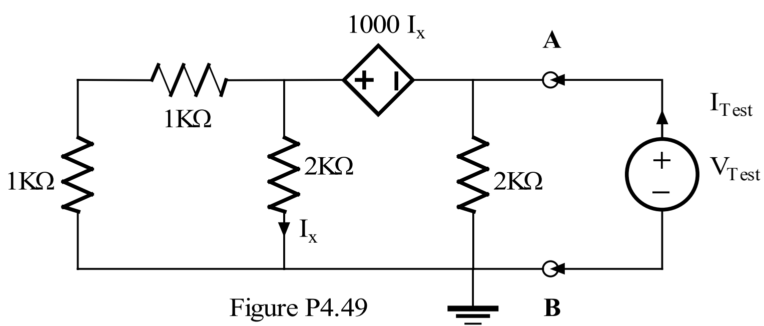


Figure P4.49

Apply a test source, V_{Test}

$R_{AB} = V_{Test} / I_{Test}$

$Ix = \frac{Vx}{2K} \qquad Vx - V_{Test} = 1000Ix = \frac{Vx}{2}$

$So, \qquad Vx = 2V_{Test}$

At the reference node,

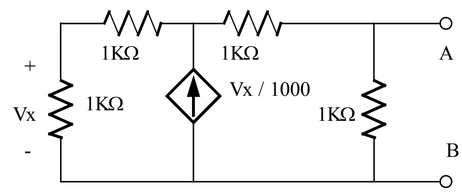
$I_{Test} = \frac{V_{Test}}{2K} + \frac{Vx}{2K} + \frac{Vx}{1K+1K} = \frac{1}{2K}[V_{Test} + 2Vx] = \frac{1}{2K}[5V_{Test}]$

$R_{AB} = \frac{V_{Test}}{I_{Test}} = \frac{2K}{5} = 400\Omega$

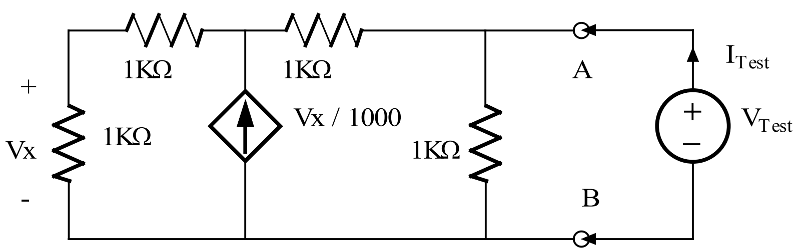
$\boxed{R_{AB} = 400\Omega}$

Problem 4.50

Find the Thevenin equivalent of the network shown at the terminals A-B.



Suggested Solution



Apply a test source,

V_{Test}, $R_{AB} = V_{Test} / I_{Test}$

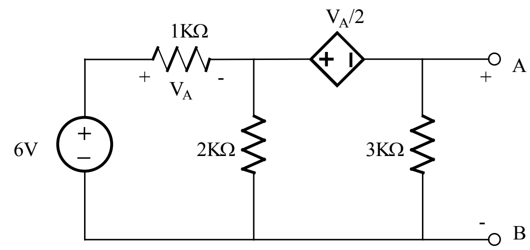
$Vx = Vy[\frac{1K}{1K+1K}] \Rightarrow Vx = \frac{Vy}{2}$

At Vy: $\frac{Vx}{1K} = \frac{Vy}{2K} + \frac{V_{Test} - Vy}{2K} \Rightarrow$ yields $Vy = V_{Test}$

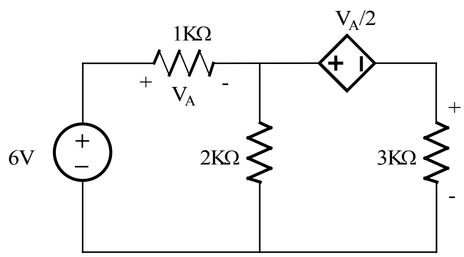
At V_{Test}: $I_{Test} = \frac{V_{Test}}{1K} + \frac{V_{Test} - Vy}{2K} \Rightarrow \boxed{\frac{V_{Test}}{I_{Test}} = R_{AB} = 1K\Omega}$

Problem 4.51

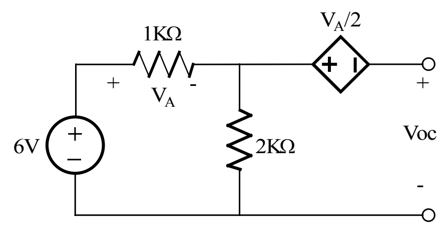
Find Vo in the network shown using Thevenin's Theorem.



Suggested Solution



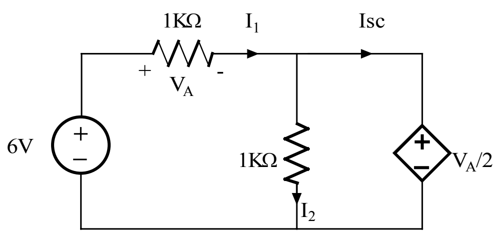
Find Voc



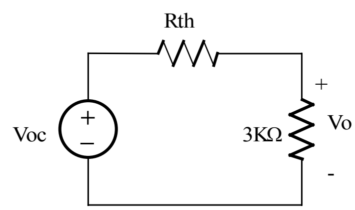
$V_A = 6\ [1K / (2K{+}1K)\ ]$

$V_A = 2V$

$Voc = -\ V_A/2 - V_A + 6 = 3V$



$V_A = 6 - V_A/2 \ \Rightarrow V_A = 4V$
$I1 = V_A / 1K = 4mA,\ \ I2 = (V_A/2)/2K = 1mA$
$Isc = I_1 - I_2 = 3mA$



Rth = Voc / Isc

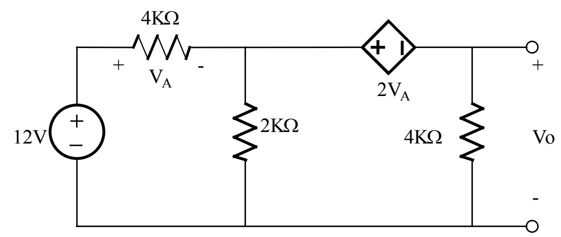
Rth = 1KΩ

$Vo = Voc[3K/(3K + Rth)] = 2.25V$

Vo = 2.25V

Problem 4.52

Find Vo in the network shown using Thevenin's Theorem.



Suggested Solution

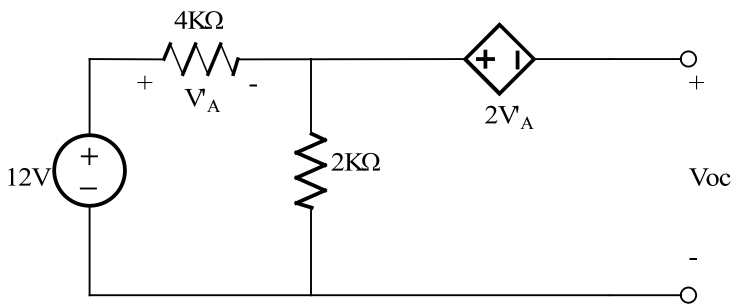
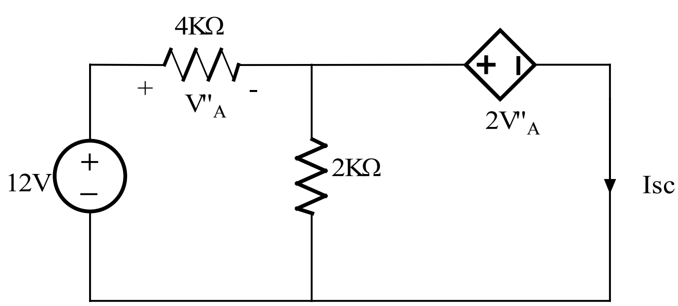


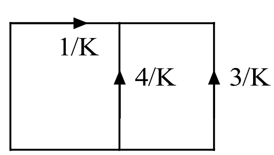
Figure P4.52

Using voltage division
$V'_A = 12(4K/6K) = 8V$

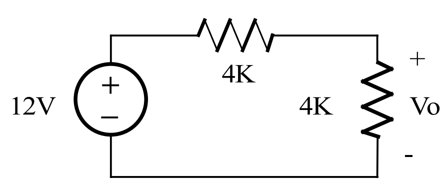


KVL outer loop
$-12 + V''_A + 2V''_A = 0$
$V''_A = 4V$

If $V''_A = 4V$



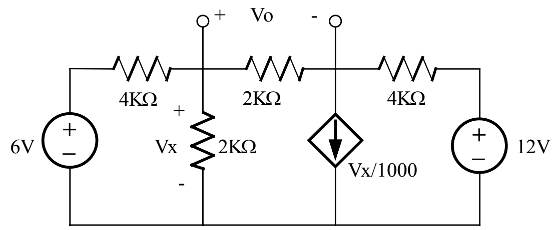
Isc = - 3mA and Rth = Voc/Isc = (-12) / (-3m) = 4KΩ



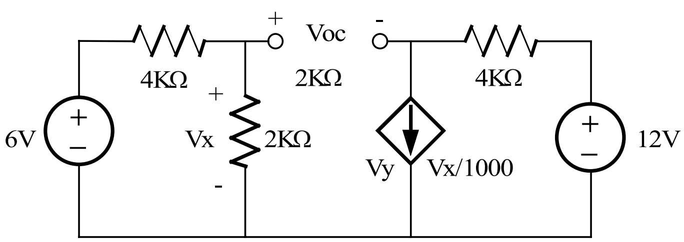
Vo = - 6V

Problem 4.53

Find Vo in the circuit shown using Thevenin's Theorem.



Suggested Solution

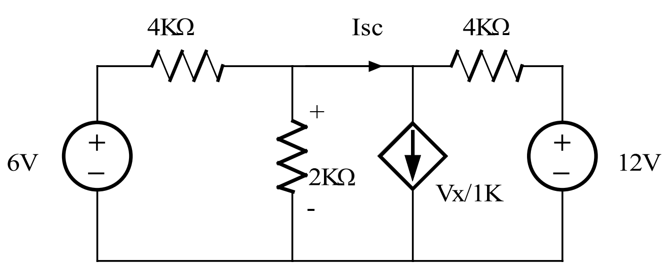


$Vx = 6\ [\ 2K / (4K + 2K)\ ] = 2V$

$Vy = 12 - 4K\ (\ Vx/1000\ ) = 4V$

$Voc = Vx + Vy = -2V$

Find Isc

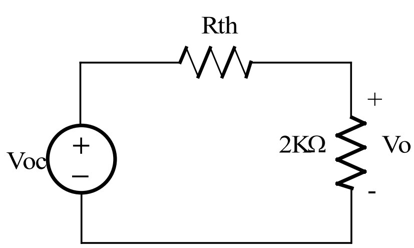


$Isc = Vx/1K + (Vx - 12) / 4K$

$(6 - Vx)/4K = Vx / 2K + Isc$

$Vx = 9/4\ V$

$Isc = -0.19\ mA$



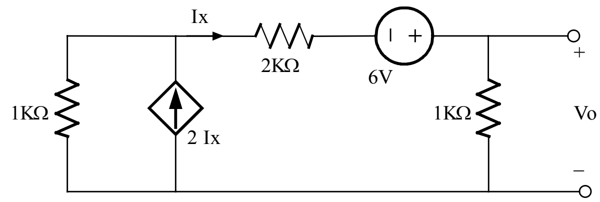
$Rth = Voc / Isc = 10.67\ K\Omega$

$Vo = Voc[\ 2K / (\ 2K + Rth)\ ]$

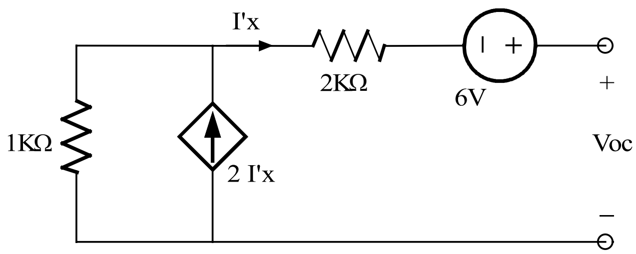
Vo = - 0.32 V

Problem 4.54

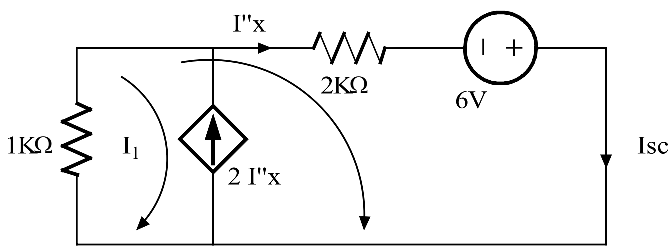
Use Thevenin's Theorem to find Vo in the circuit shown.



Suggested Solution



I'x = 0 , Voc = 6V

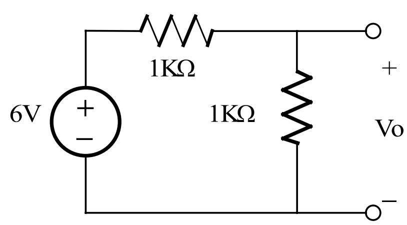


I''x = Isc, I_1 = - 2 Isc

2K Isc + 1K Isc +1K I_1 - 6 = 0

Isc = 6mA

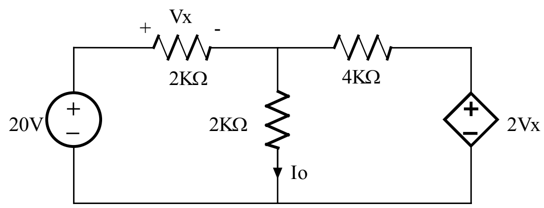
Rth = Voc / Isc = 1KΩ



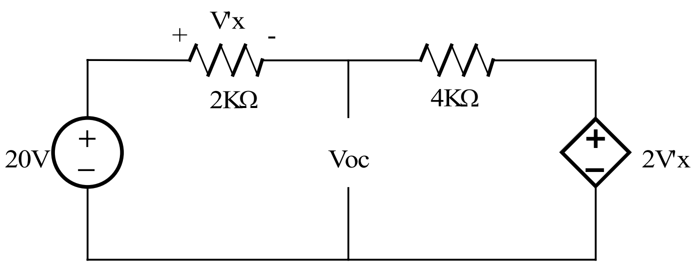
Vo = 3V

Problem 4.55

Use Thevenin's Theorem to Find Io in the circuit shown.



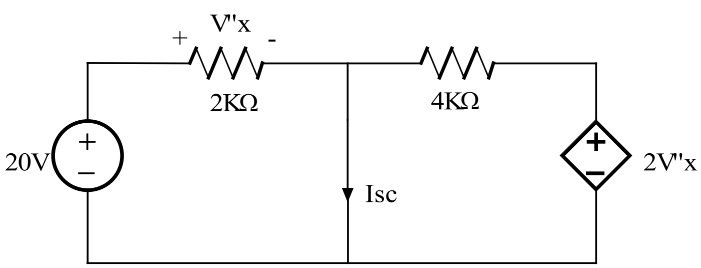
Suggested Solution



$- 20 + 2K\,I + 4K\,I + 2\,Vx = 0$
$V'x = 2K\,(I)$

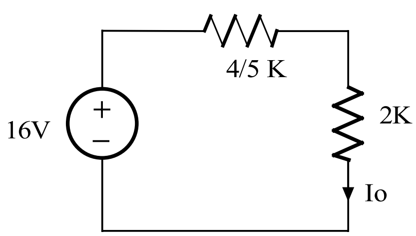
$I = 2mA$

$Voc = 20 - 2m\,(2K) = 16V$



$V''x = 20$
$Isc = 20 / 2K + 40 / 4K = 20\ mA$

$Rth = Voc / Isc = 4/5\ K\Omega$

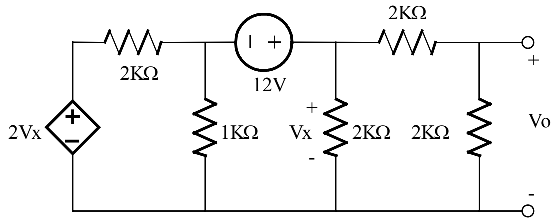


$Io = 16 / (2 + 4/5)K = 40 / 7K = 5.71\ mA$

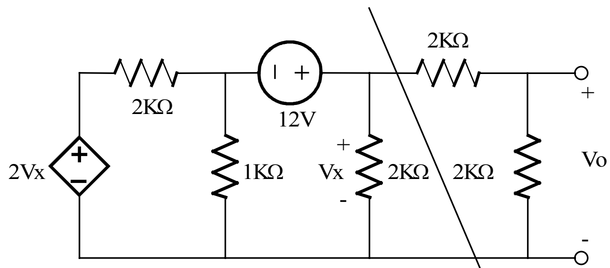
Io = 5.71 mA

Problem 4.56

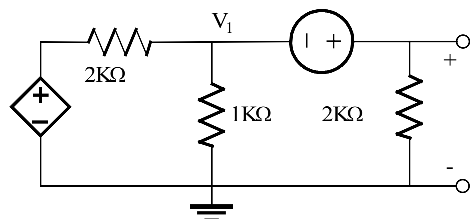
Find Vo in the network shown using Thevenin's Theorem.



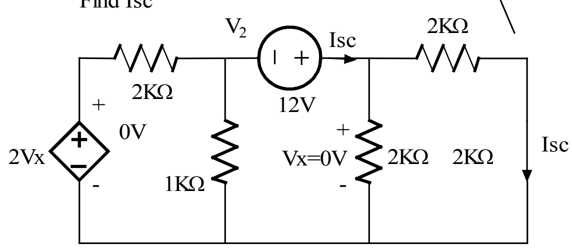
Suggested Solution



Find Voc

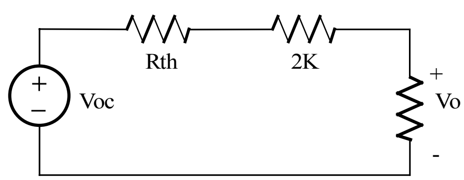


Find Isc



$(Vx - V_1)/2K = V_1/1K + Vx/2K$
yields,
$Vx = Voc = 18$ V

Now, $Vx = 0!$ and $V_2 = -12V$
$V_2/2K + V_2/1K + Isc = 0$, $Isc = 18$ mA



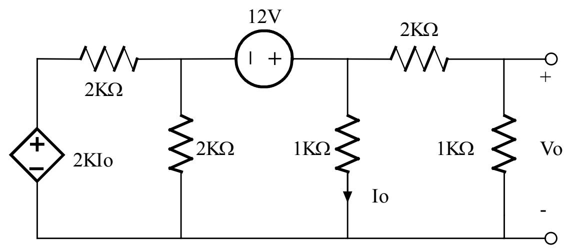
$Rth = Voc/Isc = 1K$

$Vo = Voc\ [\ 2K / (\ 2K + 2K + Rth)] = 7.2V$

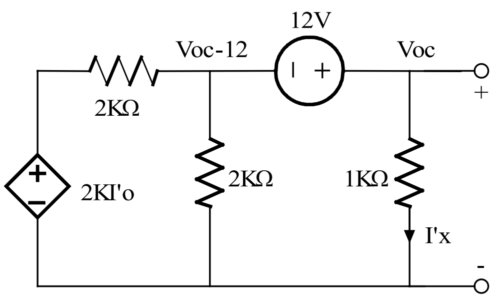
Vo = 7.2V

Problem 4.57

Use Thevenin's Theorem to find Vo in the network shown.



Suggested Solution

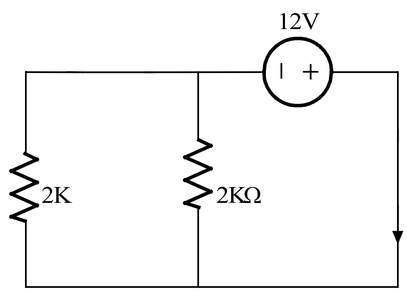


Supernode - KCL

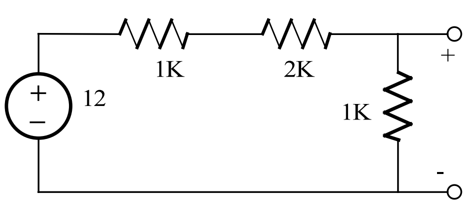
(Voc - 12 - 2KIx) / 2K + (Voc - 12) / 2K + Voc / 1K = 0

I'x = Voc / 1K   Then Voc = 12 V

1K Resistor is shorted
I''x = 0



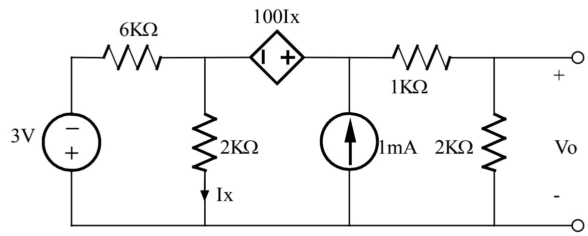
Isc = 12mA   and   Rth = Voc / Isc = 1KΩ



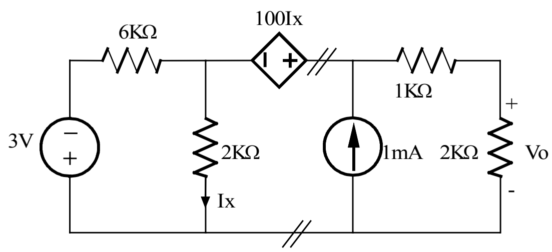
Vo = 12(1K/4K) = 3V

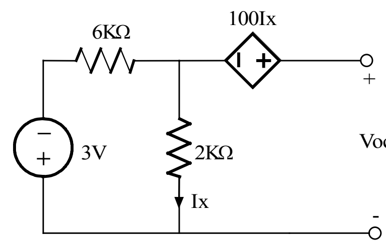
Problem 4.58

Find Vo in the network shown using Thevenin's Theorem.

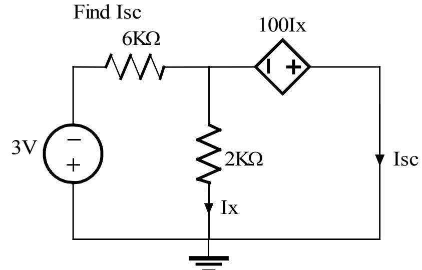


Suggested Solution





Find Isc

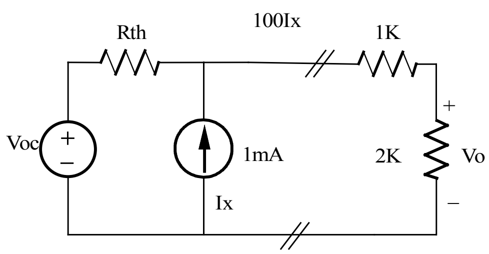


Ix = - 3/8K = - 0.38 mA

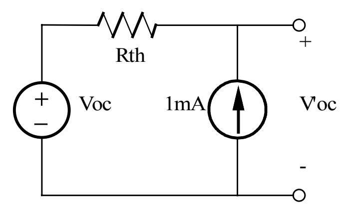
3 + (6K)Ix - (1K)Ix + Voc = 0

Voc = - 1.13V
Rth = Voc / Isc = 2.25 KΩ

Vx = 2000Ix
Vx = - 1000 Ix
Vx = 0 and Ix = 0A
Now, -3 / 6K = Isc => Isc = -0.5mA

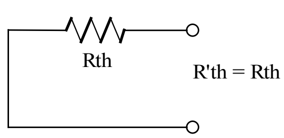


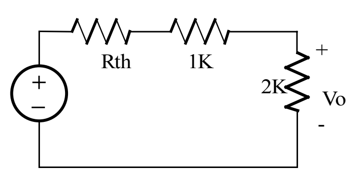
Find Voc



Voc = -1.13V, Rth = 2.25K

V'oc = (1m)Rth + Voc
V'oc = 1.13V



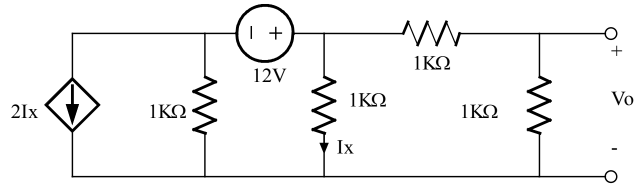


V'oc = V'oc[2K / (2K+1K+R'th)]

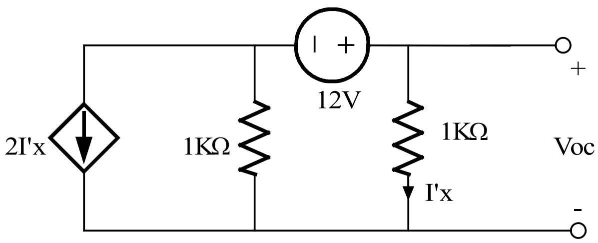
Vo = 0.43V

Problem 4.59

Use Thevenin's Theorem to Find Vo in the circuit shown.



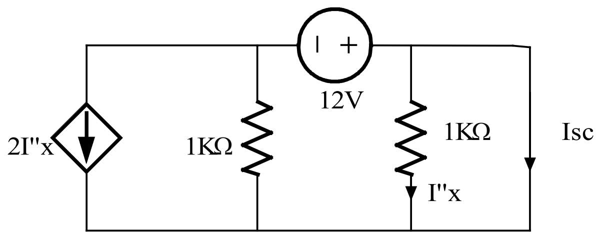
Suggested Solution



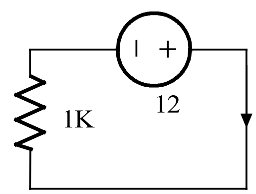
(Voc - 12) / 1K + 2I'x + Voc/1K = 0

I'x = Voc / 1K

Voc = 3V

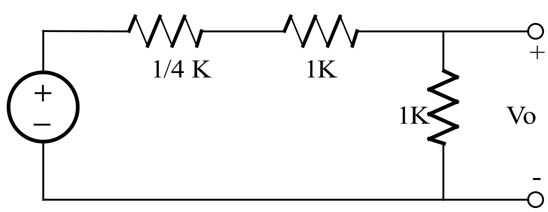


I''x = 0, The network is reduced to



Isc = 12mA

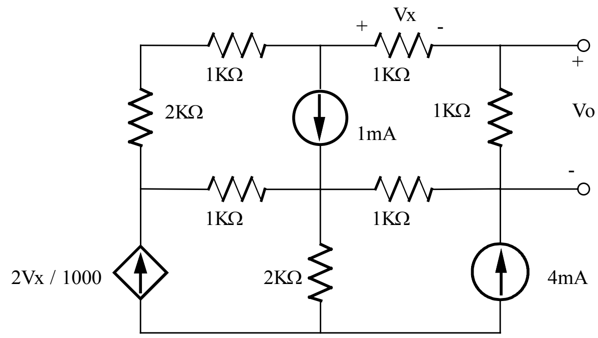
Rth = Voc / Isc = 3 / 12m = 0.25 KΩ



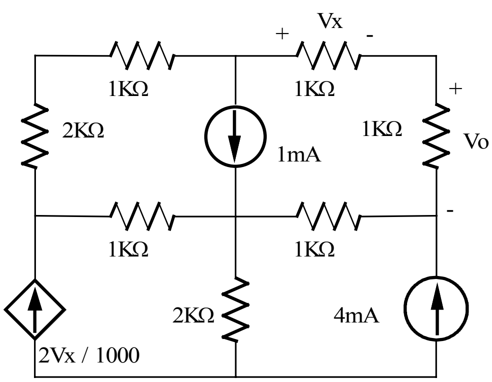
Vo = [3 / (2 + 1/4)K] 1K = 4/3 V

Vo = 4/3 V

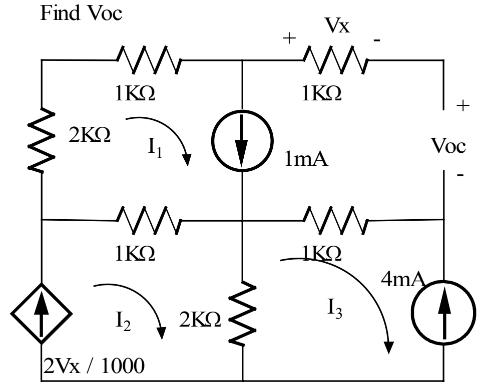
Problem 4.60



Suggested Solution

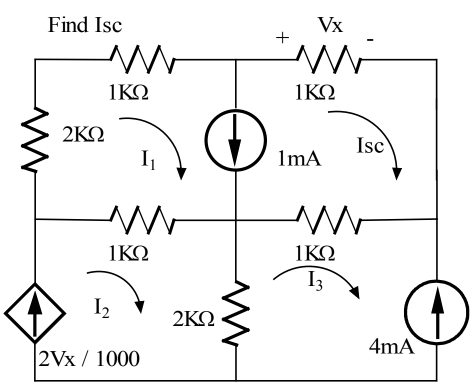


Find Voc

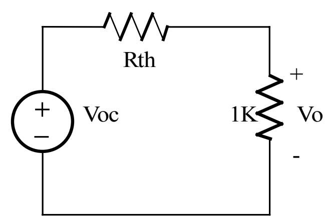


$V_x = 0$
$I_1 = 1mA$
$I_3 = -4mA$
$I_2 = 0$

Find Isc



$(3K)I_1 + V_{oc} - (1K)\, I_3 + 1K\,(I_1 - I_2) = 0$
$V_{oc} = -8V$



$R_{th} = V_{oc} / I_{sc}$

$R_{th} = 8K$

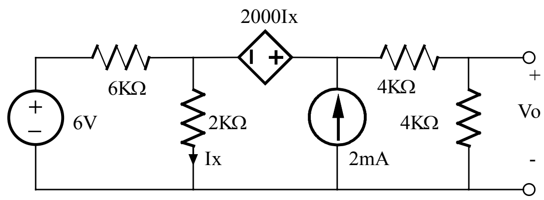
$V_o = V_{oc}\,[\,1K / (1K + R_{th})\,]$

$V_o = -0.89$ V

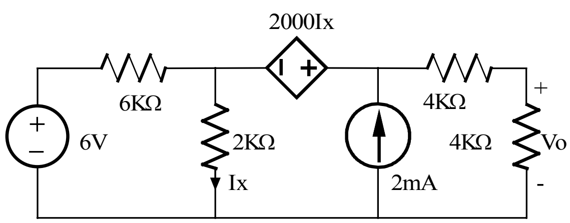
$I_1 - I_{sc} = 1mA$, $I_3 = -4$ mA
$V_x = -I_{sc}\,(1K)$, $I_2 = 2V_x / 1K$
$3KI_1 + (1K)\, I_{sc} + 1K\,(I_{sc} - I_3) + 1K\,(I_1 - I_2) = 0$
yields, $I_{sc} = -1mA$

Problem 4.61

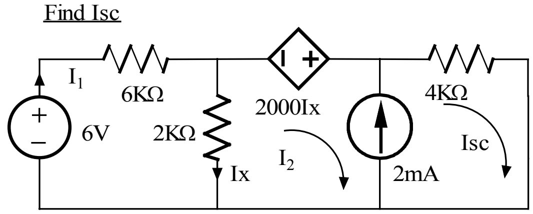
Use Norton's Theorem to find Vo in the network shown.

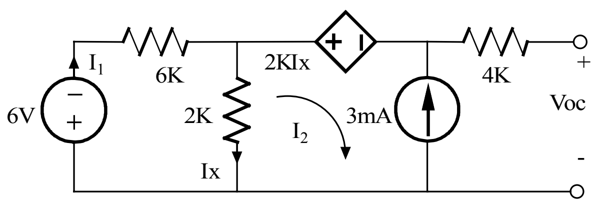


Suggested Solution



Find Isc



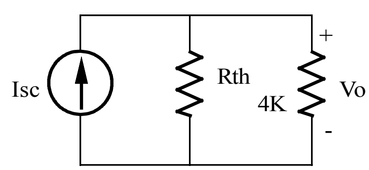


$6 + 6K\,I_1 + 2K(I_1 - I_2) = 0$
$2K\,(I_2 - I_1) - 2K\,Ix + 4K\,Isc = 0$
$Ix = I_1 - I_2 \quad Isc - I_2 = 3mA$
yields: $Isc = 0.86mA$

Same equations as those used to find Isc except Isc -> 0. Also,

$Voc = 2K\,Ix + 2K\,Ix$

yields: $Voc = 6V$

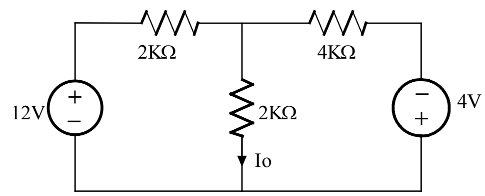


$Rth = Voc\ /\ Isc = 7K$
$Vo = Isc[\ Rth\ ||\ 4K\ ]$

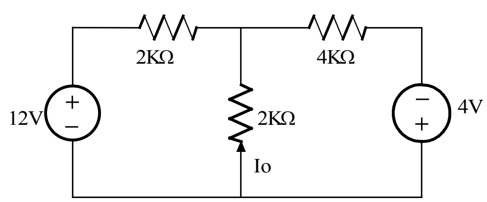
$Vo = 2.18V$

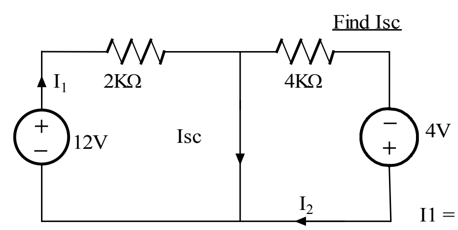
Problem 4.62

Find Io in the network shown using Norton's Theorem

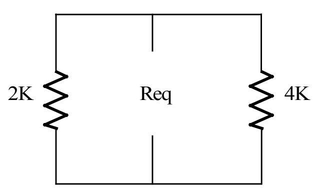


Suggested Solution



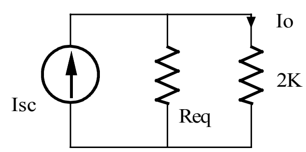


$I1 = 12 / 2K = 6mA$
$I2 = 4 / 4K = 1mA$
$Isc = I1 - I2 = 5mA$



$Req = 2K \,||\, 4K$

$Req = 1.33\ K\Omega$

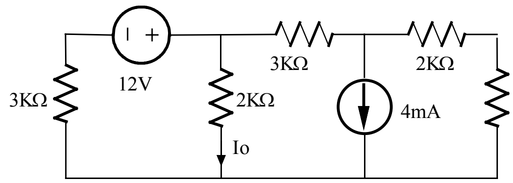


$Io = Isc[\ Req/(\ Req + 2K)\ ]$

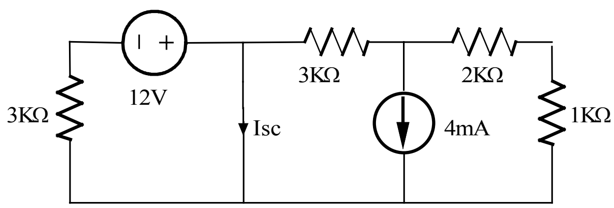
Io = 2mA

Problem 4.63

Use Norton's Theorem to find Io in the circuit shown.

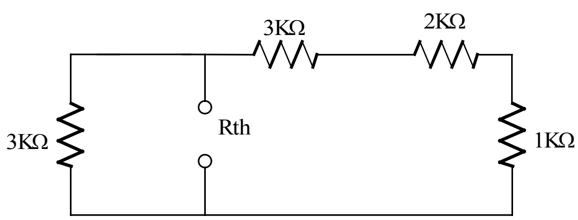


Suggested Solution

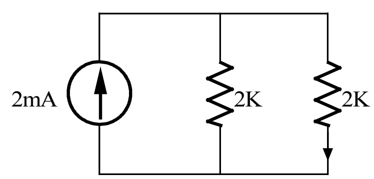


Isc = 12 / 3K - 4m[3K / (3K + 3K)]

Isc = 2mA



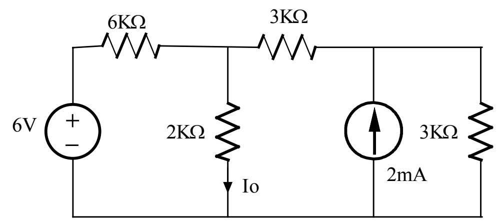
Rth = 3K || 6K = 2KΩ



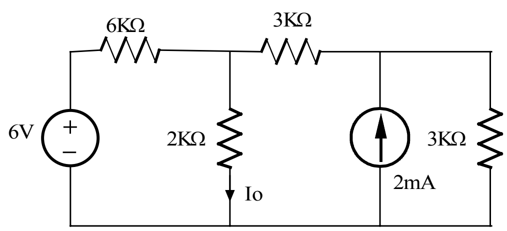
Io = 1mA

Problem 4.64

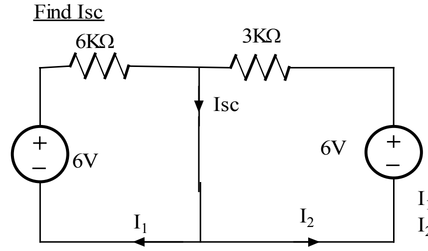
Find Io in the network shown using Norton's Theorem.



Suggested Solution



Find Isc

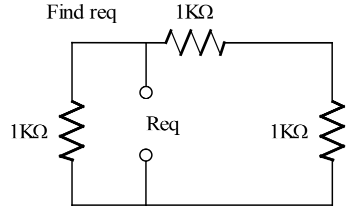


$I_1 = 6 / 6K = 1mA$

$I_2 = 6/ 6K = 1mA$

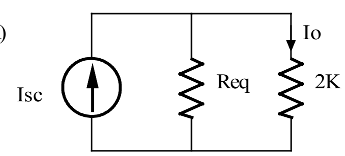
$Isc = I_1 + I_2 = 2mA$

Find req



Req = 6K || (3K + 3K)

Req = 3KΩ

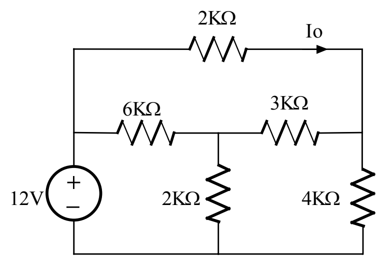


Io = Isc [Req / (Req + 2K)]

Io = 1.2 mA

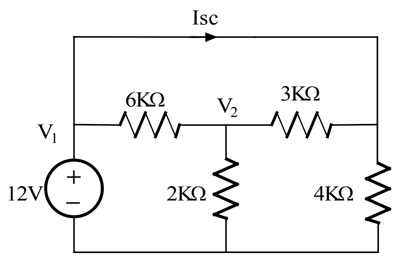
Problem 4.65

Find Io in the network shown using Norton's Theorem.



Suggested Solution

Find Isc

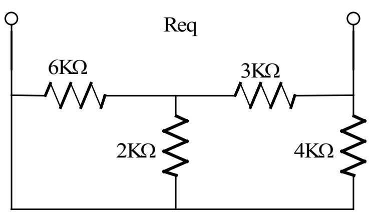


$V_1 = 12V$

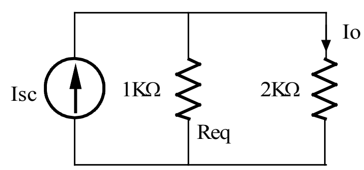
$(V_1 - V_2) / 6K + (V_1 - V_2) / 3K = V_2 / 2K$

yields, $V_2 = 6V$

$Isc = (V_1 - V_2) / 3K + V_1 / 4K = 5mA$



Req = 4K || [3K + (6K || 2K)]
Req = 2.12 K

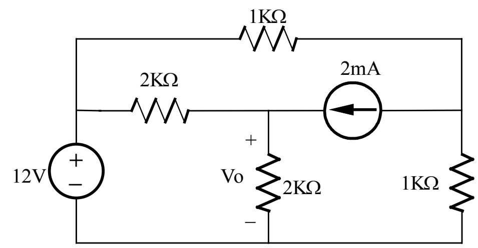


Io = Isc[Req / (Req + 2K)]

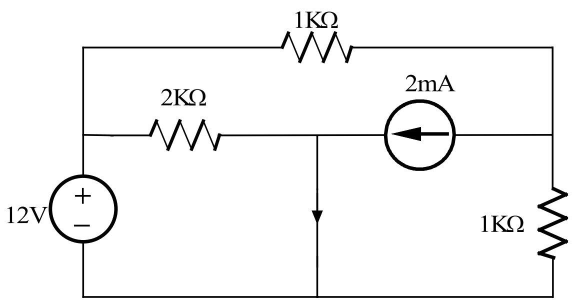
Io = 2.57mA

Problem 4.66

Use Norton's Theorem to find Vo in the network shown.

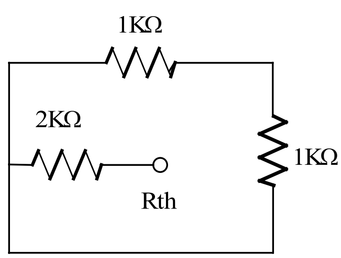


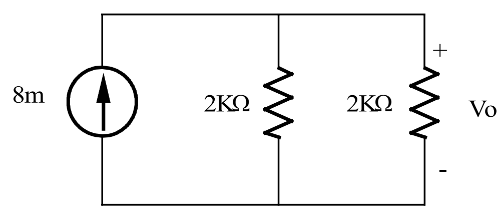
Suggested Solution



Isc = 12 / 2K + 2 / 1K = 8mA

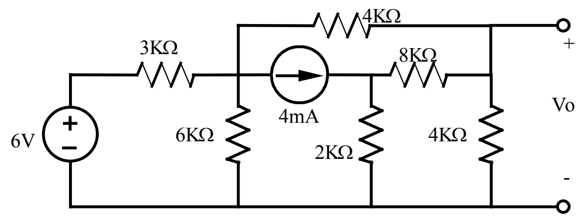
Rth = 2K



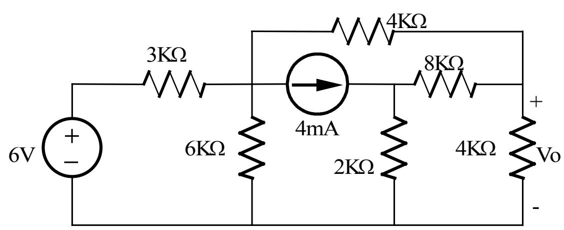


Vo = 8V

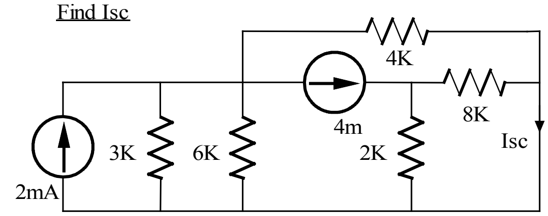
Problem 4.67



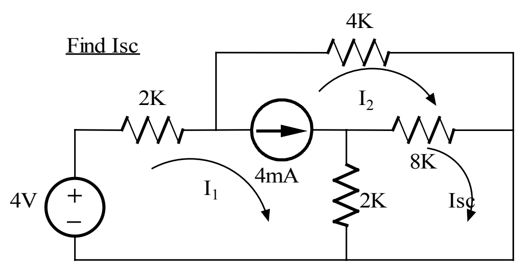
Suggested Solution



Find Isc

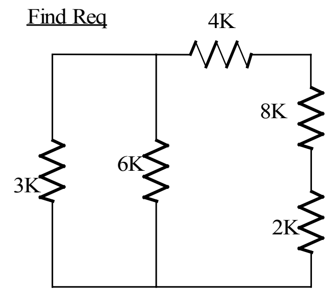


Find Isc

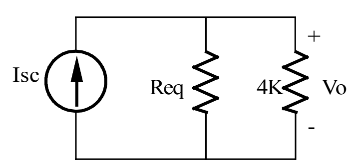


$I_1 - I_2 = 4mA$, $2K(Isc - I_1) + 8K(Isc - I_2) = 0$
$4 = (2K)\, I_1 + (4K)\, I_2$
yields: Isc = 0.1333 mA

Find Req



Req = (8K + 2K) || [4K + (6K || 3K)]
Req = 3.75 K

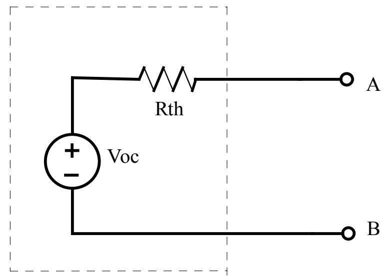


Vo = Isc [Req || 4K] = 258mV

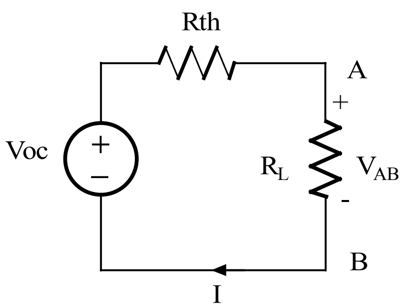
Vo = 258 mV

Problem 4.68

Given the linear circuit shown, it is known that when a 2-KΩ load is connected to the terminals A-B, the load current is 10mA. If a 10-KΩ load is connected to the terminals the load current is 6mA. Find the current in a 20KΩ load.



Suggested Solution



$(Rth + R_L)\ I = Voc$

if $R_L = 2K\Omega$, $I = 10mA \Rightarrow Voc = 20 + 0.01Rth$

if $R_L = 10K\Omega$, $I = 6mA \Rightarrow Voc = 60 + 0.006Rth$

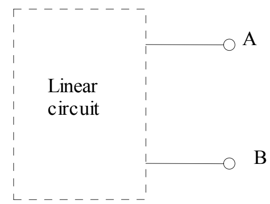
yield $Voc = 120V$ and $Rth = 10K\Omega$

If $RL = 20K\Omega$, $I = Voc / (R_L + Rth)$

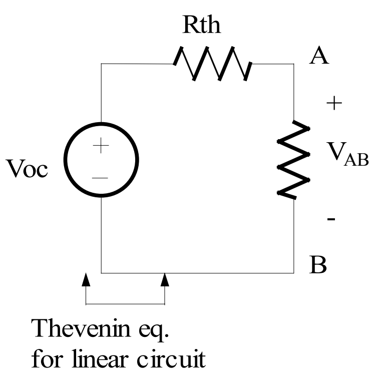
$$I = 4\ mA$$

Problem 4.69

If an 8-KΩ load is connected to the terminals of the network shown, V_{AB} = 16V. If a 2-KΩ load is connected to the terminals V_{AB} = 8V. Find V_{AB} if a 20KΩ load is connected to the terminals.



Suggested Solution



$V_{AB} = Voc[\, R_L / (R_L + Rth)\,] \Rightarrow Voc = V_{AB}\,[\,1 + Rth / R_L\,]$

If $R_L = 8K\Omega$, $V_{AB} = 16V \Rightarrow Voc = 16\,[\,1 + Rth / 8K\,]$

If $R_L = 2K\Omega$, $V_{AB} = 8V \Rightarrow Voc = 8\,[\,1 + Rth / 2K\,]$

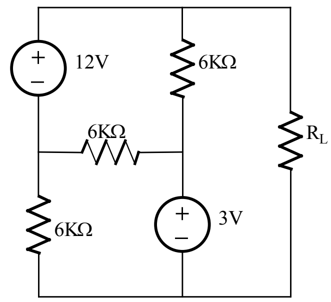
yield: $Rth = 4K\Omega$ and $Voc = 24V$

If $RL = 20K\Omega$, $V_{AB} = 24\,[\,20 / (20 + 4)\,]$

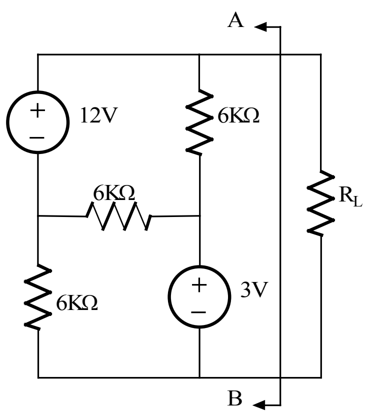
$$V_{AB} = 20\ V$$

Problem 4.70

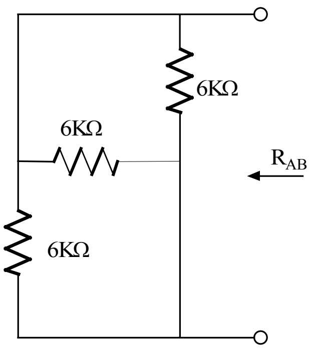
Find R_L for maximum power transfer and the maximum power that can be transferred in the network shown.



Suggested Solution



Find R_{AB}

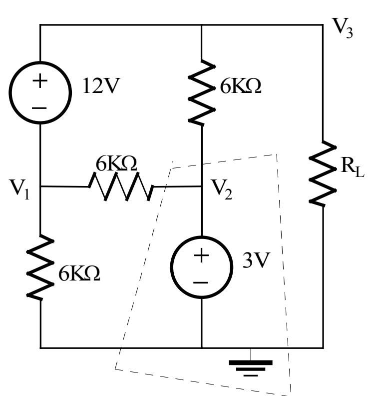


All 3 Resistors are attached to both node A and node B, so,

$R_{AB} = 6K \| 6K \| 6K$

$R_{AB} = 2K\Omega$

Find maximum load tranfer



$V_3 - V_1 = 12V$

$V_2 = 3V$

At supernode:

$(V_3 - V_2) / 6K + (V_1 - V_2) / 6K + V_3 / 2K + V_1 / 6K = 0$

Yields:

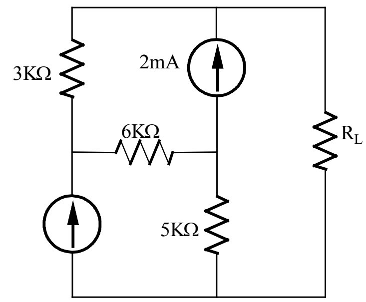
$V_3 = 5V$

$P_L = V^2_3 / R_{AB}$

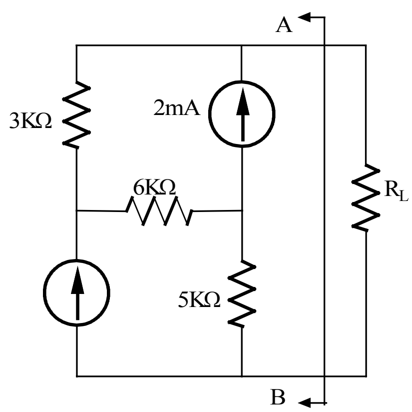
PL = 12.5 mW

Problem 4.71

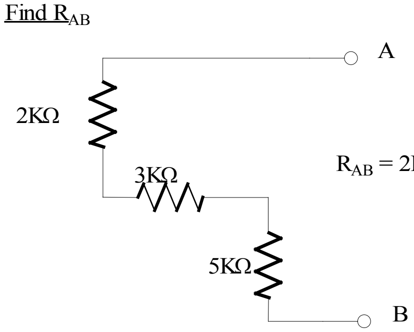
Find R_L for maximum power transfer and the maximum power that can be transferred in the network shown.



Suggested Solution

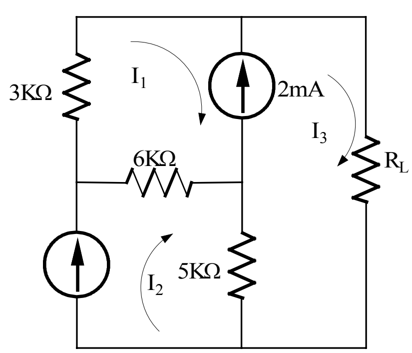


Find R_{AB}



$R_{AB} = 2K + 3K + 5K = 10K\Omega$

Find maximum load tranfer



$I_3 - I_1 = 2mA \quad I_2 = 1mA$

$(2K)\ I_1 + (10K)\ I_3 + 5K\ (I_3 - I_2) + 3K\ (I_1 - I_2) = 0$

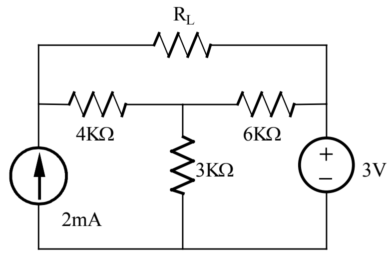
yields: I3 = 0.9mA

$P_L = I^2_3\ R_{AB}$

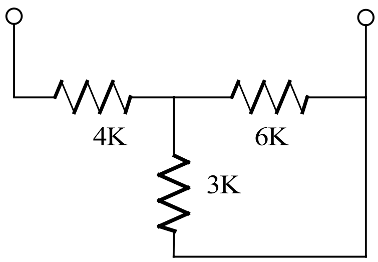
PL = 8.1 mW

Problem 4.72

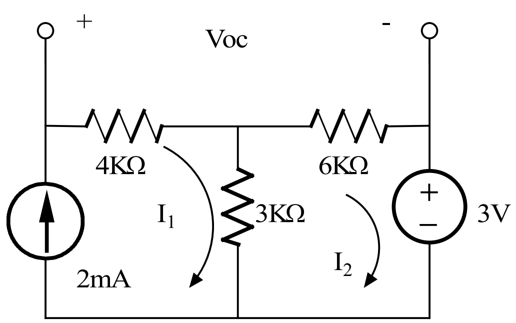
In the network shown find R_L for maximum power transfer and the maximum power that can be transferred to this load.



Suggested Solution



$Rth = 4K + 3K \| 6K = 6K\Omega$

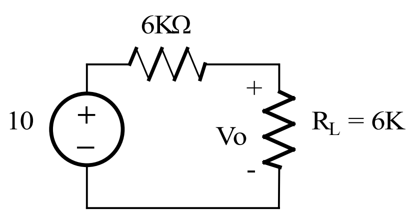


$I_1 = 2m$

$3K(I_2 - I_1) + 6K I_2 + 3 = 0$

$I_2 = 1/3$ mA

$Voc = 4K(2m) + 6K(1/3\ m) = 10V$



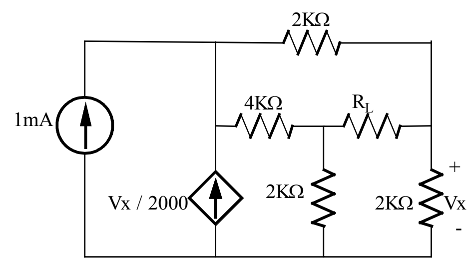
Vo = 5V

$PL = V^2o / R_L$

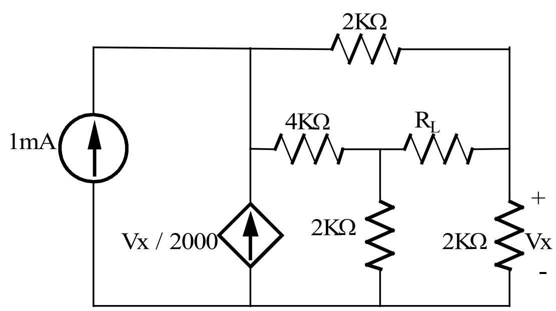
$P_L = 25/6$ mW

Problem 4.73

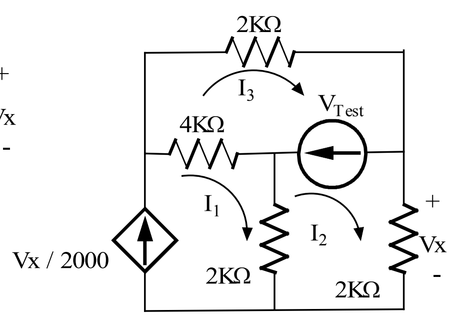
Find R_L for maximum power transfer and the maximum power that can be transferred in the network shown.



Suggested Solution

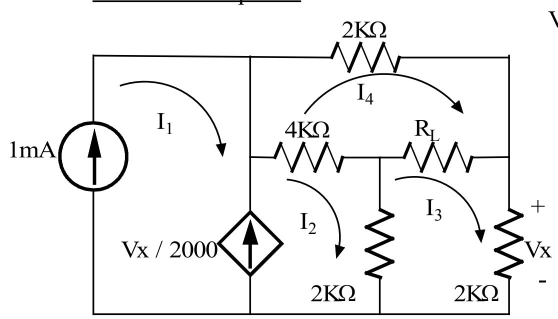


To find Req, replace RL with a 1mA current source,
$Req = V_{Test} / 1mA$



$I_1 = Vx / 2K$, $I_2 = Vx / 2K$
$I_3 - I_2 = 1mA$
$(2K)I_3 + (2K)I_2 + 2K(I_2 - I_1) + 4K(I_3 - I_1) = 0$
$V_{Test} = 4K(I_3 - I_1) + (2K)I_3$
yields: $V_{Test} = 3V$
$Req = V_{Test} / 1mA = 3K$

<u>Find max load power</u>



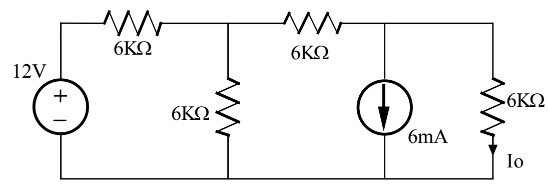
$I_1 = 1mA$, $I_2 - I_1 = Vx / 2K$, $I_3 = Vx / 2K$
$4K (I_4 - I_2) + 2K I_4 + 3K (I_4 - I_3) = 0$
$2K I_3 + 2K (I_3 - I_2) + 3K (I_3 - I_4) = 0$

yields: $I_3 = 1.25mA$ and $I_4 = 1.42mA$ => $P_L = 3K (I_4 - I_3)^2$

$P_L = 83.3\ \mu W$

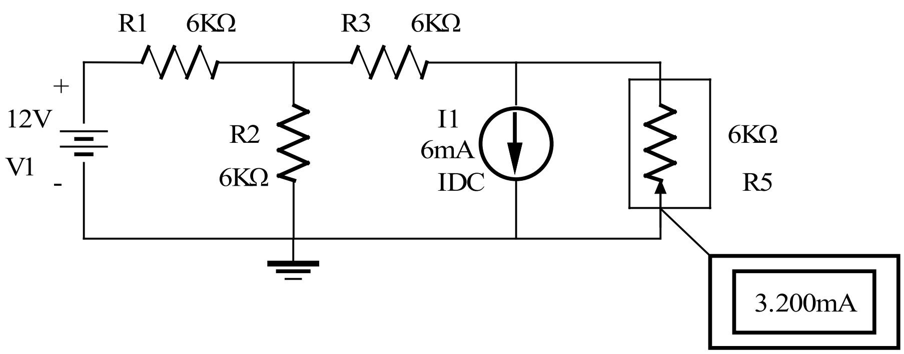
Problem 4.74

In the network shown find I_o using PSPICE.



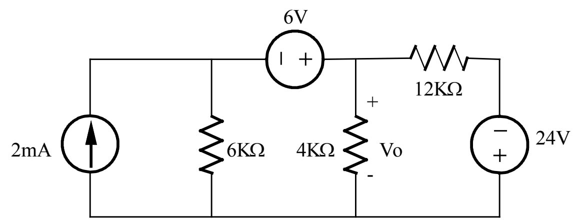
Suggested Solution

$I_o = 3.2$ mA.



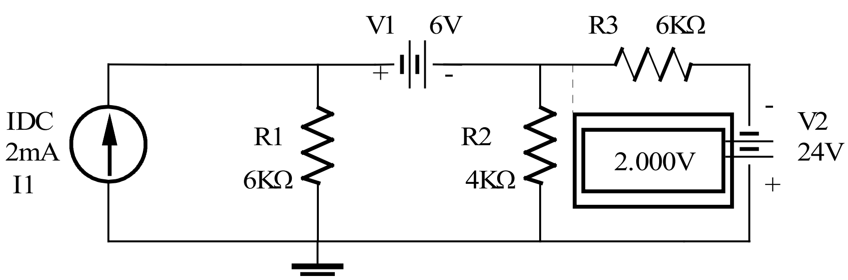
Problem 4.75

In the network shown determine V_o using PSPICE.



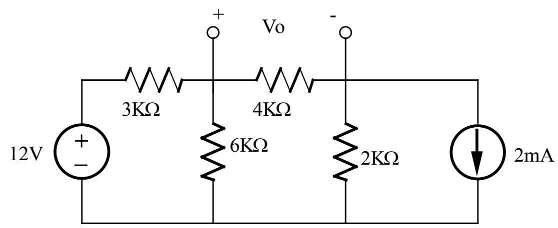
Suggested Solution

Vo = 2V



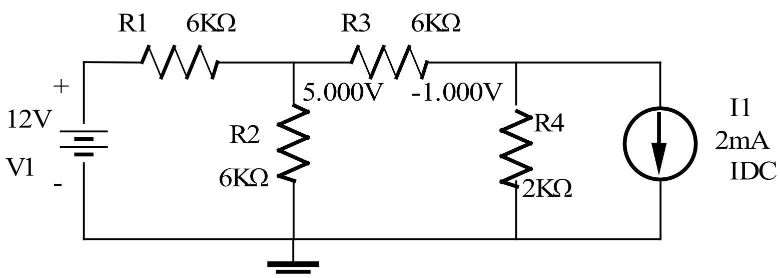
Problem 4.76

Find V_o in the network shown using PSPICE.



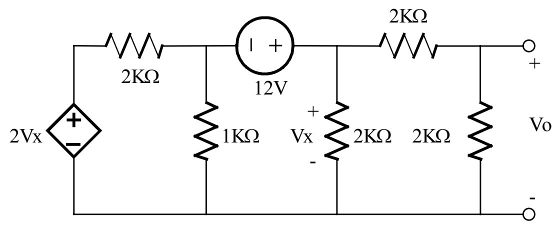
Suggested Solution

$V_o = 5 - (-1) = 6$ V.



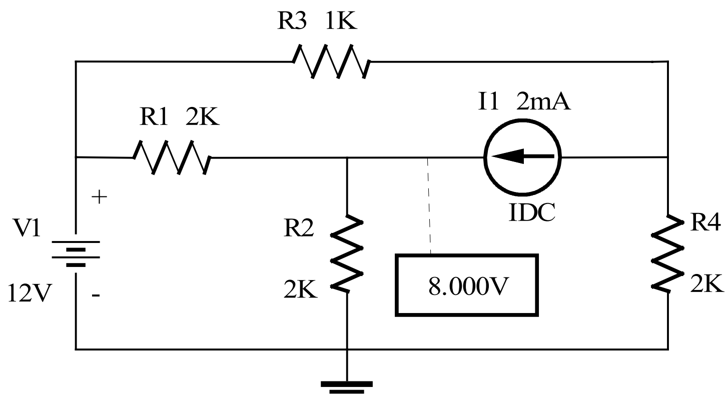
Problem 4.77

Find V_o in the network shown using PSPICE.



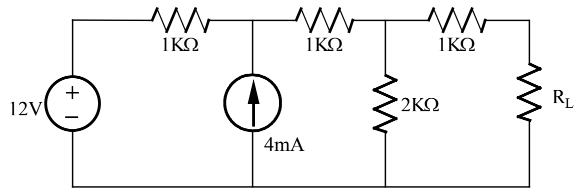
Suggested Solution

Vo = 8V

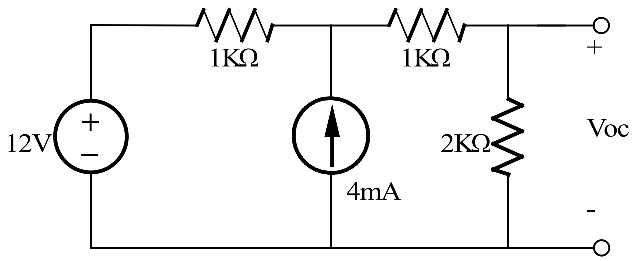


Problem 4FE-1

Determine the maximum power that can be delivered to the load R_L in the network shown.



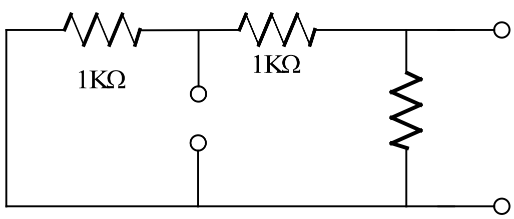
Suggested Solution



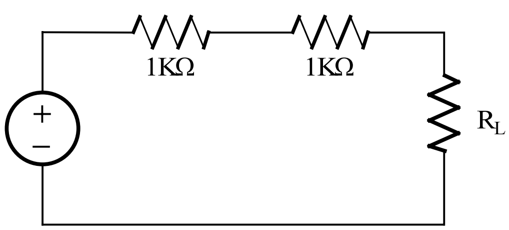
$(Vx - 12) / 1K - 4m + Vx / 3K = 0$

$Vx = 12V$

$Voc = Vx [2K / (2K + 1K)] = 8V$



$Rth = 2K \| 2K = 1K$



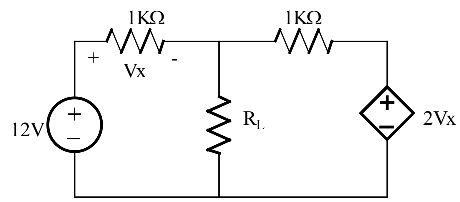
$R_L = 2K$

$P_{RL} = (8 / 4K)2 (2K)$

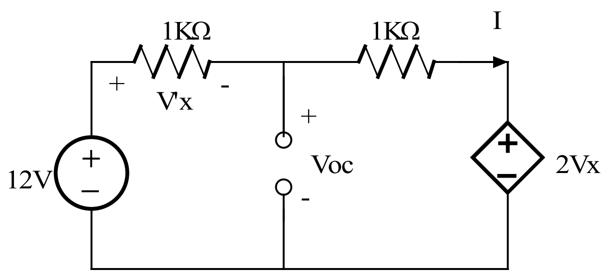
$P_{RL} = 8mW$

Problem 4FE-2

Find the value of the load R_L in the network shown that will achieve maximum power transfer, and determine the value of the maximum power.



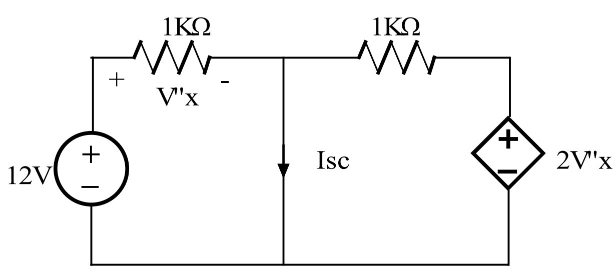
Suggested Solution



$- 12 + 2K\,I + 1K\,I + 2V'x = 0$

$V'x = 2K\,I$

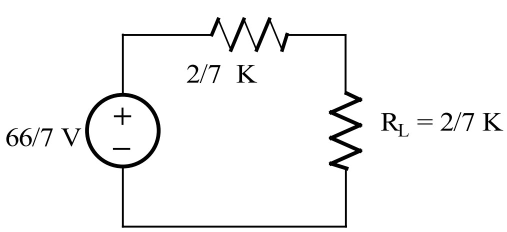
$Voc = 12 - 2K\,I = 66/7$ V



$Vx = 12$

$Isc = 12 / 2K + 24 / 1K = 30mA$

$Rth = Voc / Isc = 2/7$ K

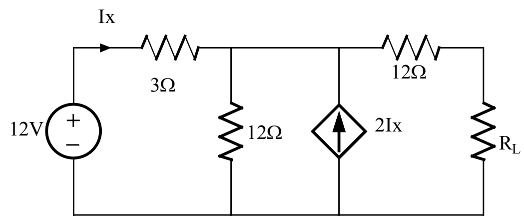


$P = [(60/7)/(4/7\ K)]^2\ (2/7\ K)$

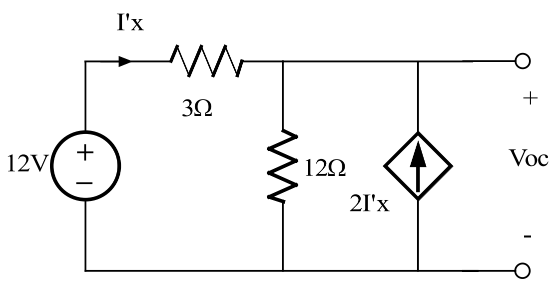
P = 64.3 mW

Problem 4FE-3

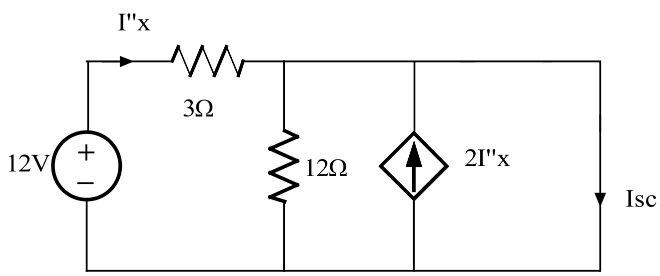
Find the value of R_L in the network shown for maximum power transfer to this load.



Suggested Solution



(Voc - 12) / 3 + Voc / 12 - 2 I'x = 0
I'x = (12 - Voc) / 3
Voc = 144/13 V



I"x = 12/3 = 4mA

Then

Isc = I"x + 2I"x = 12A

Rth = Voc / Isc = (144/13)/12 = 12/13 Ω

Then R_L = 12 + 12/13

RL = 12.92Ω for MPT

Problem 5.1

A 12µF capacitor has an accumulated charge of 480µC. Determine the voltage across the capacitor after 4 s.

Suggested Solution

$v(t_2) - v(t_1) = \frac{1}{C}\int_{t_1}^{t_2} i(t)dt$ $\quad C = 100\mu F \quad i(t) = 1mA = I$

$v(t_1) = 0.$ so, $v(t_2) = \frac{I}{C}(t_2 - t_1)$ where $t_2 - t_1 = 4\sec$

$$\boxed{v(t_2) = 40V}$$

Problem 5.2

A 12μF capacitor has an accumulated charge of 480μC. Determine the voltage across the capacitor.

Suggested Solution

$C = \frac{Q}{C} \quad Q = 480\mu C \qquad \boxed{C = 12\mu F} \qquad \boxed{V = 40V}$

Problem 5.3

A capacitor has an accumulated charge of 600μC with 5V across it. What is the value of capacitance?

Suggested Solution

$C = \frac{Q}{V}$ $\quad Q = 600\mu C$ $\quad \boxed{V = 5V}$ $\quad \boxed{C = 120\mu F}$

Problem 5.4

A 25- μ F capacitor initially charged to –10V is charged by a constant current of 2.5 μ A. Find the voltage across the capacitor after 2 ½ minutes.

Suggested Solution

$$V = \frac{1}{C}\int_0^t idt + v(0)$$

$$= \frac{1}{25x10^{-6}}\int_0^{150} 2.5x10^{-6}\,dt - 10$$

$$= \frac{1}{10}\int_0^{150} dt - 10 = \frac{1}{10}t\,\Big|_0^{150} - 10$$

$$\boxed{V = 15 - 10 = 5V}$$

Problem 5.5

The energy that is stored in a 25-μF capacitor is $w(t)=12\sin^2 377t$ J. Find the current in the capacitor.

Suggested Solution

$$w(t) = \tfrac{1}{2}Cv^2(t) = 12\sin^2 377t$$

$$C = 25\ \mu\text{F} \quad \Rightarrow \quad v^2(t) = \frac{2(12)}{25\times10^{-6}}\sin^2 377t = 9.6\times10^5 \sin^2 377t$$

$$v(t) = \pm 979.8\sin 377t\ \text{V}$$

$$i(t) = C\frac{dv(t)}{dt} = \pm 25\times10^{-6}(979.8)(377)\cos 377t = \pm 9.23\cos 377t\ \text{A}$$

Problem 5.6

An uncharged 10-uF capacitor is charged by the current I(t)=10cos377t mA. Find (a) the expression for the voltage across the capacitor and (b) the expression for the power.

Suggested Solution

$i(t) = 10\cos(377t)mA \qquad C = 10\mu F \qquad v(0) = 0V$

$a)$

$$v(t) = \tfrac{1}{C}\int_0^t i(t)dt = \tfrac{0.01}{C}\tfrac{1}{377}\sin(377t)\,|_0^t$$

$$\boxed{v(t) = 2.65\sin(377t)\ \ V}$$

$b)$

$$P(t) = v(t)i(t) = (2.65)(0.01)\sin(377t)\cos(377t)$$

but $\cos(x)\sin(x) = \tfrac{1}{2}\sin(2x)$ so,

$$\boxed{p(t) = 13.3\sin(754t)\ \ mW}$$

Problem 5.7

The voltage across a 100-uF capacitor is given by the expression v(t)=120sin(377t) V. Find (a)the current in the capacitor and (b) the expression for the energy stored in the element.

Suggested Solution

$$v(t) = 120\sin(377t)\text{ V}, \qquad C = 100\ \mu\text{F}$$

$$a)\ i(t) = C\frac{dv}{dt} = (100\times 10^{-6})(120)(377)\cos(377t) = 4.52\cos(377t)\text{ A}$$

$$b)\ w(t) = \frac{1}{2}Cv^2(t) = \frac{1}{2}\left(10^{-4}\right)\left(120\right)^2\sin^2\left(377t\right)$$

$$= 720\sin^2\left(377t\right)\text{ mJ}$$

$$= 360 - 360\cos 754t\text{ mJ}$$

Problem 5.8

A capacitor is charged by a constant current of 2mA and results in a voltage increase of 12V in a 10-s interval. What is the value of the capacitance?

Suggested Solution

$$v(t2) - v(t1) = \tfrac{1}{C}\int_{t1}^{t2} i(t)dt \qquad i(t) = 2mA = I$$

$$v(t_2) - v(t_1) = 12 = \tfrac{I}{C}(t_2 - t_1) = \tfrac{I}{C}(10)$$

$$C = (2m)(10)/12 = 1.67mF$$

$$\boxed{C = 1.67mF}$$

Problem 5.9

The current in a 100μF capacitor is shown in Figure P5.9. Determine the waveform for the voltage across the capacitor if it is initially uncharged.

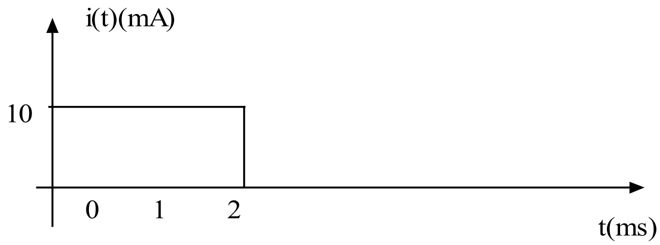
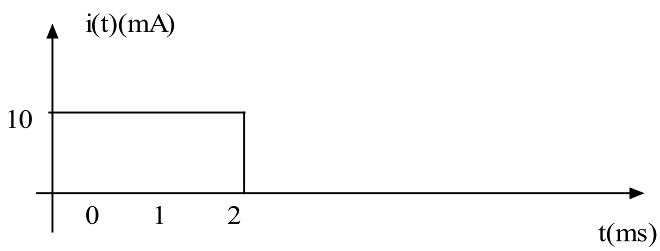


Figure P5.9

Suggested Solution

$C = 100\mu F \qquad v(t) = \frac{1}{C}\int i(t)dt$

Time(ms)	i(t)(mA)	v(t)(V)
$0 \le t \le 2$	10	$100t$
$t > 0$	0	$0 + v(2m) = 0.2V$



Problem 5.10

The voltage across a 100-μF capacitor is shown in Figure P5.10. Compute the waveform for the current in the capacitor

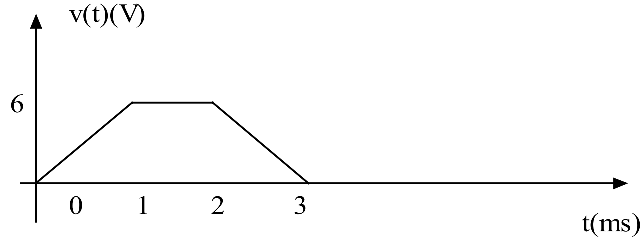


Figure P5.10

Suggested Solution

$C = 100\mu F \qquad i(t) = Cc$

Time(ms)	$\frac{dv}{dt}$(V/ms)	i(t)(mA)
$0 \le t \le 1$	6	600
$1 \le t \le 2$	0	0
$2 \le t \le 3$	-6	-600

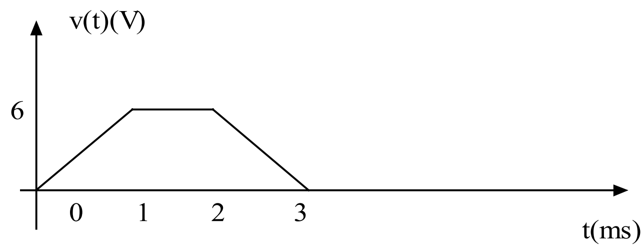


Figure P5.10

Problem 5.11

The voltage across a 6μF capacitor is shown in Figure P5.11. Compute the waveform for the current in the capacitor

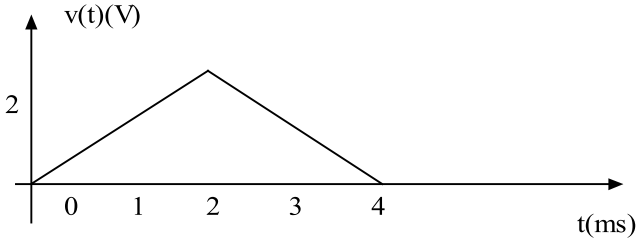
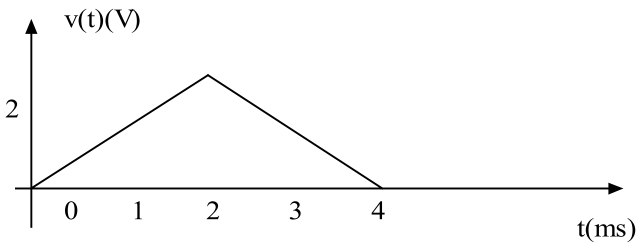


Figure P5.11

Suggested Solution

$C = 6\mu F \qquad i(t) = C\,\frac{dv(t)}{dt}$

Time (ms)	$\frac{dv}{dt}$(V/ms)	i(t) (mA)
$0 \le t \le 2$	61	6
$2 \le t \le 4$	-1	-6



Problem 5.12

The voltage across a 50-μF capacitor is shown in Figure P5.12. Determine the current waveform.

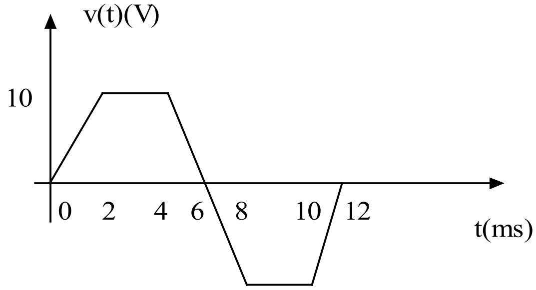
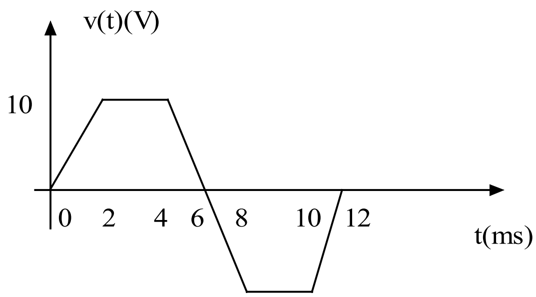


Figure P5.12

Suggested Solution

$C = 50\mu F \qquad i(t) = C\frac{dv}{dt}$

Time(ms)	$\frac{dv}{dt}$(V/ms)	i(t)(mA)
$0 \le t \le 2$	5	250
$2 \le t \le 4$	0	0
$4 \le t \le 8$	-5	-250
$8 \le t \le 10$	0	0
$10 \le t \le 12$	5	250
$t > 12$	0	0



Problem 5.13

The voltage across a 2-μF capacitor is given by the waveform in Figure P5.13. Compute the current waveform.

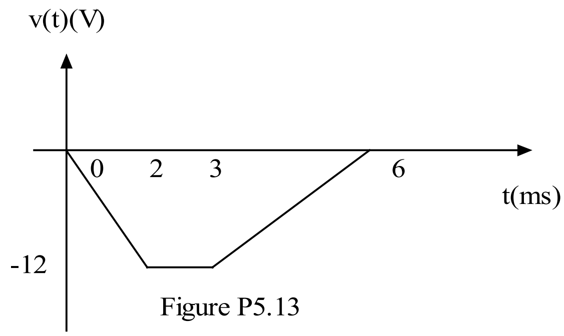
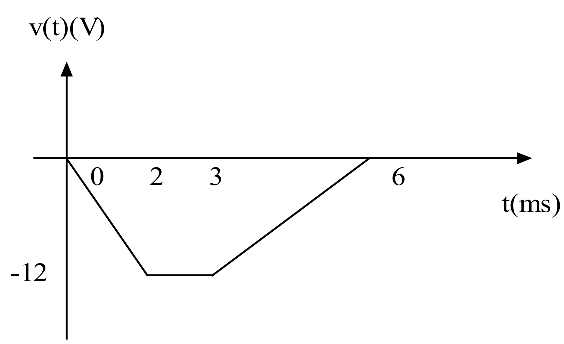


Figure P5.13

Suggested Solution

$C = 2\mu F \qquad i(t) = C\frac{dv}{dt}$

Time (ms)	$\frac{dv}{dt}$ (V/ms)	i(t) (mA)
$0 \le t \le 2$	−6	-12
$2 \le t \le 3$	0	0
$3 \le t \le 6$	4	8
$t > 6$	0	0



Problem 5.14

The voltage across a 0.1-F capacitor is given by the waveform in Figure P5.14. Find the waveform for the current in the capacitor

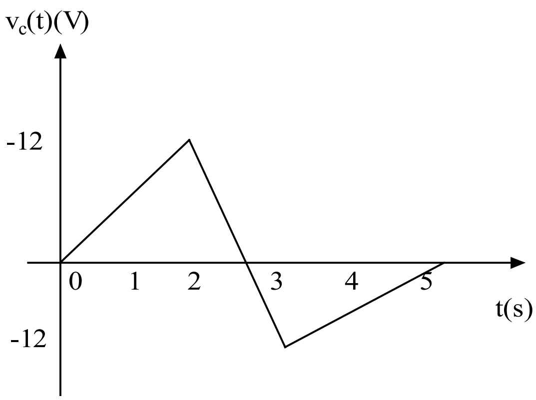
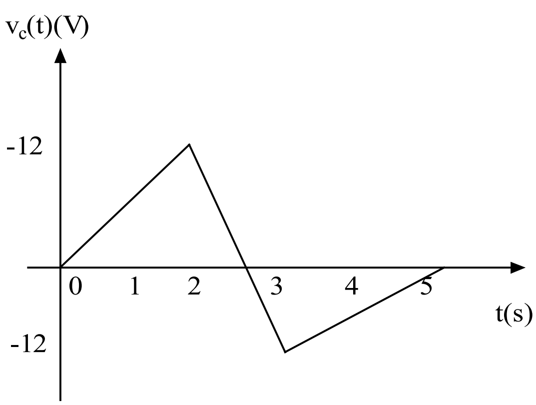


Figure P5.14

Suggested Solution

$C = 0.1F \qquad i(t) = C\frac{dv}{dt}$

Time(s)	$\frac{dv}{dt}$(V/ms	i(t)(A)
$0 \le t \le 2$	6	0.6
$2 \le t \le 3$	-24	-2.4
$3 \le t \le 5$	6	0.6
$t > 5$	0	0



Problem 5.15

The waveform for the current in a 200-μF capacitor is shown in Figure P5.15. Determine the waveform for the capacitor voltage.

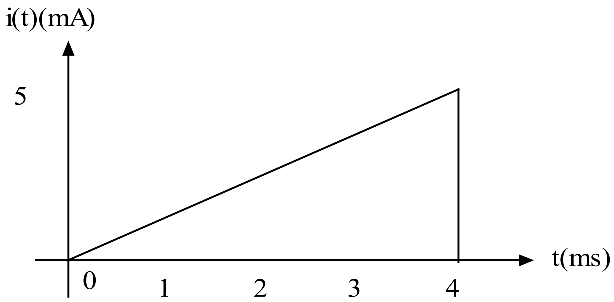


Figure P5.15

Suggested Solution

$C = 200\mu F \qquad v(t) = \frac{1}{C}\int i(t)dt$

$\underline{0 \le t \le 4ms}$

$i(t) = 1.25t \quad A$

$v(t) = 3125t^2 + v(0)$

assuming $v(0) = 0V$,

$v(t) = 3125t^2$

$\underline{t > 4ms}$

$i(t) = 0A$

$v(t) = 0 + v(4m)$

$v(t) = 0 + 3125(4m)^2 = 50mV$

$$\boxed{v(t) = \begin{cases} 3125t^2 V & 0 \le t \le 4ms \\ 50mV & t > 4ms \end{cases}}$$

Problem 5.16

Draw the waveform for the current in a 12-μF capacitor when the capacitor voltage is as described in Figure 5.16

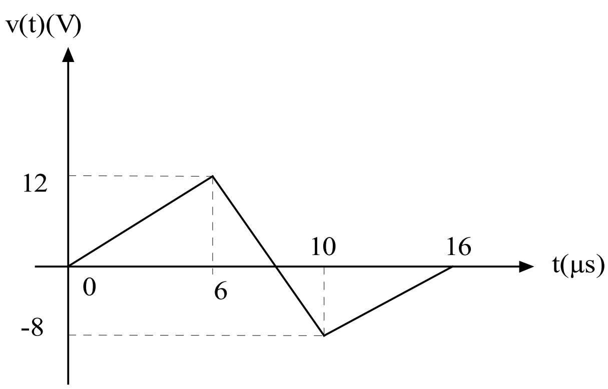
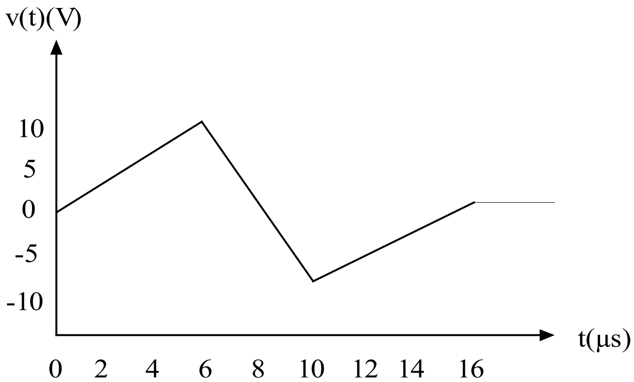


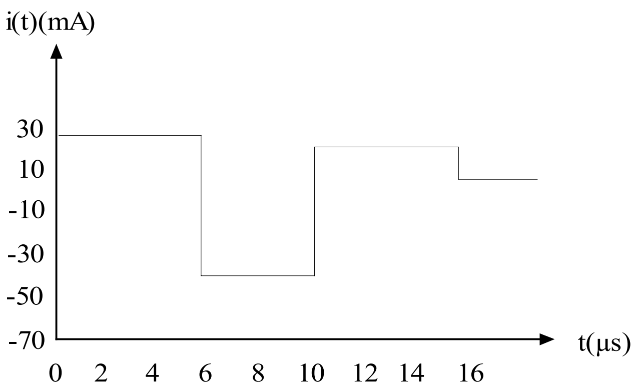
Figure P5.16

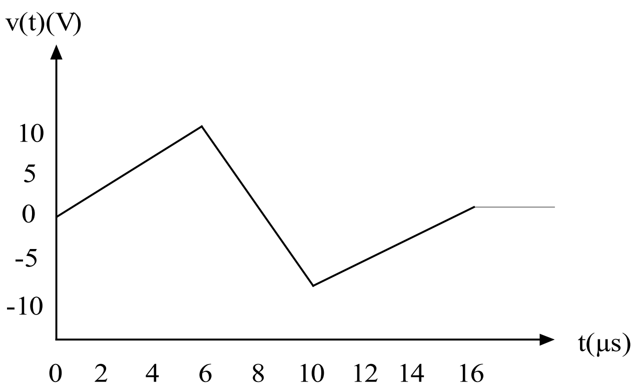
Suggested Solution

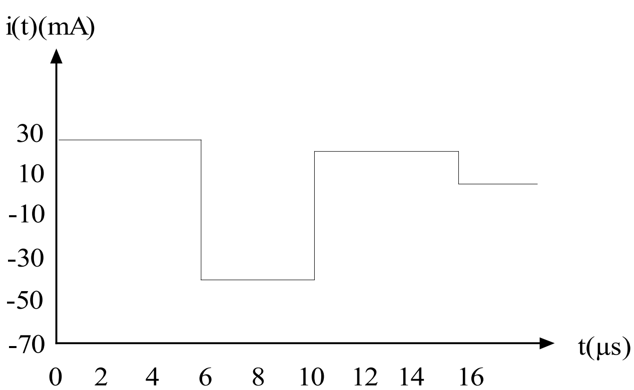
$C = 12\mu F \qquad i(t) = C\frac{dv}{dt}$

Time (μs)	$\frac{dv}{dt}$(V/μs)	i(t) (A)
$0 \le t$	2	24
$6 \le t \le 10$	-5	-60
$10 \le t \le 16$	1.33	16
$t > 16$	0	0


v(t)(V)
10
5
0
-5
-10
t(μs)
0 2 4 6 8 10 12 14 16


i(t)(mA)
30
10
-10
-30
-50
-70
t(μs)
0 2 4 6 8 10 12 14 16


v(t)(V)
10
5
0
-5
-10
t(μs)
0 2 4 6 8 10 12 14 16


i(t)(mA)
30
10
-10
-30
-50
-70
t(μs)
0 2 4 6 8 10 12 14 16

Problem 5.17

Draw the waveform for the current in a 3-μF capacitor when the voltage across the capacitor is given in Figure P5.17

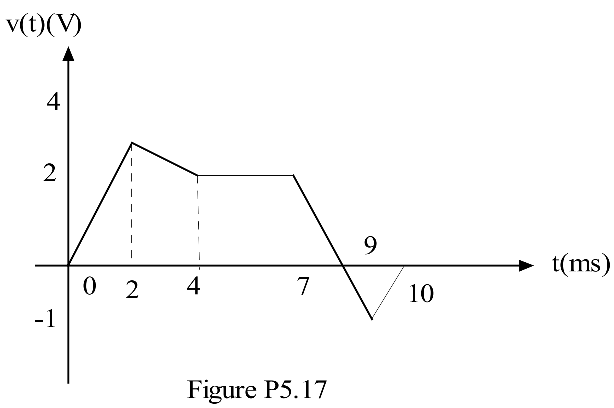
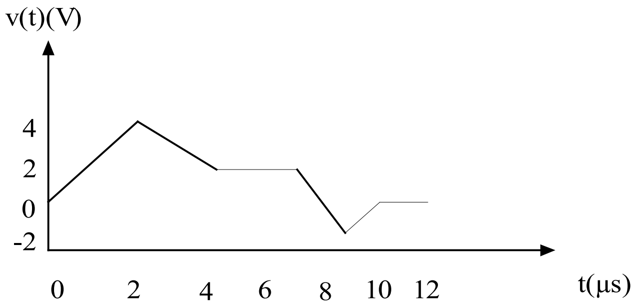


Figure P5.17

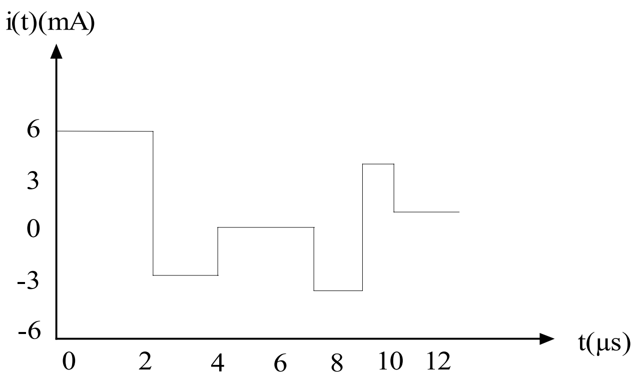
Suggested Solution

$C = 3\mu F \qquad i(t) = C\frac{dv}{dt}$

Time(ms)	$\frac{dv}{dt}$(V/μs)	i(t)(A)
$0 \le t \le 2$	2	6
$2 \le t \le 4$	-1	-3
$4 \le t \le 7$	0	0
$7 \le t \le 9$	$\frac{-3}{2}$	-4.5
$9 \le t \le 10$	1	3
$t > 0$	0	0



v(t)(V)
4
2
0
-2
0
2
4
6
8
10
12
t(μs)



i(t)(mA)
6
3
0
-3
-6
0
2
4
6
8
10
12
t(μs)

Problem 5.18

The waveform for the current in a 100-μF initially uncharged capacitor is shown in Figure P5.18. Determine the waveform for the capacitor's voltage.

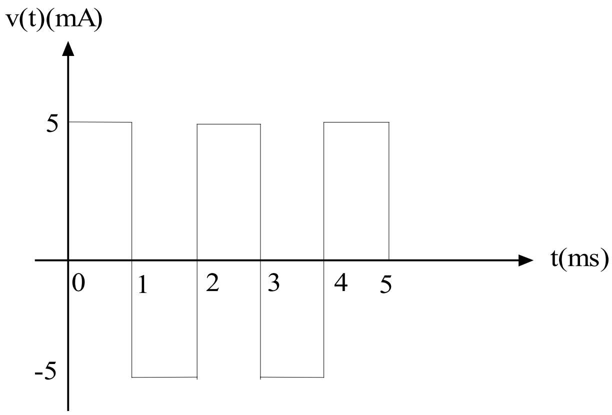
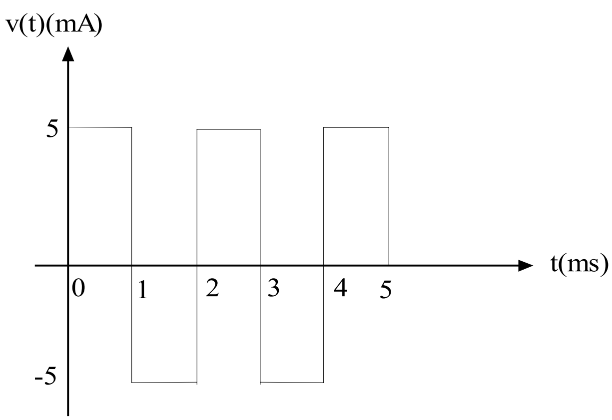


Figure P5.18

Suggested Solution



$C = 100\mu F \qquad v(t) = \frac{1}{C}\int i(t)dt$

$v(0)=0V$

$0 \le t \le 1ms \qquad i(t) = -5mA$

$v(t)=50t$ V

$1ms \le t \le 2ms \qquad i(t) = -5mA$

$v(t) = -50(t-1m) + v(1m)$

$v(t) = -50t + 0.1$ V

$2ms \le t \le 3ms \qquad i(t) = 5mA$

$v(t) = 50(t+2m) + v(2m)$

$v(t) = 50t - 0.1$ V

$3ms \le t \le 4ms \qquad i(t) = -5mA$

$v(t) = -50(t-3m) + v(3m)$

$v(t) = -50t + 0.2$ V

$4ms \le t \le 5ms \qquad i(t) = 5mA$

$v(t) = 50(t-4m) + v(4m)$

$v(t) = 50t - 0.2$ V

$t > 5ms \qquad i(t) = 0$

$v(t) = 50mV$

$$v(t) = \begin{cases} 50t & \text{V} \quad 0 \le t \le 1ms \\ -50t + 0.1 & \text{V} \quad 1ms \le t \le 2ms \\ 50t - 0.1 & \text{V} \quad 2ms \le t \le 3ms \\ -50t + 0.2 & \text{V} \quad 3ms \le t \le 4ms \\ 50t - 0.2 & \text{V} \quad 4ms \le t \le 5ms \\ 50 & mV \quad t > 5ms \end{cases}$$

Problem 5.19

The voltage across a 6-μF capacitor is given by the waveform in Figure P5.19. Plot the waveform for the capacitor current.

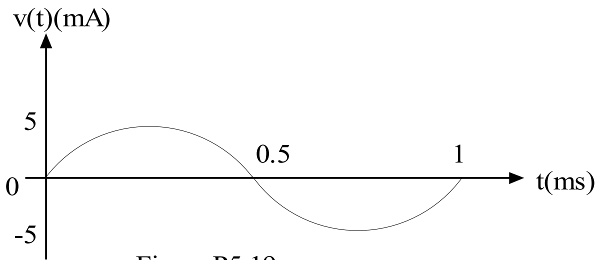


Figure P5.19

Suggested Solution

$C = 6\mu F \qquad i(t) = C\frac{dv}{dt}$

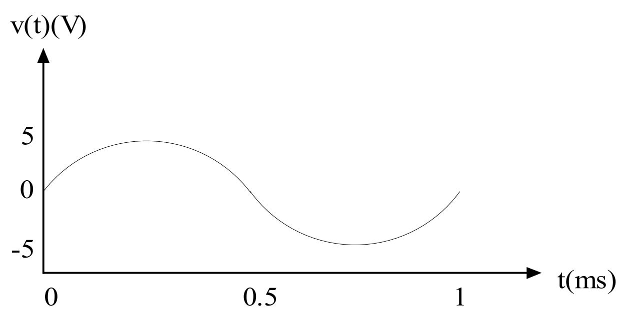
if $v(t) = 10\sin(\omega t)V$

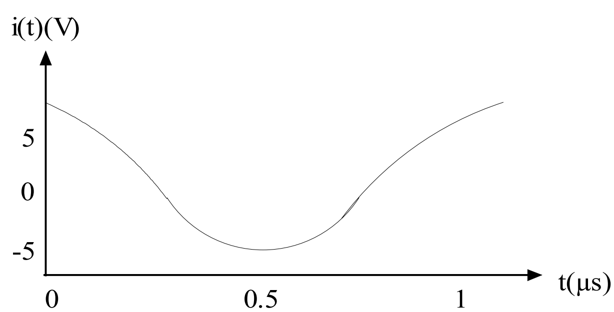
$\omega = 2\pi f = 2\pi / T$ where $T = 1m$ sec,

$v(t) = 10\sin(2000\pi t)V$

$i(t) = (6\mu)(10)(2000\pi)\cos(2000\pi f)$

$\boxed{i(t) = 377\cos(2000\pi t)mA}$





Problem 5.20

The current in an inductor changes from 0 to 200mA in 4ms in 4 ms and induces a voltage of 100mV. What is the value of the inductor?

Suggested Solution

$v(t) = L\frac{di}{dt}$

$\Delta i = 200mA \qquad \Delta t = 4ms \qquad v_{ind} = 100mV$

Assuming the current change is linear,

$v_{ind} = L\frac{\Delta i}{\Delta t} \Rightarrow \quad L = v_{ind}(\frac{\Delta i}{\Delta t}) = 2mH$

$\boxed{L = 2mH}$

Problem 5.21

The current in a 100-mH inductor is i(t)=2sin(377t)A. Find (a) the voltage across the inductor and (b) the expression for the energy stored in the element

Suggested Solution

$L = 100mH \qquad i(t) = 2\sin(377t)A$

$a)$

$v(t) = L\frac{di}{dt} = (0.1)(2)(377)\cos(377t) = \boxed{75.4\cos(377t)V}$

$b)$

$w(t) = \frac{L}{2}i^2(t) = \frac{0.1}{2}(2)2\sin^2(377t) = 0.2\sin^2(377t)$

$but \qquad \sin^2 x = \frac{1}{2} - \frac{1}{2}\cos(2x)$ so

$\boxed{w(t) = 0.1 - 0.1\cos(754t) \;\; J}$

Problem 5.22

A 10-mH inductor has a sudden current change from 200mA to 100mA in 1ms. Find the induced voltage.

Suggested Solution

$L = 10mH \qquad \Delta i = 100mA - 200mA = -100mA \qquad \Delta t = 1ms$

$v(t) = L\frac{di}{dt}$ assuming the current changed linearly

$v_{IND} = (0.01)\frac{\Delta i}{\Delta t} = -1V$

$\boxed{v_{IND} = -1V}$

Problem 5.23

The induced voltage across a 10-mH inductor is v(t)=120cos(377t) V. Find (a) the expression for the inductor current and (b) the expression for the power.

Suggested Solution

$L = 10mH \qquad v(t) = 120\cos(377t)V \qquad$ Assume $i(0) = 0.$

$a)$

$$i(t) = \tfrac{1}{L}\int v(t)dt = \tfrac{120}{0.01}\tfrac{1}{377}\sin(377t) = \boxed{31.83\sin(377t)A}$$

$b)$

$$p(t) = v(t)i(t) = (120)(31.83)\cos(377t)\sin(377t)$$

but $\quad \sin(x)\cos(x) = \tfrac{1}{2}\sin(x) \quad$ so,

$$\boxed{p(t) = 1910\sin(754t)W}$$

Problem 5.24

The current in a 25-mH inductor is given by the expressions

i(t)=0	t<0
$i(t)=10(1-te^{-4t})$mA	t>0

Find (a) the voltage across the inductor, (b) the expression for the energy stored in it.

Suggested Solution

$L = 25mH$

$$a)\quad i(t) = \begin{cases} 0 & t < 0 \\ 10(1-e^{-t})mA & t > 0 \end{cases}$$

$v(t) = L\frac{di}{dt}$

$t < 0: \qquad v(t) = 0$

$t > 0: \qquad v(t) = (0.025)(0.01)e^{-t} = 250e^{-t}\mu V$

$$\boxed{v(t) = \begin{cases} 0 & t < 0 \\ 250e^{-t}\mu V & t > 0 \end{cases}}$$

$b)$

$$\boxed{w(t) = \tfrac{L}{2}i^2(t) = 1.25[1-2e^{-t}+e^{-2}t]\mu J}$$

Problem 5.25

Given the data in the previous problem, find the voltage across the inductor and the energy stored in it after 1 s.

Suggested Solution

$v(t) = 250e^{-t}\,\mu V \qquad w(t) = 1.25[1-2e^{-t}+e^{-2t}]\,\mu J$

$at \quad t = 1\,\text{sec}. \qquad \boxed{v(1) = 91.97\,\mu V \qquad w(1) = 0.5\,\mu J}$

Problem 5.26

The current in a 50-mH inductor is given by the expressions

i(t)=0	t<0
$i(t)=2te^{-4t}$ A	t>0

Find (a) the voltage across the inductor, (b) the time at which the current is a maximum, and (c) the time at which the voltage is a minimum

Suggested Solution

$$L = 50mH \qquad i(t) = \begin{cases} 0 & t < 0 \\ 2te^{-t}A & t > 0 \end{cases}$$

$a)\ \ v(t) = L\frac{di(t)}{dt} = (0.05)(2e^{-4\,t} - 8te^{-4t})$ for $t > 0$

$$v(t) = \begin{cases} 0 & t < 0 \\ 0.1e^{-4t}(1-4t)V & t > 0 \end{cases}$$

$b)$

The current will be at its maximum when di/dt = 0, or, v(t) = 0

$v(t_{\max}) = 0 = 0.1e^{-4t_{mx}}(1-4t_{\max}) \rightarrow t_{\max} = 0.25\sec$

$c)$

The voltage will be at its max. or min. when dv/dt = 0

$\frac{di}{dt}|_{t=t_{\min}} = [0.1e^{-4t}(-4) + 0.1e^{-4t}(4)(4t-1)]|_{t=\min} = 0$

$yields:$ $\boxed{t_{\min} = 0.5 \text{ sec.}}$

Problem 5.27

The current

i (t) = 0	t<0
i (t) = $100te^{-t/10}$ A	t>0

flows through a 150-mH inductor. Find both the voltage across the inductor and the energy stored in it after 5 seconds.

Suggested Solution

$$v(t) = L\frac{di}{dt}$$

$$v(t) = 100x10^{-3}\frac{d}{dt}(100e^{-t/10})$$

$$v(t) = \frac{100}{1000}(\frac{100}{10})(-e^{-t/10})$$

$$v(t) = -1.5e^{-t/10}$$

$$v(s) = -1.5e^{-1/2} = -0.91V$$

$$w(t) = \frac{1}{2}Li^2$$

$$w(t) = \frac{1}{2}(50x10^{-3})(10^4 e^{-\frac{-2t}{10}})$$

$$w(t) = \frac{1}{2}(50)(10)e^{-\frac{-t}{5}})$$

$$w(t) = 250e^{-\frac{-t}{5}}$$

$$\boxed{w(5) = 91.97J}$$

Problem 5.28

The current in a 10-mH inductor is shown in Figure P5.28. Find the voltage across the inductor.

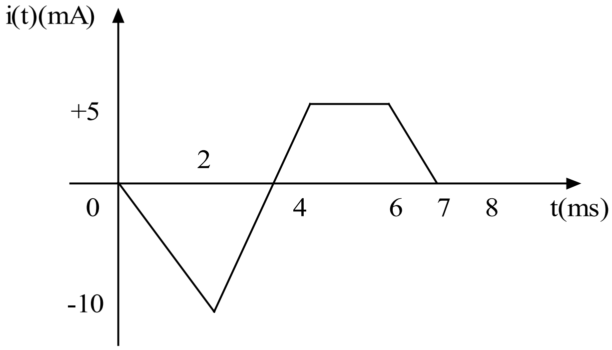


Figure P5.28

Suggested Solution

$L = 10mH \qquad v(t) = C\frac{di}{dt}$

Time (ms)	$\frac{di}{dt}$(A/ms)	v(t) (mV)
$0 \le t \le 2$	-5	-50
$2 \le t \le 4$	7.5	75
$4 \le t \le 6$	0	0
$6 \le t \le 7$	-5	-50
$t > 7$	0	0

Problem 5.29

The current in a 50-mH inductor is given in Figure P5.29. Sketch the inductor voltage.

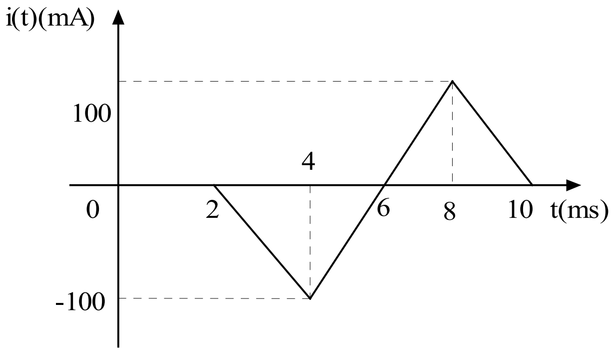
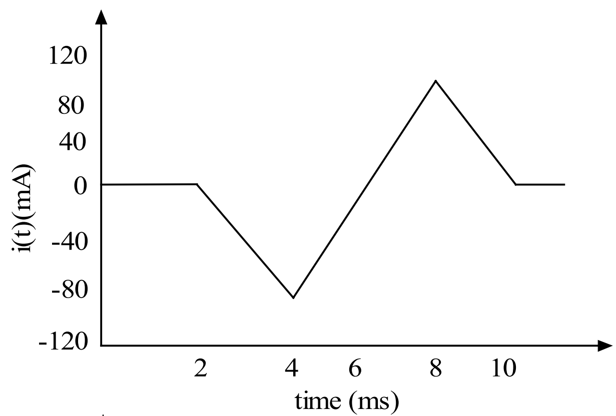


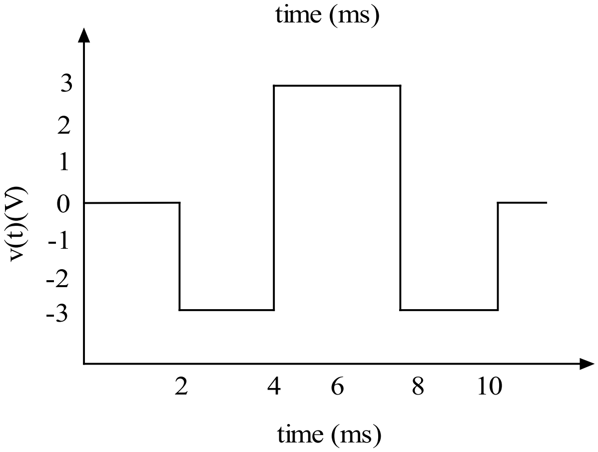
Figure P5.29

Suggested Solution

$L = 50mH \qquad v(t) = L\frac{di}{dt}$

Time(ms)	$\frac{di}{dt}$(A/s)	v(t)(V)
$0 \le t \le 2$	0	0
$2 \le t \le 4$	-50	-2.5
$4 \le t \le 8$	50	2.5
$8 \le t \le 10$	-50	-2.5
$t > 10$	0	0


120
80
40
0
-40
-80
-120
i(t)(mA)
2
4
6
8
10
time (ms)


3
2
1
0
-1
-2
-3
v(t)(V)
2
4
6
8
10
time (ms)

Problem 5.30

The current in a 16-mH inductor is given by the waveform in Figure P5.30. Find the waveform of the voltage across the inductor.

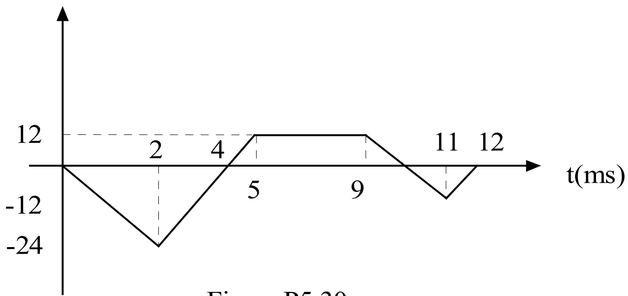
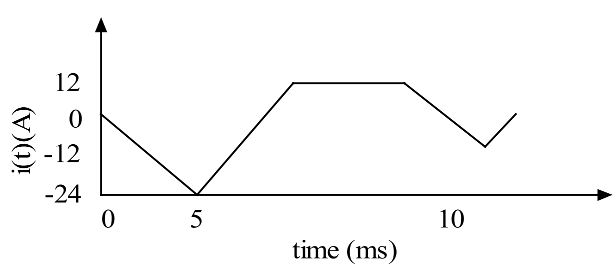


Figure P5.30

Suggested Solution

$L = 16mH \qquad v(t) = L\frac{di}{dt}$

Time (ms)	$\frac{di}{dt}$(A/ms)	v(t) (V)
$0 \le t \le 2$	-12	-192
$2 \le t \le 5$	$+12$	192
$5 \le t \le 9$	0	0
$9 \le t \le 11$	-12	-192
$11 \le t \le 12$	$+12$	192
$t > 12$	0	0



Problem 5.31

Draw the waveform for the voltage across a 10-mH inductor when the inductor current is given by the waveform shown in Figure P5.31

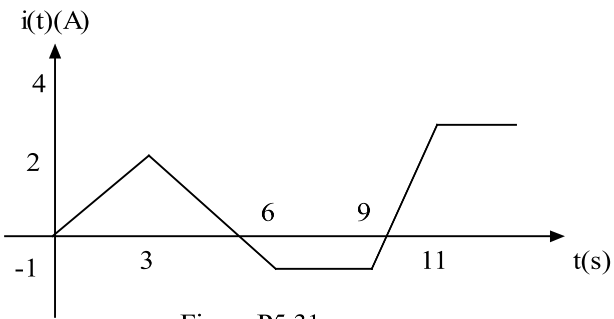
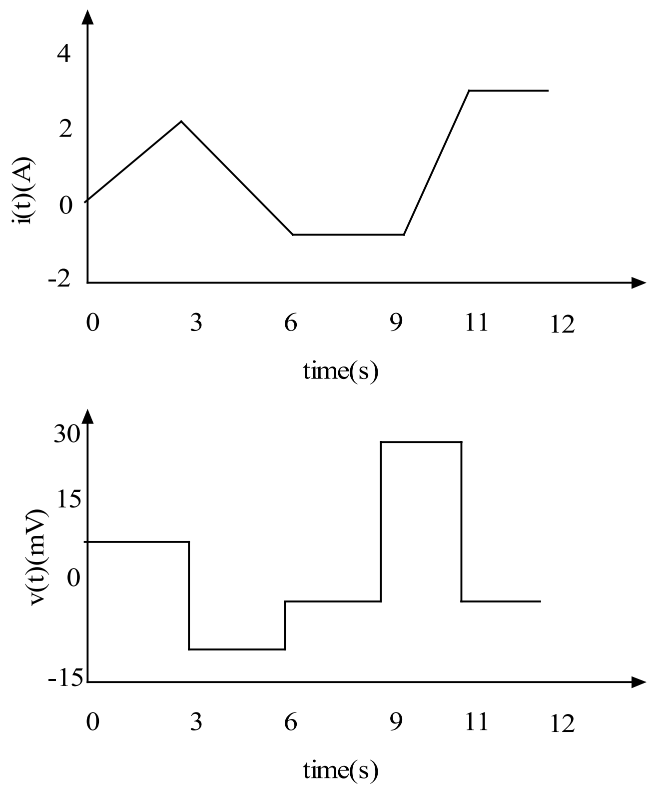


Figure P5.31

Suggested Solution

$L = 10mH$ $\qquad$ $v(t) = L\frac{di}{dt}$

Time(s)	$\frac{di}{dt}$(A/s)	v(t)(mV)
$0 \le t \le 3$	2/3	6.67
$3 \le t \le 6$	-1	-10
$6 \le t \le 9$	0	0
$9 \le t \le 11$	2.5	25
$t > 11$	0	0



i(t)(A)
4
2
0
-2
0
3
6
9
11
12
time(s)
v(t)(mV)
30
15
0
-15
0
3
6
9
11
12
time(s)

Problem 5.32

The voltage across a 10-mH inductor is shown in Figure P5.32. Determine the waveform for the inductor current

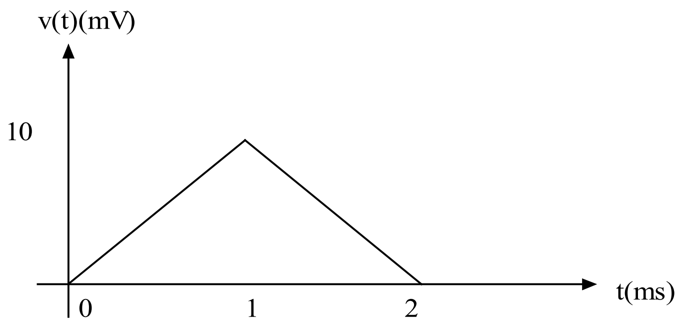


Figure P5.32

Suggested Solution

$L = 10mH \qquad i(t) = \frac{1}{L}\int v(t)dt$

$\underline{0 \le t \le 1ms} \qquad v(t) = 10t \;\; mV$

assuming $i(0) = 0A, \quad i(t) = 500t^2 A$

$1 \le t \le 2ms \qquad v(t) = 10^{-2} - 10(t - 10^{-3})V$

$i(t) = \int (2 - 1000t)dt + K$

where K = integration constant

$i(t) = 2t - 500t^2 + K$ A

Both equations for i(t) must be equal at t=1ms.

$500(10^{-3})^2 = 2(10^{-3}) - 500(10^{-3})^2 + K \Rightarrow \quad K = -1mA$

So $\quad i(t) = 2t - 500t^2 - 10^{-3} A$

$$i(t) = \begin{cases} 500t^2 \;\; A & 0 \le t \le 1ms \\ 2t - 500t^2 - 10^{-3} A & 1 \le t \le 2ms \\ 1 \;\;\; mA & t > 2ms \end{cases}$$

Problem 5.33

The waveform for the voltage across a 20-mH inductor is shown in Figure P5.33. Compute the waveform for the inductor current.

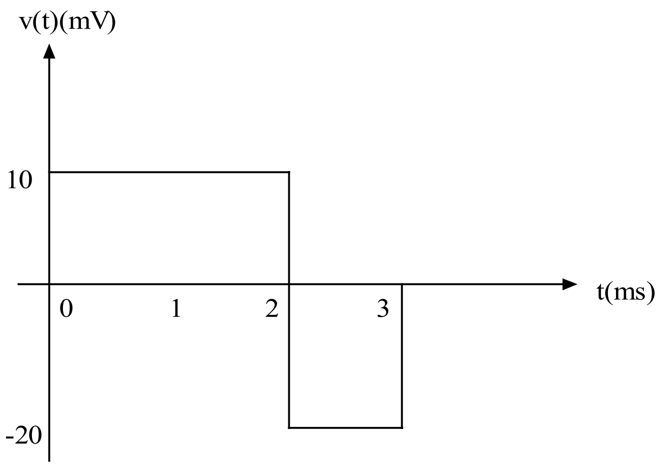


Figure P5.33

Suggested Solution

$L = 20mH \qquad i(t) = \frac{1}{L}\int v(t)dt$

$\underline{0 \le t \le 2ms} \qquad v(t) = 10mV$

if $\quad i(0) = 0A, \quad i(t) = 0.5t \quad A$

$\underline{2 \le t \le 3ms} \qquad v(t) = -20mV$

$i(t) = -(t - 2m) + i(2m)$

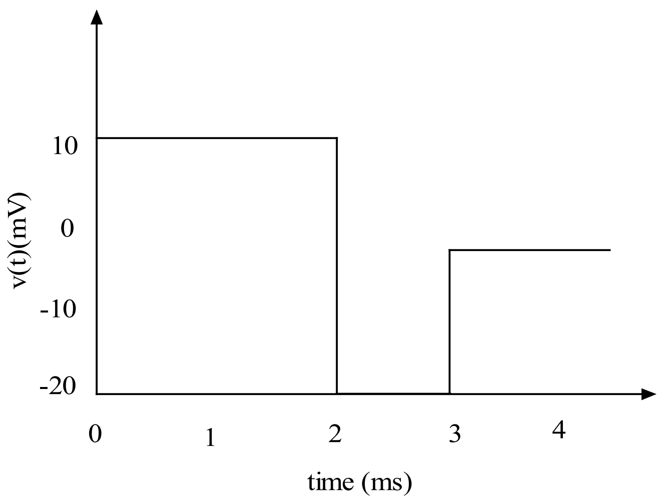
$\quad = -t + 2m + 1m$

$i(t) = (3x10^{-3} - t) \quad A$

$\underline{t > 3ms} \qquad v(t) = 0V$

$i(t) = 0 + i(3m) = 0A$

$$i(t) = \begin{cases} 0.5t & A & 0 \le t \le 2ms \\ 3x10^{-3} - t & A & 2 \le t \le 3ms \\ 0 & A & t > 3ms \end{cases}$$



10
0
-10
-20
v(t)(mV)
0
1
2
3
4
time (ms)

Problem 5.34

The voltage across a 2-H inductor is given by the waveform shown in Figure P5.34. Find the waveform for the current in the inductor.

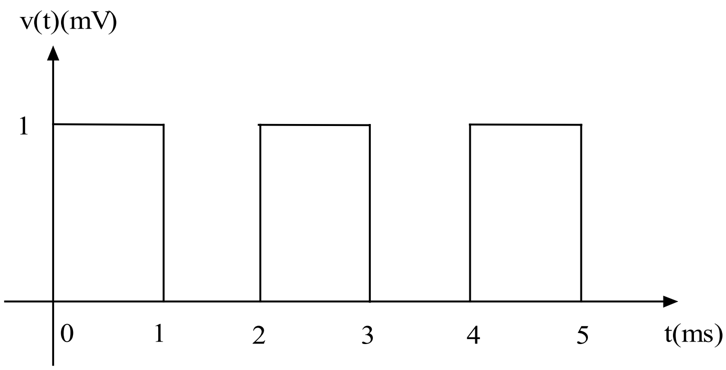


Figure P5.34

Suggested Solution

$L = 2H \qquad i(t) = \frac{1}{L}\int v(t)dt$

$\underline{0 \le t \le 1ms}$ assume $i(0) = 0A$

$v(t) = 1mV$

$i(t) = 500t \ \ \mu A$

$\underline{1 \le t \le 2mA}$ $v(t) = 0V$

$i(t) = i(1ms) = \underline{0.5\mu A}$

$\underline{2ms \le t \le 3ms}$ $v(t) = 1mV$

$i(t) = 500(t - 2m) + K_1 \mu A$

K1 is an integrator constant,

$i(2m) = 0.5\mu A = 500(2m - 2m) + K_1$

$K_1 = 0.5\mu A$

so, $i(t) = \underline{500t - 0.5\mu A}$

$\underline{3ms \le t \le 4ms}$ $v(t) = 0$

$i(t) = 0 + i(3m) = \underline{1\mu A}$

$\underline{4ms \le t \le 5ms}$ $v(t) = 1mV$

$i(t) = 500(t - 4m) + K_2 \mu A$

$i(4m) = 1\mu A = 500(4m - 4m) + K_2$

$K2 = 1\mu A$

$i(t) = 500t - \underline{1\mu A}$

$\underline{t > 5ms}$ $i(t) = \underline{1.5\mu A}$

$$i(t) = \begin{cases} 500t & \mu A & 0 \le t \le 1ms \\ 0.5 & \mu A & 1 \le t \le 2ms \\ 500t - 0.5 & \mu A & 2 \le t \le 3ms \\ 1.0 & \mu A & 3 \le t \le 4ms \\ 500t - 1 & \mu A & 4 \le t \le 5ms \\ 1.5 & \mu A & t > 5ms \end{cases}$$

Problem 5.35

Find the possible capacitance range of the following capacitors.

a) 0.068 μF with a tolerance of 10%
b) 120pF with a tolerance of 20%
c) 39μF with a tolerance of 20%

Suggested Solution

a) Minimum capacitor value = 0.9C = 61.2nF
Maximum capacitor value = 1.1C = 74.8nF

b) Minimum capacitor value = 0.8C = 96pF
Maximum capacitor value = 1.2C = 144pF

c) Minimum capacitor value = 0.8C = 31.2μF
Maximum capacitor value = 1.2C = 46.8μF

Problem 5.36

The capacitor in Figure P5.36a is 51 nF with a tolerance of 10%. Given the voltage waveform in Figure P5.36b graph the current i(t) for the minimum and maximum capacitor values.

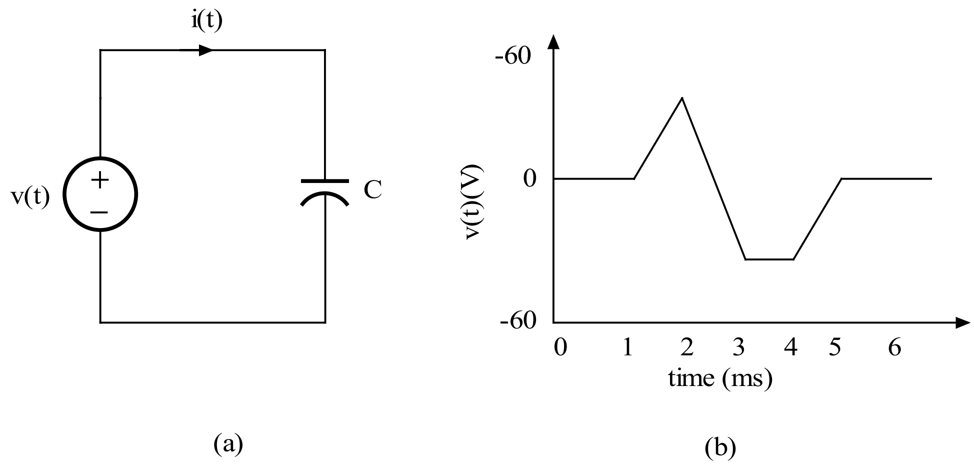


Figure P5.36

Suggested Solution

Maximum capacitor value = 1.1C = 56.1 nF
Minimum capacitor value = 0.9C = 45.9 nF

The capacitor voltage and current are related by the equation

$$i(t) = C\frac{dv(t)}{dt}$$

Problem 5.37

Find the possible inductance range of the following inductors

a) 10 mH with a tolerance of 10%
b) 2.0 nH with a tolerance of 5%
c) 68 μH with a tolerance of 10%

Suggested Solution

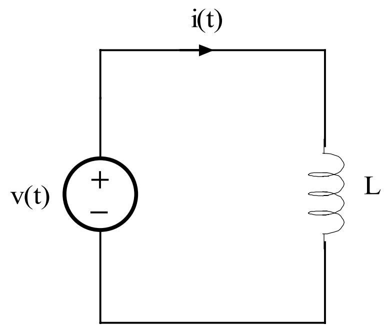
a)
Minimum inductor value = 0.9L = 0.9 mH
Maximum inductor value = 1.1L = 1.1 mH

b)
Minimum inductor value = 0.95 = 1.9 nH
Maximum inductor value = 1.05L = 2.1 nH

c)
Minimum inductor value = 0.9L = 61.2 nH
Maximum inductor value = 1.1L = 74.8 nH

Problem 5.38

The inductor in Figure P5.38a is 330μH with a tolerance of 5%. Given the current waveform in Figure P5.38b, graph the voltage v(t) for the minimum and maximum inductor values.



(a)

Figure P5.38

Suggested Solution

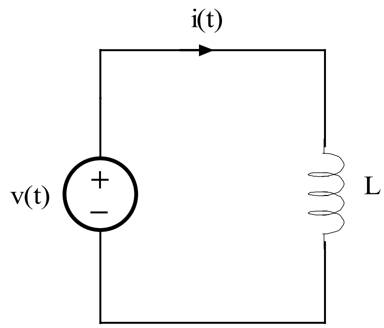
Maximum inductor value = 1.05L = 346.5 μH
Minimum inductor value = 0.95L = 313.5 μH

The inductor voltage and current are related by the equation

$$v(t) = L\frac{di(t)}{dt}$$

Problem 5.39

The inductor in Figure P5.39a is 4.7 μH with a tolerance of 20%. Given the current waveform in Figure P5.39b, graph the voltage v(t) for the minimum and maximum inductor values.



(a)

Figure P5.39

Suggested Solution

Maximum inductor value = 1.2L = 5.64 μH
Minimum inductor value = 0.8L = 3.76 μH

The inductor voltage and current are related by the equation

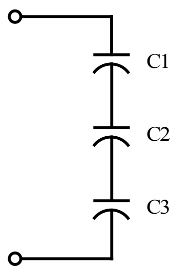
$$v(t) = L\frac{di(t)}{dt}$$

Problem 5.40

What values of capacitance can be obtained by interconnecting a 4-μF capacitor, a 6-μF capacitor, and a 12-μF capacitor?

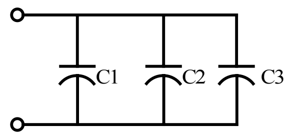
Suggested Solution

Combo A



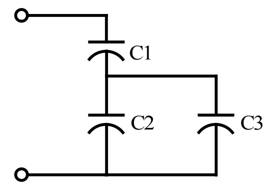
$C_A = 1 / (1/C_1 + 1/C_2 + 1/C_3) = 2\mu F$

Combo B



$C_B = C_1 + C_2 + C_3 = 22\mu F$

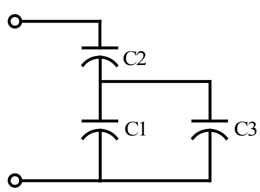
Combo C



$C_C = [C_1 (C_2 + C_3)] / [C_1 + (C_2 + C_3)]$

$C_C = 3.27\mu F$

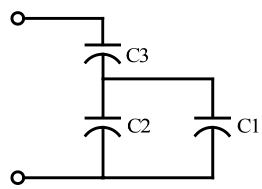
Combo D



$C_D = [C_2 (C_1 + C_3)] / [C_2 + (C_1 + C_3)]$

$C_D = 4.36\mu F$

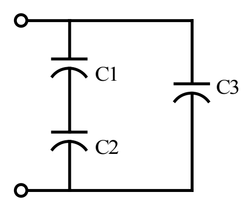
Combo E



$C_C = [C_3 (C_2 + C_1)] / [C_3 + (C_2 + C_1)]$

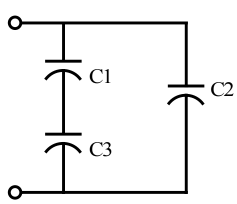
$C_C = 5.45\mu F$

Combo F



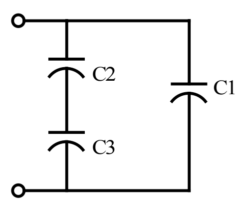
$C_F = C_3 + C_1 C_2 / (C_1 + C_2) = 14.4\mu F$

Combo G



$C_G = C_2 + C_1 C_3 / (C_1 + C_3) = 9\mu F$

Combo H



$C_H = C_1 + C_2 C_3 / (C_2 + C_3) = 8\mu F$

Possibilities:

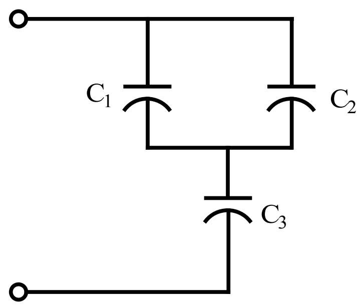
CA = 2μF
CB = 22μF
CC = 3.27μF
CD = 4.36 μF
CE = 4.45μF
CF = 14.4μF
CG = 9 μF
CH = 8 μF

Problem 5.41

Given a 1, 3, and 4-μF capacitor, can they be interconnected to obtain an equivalent 2-μF capacitor?

Suggested Solution

C1 = 1μF C2 = 3μF C3 = 4μF



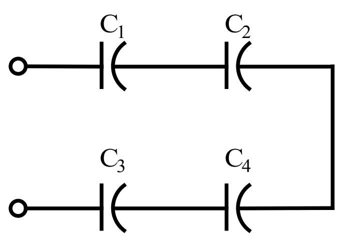
$$C_{eq} = \frac{(C_1 + C_2)C_3}{C_1 + C_2 + C_3} = \frac{(1\mu + 3\mu)4\mu}{1\mu + 3\mu + 4\mu} = \underline{2\mu F}$$

Problem 5.42

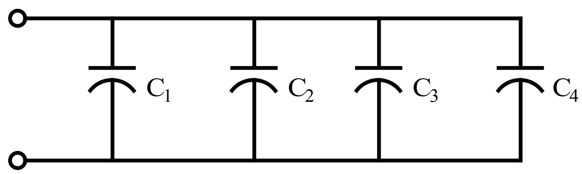
Given four 2-μF capacitors, find the maximum value and minimum value that can be obtained by interconnecting the capacitors in series/parallel combinations.

Suggested Solution

Minimum combo



Maximum combo



$$C_{min} = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\frac{1}{C_4}} = 0.5\mu F$$

$$C_{max} = C_1 + C_2 + C_3 + C_4 = 8\mu F$$

$$\boxed{\begin{array}{l} C_{min} = 0.5\mu F \\ C_{max} = 8\mu F \end{array}}$$

Problem 5.43

The two capacitors in Figure P5.42 were charged and then connected as shown. Determine the equivalent capacitance, the initial voltage at the terminals, and the total energy stored in the network.

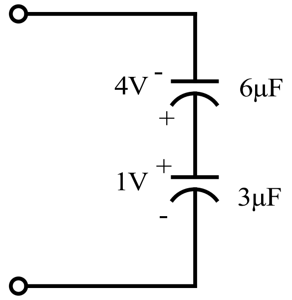
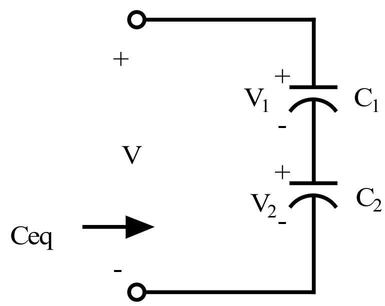


Figure 5.43

Suggested Solution



$C_1 = 6\mu F \quad V_1 = -4V$

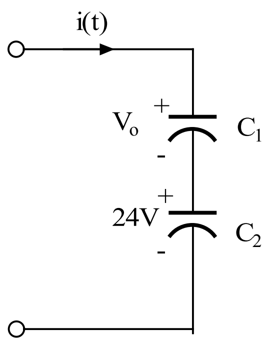
$C_2 = 3\mu F \quad V_2 = 1V$

$$C_{eq} = C_1 C_2 / (C_1 + C_2) = 2\mu F$$
$$V = V_1 + V_2 = -3V$$
$$W = \tfrac{1}{2} C_1 V^2{}_1 + \tfrac{1}{2} C_1 V^2{}_2$$
$$W = 49.5\mu J$$

$$\boxed{\begin{aligned} C_{eq} &= 2\mu F \\ V &= -3V \\ W &= 49.5\mu J \end{aligned}}$$

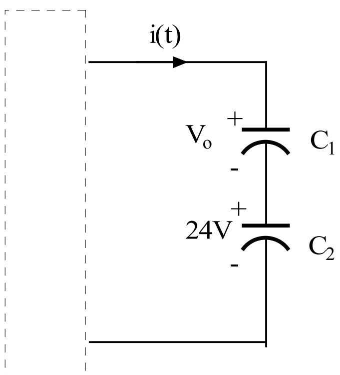
Problem 5.44

Two capacitors are connected in series as shown in Figure P5.44. Find Vo



$C_1 = 12\mu F \quad C_2 = 6\mu F$

Suggested Solution



$C_1 = 12\mu F \quad C_2 = 6\mu F$

$$vc = \frac{1}{C}\int i(t)dt$$

Since same current charged both caps,

$$C_1 V_o = C_2(24)$$

$$V_o = 24(C_2 / C_1)$$

$$\boxed{V_o = 12V}$$

Problem 5.45

Three capacitors are connected as shown in Figure P5.45. Find V_1 and V_2.

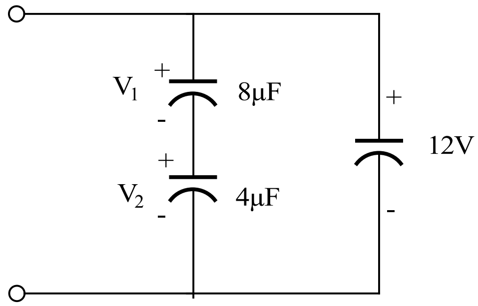
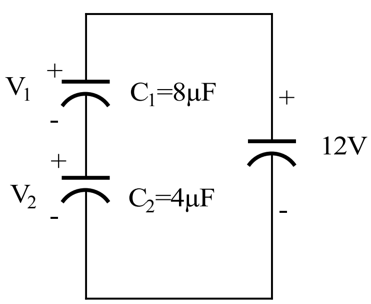


Figure 5.45

Suggested Solution



$V_1 + V_2 = 12V$

Assuming C_1 and C_2 are charged by the same surrent,

$C_1V_1 = C_2V_2$ (same as $Q_1 = Q_2$)

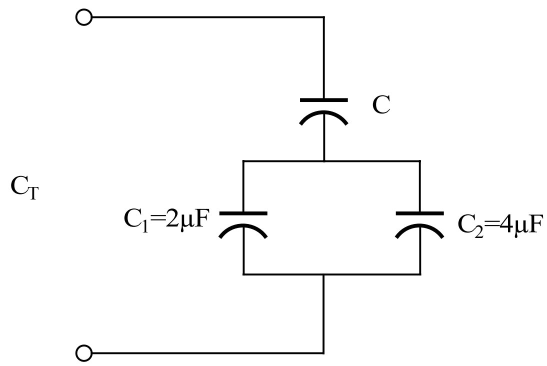
$$V_1 = 12\frac{C_2}{C_1 + C_2} = 4V$$

$$V_2 = 12\frac{C_1}{C_1 + C_2} = 8V$$

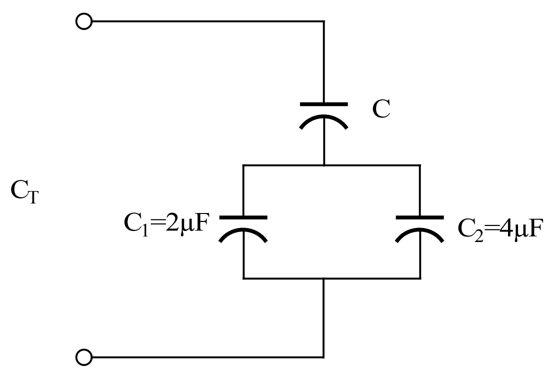
$$\boxed{\begin{array}{l} V_1 = 4V \\ V_2 = 8V \end{array}}$$

Problem 5.46

Select the value of C to produce the desired total capacitance of C_T=2μF in the circuit in Figure P5.46



Suggested Solution



$$C_T = \frac{C(C_1 + C_2)}{C + C_1 + C_2} \Rightarrow 2 = \frac{6C}{6 + C}$$

$$12 + 2C = 6C \Rightarrow 4C = 12$$

$$\boxed{C = 3\mu F}$$

Problem 5.47

Select the value of C to produce the desired total capacitance of C_T=1μF in the circuit in Figure P5.47.

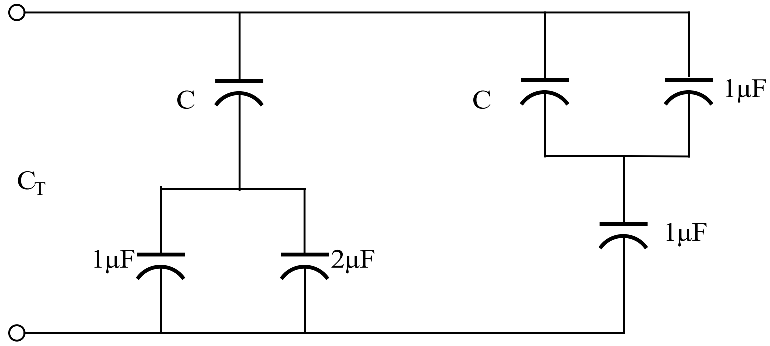


Figure P5.57

Suggested Solution

$$CT = 1 = \frac{3C}{3+C} + \frac{(1+C)(1)}{(1+C)+1}$$

$$1 = \frac{3C}{3+C} + \frac{1+C}{2+C}$$

$$(3+C)(2+C) = 3C(2+C) + (3+C)(1+C)$$

$$0 = 3C^2 + 5C - 3$$

$$C1, C2 = \frac{-5 \pm \sqrt{25-4(3)(-3)}}{2(3)}$$

$$\boxed{C = 0.47\mu F}$$

Problem 5.48

Find the equivalent capacitance at terminals A-B in Figure P5.48

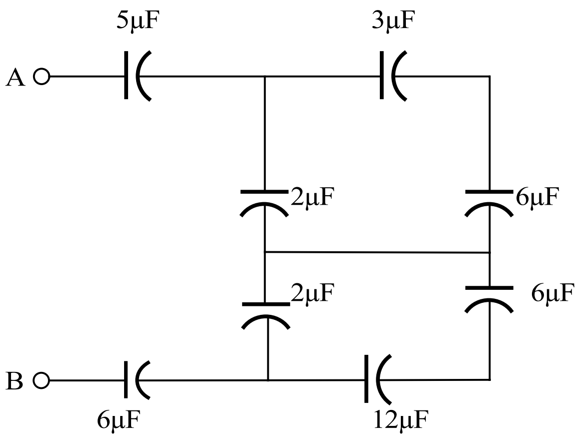
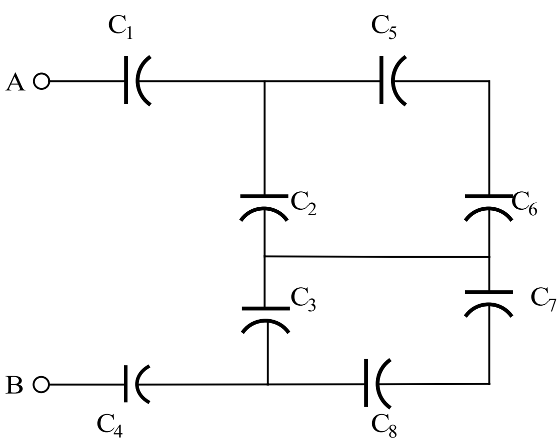


Figure 5.48

Suggested Solution



$C_1=5\mu F$	$C_4=7\mu F$	$C_6=6\mu F$
$C_2=2\mu F$	$C_5=3\mu F$	$C_7=6\mu F$
$C_3=2\mu F$	$C_8=12\mu F$	

$$C_{eq1} = \frac{C_5 C_6}{C_5 + C_6} = 2\mu\text{F}$$

$C_{eq2}=C_2+C_{eq1}=4\mu\text{F}$

$$C_{eq3} = \frac{C_7 C_8}{C_7 + C_8} = 4\mu\text{F}$$

$C_{eq4}=C_3+C_{eq3}=6\mu\text{F}$

$$\frac{1}{C_{eq5}} = \frac{1}{C_{eq2}} + \frac{1}{C_{eq4}} \Rightarrow C_{eq5} = 2.4\mu\text{F}$$

$$\frac{1}{C_{AB}} = \frac{1}{C_1} + \frac{1}{C_4} + \frac{1}{C_{eq5}} \Rightarrow \boxed{C_{AB} = 1.32\mu\text{F}}$$

Problem 5.49

Determine the total capacitance of the network in Figure P5.49

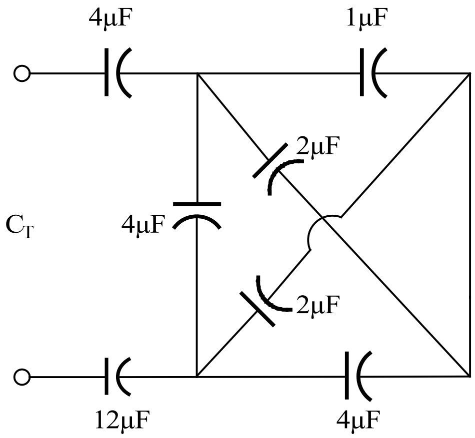
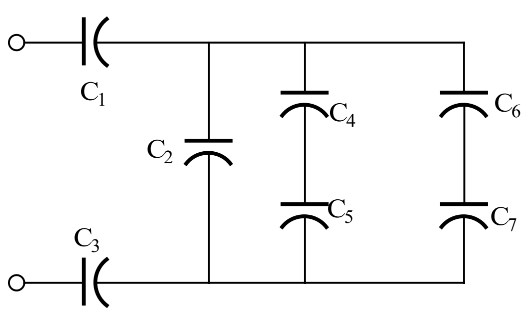


Figure P5.49

Suggested Solution



C_1=4μF C_4=1μF C_6=2μF
C_2=4μF C_5=2μF C_7=4μF
C_3=12μF

$$C_{AB} = \frac{(C_4 + C_6)(C_5 + C_7)}{(C_4 + C_6) + (C_5 + C_7)}$$

$$C_{AB} = 2\mu F$$

$$\frac{1}{C_T} = \frac{1}{C_2 + C_{AB}} + \frac{1}{C_1} + \frac{1}{C_3} = \frac{1}{6} + \frac{1}{4} + \frac{1}{12}$$

$$\boxed{CT = 2\mu F}$$

Problem 5.50

Find C_T in the network in Figure P5.50 if (a) the switch is open and (b) the switch is closed.

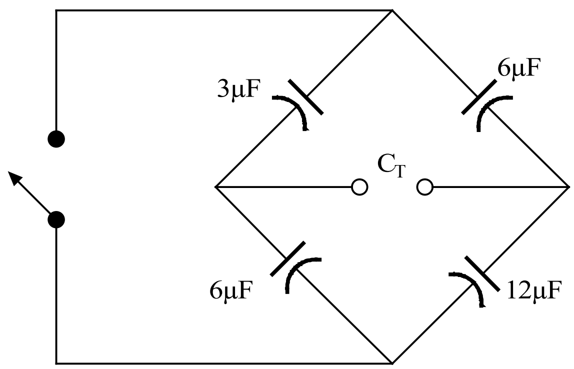
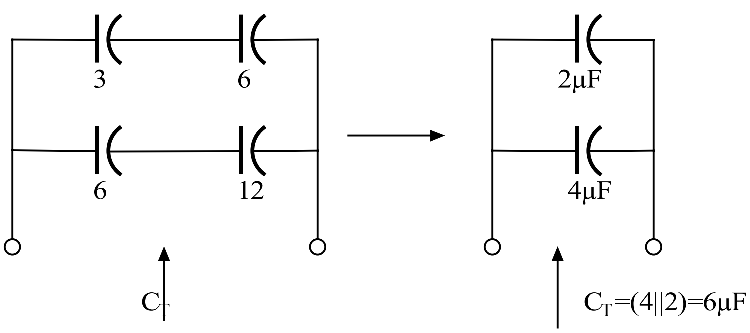


Figure P5.50

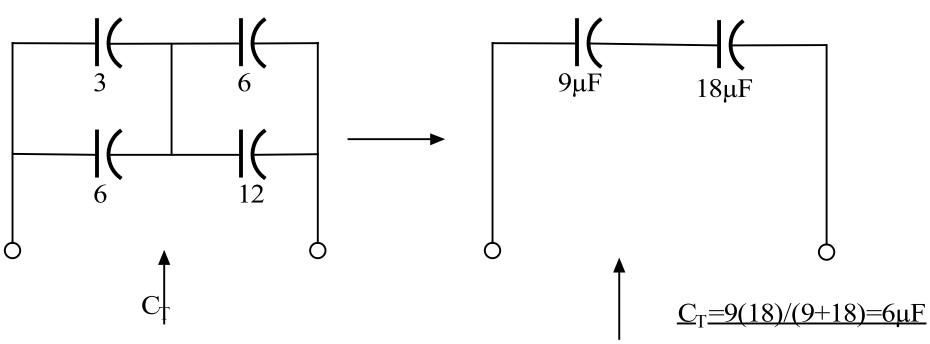
Suggested Solution

The networks can be reduced as follows:

a)



b)



Problem 5.51

Find the total capacitance C_T of the network in Figure P5.51.

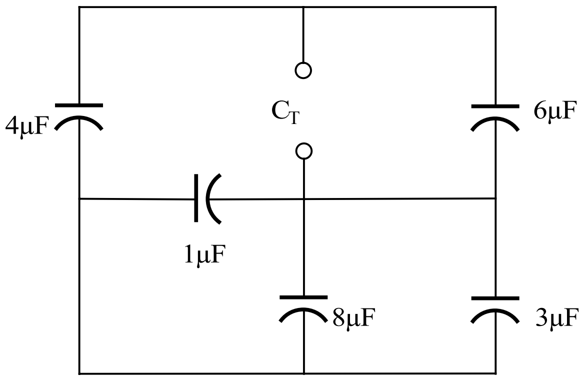
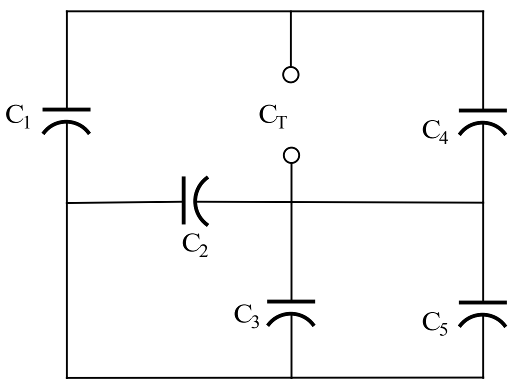


Figure P5.51

Suggested Solution



$C_1 = 4\mu F$

$C_2 = 1\mu F$

$C_3 = 8\mu F$

$C_4 = 6\mu F$

$C_5 = 3\mu F$

$C_{eq1} = C_2 + C_3 + C_5 = 12\mu F$
$C_{eq2} = C_1 C_{eq1}/(C_1 + C_{eq1}) = 3\mu F$
$C_T = C_4 + C_{eq2} = 9\mu F$

$C_T = 9\mu F$

Problem 5.52

Compute the equivalent capacitance of the network in Figure P5.52 if all the capacitors are 5μF.

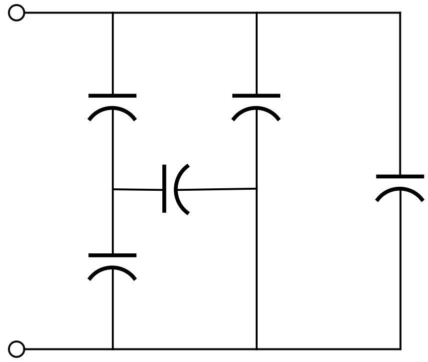
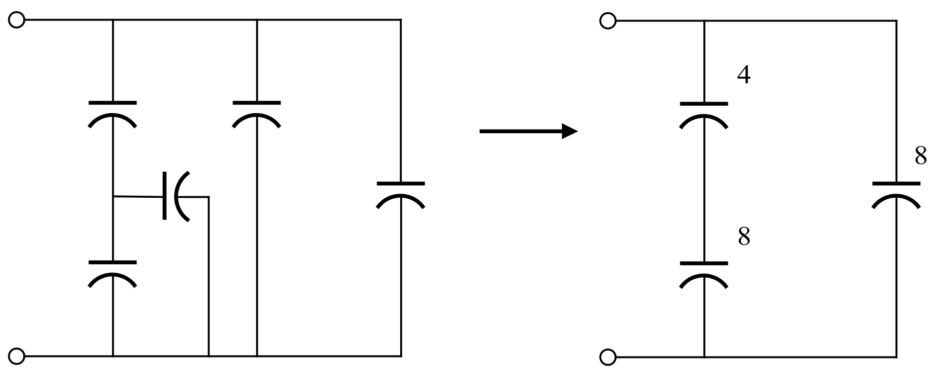


Figure P5.52

Suggested Solution



$C_{eq} = 4(8)/12 + 8 = 32/3\ \mu F$

Problem 5.53

If all the capacitors in Figure P5.53 are 6 μF, find C_{eq}

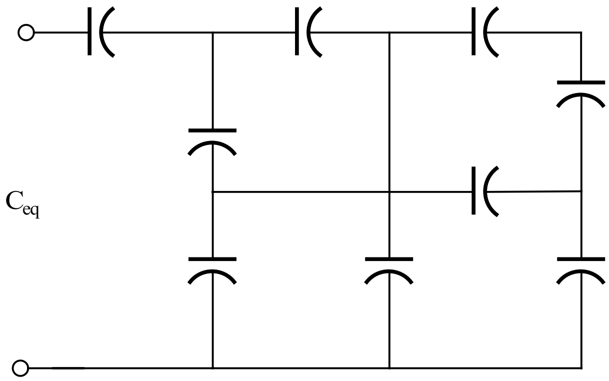
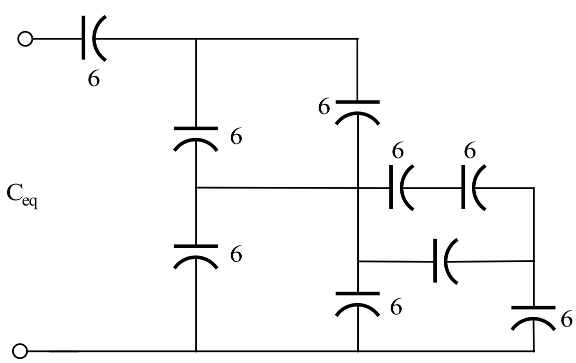
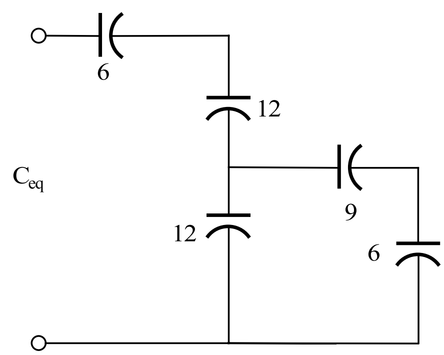
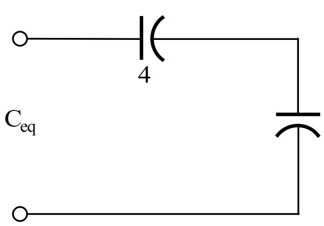


Figure P5.53

Suggested Solution







12 + 54/15 = 15.6

Ceq = 4(15.6)/(19.6)=3.18μF

Problem 5.54

Given the capacitors in Figure 5.54 are C1=2.0uF with a tolerance of 2% and C2=2.0uF with a tolerance of 20%, find

a) the nominal value of CEQ
b) the minimum and maximum possible values of CEQ
c) the percent errors of the minimum and maximum values

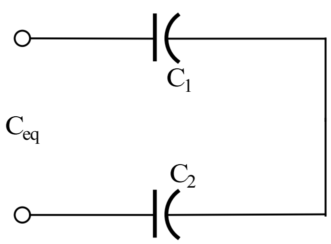


Figure P5.54

Suggested Solution

a) The nominal value is

$$C_{EQ} = C_1 + C_2 + C_3 = (0.1 + 0.33 + 1)x10^{-6} = \underline{1.43\mu F}$$

b) The minimum value of CEQ is

$$C_{EQ,\min} = C_{1,\min} + C_{2,\min} + C_{3,\min} = (0.1*0.9 + 0.33*0.8 + 1*0.9)x10^{-6} = \underline{1.254\mu F}$$

The maximum value of CEQ is

$$C_{EQ,\max} = C_{1,\max} + C_{2,\max} + C_{3,\max} = (0.1*1.1 + 0.33*1.2 + 1*1.1)x10^{-6} = \underline{1.606\mu F}$$

c) The percent irror of the minimum value is

$$\frac{1.254 - 1.43}{1.43} x100 = \underline{-12.3\%}$$

The percent irror of the maximum value is

$$\frac{1.606 - 1.43}{1.43} x100 = \underline{12.3\%}$$

Problem 5.55

Given the capacitors in Figure 5.55 are C_1=0.1μF with a tolerance of 2% and C_2=0.33μF with a tolerance of 20% and 1 μF with a tolerance of 10% . Find the following.

a) the nominal value of C_{EQ}
b) the minimum and maximum possible values of C_{EQ}
c) the percent errors of the minimum and maximum values

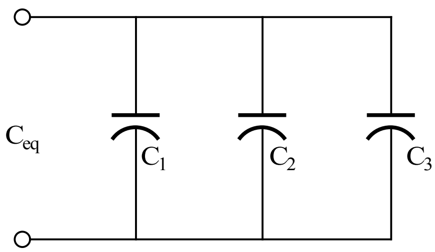


Figure P5.55

Suggested Solution

a) The nominal value is

$$C_{EQ} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(2.0)(2.0)}{2.0 + 2.0} x10^{-6} = \underline{1.0\mu F}$$

b) The minimum value of CEQ is

$$C_{EQ,\min} = \frac{C_{1,\min} C_{2,\min}}{C_{1,\min} + C_{2,\min}} \frac{(2.0 * 0.98)(2.0 * 0.8)}{(2.0 * 0.98) + (2.0 * 0.8)} x10^{-6} = \underline{0.881\mu F}$$

The maximum value of CEQ is

$$C_{EQ,\max} = \frac{C_{1,\max} C_{2,\max}}{C_{1,\max} + C_{2,\max}} \frac{(2.0 * 1.02)(2.0 * 1.2)}{(2.0 * 1.02) + (2.0 * 1.2)} x10^{-6} = \underline{1.103\mu F}$$

c) The percent irror of the minimum value is

$$\frac{0.881 - 1.0}{1.0} x100 = \underline{-11.9\%}$$

The percent irror of the maximum value is

$$\frac{1.103 - 1}{1} x100 = \underline{10.3\%}$$

Problem 5.56

Select the value of L that produces a total inductance of L_T = 10mH in the circuit in Figure P5.56

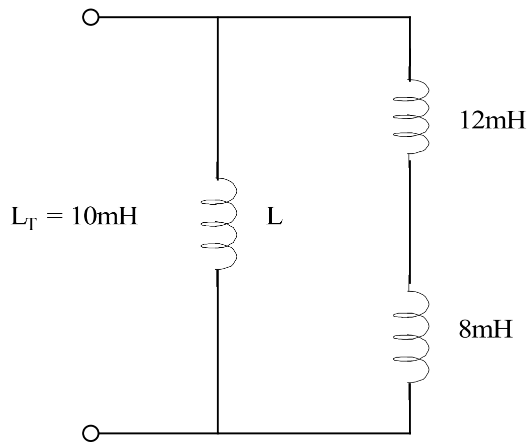
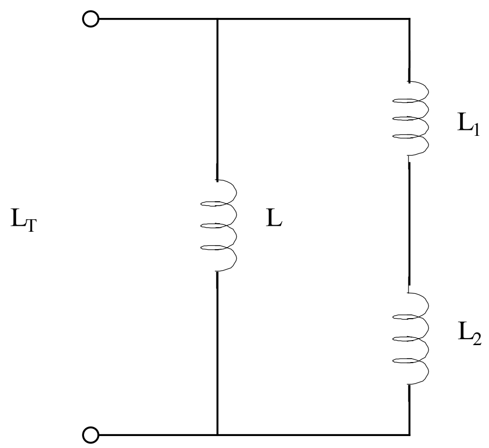


Figure P5.56

Suggested Solution



$L_T = (L_1 + L_2)\,L \;/\; (L_1 + L_2 + L)$

$10 = 20\,L / (20 + L)$

$200 + 10L = 20L$

$\underline{L = 20mH}$

Problem 5.57

Find the value of L in the network in Figure P5.57 so that the total inductance L_T will be 2 mH.

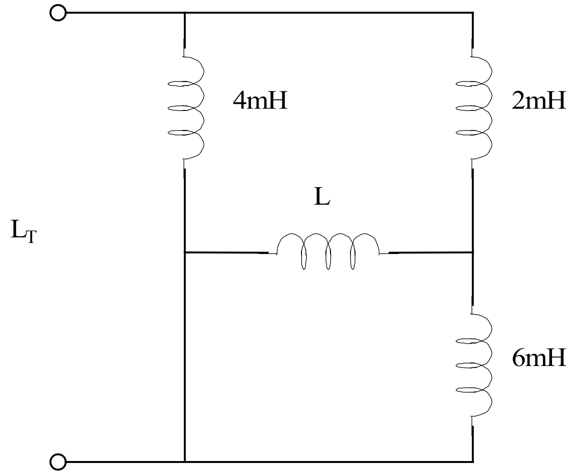
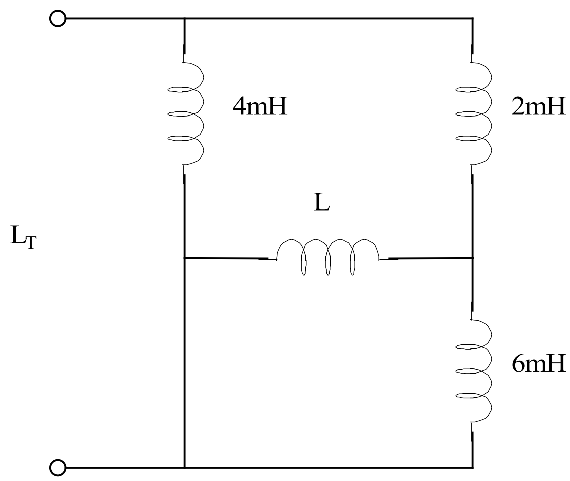


Figure P5.57

Suggested Solution



All L's in mH

$$\frac{[\frac{6L}{6+L}+2]4}{\frac{6L}{6+L}+2+4}=2$$

$$\frac{\frac{24L}{6+L}+8}{\frac{6L}{6+L}+6}=2$$

$$24L+8(6+L)=(6L+(6+L))2$$

$$24L+48+8L=12L+72+12L$$

$$8L=24$$

$$\boxed{L=3mH}$$

Problem 5.58

Find the value of L in the network in Figure 5.58 so that the value of L_T will be 2 mH.

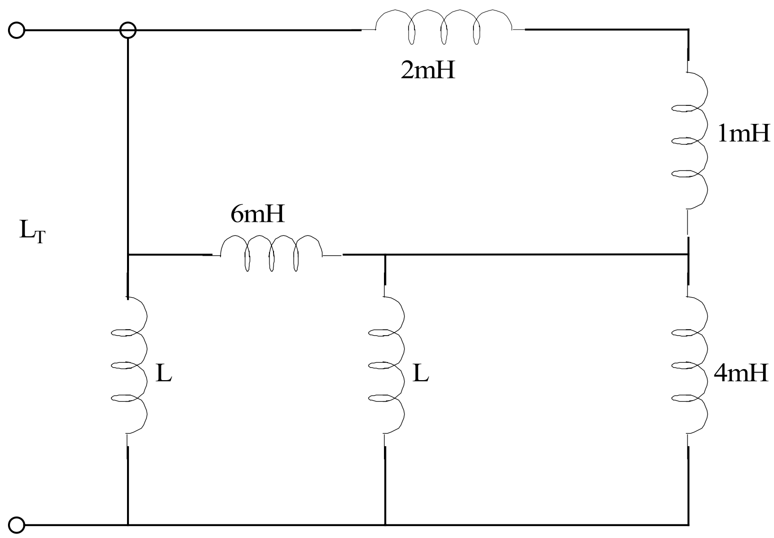
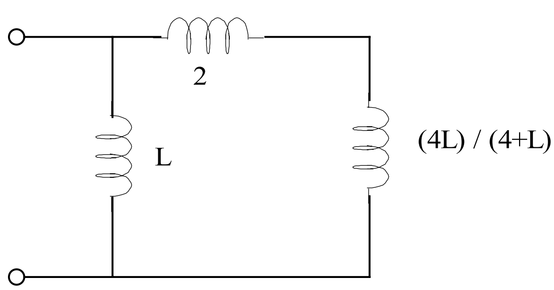


Figure P5.58

Suggested Solution



$$\frac{[\frac{4L}{4+L}+2]L}{\frac{4L}{4+L}+2+L}=2$$

$$6L^2+8L=2L^2+20L+16$$

$$(L+4)(L+1)=0$$

$$\boxed{L=4mH}$$

Problem 5.59

Determine the inductance at terminals A-B in the network in Figure P5.59.

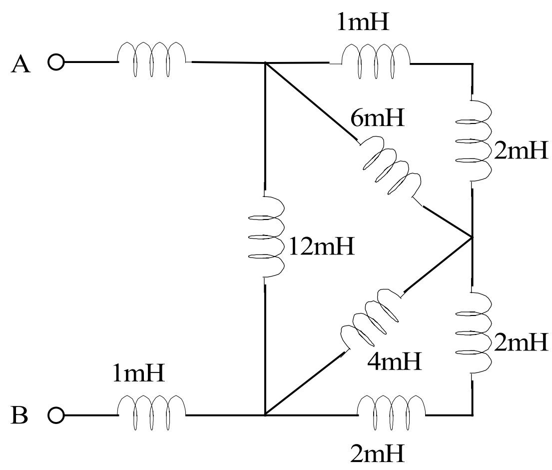
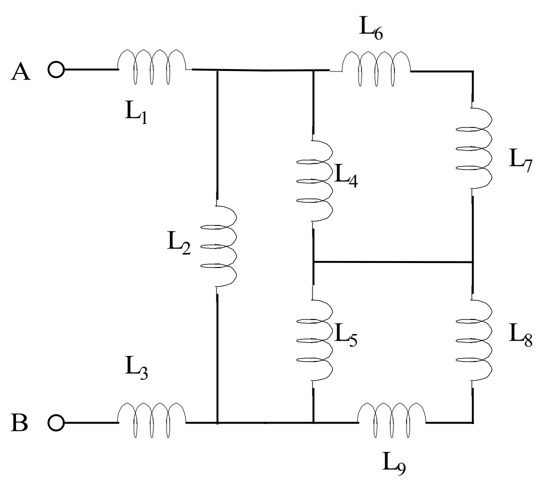


Figure 5.59

Suggested Solution



$L_1 = L_3 = L_6 = 1mH \quad L_2 = 12mH$

$L_4 = 6mH \quad L_5 = 4mH$

$L_7 = L_8 = L_9 = 2mh$

$$L_{eq1} = L_6 + L_7 = 3mH$$

$$L_{eq2} = L_4 L_{eq1} / (L_4 + L_{eq1}) = 2mH$$

$$L_{eq3} = L_8 + L_9 + 4mH$$

$$L_{eq4} = L_{eq3} L_5 / (L_{eq3} + L_5) = 2mH$$

$$L_{eq5} = \frac{L_2 (L_{eq2} + L_{eq4})}{L_2 + L_{eq2} + L_{eq4}} = 3mH$$

$$L_{AB} = L_1 + L_3 + L_{eq5} = 5mH$$

$$\boxed{L_{AB} = 5mH}$$

Problem 5.60

Compute the equivalent inductance of the network in Figure P5.60 if all inductors are 5 mH

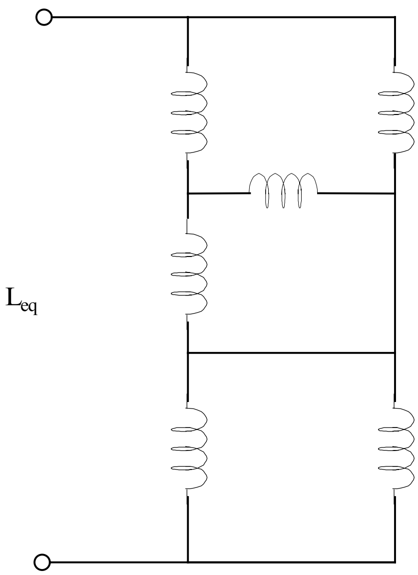
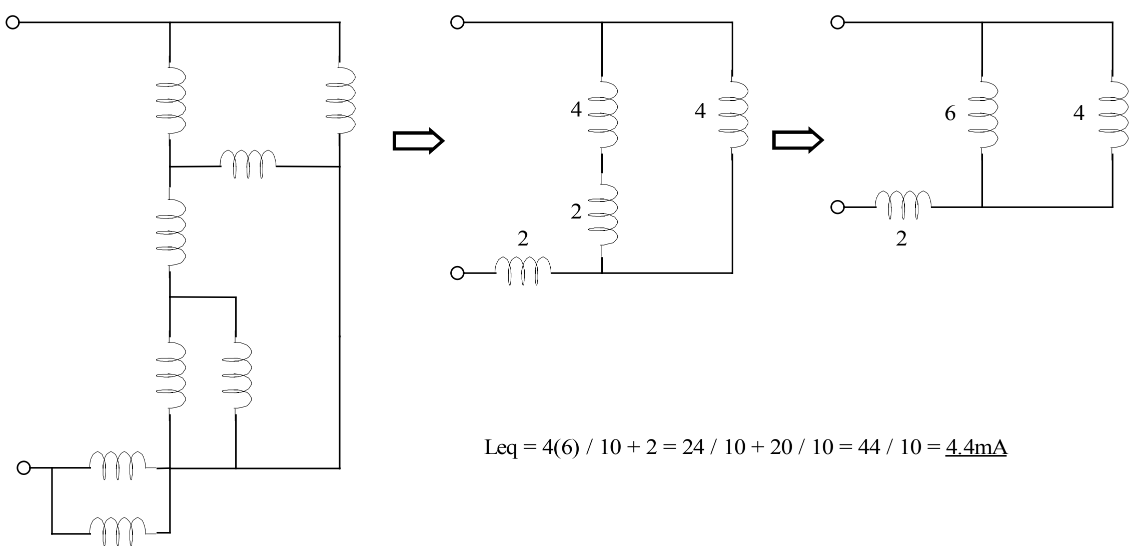


Figure 5.60

Suggested Solution

Redrawing the network



$L_{eq} = 4(6) / 10 + 2 = 24 / 10 + 20 / 10 = 44 / 10 = \underline{4.4mA}$

Problem 5.61

Determine the inductance at terminals A-B in the network in Figure P5.61

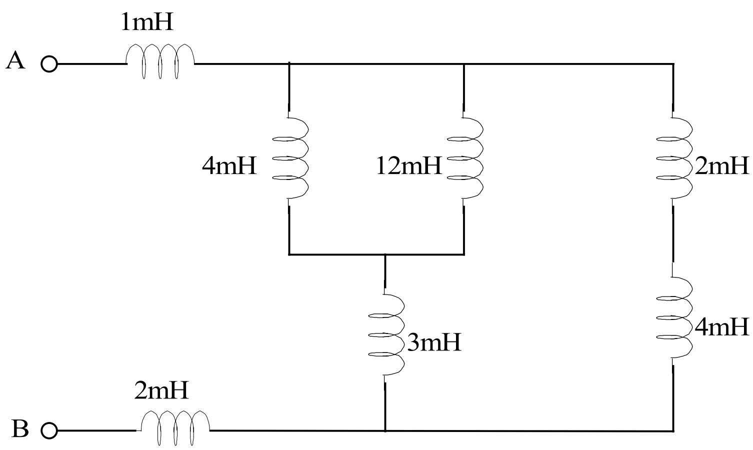
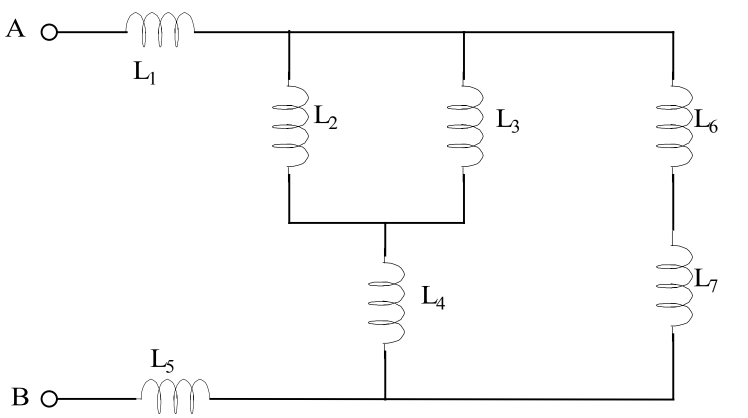


Figure 5.61

Suggested Solution



$L_1 = 1mH \quad L_2 = L_7 = 4mH \quad L_3 = 12mH$

$L_4 = 3mH \quad L_5 = L_6 = 2mH$

$$L_{eq1} = L_6 + L_7 = 6mH$$

$$L_{eq2} = L_2 L_3 /(L_2 + L_3) = 3mH$$

$$L_{eq3} = L_{eq2} + L_4 = 6mH$$

$$L_{eq4} = L_{eq1} L_{eq3} /(L_{eq1} + L_{eq3})$$

$$L_{eq4} = 3mH$$

$$L_{AB} = L_1 + L_{eq4} + L5$$

$$\boxed{L_{AB} = 6mH}$$

Problem 5.62

Find the total inductance at the terminals of the network in Figure P5.62

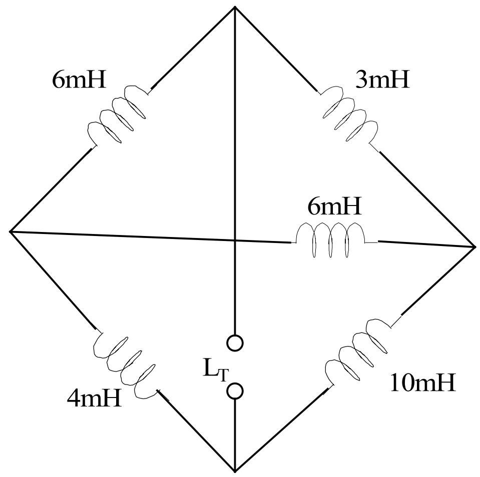
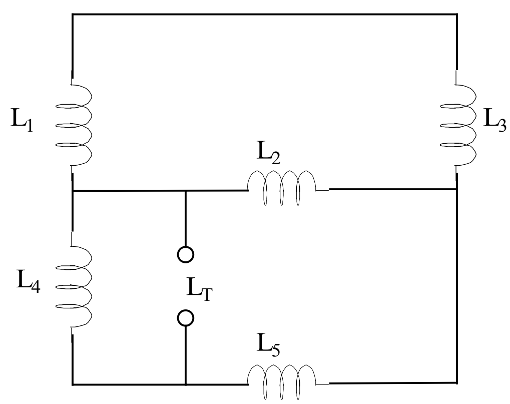


Figure 5.62

Suggested Solution



L_1 = 6mH

L_2 = 6mH

L_3 = 3mH

L_4 = 4mH

L_5 = 10mH

L_1 is shorted out!

$$L_{eq1} = L_2 L_3 /(L_2 + L_3) = 2mH$$

$$L_{eq2} = L_{eq1} + L_5 = 10m + 2m = 12mH$$

$$L_T = L_4 L_{eq2} /(L_4 + L_{eq2})$$

$$\boxed{L_T = 3mH}$$

Problem 5.63

Given the network shown in Figure P5.63 find (a) the equivalent inductance at terminals A-B with terminals C-D short circuited. And (b) the equivalent inductance at terminals C-D with terminals A-B open circuited.

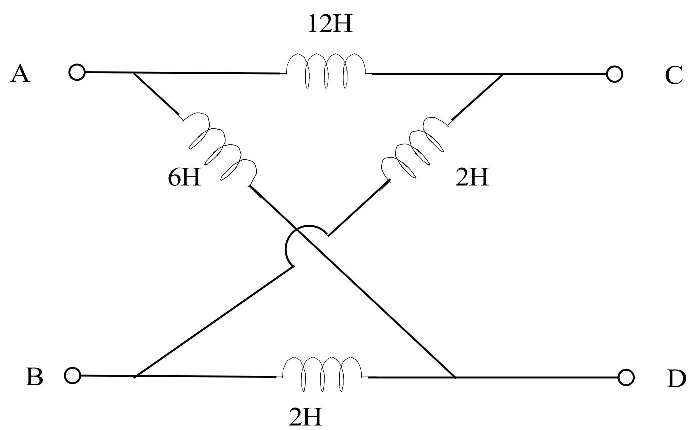
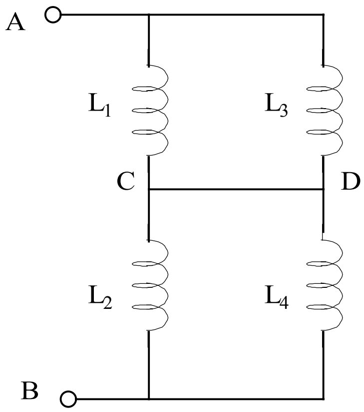


Figure P5.63

Suggested Solution



$L_1 = 12H$

$L_2 = 2H$

$L_3 = 6H$

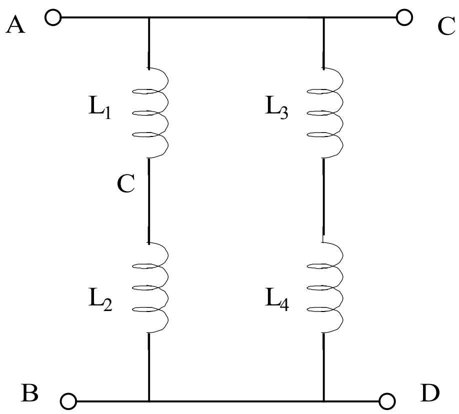
$L_4 = 2H$

$L_{eq1} = L_1L_3/(L_1 + L_3) = 4H$

$L_{eq2}=L_2L_4/(L_2 + L_4) = 1H$

$L_{AB} = L_{eq1} + L_{eq2}$

$\underline{L_{AB} = 5H}$



$L_{eq3} = L_1 + L_3 = 18H$

$L_{eq4} = L_2 + L_4 = 4H$

$1/L_{CD} = 1/L_{eq3} + 1/L_{eq4}$

$\underline{L_{CD} = 3.27\ H}$

Problem 5.64

For the network in Figure P5.64 choose C such that

$$v_o = -10\int v_s dt$$

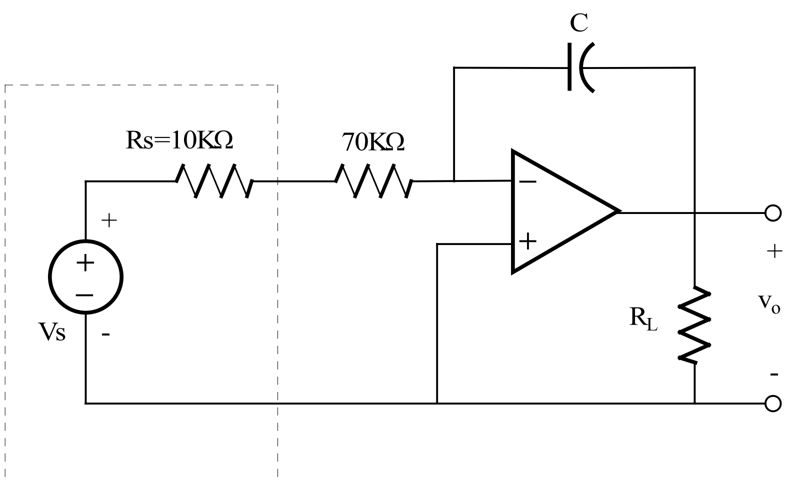
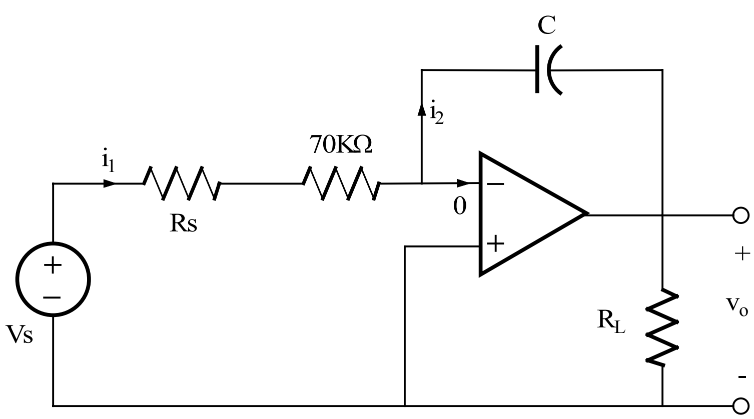


Figure P5.64

Suggested Solution



$Rs = 10K\Omega$

$v_o = -10\int v_s dt$

$\mathrm{R}_{eq} = Rs + 70K = 80K$

Using ideal op-amp assumptions,

$i_1 = i_2$

$$\frac{v_s - 0}{\mathrm{R}_{eq}} = -C\frac{dv_o}{dt} \Rightarrow v_o = -\frac{1}{\mathrm{R}_{eq}C}\int v_s dt$$

$So,$

$$\mathrm{R}_{eq}C = \frac{1}{10} \Rightarrow \boxed{C = 1.25\mu F}$$

Problem 5.65

For the network in Figure P5.65, vs(t)=120cos377t V. Find Vo(t)

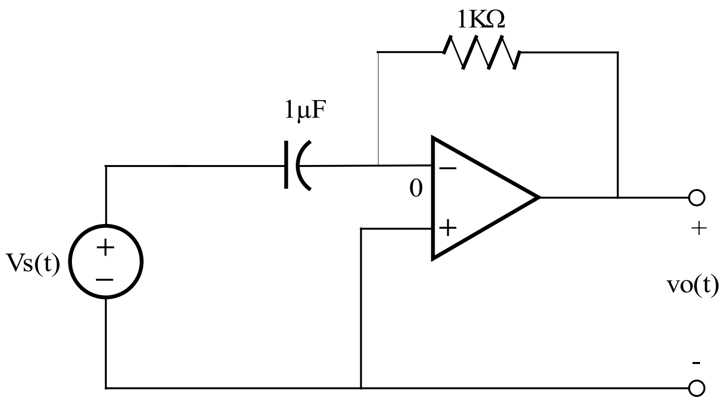
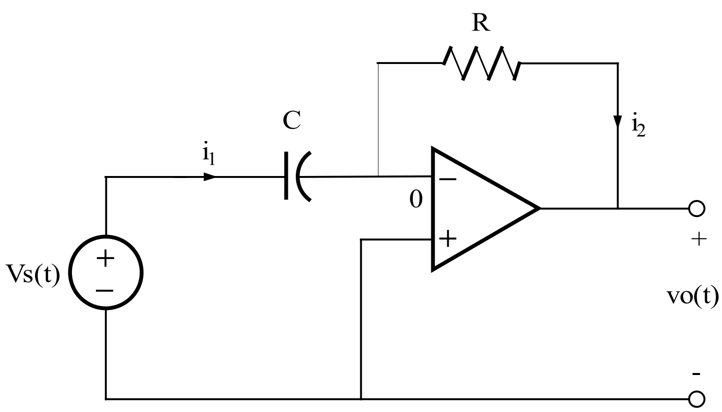


Figure P5.65

Suggested Solution



$C = 1\mu F \qquad R = 1K\Omega$

$v_s = 120\cos 377t$ V

$i_1 = i_2$ (ideal op-amp assumptions)

$$C\frac{dv_s}{dt} = -\frac{v_o}{R} \Rightarrow v_o = -RC\frac{dv_s}{dt}$$

$$v_o(t) = +(1k)(1\mu)(120)(377)\sin(377)t$$

$$\boxed{v_o(t) = 45.24\sin(377t)\ V}$$

Problem 5.66

For the network in Figure P5.66, $v_s(t)$=115sin377t V. Find $v_o(t)$

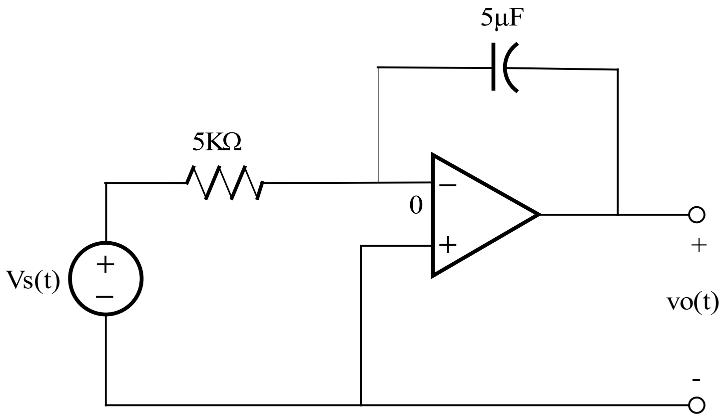
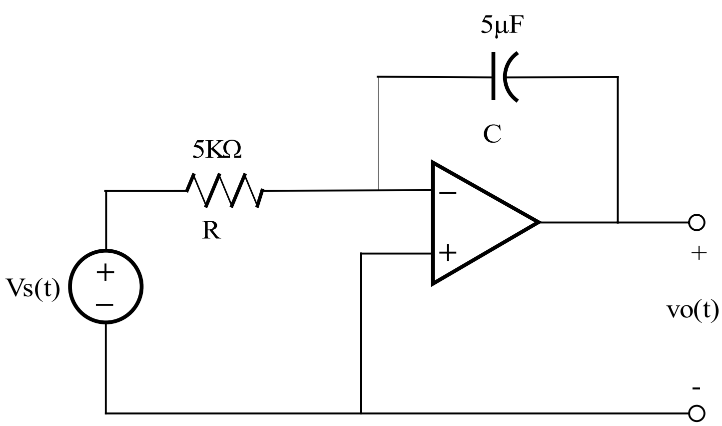


Figure P5.66

Suggested Solution



$$v_o(t) = -\frac{1}{RC}\int v_s dt$$

$$= \frac{115}{(5K)(5\mu)(377)}\cos(377t)$$

$$\boxed{v_o(t) = 12.20\cos(377t)\ V}$$

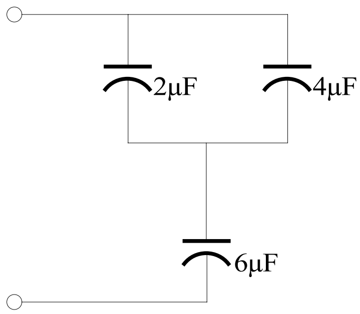
Problem 5FE-1

Given three capacitors with values 2μF, 4μF and 6μF, can the capacitors be interconnected so that the combination is an equivalent 3μF?

Suggested Solution

Yes.

3μF



Problem 5FE-2

The current pulse shown in Figure 5PFE-2 is applied to a 1μF capacitor. Determine the charge on the capacitor and the energy stored.

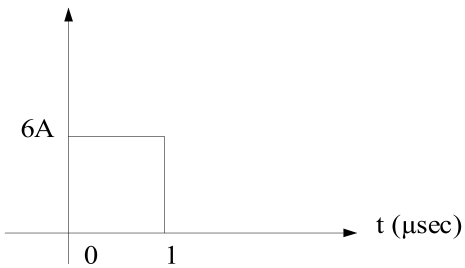


Figure PFE-2

Suggested Solution

The capacitor voltage is

$$v(t) = \frac{1}{C}\int_0^T i(t)dt = \frac{1}{10^{-6}}\int_0^{10^{-6}} 6dt = 6V$$

$$Q = CV = 10^{-6} \times 6 = 6\mu C$$

$$\underline{W} = \frac{1}{2}CV^2 = \frac{1}{2}(10^{-6})(6)^2 = \underline{18\mu J}$$

Problem 5FE-3

In the network shown in Figure 5PFE-3, determine the energy stored in the unknown capacitor Cx.

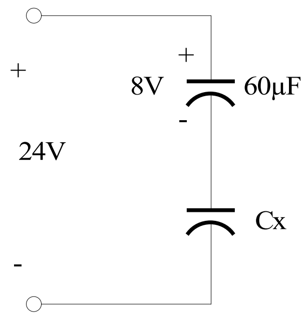
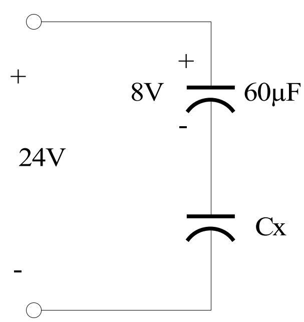


Figure PFE-3

Suggested Solution



$Q = CV = (60x10^{-6})(8) = 480\mu C$

$Vx = 24 - 8 = 16V$

$Cx = Q/Vx = (480x10^{-6}) / 16 = 30\mu F$

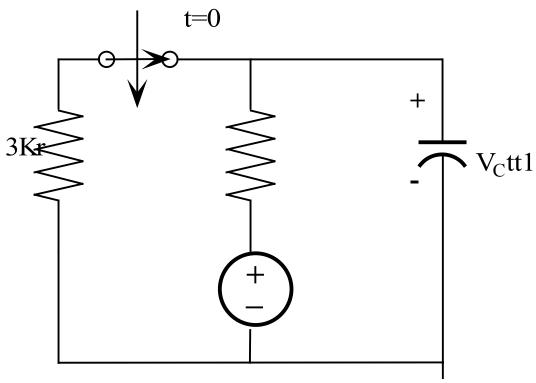
$W = 1/2\ CV^2$

$\quad = 1/2\ (30x10^{-6})\ (16)^2$

$\underline{W = 3.84mJ}$

Problem 6.1

Use the differential equation approach to find $V_C(t)$ for t>0 in the circuit in Fig. P6.1



Suggested Solution

$$V_c(0-) = 12(\frac{3k}{9k}) = 4V$$

$$\frac{V_c(t)-12}{6k} + \frac{cdv_c(t)}{dt} = 0$$

$$OR$$

$$\frac{dV_c(t)}{dt} + \frac{V_c(t)}{0.6} = 12$$

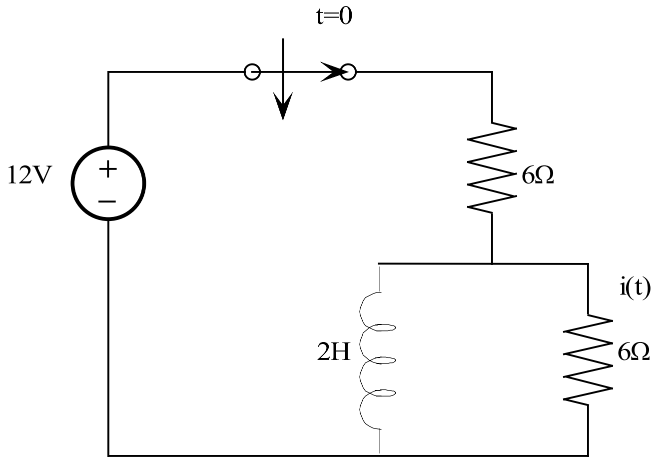
$$V_c(t) = k_1 + k_2 e^{-t/0.6}$$

$$Att = ok_1 + k_2 = 4 \; and \; at \; t = \infty k_1 = 12 \therefore k_2 = -8$$

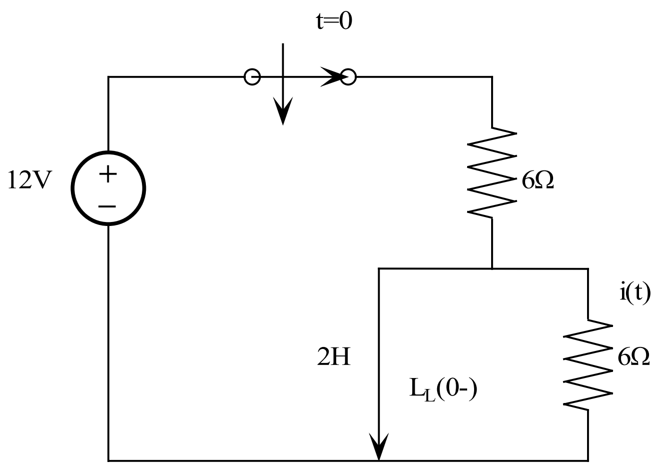
$$Hence V_c(t) = 12 - 8e^{-t/0.6} Vt > 0$$

Problem 6.2

Use the differential equation approach to find i(t) for t > 0 in the network in Fig P6.2 as shown.



Suggested Solution



$$i_L(0-) = 2A$$

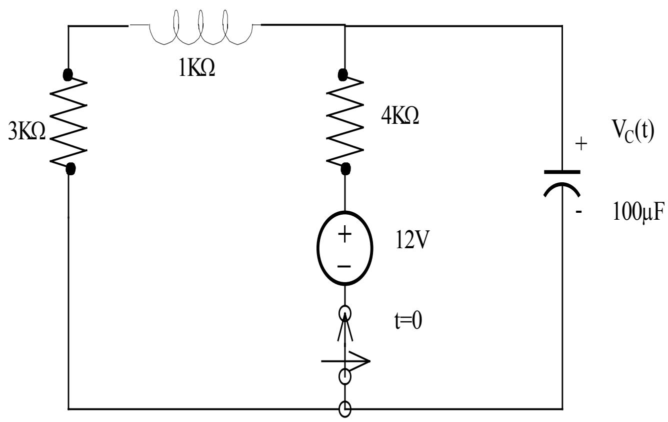
$$Ld\frac{i(t)}{dt} + Ri(t) = 0$$

$$i(t) = K_2 e^{-R/2t} = K_2 e^{-3t}$$

$$i(0) = 2 = K2 \therefore \text{ i(t)=2e}^{-3t} A \quad \text{t>0}$$

Problem 6.3

Use the differential equation approach to find $V_C(t)$ for t>0 in the circuit in Fig. P6.3



Suggested Solution

$$V_C(0-) = 12(\frac{4K}{8K}) = 6V$$

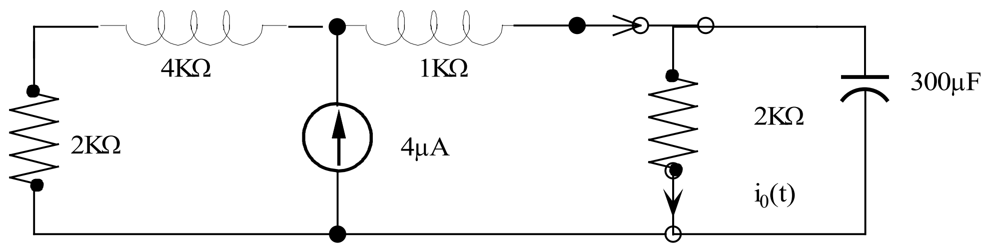
$$\frac{V_C(t)}{4K} + Cd\frac{V_C(t)}{dt} = 0 \quad \Rightarrow \; \mathrm{d}\frac{V_C(t)}{\mathrm{dt}} + \frac{V_C(t)}{0.4} = 0$$

$\therefore V_C(t) = K_2 e^{-t/0.4} V$ and since $V_C(0) = 6 = K_2$

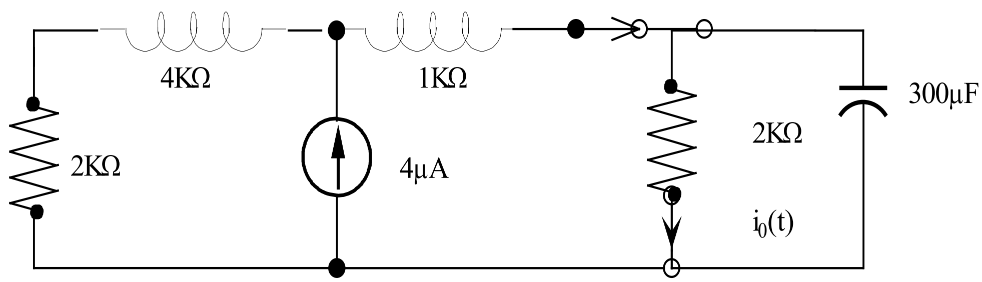
$V_C(t) = 6e^{-t/0.4} V$ t>0

Problem 6.4

Use the differential equation approach to find $i_0(t)$ for t>0 in the network in Fig P6.4



Suggested Solution



$$i_0(0-)\ \frac{6}{\mathrm{K}}(\frac{6K}{9K}) = \frac{4}{K}A$$

$$V_C(0-) = 2K(\frac{4}{K}) = 8V$$

$$for\ t > 0 \qquad \mathrm{C}\frac{\mathrm{dV_c}(t)}{dt} + \frac{\mathrm{V_c}(t)}{2K} = 0$$

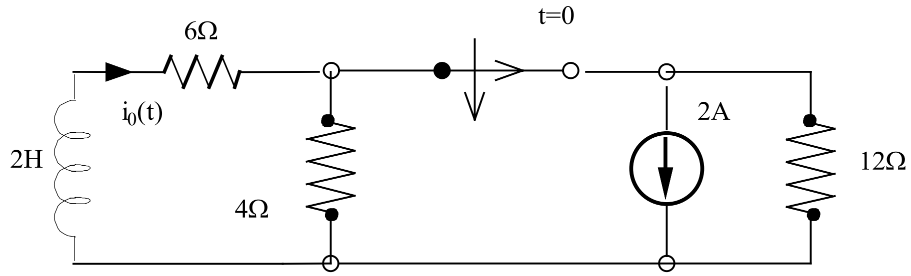
$$\frac{\mathrm{dV_c}(t)}{dt} + \frac{\mathrm{V_c}(t)}{0.6} = 0$$

$$\mathrm{V_c}(t) = K_2 e^{-t/0.6} V$$

$$\mathrm{V_c}(t) = 8 = K_2 \quad \therefore \quad \mathrm{V_c}(t) = 8^{-t/0.6} V \quad \text{t>o}$$

Problem 6.5

In the network in Fig P6.5, find $i_0(t)$ for $t>0$ using the differential equation approach.



Suggested Solution

$$i_L(0^-) = 2\left(\frac{3}{3+6}\right) = \frac{2}{3} \text{ A} \quad \text{using current division}$$

$$L\frac{di_o(t)}{dt} + 10i_o(t) = 0 \qquad 10 = 6+4$$

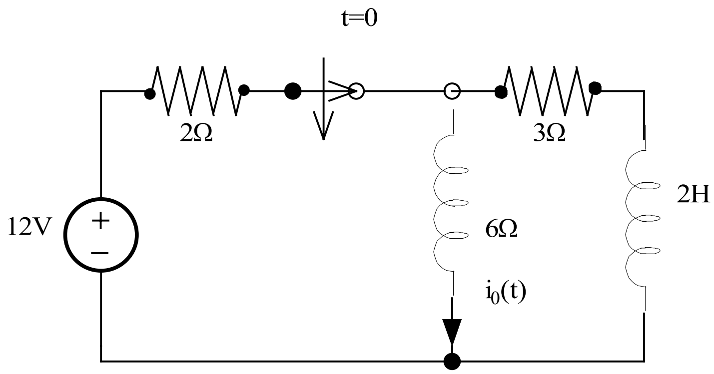
$$\therefore\ i_o(t) = K_2 e^{-5t} \text{ A} \qquad 5 = \frac{10}{L} = \frac{10}{2}$$

So, $i_o(0) = \frac{2}{3} = K_2$

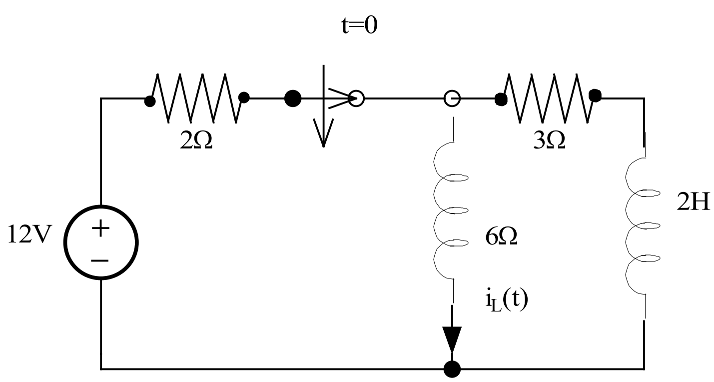
Then, $i_o(t) = \frac{2}{3}e^{-5t}$ A, $t > 0$

Problem 6.6

In the circuit in Fig P6.6, find $i_0(t)$ for t>0 using the differential equation approach.



Suggested Solution



$$i_L(0-) = \frac{12}{2+6/13}\left(\frac{6}{3+6}\right) = 2A$$

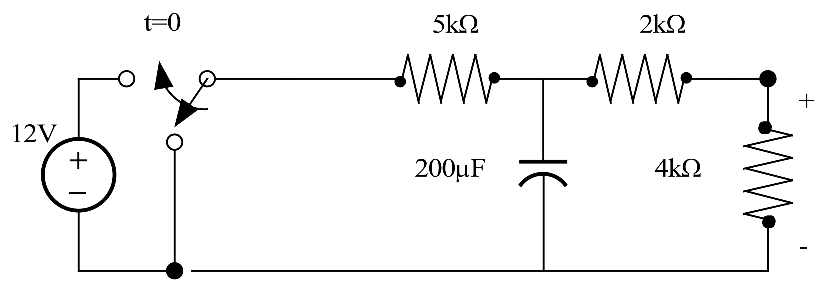
$$for\ t>0 L\ \frac{di_L(t)}{dt} + Ri_L(t) = 0 \ \text{ or } \ \frac{\mathrm{di_L}(t)}{c(t)} + \frac{9}{2}i_L(t) = 0$$

$$i_L(t) = K_2 e^{-4.5t} A \qquad \mathrm{i_L}(0) = 2 = K_2$$

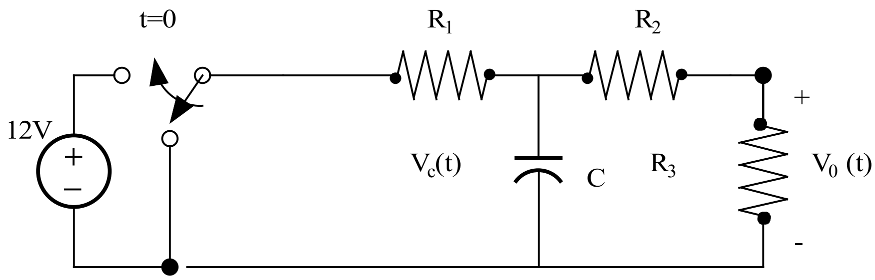
$$i_L(t) = 2e^{-4.5t} A \ \text{ but } \mathrm{i}_0(t) = -i_L(t) = -2e^{-4.5t} A \ \ \text{t>0}$$

Problem 6.7

Use the differential equation approach to find $V_0(T)$ for t>0 in the circuit in Fig P6 and plot the response including the time interval just prior to switch action.



Suggested Solution



$$For\ t<0,\ V_C(0-) = V_C(0+) = 0\ and\ V_0(0-) = 0V$$

$$R_1 = 5K\Omega \quad R_2 = 2K\Omega$$

$$R_3 = 4K\Omega \ \text{C=0.2mF}$$

For $t \geq 0$

$$V_C(0+) = 0V \qquad V_0(0+)R_3 = 0V$$

$$By\ KCL: \quad \frac{12\text{-}V_2}{R_1} = \frac{V_2}{R_2 + R_3} + C\ \frac{dV_2}{dt}$$

$$By\ KVL: \quad V_0(t) = V_2\left(\frac{R_3}{R_2 + R_3}\right) \qquad 12 = V_2\left[1 + \frac{R_1}{R_2 + R_3}\right] + R_C\ \frac{dV_2}{dt}$$

$$or\ V_2 = 1.5\ V_0(t) \quad \Rightarrow \quad \frac{dV_0}{dt} + \frac{11}{6}V_0 - 8 = 0 \quad \text{Eq.1}$$

Assume,

$$V_0(t) = K_1 + K_2 e^{\frac{-t}{}} - 8 = 0 \quad \Rightarrow \quad \{K_1 = \frac{48}{11}\ \ V$$

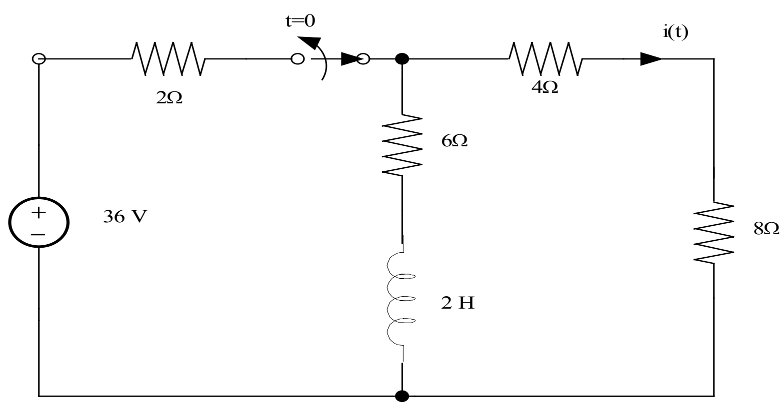
$$\text{Now,} \quad V_0(t) = \frac{48}{11} + K_2 e^{\frac{-11t}{6}} V$$

$$V_0(0+) = 0 = \frac{48}{11} + K_2 \quad \Rightarrow \quad K_2 = \frac{-48}{11}$$

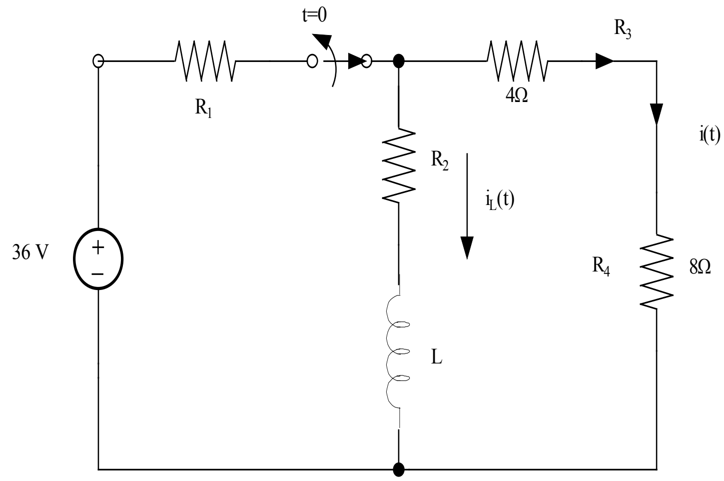
$$V_0(t) = \frac{48}{11}\left(1 - e^{\frac{-11t}{6}}\right)V \quad t>0 \ \text{ and } V_0(t) = 0 \quad t<0$$

Problem 6.8

Use the differential equation approach to find i(t) for t > 0 in the circuit in Fig. P8 and plot the response including the time interval just prior to opening the switch.



Suggested Solution



$R_1 = 2\Omega$ For t<0

$R_2 = 6\Omega$ $i_L(0\text{-}) = i_L(0+) = 4A$

$R_3 = 4\Omega$ $i(0\text{-}) = 2A$

$R_4 = 8\Omega$

$L = 2H$

By KVL: $i(t)[R_2 + R_3 + R_4] + L \dfrac{d_i(t)}{dt} = 0$

or, $\dfrac{di(t)}{dt} + 9i(t) = 0$

Assume $i(t) = K_1 + K_2 e^{\frac{-t}{}}$

Now, $\dfrac{-K_2}{} e^{\frac{-t}{}} + 9K_1 + 9K_2 e^{\frac{-t}{}} = 0$

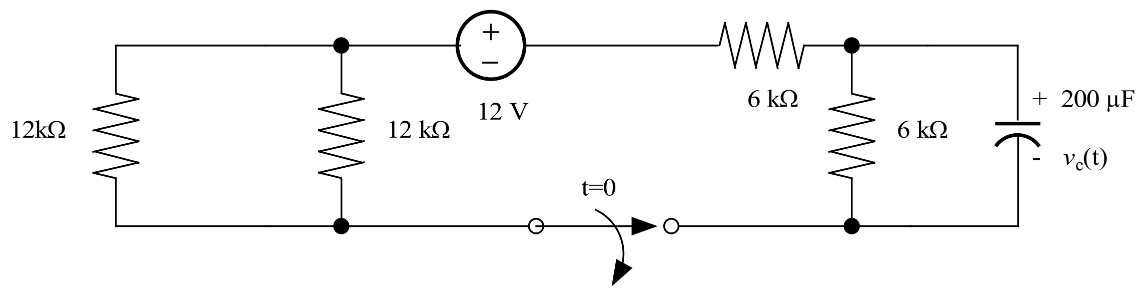
$So, K_1 = 0A \; and \; \tau = \dfrac{1}{9}$ sec. Also, $I(0+) = LK_1 + K_2 = -4 \Rightarrow K_2 = -4$

$i(t) = -4e^{-9t}$ t>0

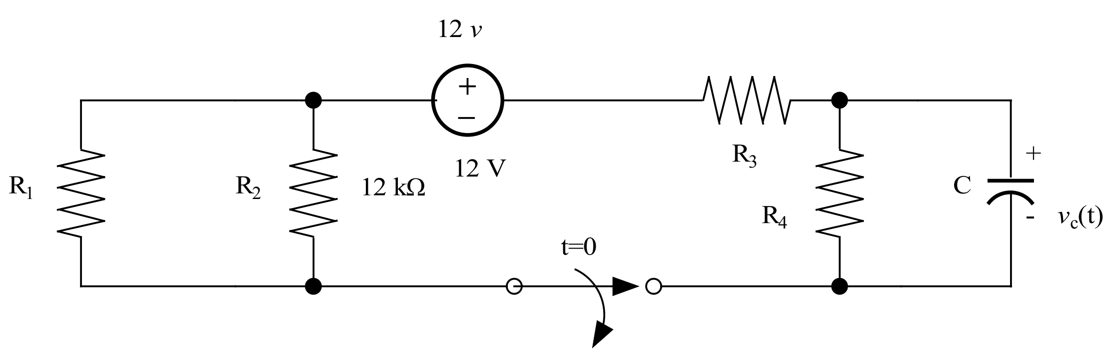
$i(t) = 2$ t<0

Problem 6.9

Use the differential equation approach to find $v_c(t)$ for $t > 0$ in the circuit in Fig P6.9 and plot the response including the time interval just prior to opening the switch.

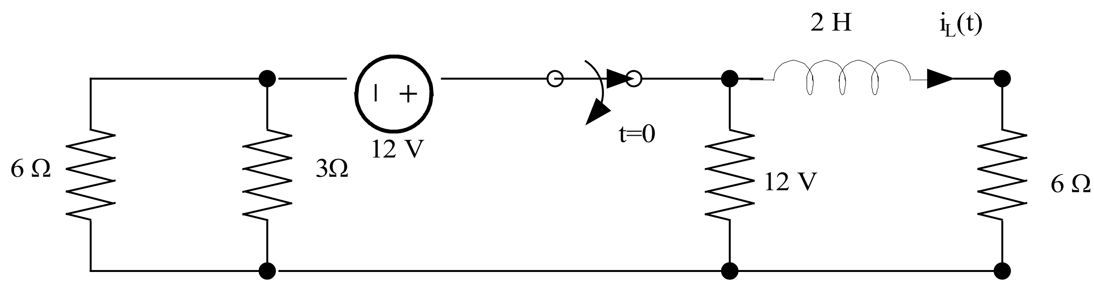


Suggested Solution



Problem 6.10

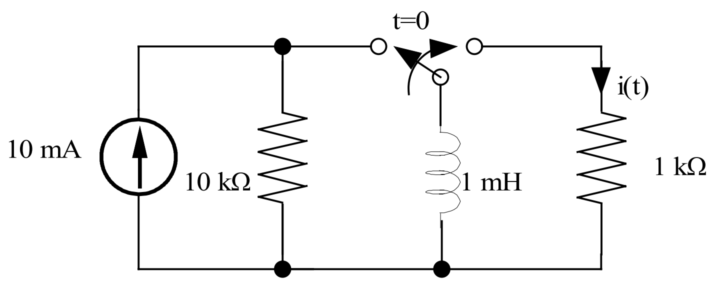
Use the differential equation approach to find I_L (t) for t > 0 in the circuit in Fig. P6.10 and plot the response including the time interval just prior to opening the switch.



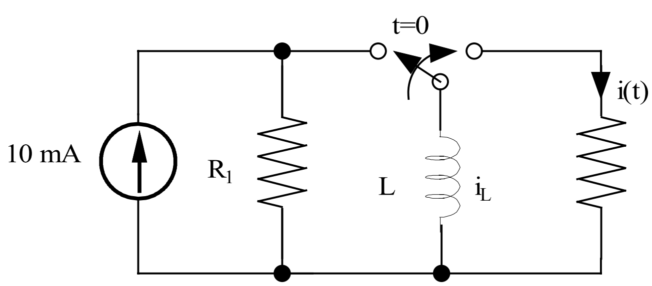
Suggested Solution

Problem 6.11

Use the differential equation approach to find i(t) for t>0 in the circuit in Fig. P6.4 and plot the response including the time interval just prior to switch movement.



Suggested Solution



$R_1 = 10k\Omega$

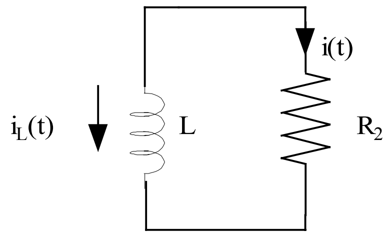
$R2 = 1k\Omega$

$L = 1\ mH$

At $t=0^-$ and $t=0^+$

$i_L(0^-) = i_L(0^+) = 10$ mA

$i(0^-) = 0A$



$i_L(0^+)$=-$i_L(0^+)$=-10mA

By KVL: $R_2 i(t) + L\dfrac{di(t)}{dt} = 0$

or, $\dfrac{di(t)}{dt} + \dfrac{R_2}{L}\ i(t) = 0$

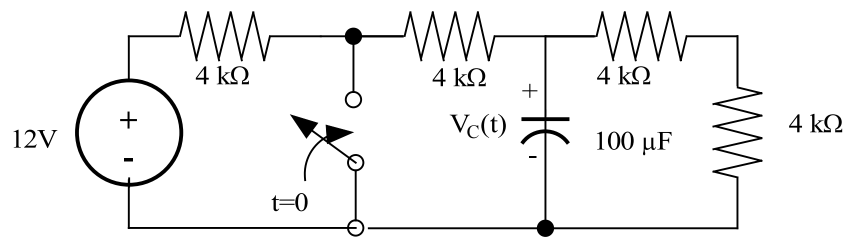
Assuming $i(t) = K_1 + K_2 e^{\frac{-t}{\tau}}$

$$Now, -\frac{K_2}{\tau} e^{\frac{-t}{\tau}} + \frac{R_2 K_1}{L} + \frac{R_2 K_2}{L} e^{\frac{-t}{\tau}} = 0 \Rightarrow \begin{cases} K_1 = 0 \\ \tau = \dfrac{L}{R_2} = 0.1\mu s \end{cases}$$

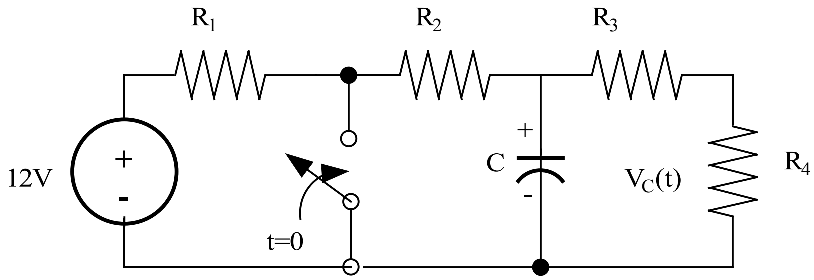
$i(t) = -10e^{-10^7 t}$ mA t>0 and i(t)=0 t<0

Problem 6.12

Use the differential equation approach to find $V_C(t)$ for t >0 in the circuit in Fig. P6.12 and plot the response including the time interval just prior to closing the switch.



Suggested Solution



All R=4kΩ $\quad V_C(0^-) = V_C(0^+) = 12\left[\dfrac{R_3+R_4}{R_1+R_2+R_3+R_4}\right]$

$C = 0.1$ mF

$$V_C(0) = 6V$$

For $t > 0$ By KCL: $\dfrac{V_C(t)}{\frac{8}{3}k} + C\,\dfrac{dV_C(t)}{dt} = 0$

$$\frac{dV_C(t)}{dt} + \frac{15}{4}V_C(t) = 0$$

Assume, $V_C(t) = K_1 + K_2 e^{\frac{-t}{\tau}}$

$$\frac{-K_2}{\tau}e^{\frac{-t}{\tau}} + \frac{15}{4}K_1 + \frac{15}{4}K_2 e^{\frac{-t}{\tau}} = 0$$

$$K_1 = 0 \qquad \tau = \frac{4}{15}\text{sec.}$$

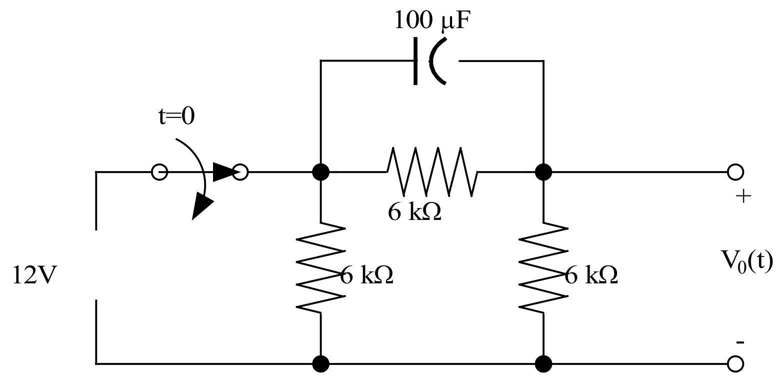
Now,

$V_C(t) = K_2 e^{\frac{-t}{\tau}} \qquad V_C(0+) = K_2 = 6$ so,

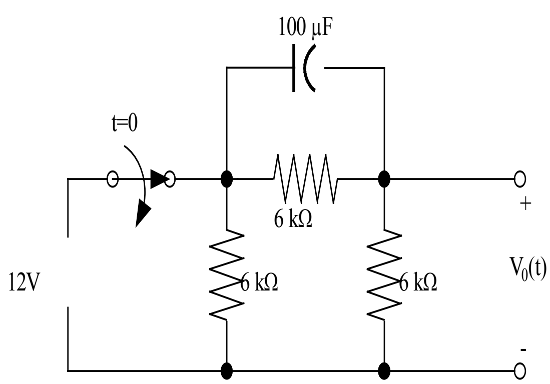
$$V_C(t) = 6e^{-(\frac{15t}{4})} \text{ v for } t > 0 \quad V_C(t) = 6 \textit{ for } t < 0$$

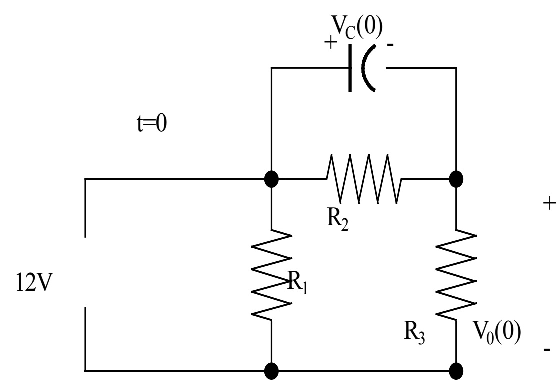
Problem 6.13

Use the differential equation approach to find $V_c(t)$ for $t > 0$ in the circuit in Fig. P.13 and plot the response including the time interval just prior to opening the switch.



Suggested Solution





$R_1 = 6k\Omega$ For t < 0

$R_2 = 6k\Omega$ $Vc(0^-)=V_C(0^+)$

$R_3 = 6k\Omega$ $V_C = 12\left[\dfrac{R_2}{R_2+R_3}\right]$

$C = 0.1mF$ $V_C(0) = 6V$

$V_C(0^-) = 6V$

At t > 0

By KCL: $C\,\dfrac{dV_c}{dt} + \dfrac{V_C}{R_2} + \dfrac{V_C}{R_1+R_3} = 0$

or, $\dfrac{dV_C(t)}{dt} + 2.5V_C(t) = 0$

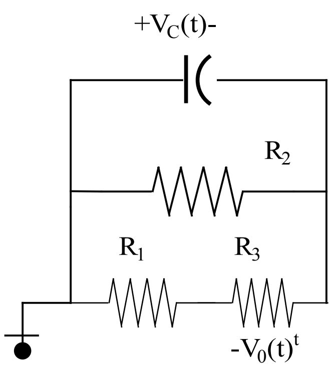
Assuming $V_C(t) = K_1 + K_2 e^{\frac{-t}{\tau}}$, *we find*

$-K_2 e^{\frac{-t}{\tau}} + 2.5K_1 + 2.5K_2 e^{\frac{-t}{\tau}} = 0 \;\Rightarrow\; K_1 = 0$ and $\tau = 0.45$

$V_C(0^+) = 6V = K_2 e^{\frac{-t}{\tau}} = K_2$ so $V_C(t) = 6e^{-2.5t}V$

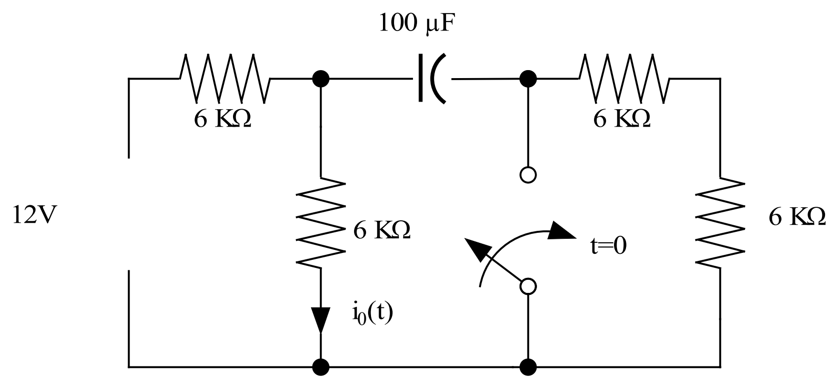
But, $V_0(t) = -V_C(t)\dfrac{[R_3]}{R_1+R_3}$ so, $V_0(t) = -3e^{-2.5}tV \quad t \geq 0$

$V_0(t) = 6V \quad t < 0$

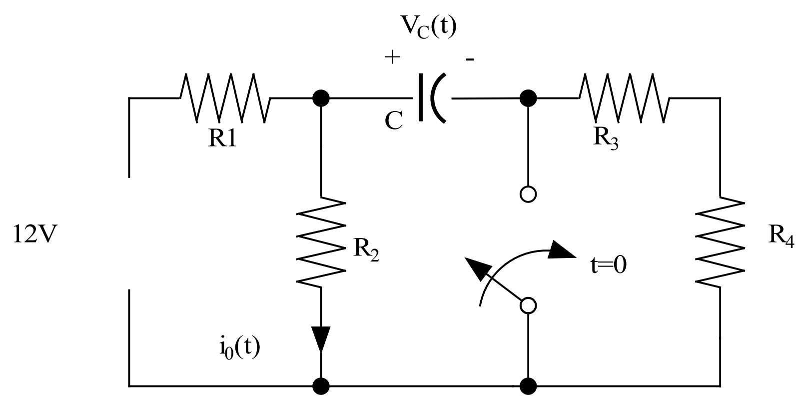


Problem 6.14

Use the differential equation approach to find $i_0(t)$ for $t > 0$ in the circuit in Fig. P6.14 and plot the response including the time interval just prior to closing the switch.



Suggested Solution



$All \;\; R = 6K\Omega$ For t < 0

C=0.1mF

$$i_0(0^-)=\frac{12}{R_1+R_2}=1mA$$

$$V_C(0^-)=V_C(0^+)$$
$$=i_0(0^-)R_2$$
$$=6V$$

For t > 0

$$\text{KCL:} \quad \frac{12-V_C(t)}{R_1}=\frac{V_C(t)}{R_2}+\frac{C\,dVC(t)}{dt}$$

$$\text{or,} \quad \frac{dV_C(t)}{dt}+\frac{10}{3}V_C(t)=20$$

Assume $VC(t)=K_1+K_2e^{\frac{-t}{\tau}}$. Now, $\frac{-K_2e^{\frac{-t}{\tau}}}{\tau}+\frac{10K_1}{3}+\frac{10K_2e^{\frac{-t}{\tau}}}{3}=20$

So, τ=0.3 s and K_1=6. Yields $V_C(t)=6+K_2e^{\frac{-t}{\tau}}$

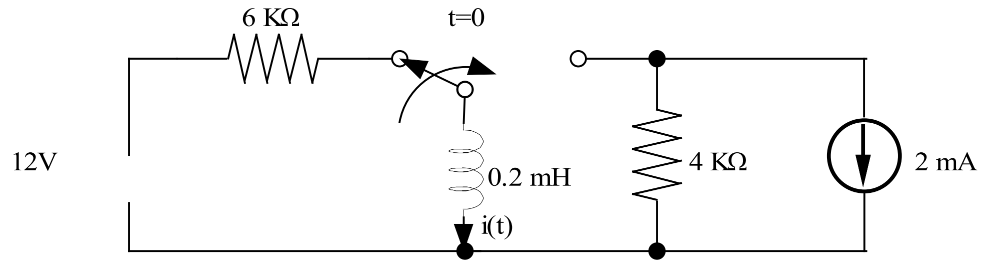
But $V_C(0^+)=6=6+K_2e^0=6+K_2 \Rightarrow K_2=0!$

$V_C(t)=6V$.

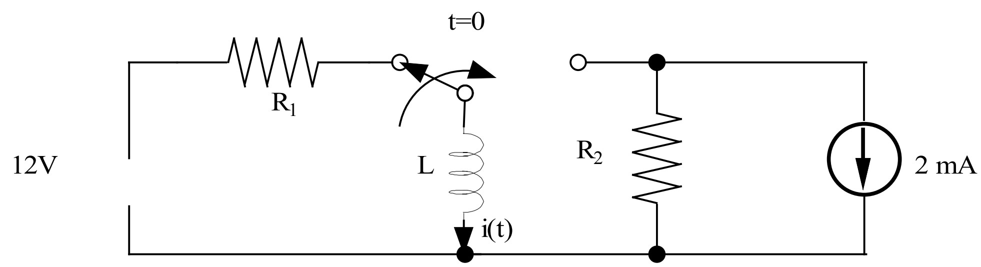
Note that $i_0(t)=\frac{V_C(t)}{R_2}=1mA$ $\boxed{i_0(t)=1mA \text{ for all } t}$

Problem 6.15

Use the differential equation approach to find i(t) for t > 0 in the circuit in Fig. P.16 and plot the response including the time interval just prior to switch movement.



Suggested Solution



$R1 = 6K\Omega$ when t < 0,

R2=4KΩ $i(0^-)=i(0^+)=\frac{12}{R_1}=2mA$

L=0.2mH

For t > 0 KVL: $L\frac{di_1(t)}{dt}+R_2(i_1-i_2)=0$ and $i_2=2mA$

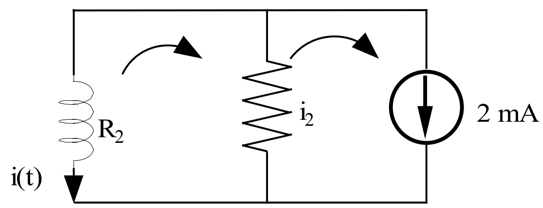
$$\frac{di_1(t)}{dt}+\frac{R_2}{L}i_1=\frac{R_2}{L}i_2$$

Assume $i_1(t)=K_1+K_2e^{\frac{-t}{\tau}}$, now, $\frac{-K_2}{\tau}e^{\frac{-t}{\tau}}+\frac{R_2}{L}K_1+\frac{R_2}{L}K_2e^{\frac{-t}{\tau}}=\frac{R_2}{L}i_2$

yields $\tau=\frac{L}{R_2}=50ns \text{ and } K_1=2mA.$

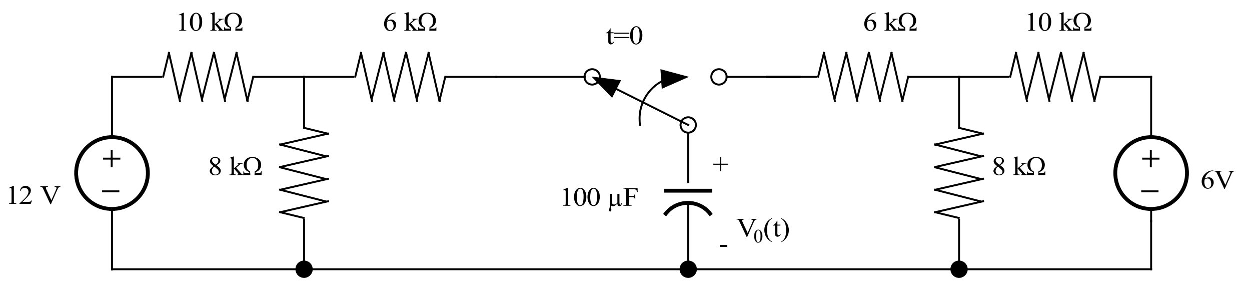
$i_1(0)=-i(0)=-2mA=K_1+K_2 \Rightarrow K_2=-4mA.\ \textit{Also},\ i(t)=-i_1(t)$

$$\boxed{i(t)=\left(4e^{-2x10^6 t}-2\right)mA \text{ for } t\geq 0 \text{ and } i(t)=2mA \quad t<0}$$

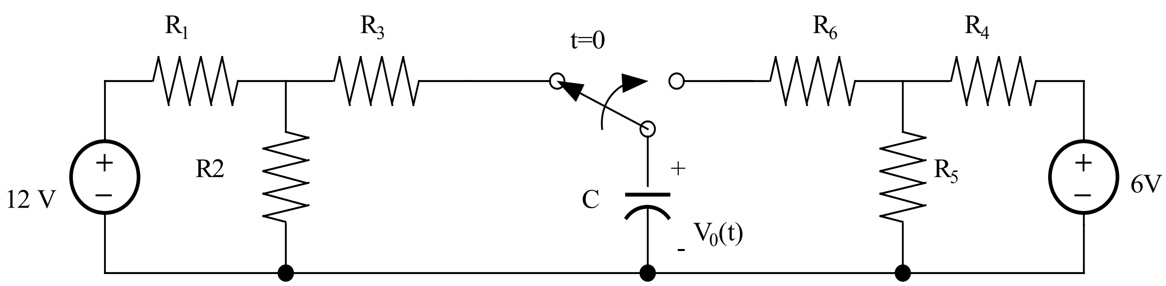

R2
i(t)
i2
2 mA

Problem 6.16

Use the differential equation approach to find $V_0(t)$ for $t > 0$ in the circuit in Fig. P.16 and plot the response including the time interval just prior to switch action.

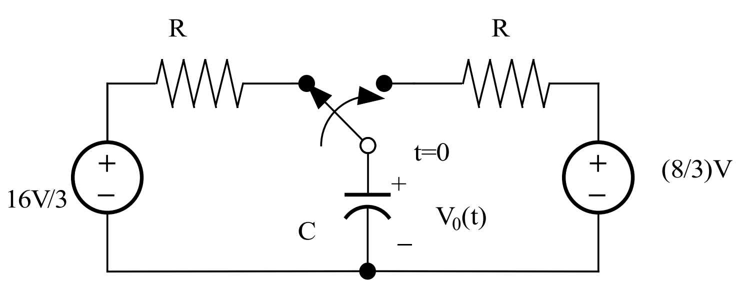


Suggested Solution



$R_1 = R_4 = 10k\Omega$ $\quad C = 0.1mF$
$R_2 = R_5 = 8k\Omega$
$R_3 = R_6 = 6k\Omega$

Perform 2 Thevenin equivalents, one at each side of the switch,



For t > 0

$$R = \frac{94}{9} k\Omega$$

By KCL: $\dfrac{\frac{8}{3} - V_0}{R} \quad C \; \dfrac{dV_0}{dt}$

Assume, $V_0(t) = K_1 + K_2 e^{\frac{-t}{\tau}}$

$$Now, \; \frac{8}{3} - K_1 - K_2 e^{\frac{-t}{\tau}} = -\left(\frac{9.4K_2}{9\tau}\right)e^{\frac{-t}{\tau}} \Rightarrow K_1 = \frac{8}{3} \text{ and } \tau = \frac{9.4}{9}$$

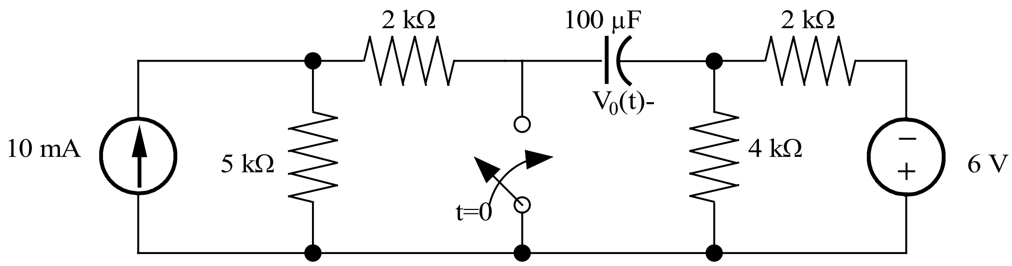
$$V_0(0^+) = \frac{16}{3} = K_1 + K_2 \Rightarrow K_2 = \frac{8}{3}$$

$$V_0(t) = 8\left[1 - e^{-0.96t}\right] V, \quad t \geq 0$$

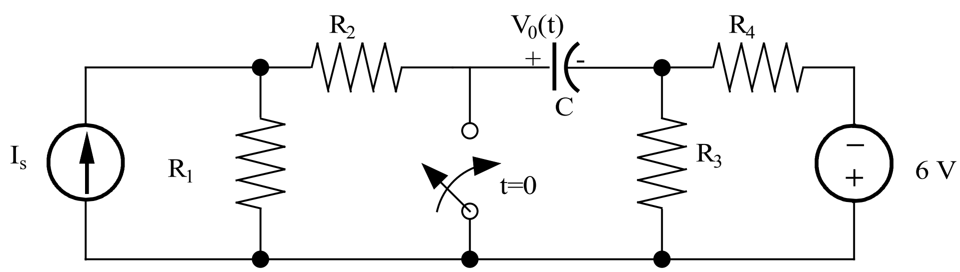
$$V_0(t) = \frac{16}{3} \quad V, \quad t < 0$$

Problem 6.17

Use the differential equation approach to find $V_0(t)$ for $t > 0$ in the circuit in Fig. P.17 and plot the response including the time interval just prior to closing the switch.



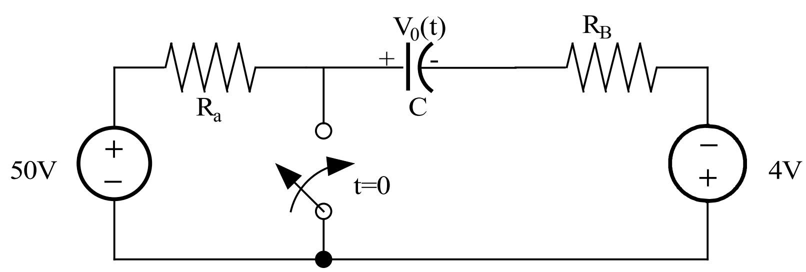
Suggested Solution



R_1=5kΩ R_3=4kΩ C=0.1 mF

R2=2kΩ R_4=2kΩ I_s=10 mA

Perform 2 Thevenin equivalents-one at each side (left/right) of the switch.



R_a=7kΩ

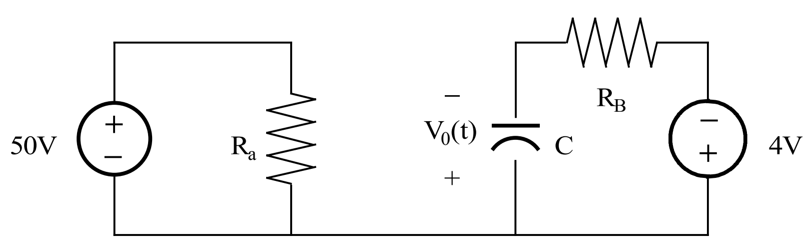
R_B=4/3 kΩ

For t < 0

No current flow,

$V_0(0\text{-})=V_0(0^+)=54V$

For t > 0



By KCL: $C\ \frac{dV_0}{dt}+\frac{-4-V_0(t)}{R_B}=0$

or, $\frac{dV_0}{dt}-7.5V_0(t)=30$

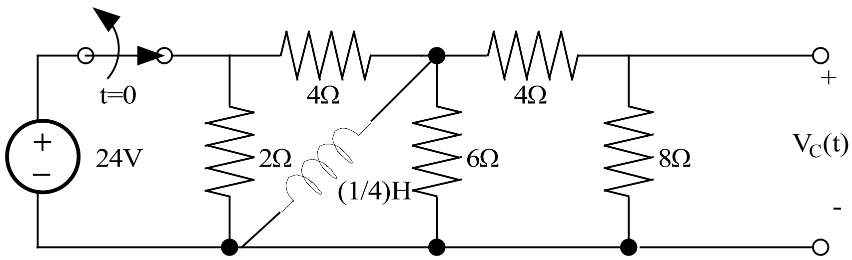
If $V_0(t)=K_1+K_2e^{\frac{-t}{\tau}}$, $-K_2e^{\frac{-t}{\tau}}-7.5K_1-7.5K_2e^{\frac{-t}{\tau}}=30$

yields, $\tau=\frac{1}{7.5}$ *and* $K_1=-4$. Nor, $V_0(0^+)=54=K_1+K_2 \Rightarrow K_2=58$

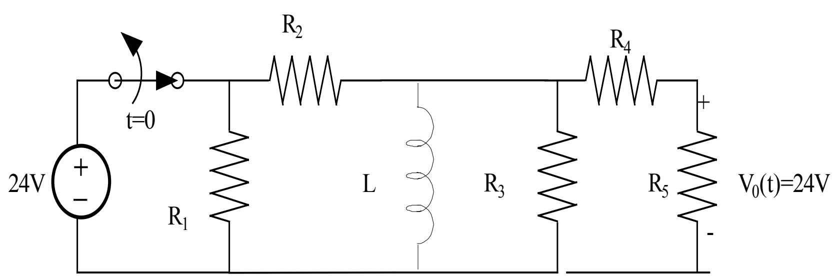
$$\boxed{\begin{array}{ll} V_0(t)=-4+58e^{-7.5t}V & t\ \geq 0 \\ \quad = 54V & t< \ 0 \end{array}}$$

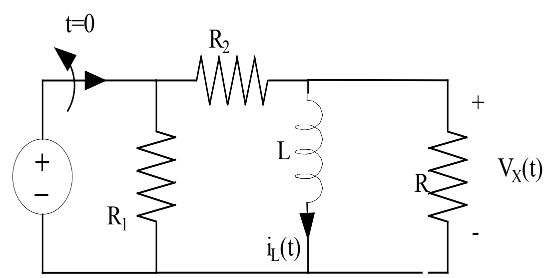
Problem 6.18

Use the differential equation approach to find $V_0(t)$ for $t > 0$ in the circuit in Fig. P6.18 and plot the response including the time interval just prior to opening the switch.



Suggested Solution





$$R_1 = 2\Omega \quad R_2{=}R_4{+}4\Omega \quad R_3{=}6\Omega \quad R_5{=}8\Omega \quad L{=}\frac{1}{4}H \quad R{=}4\Omega \quad V_0{=}\frac{2^{\mu}x}{3}$$

$\underline{At\ \ t<0:}$ $i_L(0^-){=}i_L(0^+){=}\dfrac{24}{R_2}{=}6A$ and $V_x(0^-){=}0$

$\underline{\text{At}\ \ t>0}$ $\quad\quad$ $\underline{\text{KVL:}}$ $\dfrac{Ldi_L}{dt}+R_x i_L=0 \Rightarrow \dfrac{di_L}{dt}+9.6i_L=0$

$$\text{If } i_L(t){=}K_1{+}K_2e^{\frac{-t}{\tau}}, then\ \tau=\frac{1}{9.6}S \text{ and } K_1{=}0$$

$$i_L(0^+){=}6{=}K_2 \ \Rightarrow\ i_L(t){=}6e^{-9.6t}A$$

$R_x{=}R//(R_1{+}R_2)$

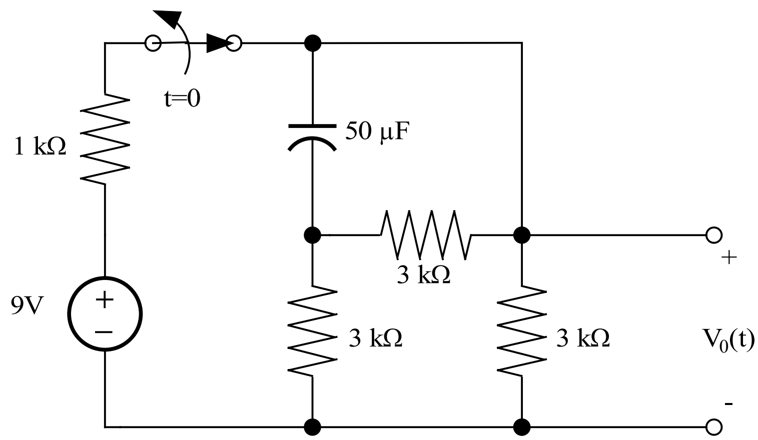
$$V_x(t){=}-R_x i_L(t){=}-1\text{-}4e^{-9.6t}V$$

$R_x{=}2.4\Omega$

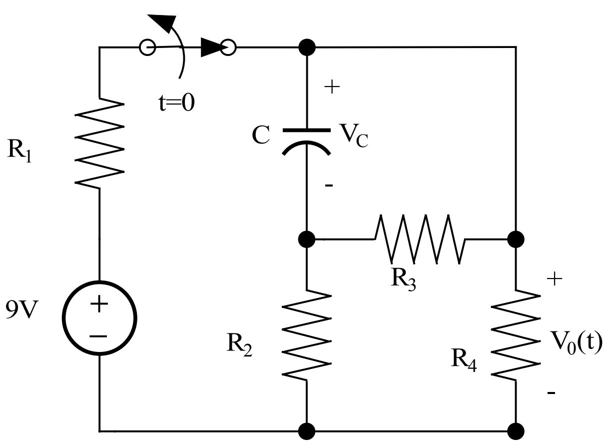
$V_0(t){=}-9.6e^{-9.6t}V$	$t>0$
$=0$	$t<0$

Problem 6.19

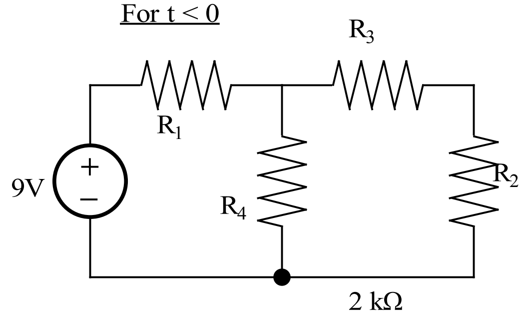
Use the differential equation approach to find $V_0(t)$ for $t > 0$ in the circuit in Fig. P.19 and plot the response including the time interval just prior to opening the switch.



Suggested Solution



For t < 0

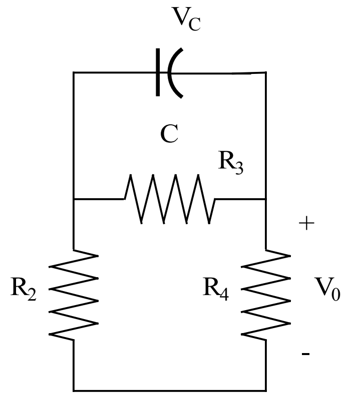


$$V_0(0^-) = 9\left(\frac{2K}{2K + R_1}\right) = 6V$$

$$V_C(0^-) = V_0(0-)\left(\frac{R_2}{R_2 + R_3}\right) = 3V$$

$$V_C(0^-) = V_C(0^-)$$

<u>For t > 0</u>



<u>KCL:</u> $C\dfrac{dV_C}{dt}+\dfrac{V_C}{R_3}+\dfrac{V_C}{R_2+R_4}=0 \Rightarrow \dfrac{dV_C}{dt}+10\ V_C=0$

If $V_C(t)=K_1+K_2e^{\frac{-t}{\tau}}$,

$$\frac{-K_2}{\tau}e^{\frac{-t}{\tau}}+10K_1+10K_2e^{\frac{-t}{\tau}}+0 \ \Rightarrow\ \tau=0.15,\ K_1=0$$

$V_C(0^+)=3=K_2 \quad \Rightarrow \quad V_C(t)=3e^{-10t}V$

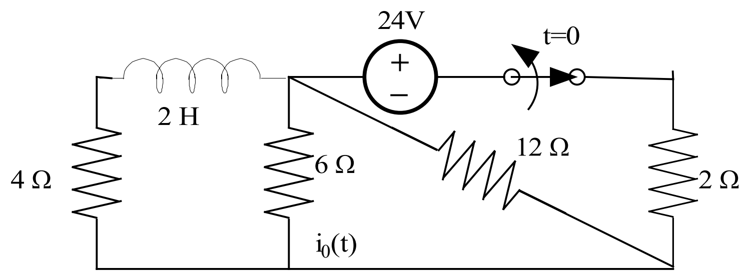
$$V_0=-V_C\left(\frac{R_4}{R_4+R_2}\right)$$

$$V_0=-\frac{V_C}{2}$$

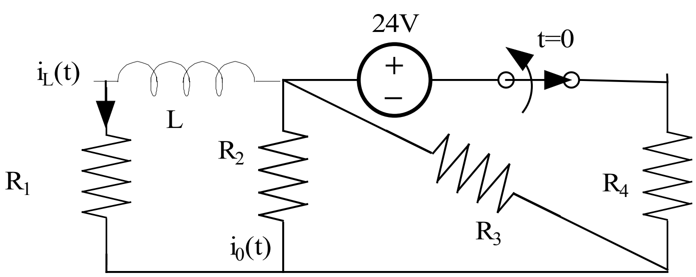
$V_0(t)=-1.5e^{-10t}V$	$t>0$
$=6V$	$t<0$

Problem 6.20

Use the differential equation approach to find $i_0(t)$ for $t > 0$ in the circuit in Fig. P.20 and plot the response including the time interval just prior to opening the switch.

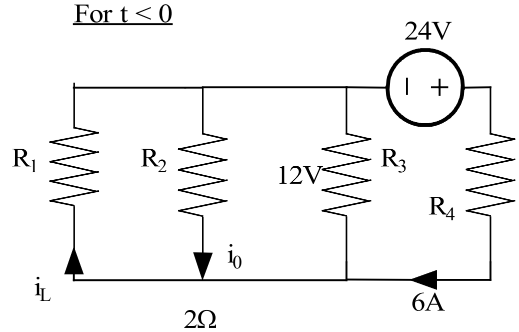


Suggested Solution



For t < 0

$R_1=4\Omega$
$R_2=6\Omega$
$R_3=12\Omega$
$R_4=2\Omega$
$R_5=2\Omega$
$L\ =2H$



$$i_0(0^-) = \frac{-12}{R_2}$$

$$i_0(0^-) = -2A$$

$$i_L(0^-) = i_L(0^+) = \frac{12}{R_1} = 3A$$

For $t > 0$

KVL: $\frac{L\, di_X}{dt} + (R+R_1) i_X(t) = 0$

or, $\frac{di_X}{dt}(t) + 4 i_X(t) = 0$

If $i_X(t) = K_1 + K_2 e^{\frac{-t}{\tau}}, \frac{-K_2 e^{\frac{-t}{\tau}}}{\tau} + 4K_1 + 4K_2 e^{\frac{-t}{\tau}} = 0$

$R = R_2 // R_3 = 4\Omega$

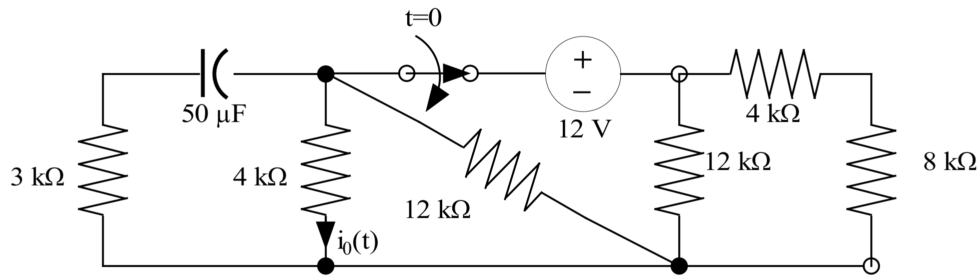
$i_0(t) = \frac{i_X(t) R_3}{R_2 + R_3} = \frac{2 i_X}{3}$ yields, τ=0.25 sec. and $K_1 = 0$

$i_X(t) = K_2 e^{-4t}$ *and* $i_X(0^+) = K_2 = i_L(0^+) = 3A$

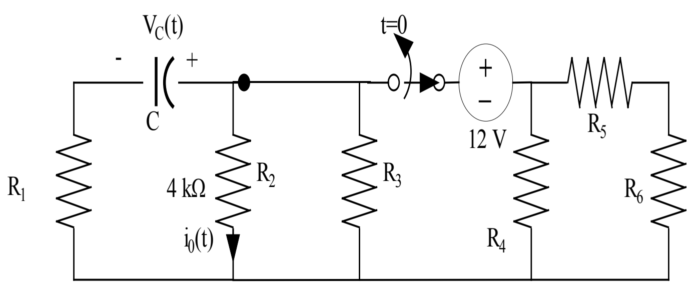
$i_0(t) = 2e^{-4t} A,\ t \geq 0$
$i_0(t) = -2\ A,\ \tau < 0$

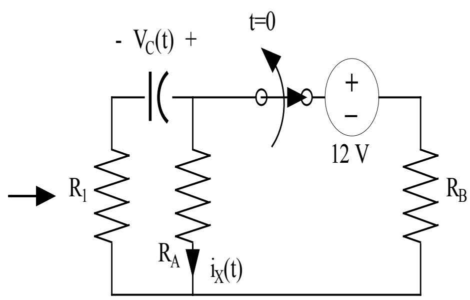
Problem 6.21

Use the differential equation approach to find $i_0(t)$ for $t > 0$ in the circuit in Fig. P.21 and plot the response including the time interval just prior opening the switch.



Suggested Solution





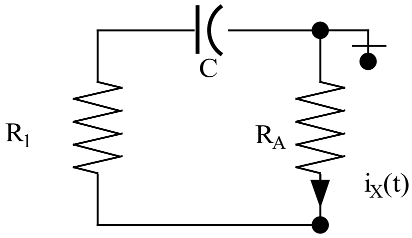
$R_1 = 3\ \text{k}\Omega \quad R_2 = R_5 = 4\ \text{k}\Omega \quad R_3 = R_4 = 12\ \text{k}\Omega \quad R_6 = 8\ \text{k}\Omega \quad C = 0.05mF \quad R_A = 3k\Omega \quad R_B = 6k\Omega$

<u>For t < 0:</u> $\quad V_C(t) = \dfrac{-12R_A}{(R_A + R_B)} = -4V = V_C(0^-) = V_C(0^+)$

$$i_X(0^-) = \frac{V_C(0-)}{R_A} = \frac{-4}{3}\ mA \quad i_0(0-) = \frac{i_X(0-)R_3}{(R_2 + R_3)} = -1\ mA$$

<u>*For t > 0:*</u> $\quad$ <u>By KCL:</u> $\quad \dfrac{C\ dV_C(t)}{dt} + \dfrac{V_C(t)}{R_1 + R_A} = 0 \Rightarrow \dfrac{dV_C}{dt} + \dfrac{10}{3} = 0$

For t > 0



$If\ \ V_C(t) = K_1 + K_2 e^{-\frac{t}{\tau}}, then\ \ \tau = 0.3\ S$ and $K_1 = 0$

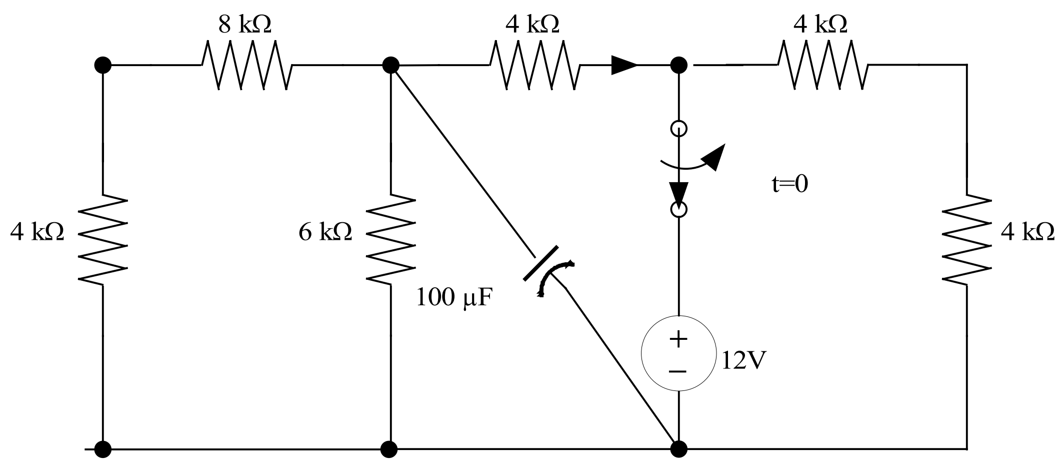
$V_C(0^+) = K_2 = -4V$ so, $V_C(t) = -4e^{-\frac{10t}{3}} V$

$But\ \ i_X(t) = \frac{-V_C(t)}{R_1 + R_A}$ and $i_0(t) = \frac{i_X(t) R_3}{R_2 + R_3} \Rightarrow$

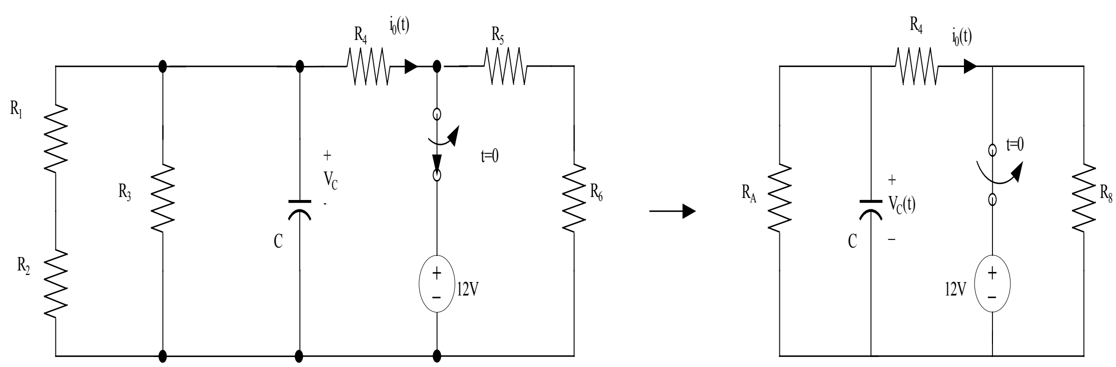
$$\boxed{\begin{array}{c} i_0(t) = 0.5e^{-\frac{10t}{\tau}} mA,\ t > 0 \\ = 1\ mA,\ t < 0 \end{array}}$$

Problem 6.22

Use the differential equation approach to find $i_0(t)$ for $t > 0$ in the circuit in Fig. P.22 and plot the response including the time interval just prior to opening the switch.

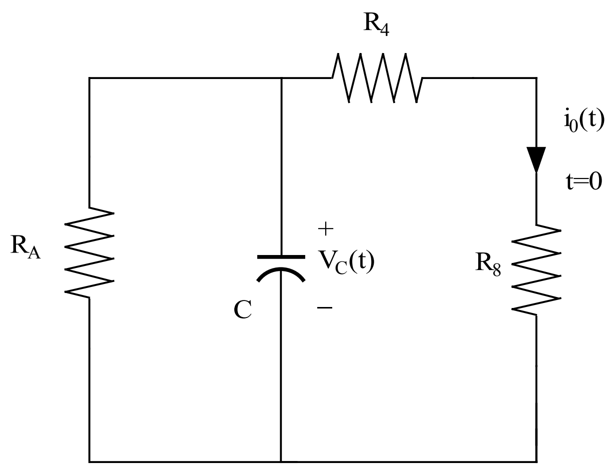


Suggested Solution



$R_1 = 8\ \text{k}\Omega \quad R_2 = R_4 = R_5 = R_6 = 4\ \text{k}\Omega \quad R_3 = 6\ \text{k}\Omega \quad C = 0.1\ \text{mF} \quad R_A = 4\ \text{k}\Omega \quad R_B = 8\ \text{k}\Omega$

<u>*For* t < 0:</u> $V_C(0^-) = V_C(0^+) = \dfrac{12R_A}{(R_A + R_4)} = 6\text{V} \quad i_0(0^-) = -1.5\ \text{mA}$



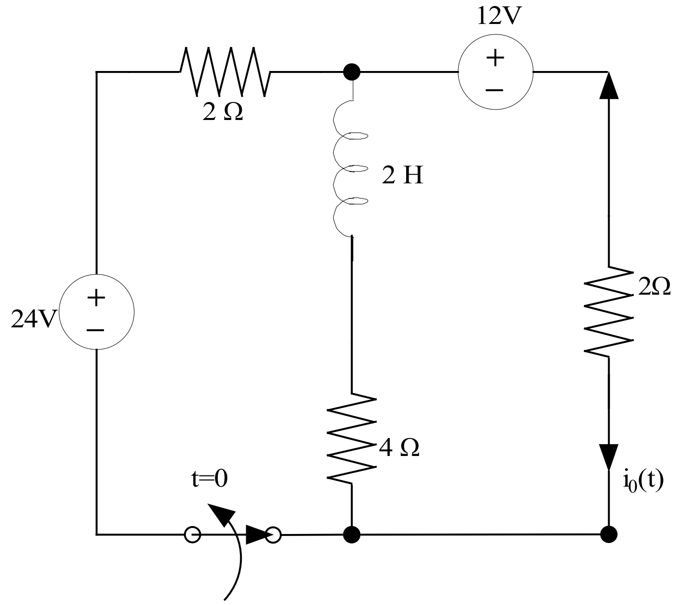
$$\underline{KCL}: \ \frac{C\ dV_C}{dt}+\frac{V_C}{R_A}+\frac{V_C}{R_4+R_B}=0 \Rightarrow \frac{dV_C}{dt}+\frac{10}{3}\ V_C=0$$

If $V_C(t)=K_1+K_2e^{\frac{-t}{\tau}}$, $\tau=0.3s$ and $K_1=0$

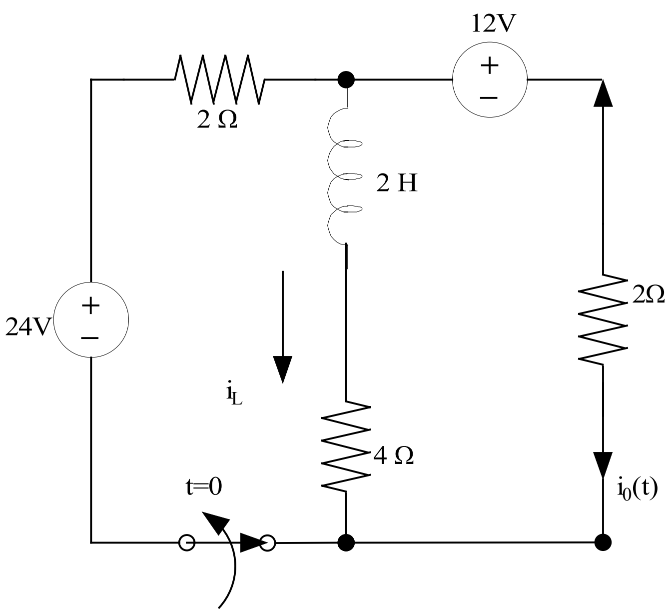
$V_C(0^+)=6=K_2$ So, $\boxed{\begin{array}{l} i_0(t)=0.5e^{\frac{-10t}{3}} \text{ mA}, \ t>0 \\ \quad =-1.5 \text{ mA}, \ t<0 \end{array}}$

Problem 6.23

Use the differential equation approach to find $i_0(t)$ for $t > 0$ in the circuit in Fig. P.23 and plot the response including the time interval just prior to opening the switch.



Suggested Solution



For $t=0^-$ Use superposition

$$i_L = \frac{24}{2+(4//2)}\left(\frac{2}{2+4}\right) - \frac{12}{2+(4//2)}\left(\frac{2}{2+4}\right) = 1.2A$$

$$i_0 = \frac{24}{2+(4//2)}\left(\frac{4}{2+4}\right) + \left(\frac{12}{2+(4//2)}\right) = 8.4A$$

For $t=0^+$: $i_0 = -L = -L(0^-) = 1.2A$

$$12 = 6\, i_0(t) + 2\frac{di_0(t)}{dt} \quad \text{so, } i_0(t) = K_1 + K_2 e^{\frac{-t}{\tau}}$$

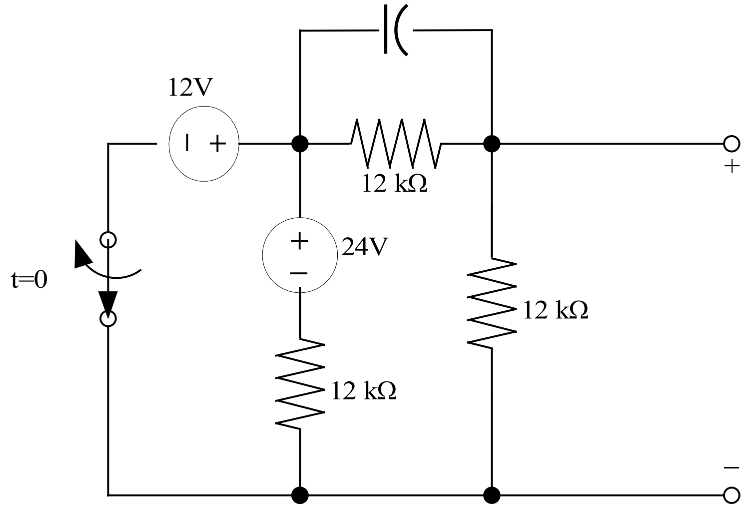
$$\textit{Now,}\quad 12=6K_1+6K_2e^{\frac{-t}{\tau}}+\left(\frac{-2K_2}{\tau}\right)e^{\frac{-t}{\tau}} \Rightarrow K_1=\frac{12}{6}=2 \textit{ and } \frac{1}{\tau}=3$$

$$i_0(0^+)=-1.2=K_1+K_2 \Rightarrow K_2=-3.2$$

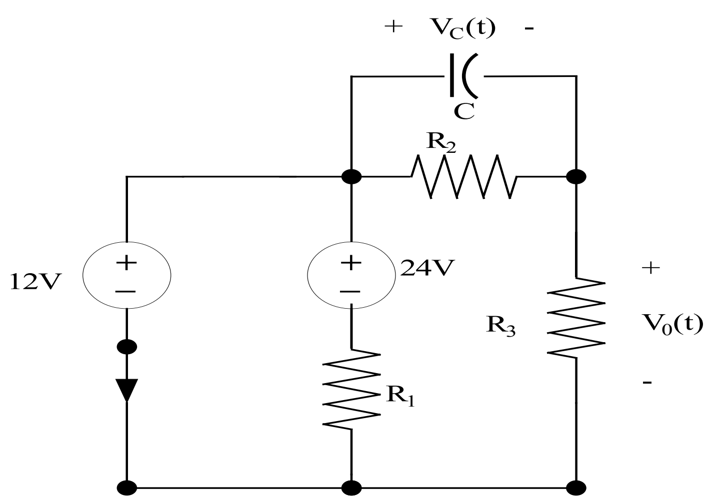
$$\boxed{i0(t)=\begin{cases}-1.2A & t<0\\ 2-3.2e^{-3t}A & t>0\end{cases}}$$

Problem 6.24

Use the differential equation approach to find $V_0(t)$ for $t > 0$ in the circuit in Fig. P.24 and plot the response including the time interval just prior to opening the switch.



Suggested Solution



For $t < 0$

$$V_C(0^-) = V_C(0^+) = 12\left(\frac{R_2}{R_2 + R_3}\right) = 6V$$

$$V_0(0^-) = 12 - V_C(0^-) = 6V$$

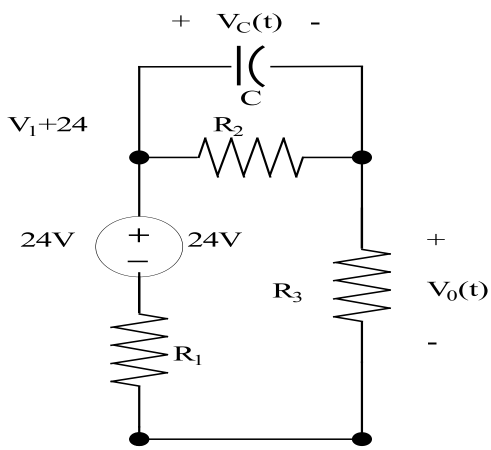
For $t > 0$

KCL:

$$C\,d\left(\frac{V1+24}{dt}\right) + 1(V_1 + 24) = \frac{V_0}{R_3}$$

$$\text{or,}\quad \frac{dV_1}{dt} + \frac{V_1}{1.2} + 20 = \frac{V_0}{1.2}$$

$$\text{Also,}\quad \frac{V_1}{R_1} + \frac{V_0}{R_3} = 0 \Rightarrow V_1 = -V_0$$



$$Now, \quad \frac{dV_0}{dt}+\frac{V_0}{0.6}=20 \qquad \text{Assume} \quad V_0(t)=K_1+K_2e^{\frac{-t}{\tau}}$$

substitute,

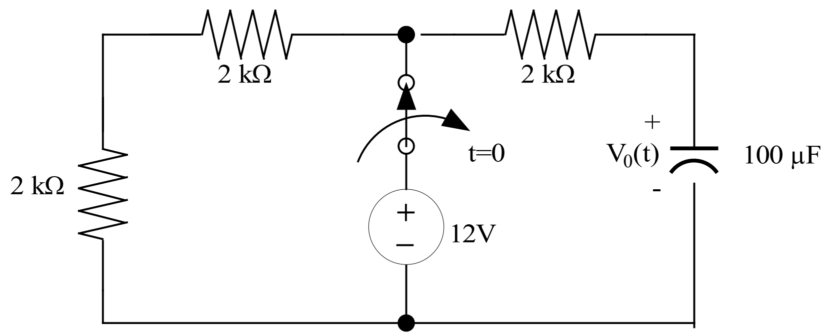
$$-K_2e^{\frac{-t}{\tau}}+\frac{K_1}{0.6}+\frac{K_2e^{\frac{-t}{\tau}}}{0.6}=20 \Rightarrow \tau=0.65 \text{ and } K_1=12$$

$$V_0(0^+)=\frac{24-6}{2}=9V=K_1+K_2 \Rightarrow K_2=-3$$

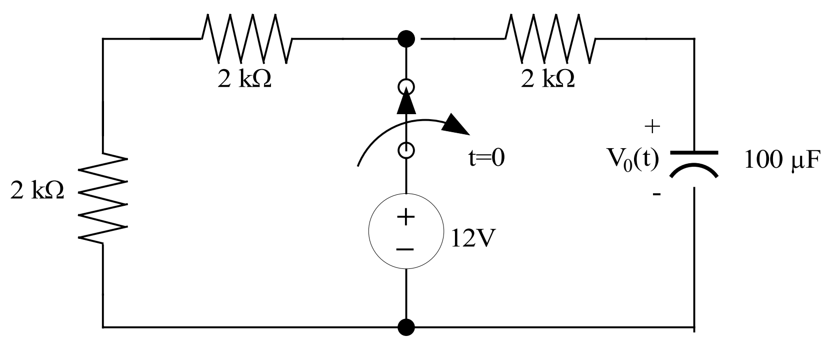
$$\boxed{V_0(t)= 12-3e^{\frac{-5t}{3}} \text{ V} \quad t>0 \quad V_0(t)=6 \quad t<0}$$

Problem 6.25

Find $V_C(t)$ for $t > 0$ in the network in Fig. 6.25 using the step-by-step method.

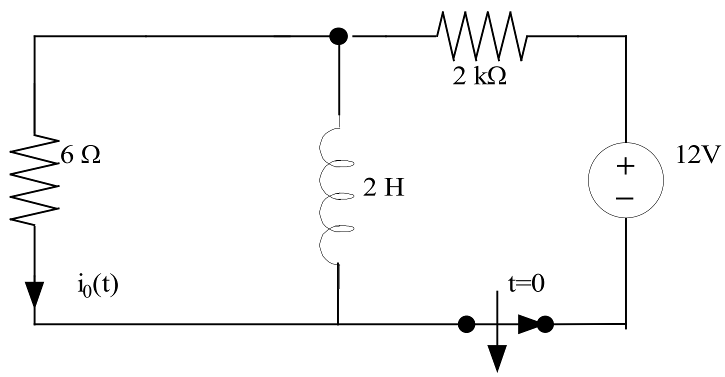


Suggested Solution

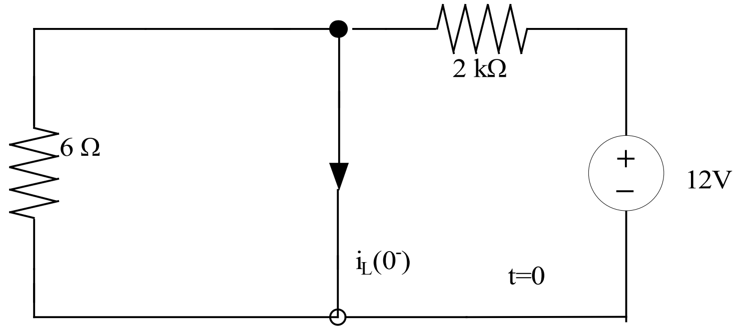


Problem 6.26

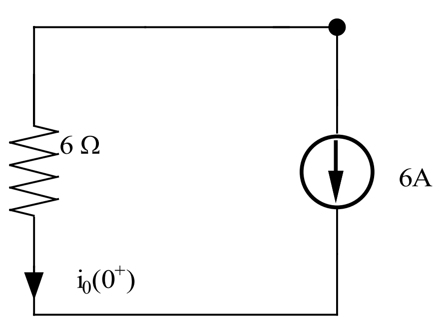
Use the step-by-step method to find $i_0(t)$ for $t > 0$ in the circuit in Fig. P.26.



Suggested Solution



$$i_L(0^-) = \frac{12}{2} = 6A$$

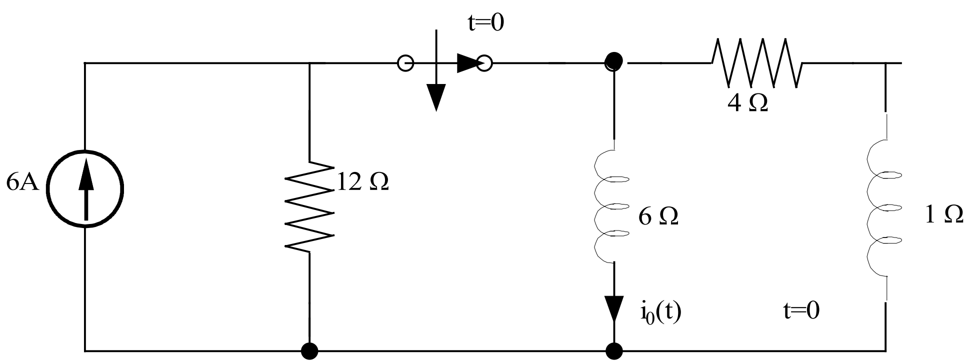


$i_0(0^+)$=-6A

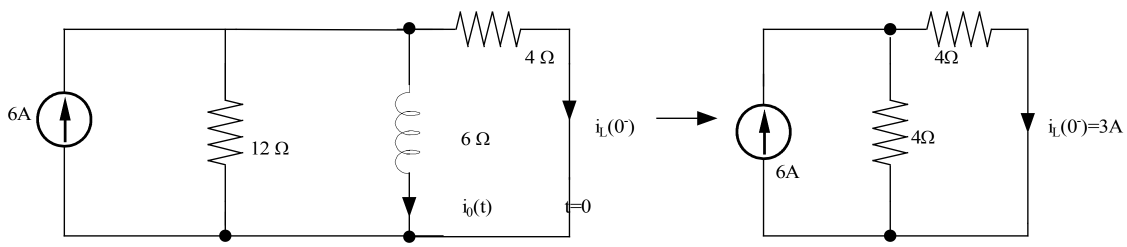
$$i - \frac{R}{L} = \frac{6}{L} = 3 \text{ sec.} \quad \therefore \ \underline{i_0(t) = -6e^{\frac{-t}{3}} A,\ t > 0}$$

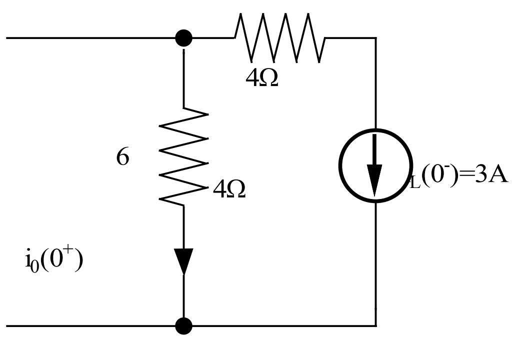
Problem 6.27

Find i0(t) for t > 0 in the network in Fig. P6.27 using the step-by-step method.



Suggested Solution





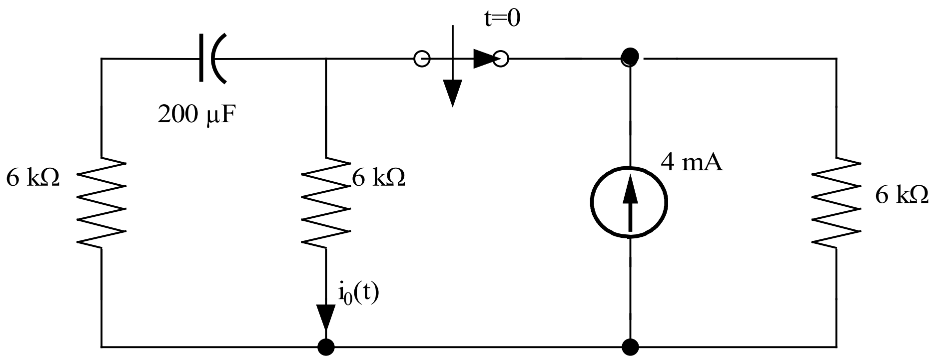
$i_0(0^+)$=-3A

$i_0(\infty)$=0 source is out

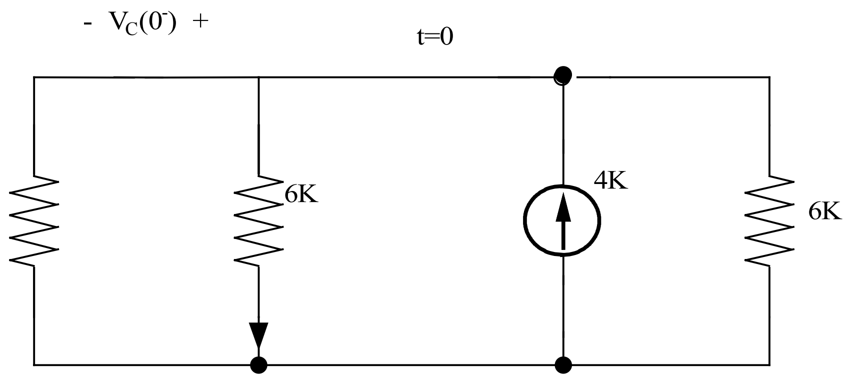
$i = R = 10 = 10$ sec $\therefore$ $i_0(t)$=-3e^{-10t}A, t>0

Problem 6.28

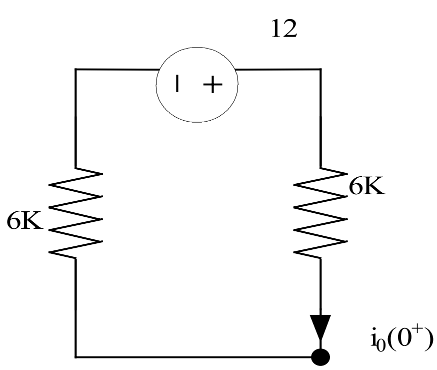
Use the step-by-step method to find $i_o(t)$ for t > 0 in the circuit in Fig. P6.28.



Suggested Solution



$$V_C(0^-)=\frac{4}{K}=(6K//6K)=12V$$



$$i_0(0^+)=\frac{12}{12K}=\frac{1}{K}A$$

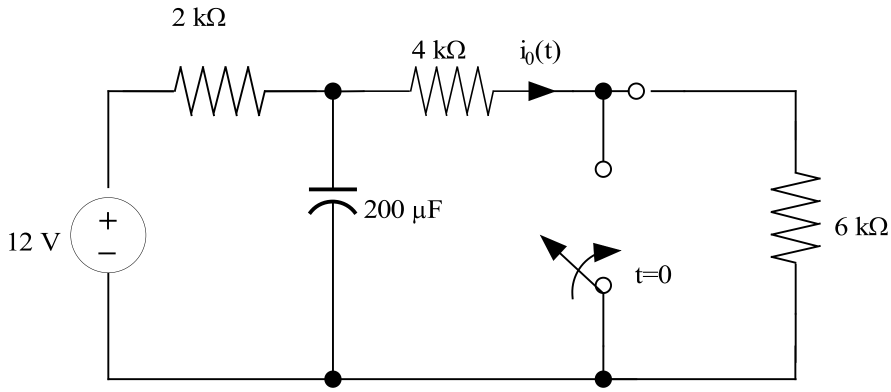
$i0(\infty)=0$ Source is out of network

$i=RC=200 \times 10^{-6} \times 12 \times 10^{3} = 2.4$ sec

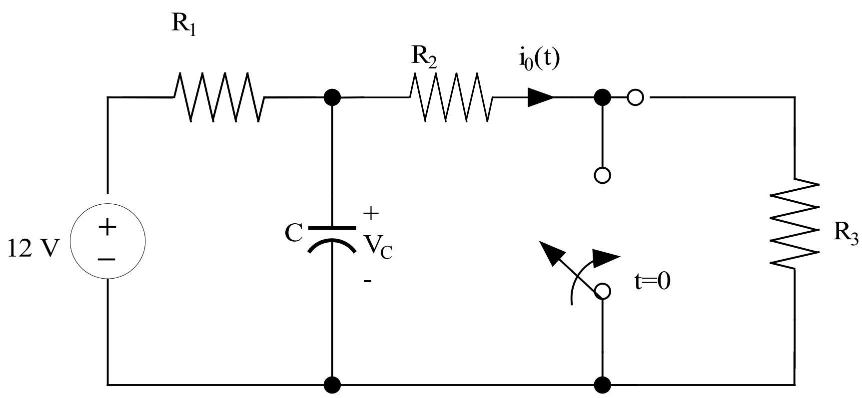
$$\boxed{i_0(t)=1e^{\frac{-t}{2.4}}\ mA,\ t>0}$$

Problem 6.29

Use the step-by-step technique to find $i_0(t)$ for $t > 0$ in the network in Fig. P6.29.



Suggested Solution



For t=0⁻

$$V_C(0^-)=V_C(0^+)=12\left[\frac{R_2+R_3}{R_1+R_2+R_3}\right]=10V$$

$R_1 = 2\ k\Omega\ \ R_2 = 4\ k\Omega\ \ R_3 = 6k\Omega\ \ C = 0.2mF$

For t=0⁺

$$i_0(0^+)=\frac{V_C(0^+)}{R_2}=2.5\text{ mA}$$

For $t \to \infty$

$i_0(t) = 12 = 2mA = K_1$

$K_1+K_2=2.5\text{mA} \Rightarrow K_2=0.5\text{mA}$

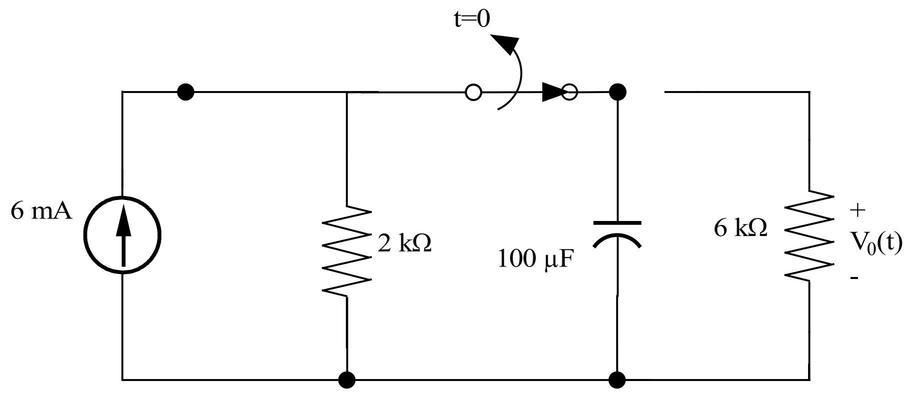
$i_0(t)=K_1+K_2e^{\frac{-t}{\tau}}$

$\tau=C\left[R_1//R_2\right]= 0.267\ \text{sec}.$

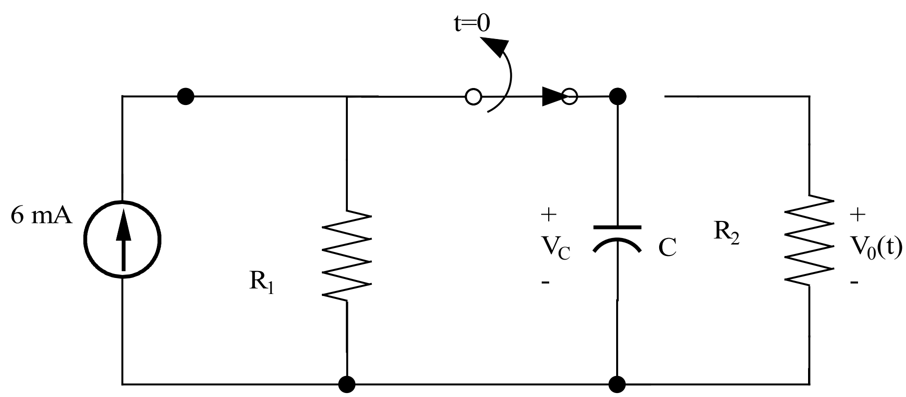
$$\boxed{i_0(t) = 2 + 0.5e^{-3.75t}\ mA}$$

Problem 6.30

Use the step-by-step method to find $V_0(t)$ for $t > 0$ in the network in Fig. P6.30.



Suggested Solution



For t=0⁻

$V_0(0^-) = V_C(0^+) = 6m(R_1 // R_2) = 9V$

For t=0⁺

R_1=2 kΩ R_2=6 kΩ C=0.1 mF

$V_C(0^+) = V_0(0^+) = 9V = K_1 + K_2$

$V_0(t) = K_1 + K_2 e^{\frac{-t}{\tau}}$

For $t \Rightarrow \infty$

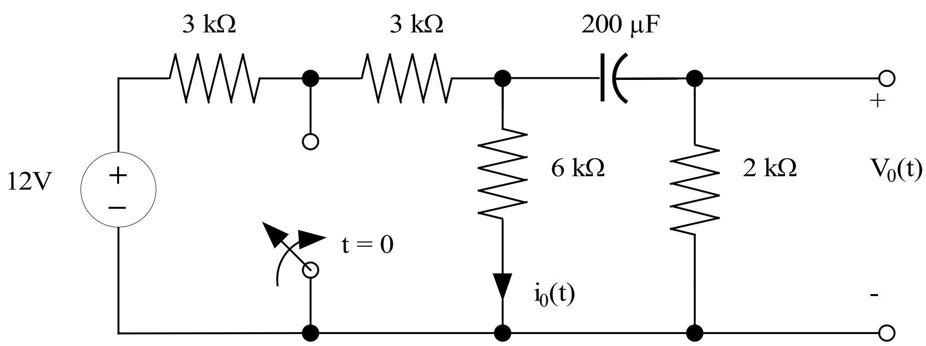
$$\boxed{V_0(t) = 9e^{-\frac{5t}{3}} V}$$

$V_0(\infty) = 0 = K_1 \Rightarrow K_2 = 9$

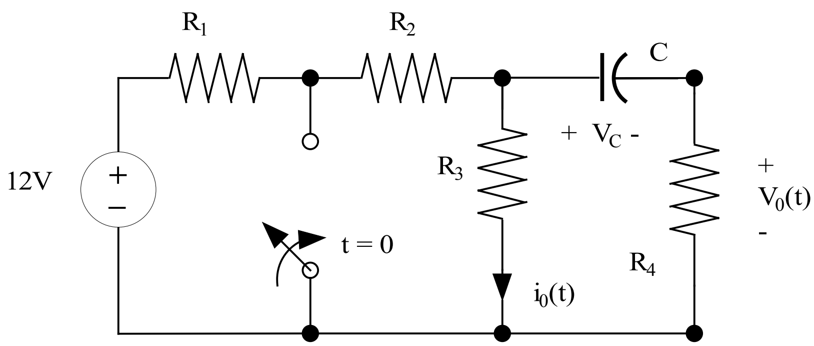
$\tau = C\, R_2 = 0.6$ sec

Problem 6.31

Use the step-by-step method to find $i_0(t)$ for $t > 0$ in the circuit in Fig. P6.31.



Suggested Solution



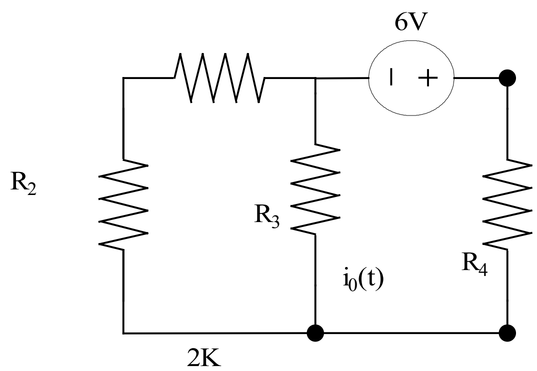
$R_1=R_2=3K \quad R_3=6k \quad R_4=2K$

$C=0.2$ mF

For $t=0^-$

$$V_C(0^-)=12\left(\frac{R_3}{R_1+R_2+R_3}\right)=6V=T_C(0^+)$$

For $t=0^+$



R_1=R_2=3K   R3=6K R_4=2K C=0.2 mF

<u>For $t \to \infty$</u>

$$i_s = \frac{6}{4K} = 1.5\text{ mA} \qquad i_0(\infty) = 0 = K_1$$

$$i0 = i_s\left(\frac{3}{3+6}\right) = 0.5mA \qquad \tau = C\ Req \quad Req = (R_2 // R_3) + R_4$$

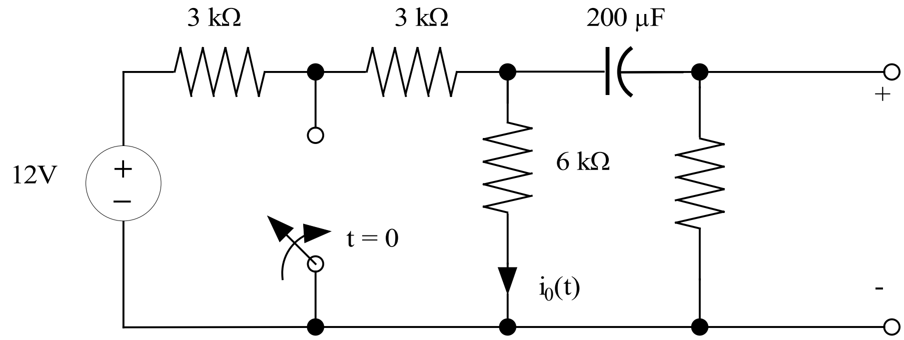
$$\tau = 0.8\text{ s}$$

So, $K_1+K_2=0.5\text{ mA}=K_2$

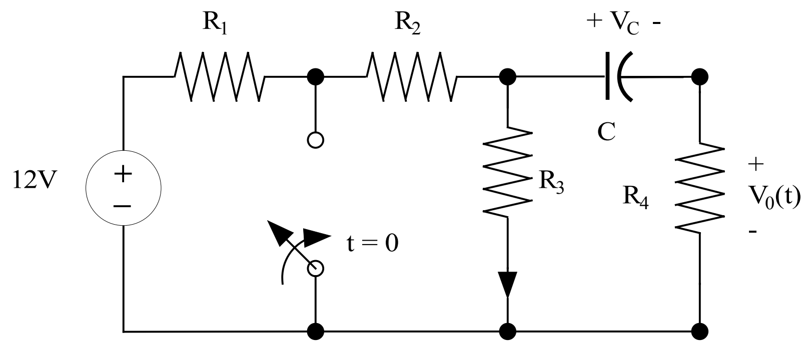
$$\boxed{i_0(t) = 0.5e^{\frac{-t}{0.8}}\,mA}$$

Problem 6.32

Find $V_0(t)$ for t > 0 in the network in Fig. P6.32 using the step-by-step technique.



Suggested Solution



For t=0$^-$

$$V_C = \frac{12\ R_3}{R_1+R_2+R3} = 6V = V_C(0^+)$$

For t=0$^+$

$R_1 = R_2 = 3K \quad R_3 = 6K \quad R_4 = 2K \quad C = 0.2mF$

$V_C(0^+)=6V$

$V_0(t)=K_1+K_2e^{\frac{-t}{\tau}}$

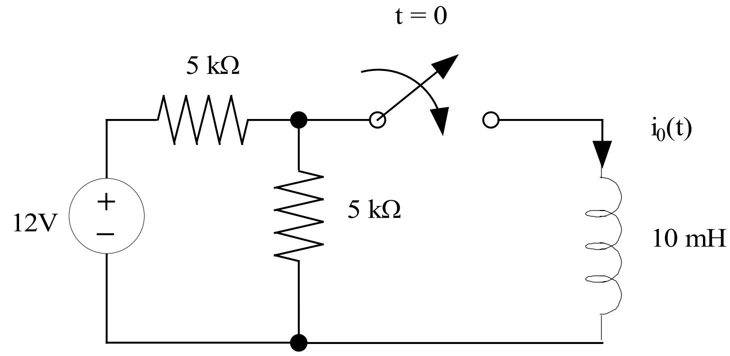
$$V_0(0^+) = -V_C\left[\frac{R_4}{R_4+(R_2//R_3)}\right] = -3V$$

$\tau = C\left[R_4 + (R_2 // R_3)\right] = 0.8$ s

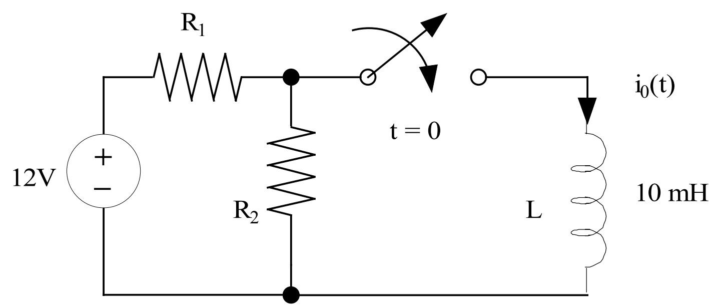
$$\boxed{V_0(t) = -3e^{\frac{-t}{0.8}}V}$$

Problem 6.33

Find $i_0(t)$ for $t > 0$ in the network in Fig. P6.33 using the step-by-step method.



Suggested Solution



$R_1=R_2=5\ k\Omega$
$L=10\ mH$

<u>For t=0$^-$</u>

$i_0(0^-)=i_0(0^+)=0A$

For t=0$^+$: $i_0(0^+)=0=K_1+K_2$

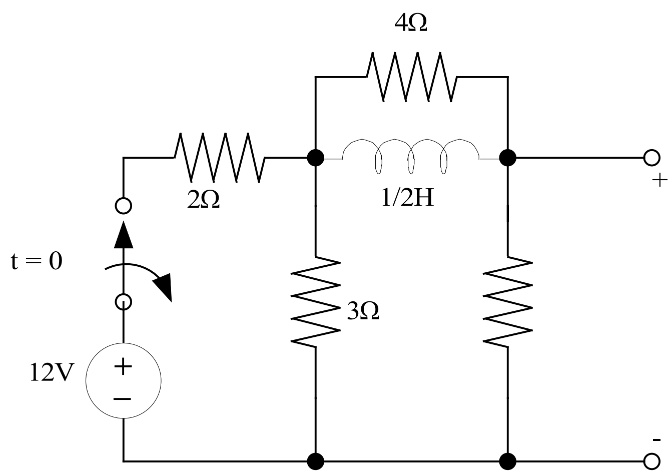
For t=∞: $i_0(\infty)=12=2.4\ mA=K_1 \Rightarrow K_2=-2.4\ mA$

$$\tau=\frac{L}{Req}=\frac{L}{R_1//R_2}=4\ \mu s$$

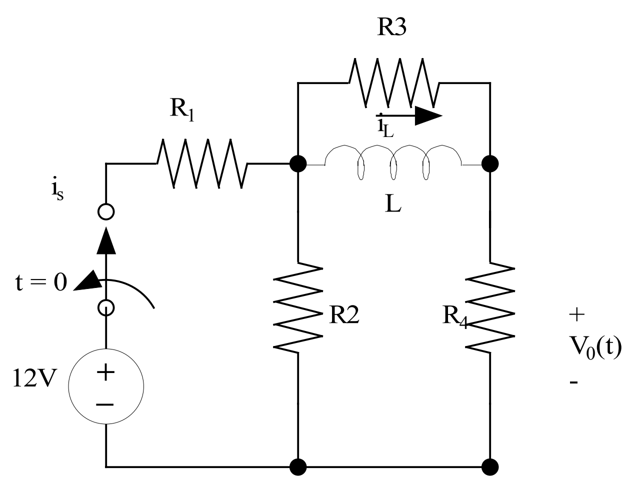
$$\boxed{i_0(t)=2.4\left(1-e^{-2.5x10^{5}t}\right)mA}$$

Problem 6.34

Find $V_0(t)$ for t > 0 in the network in Fig. P6.34 using the step-by-step method.



Suggested Solution



For t=0⁻: $i_s=\dfrac{12}{R_1+(R_2//R_4)}$ $i_L=\dfrac{i_s R_2}{R_2+R_4}=1\text{A}$

For t=0⁺: $V_0=\dfrac{-i_L(0+)R3}{R_3+R_2+R_4}$ $R_4=\dfrac{24\text{V}}{13}=K_1+K_2$

For $t\rightarrow\infty$: $V_0=0=K_1 \Rightarrow K_2=\dfrac{24\text{V}}{13}=1.85\text{V}$

$$\tau=\frac{L}{\{R_3//(R_2+R_4)\}}=0.18\text{ sec}$$

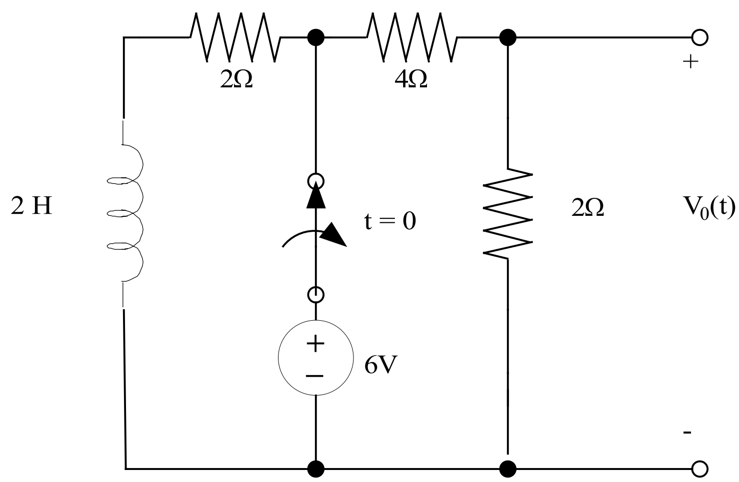
$R_1=2\Omega$ $R_2=3\Omega$ $R_3=4\Omega$

$R_4=6\Omega$ $L=\frac{1}{2}\mu$

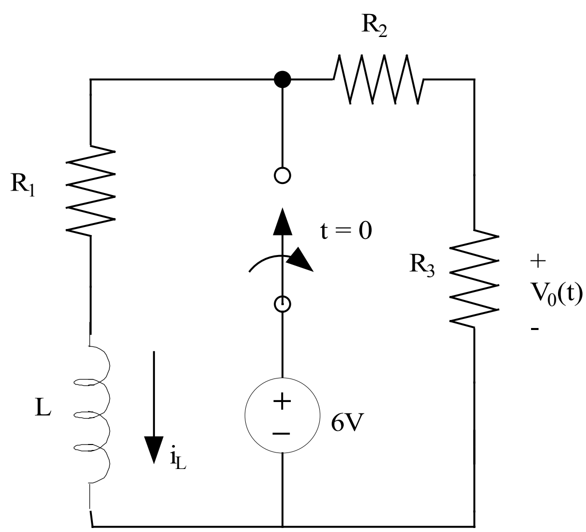
$\boxed{V_0(t)=1.85e^{-5.54t}V}$

Problem 6.35

Use the step-by-step technique to find $V_0(t)$ for $t > 0$ in the network in Fig. P.35.



Suggested Solution



$R_1=R_3=2\ \Omega$
$R_2=4\ \Omega$
$L=2H$

For $t = 0^-$

$$i_L(0^-)=i_L(0^+)=\frac{6}{R_1}=3A$$

$$V_0(t)=K_1+K_2e^{\frac{-t}{\tau}}$$

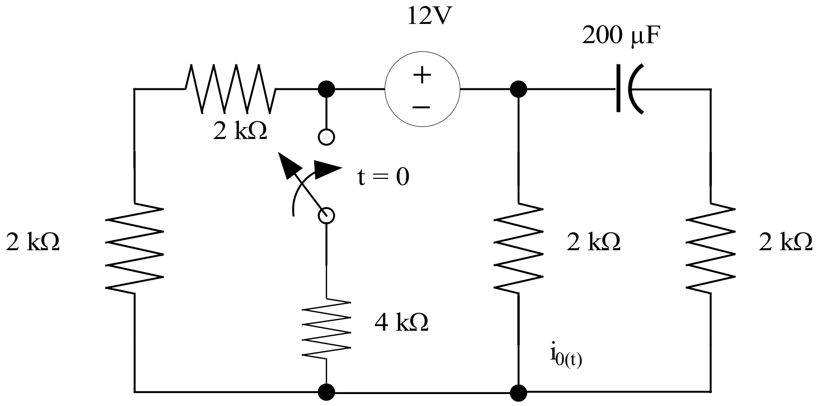
For $t = 0^+$ $i_L(0^+)=3A$, $V_0(0^+)=-i_L(0^+)R_3=-6V=K_1+K_2$

$$\tau=\frac{L}{Req}=\frac{L}{R_1+R_2+R_3}=\frac{2}{8}=\frac{1}{4}\ \text{sec}$$

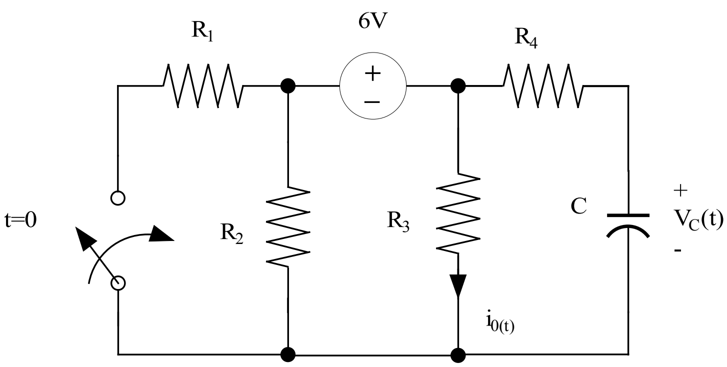
$$\boxed{V_0(t)=-6e^{-4t}V}$$

Problem 6.36

Use the step-by-step technique to find $i_0(t)$ for $t > 0$ in the networking in Fig. P6.36.



Suggested Solution



R1=R2=R3=R4=2 kΩ   C=0.2 mF

<u>For t=0⁻</u>

$$V_C(0^-) = \frac{6R3}{R_2 + R_3} = 3V$$

For $t \to \infty$ $i_0(\infty) = K_1 = \frac{6}{3k} = 2\text{ mA}$

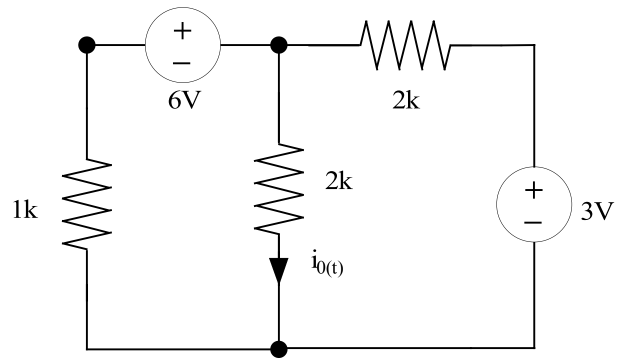
so, $K_2 = -0.25$ mA

$$\tau = C\,\mathrm{Re}\,q = C\left[2k + \left(\frac{1k}{2k}\right)\right] = \frac{1}{3}\ \text{sec}$$

$$\boxed{i_0(t) = 2 - \frac{e^{-3t}}{4}\ mA}$$

For $t=0^+$

$V_C(0+)=V_C(0-)=3V$



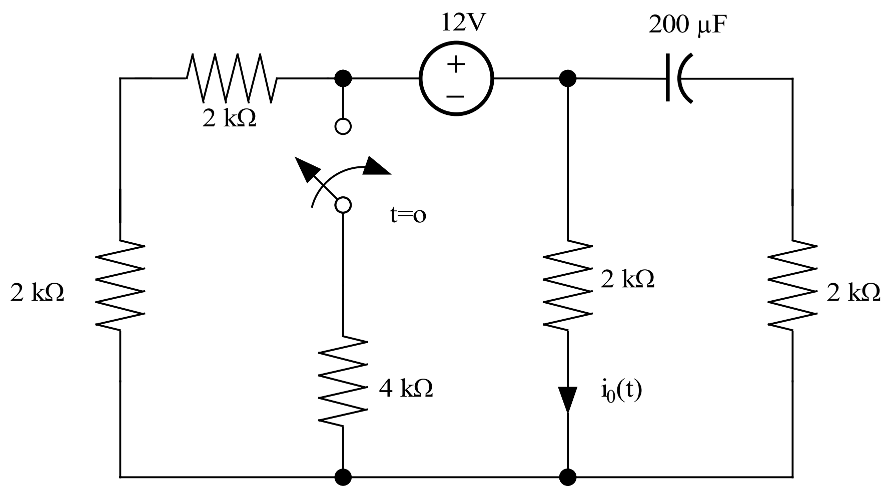
Superposition:

$$i_0 = \frac{6}{2k}\left(\frac{2}{2+2}\right) + \frac{3}{\frac{8}{3}k}\left(\frac{1}{1+2}\right)1) = 1.75mA$$

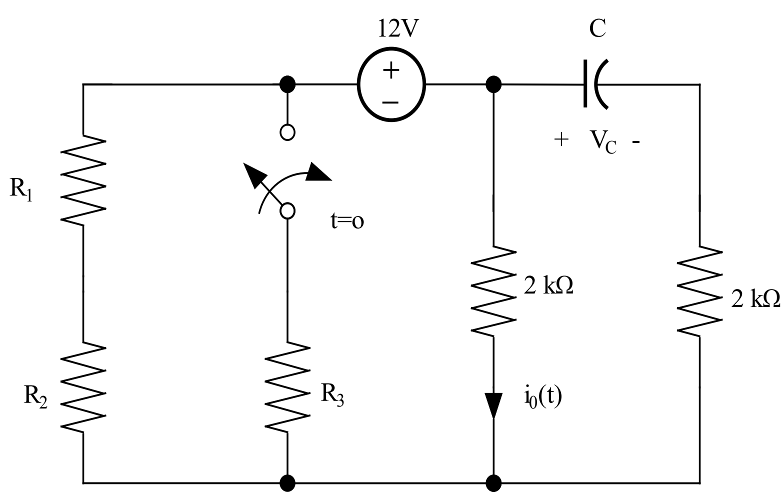
$$i_0(0^+) = K_1 + K_2 = 1.75mA$$

Problem 6.37

Find $i_0(t)$ for $t > 0$ in the networking in Fig P6.37 using the step-by-step method.



Suggested Solution



$R_1 = R_2 = R_4 = R_5 = 2\ k\Omega$

$R_3 = 4\ k\Omega,\ C = 0.2\ \text{mF}$

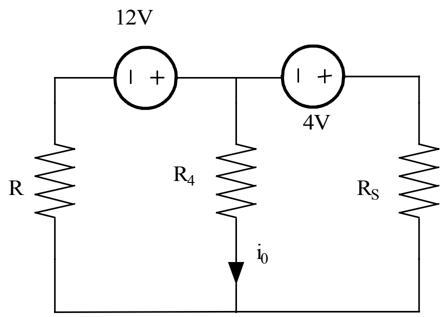
For $t=0^-$

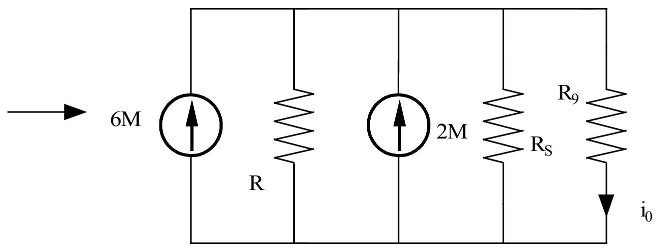
$$V_C(0^-)\frac{12R_4}{R_1+R_2+R_4}=4\text{V}$$

$$i_0(t)=K_1+K_2e^{\frac{-t}{\tau}}$$

For t > 0

$R = R_3 // (R_1 + R_2 = 2\ K\Omega$





$$i_0(0^+) = K_1 + K_2 = \frac{8M\left[R // R_S\right]}{\left[R // R_S\right] + R_4} = \frac{8}{3}\ mA$$

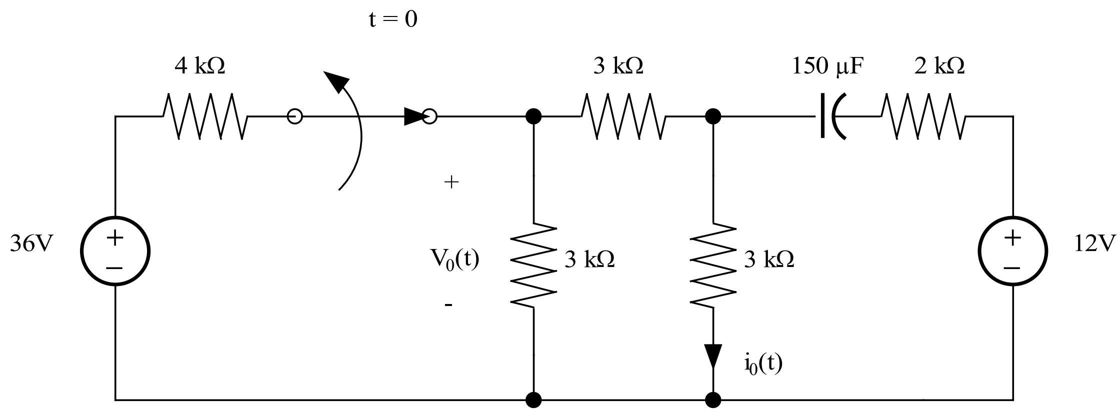
For t $\Rightarrow\ \infty$: $\quad i_0(\infty) = K_1 = \dfrac{12}{R + R_4} = 3\ mA \Rightarrow K_2 = \dfrac{-1}{3}\ mA$

$$\tau = CReq = C\left\{R_S + (R // R_4)\right\} = 0.6\ \sec$$

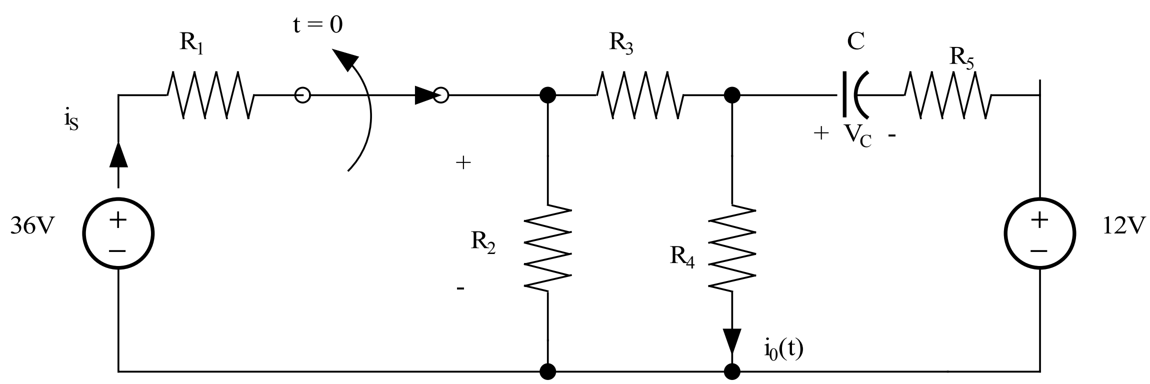
$$\boxed{i_0(t) = 3 - \frac{e^{-t0.6} mA}{3}}$$

Problem 6.38

Use the step-by-step technique to find $i_0(t)$ for $t > 0$ in the network in Fig. P6.38.



Suggested Solution



$R_1 = 4k\Omega$ $R_5 = 2\ k\Omega$ $C = 0.15\ mF$

$R_2 = R_3 = R_4 = 3\ k\Omega$

$i_0(t) = K_1 + K_2 e^{\frac{-t}{\tau}}$

$\underline{\underline{For\ t=0^-}}$

$$i_S=\frac{36}{R_1+R_2//(R_3+R_4)}=6A \qquad \mathrm{i_0=\frac{i_S R_2}{R_2+R_3+R_4}=2\ mA} \quad \mathrm{V_C=i_0R_4\text{-}12=\text{-}6V}$$

$\underline{\underline{\text{For t=0}^+}}$

$$V_C(0^+)=V_C(0^-)=-6V, \qquad R_4 i_0=\frac{\left[12+V_C(0^+)\right]\left[R_4//(R_2+R_3)\right]}{R_4//(R_2+R_3)+R_5}=3V$$

$so,\ \mathrm{i_0(0^+)=K_1+K_L=1\ mA}$

$\underline{\underline{\text{For t=}\infty}}$ $\quad \mathrm{i_0(\infty)=0=K_1 \Rightarrow K_2=1\ mA}$

$\tau\mathrm{=CReq=C}\left\{\mathrm{R_5}+\left[R_4//(R_2+R_3)\right]\right\}=0.6\ \ \mathrm{sec}$

$i_0(t)=1\ e^{\frac{-t}{0.6}}\ mA$

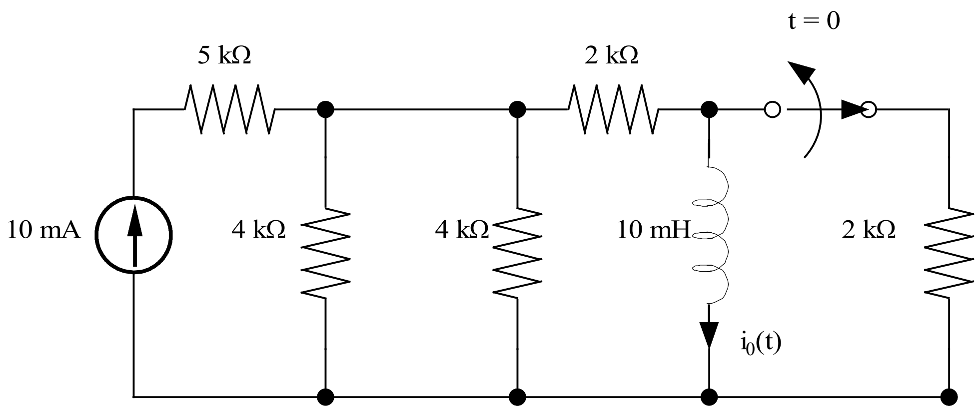
Problem 6.39

Find $V_0(t)$ for $t > 0$ in the circuit in Fig. P6.38 using the step-by-step method.

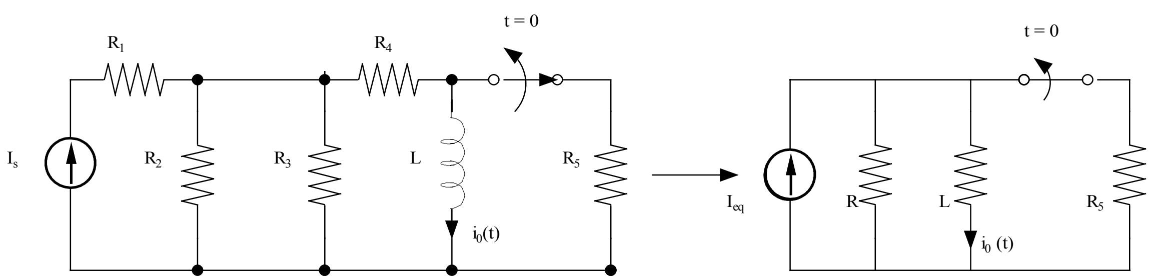
Suggested Solution

Problem 6.40

Find $i_0(t)$ for $t > 0$ in the network in Fig. P6.40 using the step-by-step method.



Suggested Solution



$R_1 = 5\ k\Omega \quad R_2 = R_3 = 4\ k\Omega \quad I_5 = 10\ mA \qquad R = R_4 + (R_1 // R_2 // R_3) = 3.43\ k\Omega$

$L = 10mH \quad R_4 = R_5 = 2\ k\Omega \qquad I_{eq} =$ 5 May 2001

For $t = 0^- : \ i_0(0^-) = i_0(0^+) = I_{eq} = 5\ mA = k_1 + k_2$

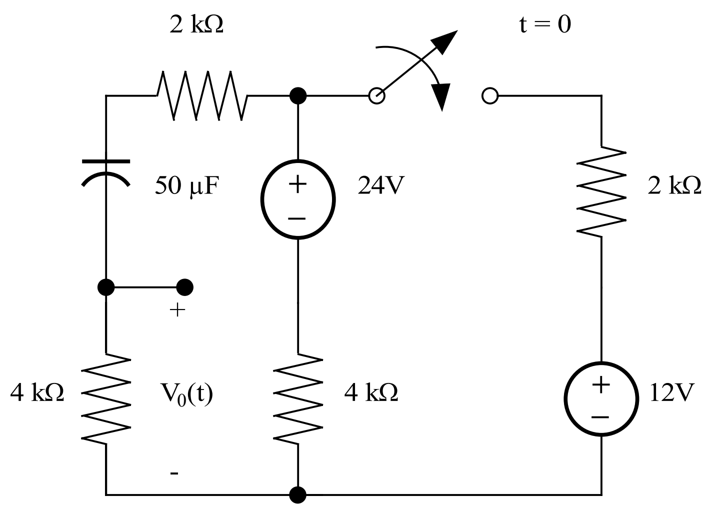
For $t = 0^+ : \ i_0(0^+) = I_{eq} = 5\ mA = k_1 + k_2$

For $t = \infty : \ i_0(\infty) = 5\ mA = k_1 \Rightarrow k_2$

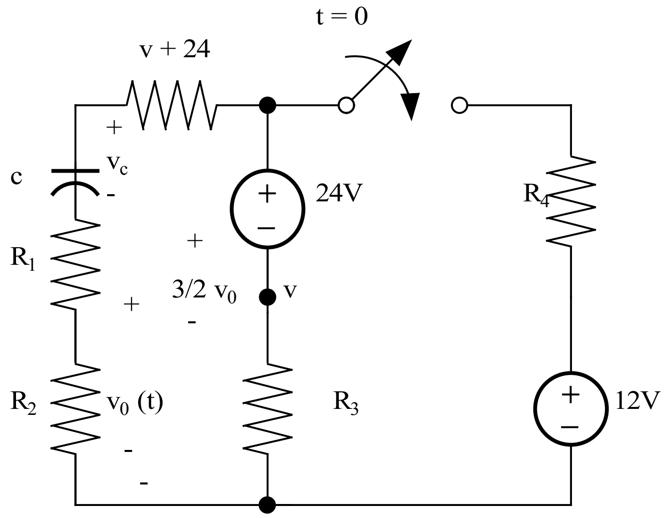
$\boxed{i_0(t) = 5mA}$

Problem 6.41

Find $V_0(t)$ for $t > 0$ in the network in Fig. P6.41 using the step-by-step method.



Suggested Solution



$R_2 = R_3 = 4\ k\Omega$, $R1 = R_4 = 2\ k\Omega$, $c = 50\ \mu F$

For $t \to \infty$: $v_0 \to 0 = k_1 \to k_2 = 4.37V$

$\tau = c\left[R_1 + R_2 + \left(R_3 // R_4\right)\right] = 0.37\sec$

For $t = 0^-$: $v_0 = 0V$, $v_c = 24V$

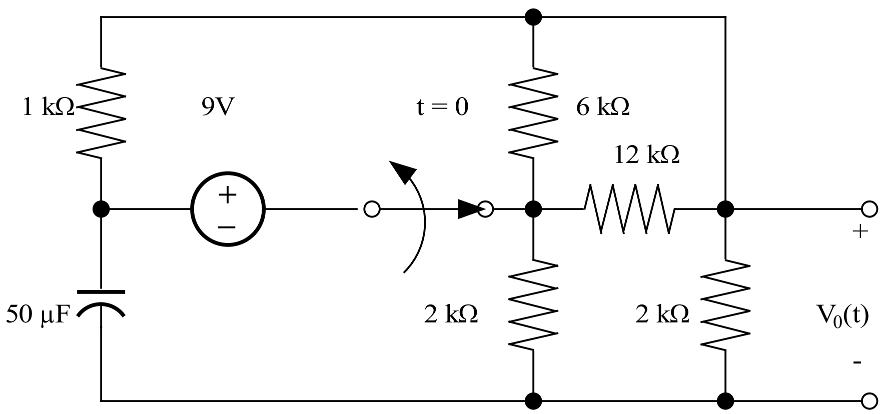
For $t = 0^+$: $\dfrac{v}{R_1 + R_2} + \dfrac{v}{R_3} + \dfrac{v + 24 - 12}{R_4} = 0$

$v = 6.55V$, $v_0 = \dfrac{2}{3}v = 4.37V = k_1 + k_2$

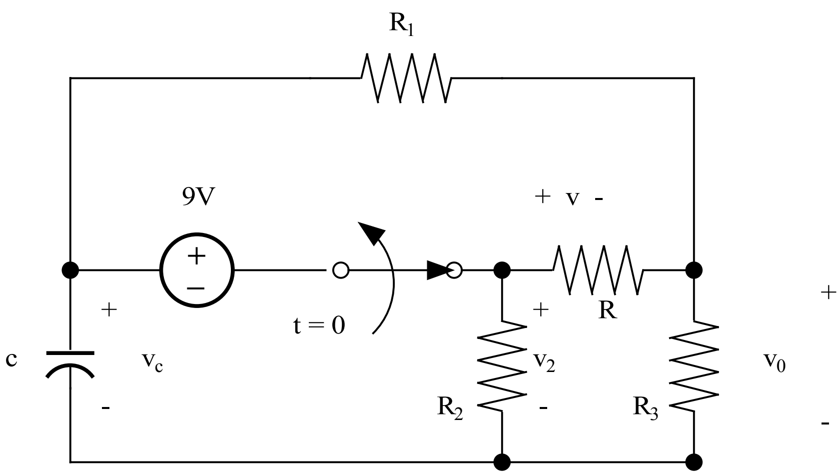
$\boxed{v_o(t) = 4.37e^{-t/0.37}V}$

Problem 6.42

Find $V_0(t)$ for t > 0 in the network in Fig. P6.42 using the step-by-step method.



Suggested Solution



$R_1 = 1\ \text{k}\Omega$, $R_2 = R_3 = k\Omega$, $c = 50\ \mu\text{F}$

R= 6 k// 12 k= 4kΩ

For $t \to \infty$: $v_0(\infty) \to 0 = k_1 \to k_2 = -3.6V$

$$\tau = c\left[R_1 + \left(R_3 // \left(R + R_2\right)\right)\right] = \frac{1}{8}\sec$$

For $t = 0^-$: $v_c = 9 - v_2, v_2 = v\left(\frac{R_2}{R_2 + R_3}\right) = \frac{v}{2}$,

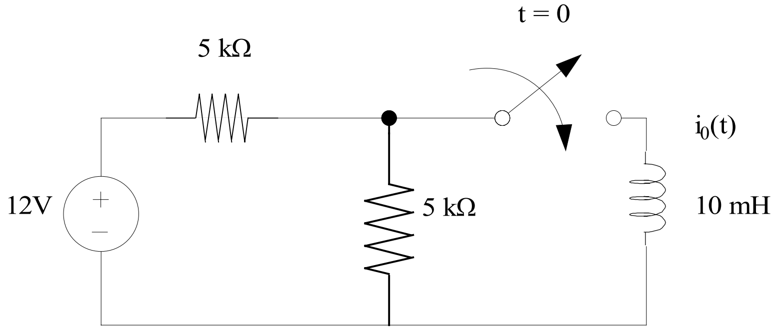
$$v = 9\left[\frac{R // \left(R_2 + R_3\right)}{R // \left(R_2 + R_3\right) + R_1}\right] = 6V$$

$So,\ v_2 = 3V,\ v_c(o^-) = 6V$

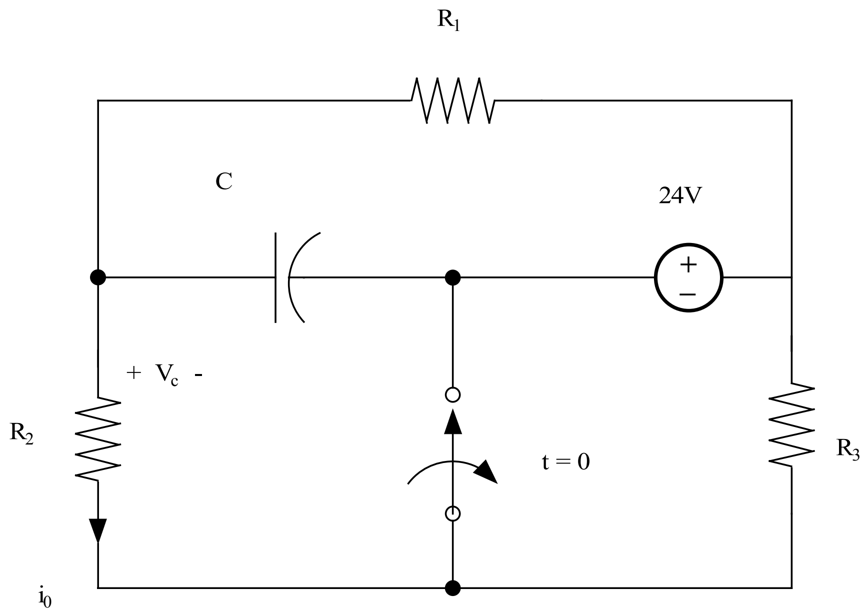
$\boxed{v_0 = -3.6e^{-8t}V}$

Problem 6.43

Find $i_0(t)$ for $t > 0$ in the circuit in Fig. P6.43 using the step-by-step method.



Suggested Solution



$R_1 = 4\ k\Omega$

$R_2 = 2\ k\Omega$

$R_3 = 10\ k\Omega$

$C = 200\ \mu F$

$\underline{\underline{\text{For } t \to \infty}}: \quad i_0 \to 0 = k_1, \qquad \text{So, } k_2 = -\frac{4}{3} mA$

$\tau = c\left[R_1 // \left(R_2 + R_3\right)\right] = 0.6\ sec$

$\underline{\underline{\text{For } t = 0^-}}: \ i_0 = \frac{24}{R_1 + R_2} = 4mA, \quad v_c = i_0 R_2 = 8V$

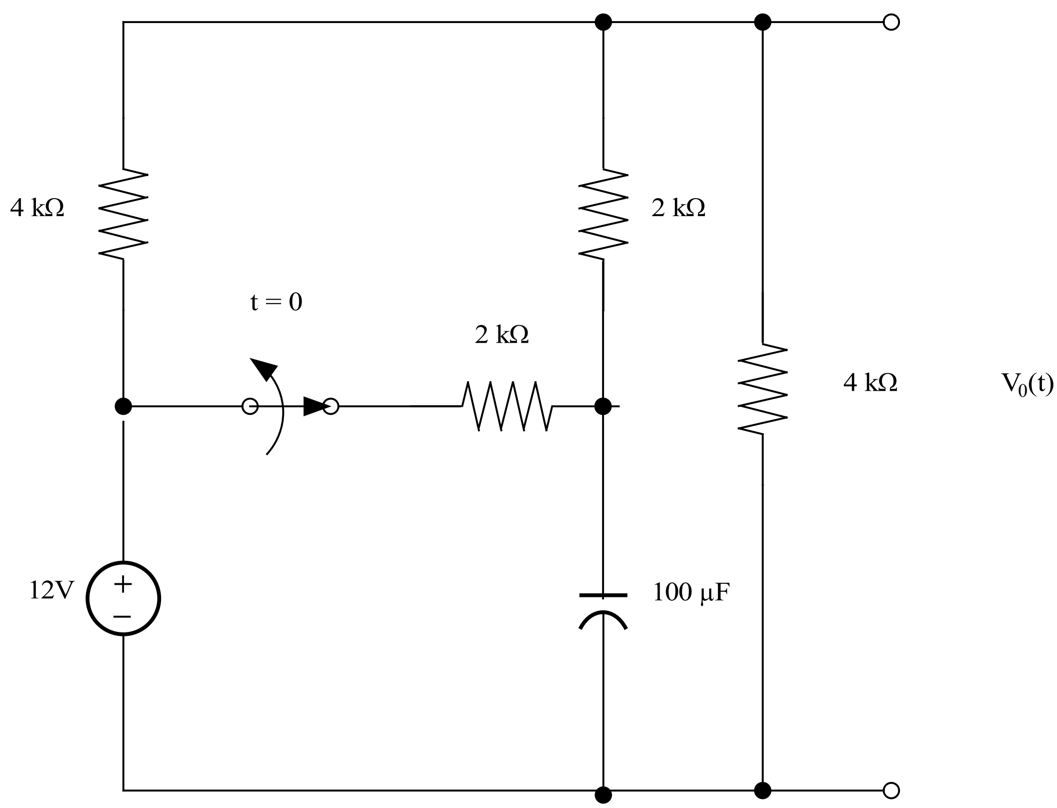
$\underline{\underline{For\ t = 0^+}}: \ i_0 = \frac{v_c(0^+) - 24}{R_2 + R_3} = \frac{8 - 24}{12k} = -\frac{4}{3} mA$

$i_o(0^+) = -\frac{4}{3} mA = k_1 + k_2$

$\boxed{i_0(t) = -1.33 e^{-t/0.6} mA}$

Problem 6.44

Find $V_0(t)$ for t > 0 in the network in Fig. P6.44 using the step-by-step method.



Suggested Solution

for t=0$^-$

$$v_o = 12\left[\frac{R_4}{R_4 + (R_1 \,||\, (R_2 + R_3))}\right]$$

$$v_2 = (12 - v_o)\left[\frac{R_2}{R_2 + R_3}\right] = 2V$$

$$v_c = (12 - v_2) = 10V$$

for t=0$^+$

$$\frac{(12 - v_o)}{R_1} + \frac{v_c(0^+) - v_o}{R_3} = \frac{v_o}{R_4}$$

$$v_c(0^+) = v_c(0^-) = 10V$$

$$12 - v_o - 2v_o + 20 = v_o \Rightarrow v_o(0^+) = 8V = k_1 + k_2$$

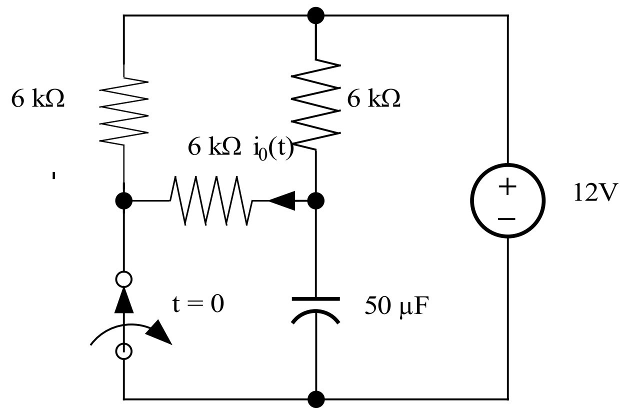
for t $\Rightarrow \infty$:

$$v_o = 12\left(\frac{R_4}{R_4 + R_1}\right) = 6V = k_1 \Rightarrow k_2 = 2V$$

$$v_o(t) = 6 + 2e^{-2.5t}v$$

Problem 6.45

Use the step-by-step method to find $i_0(t)$ for $t > 0$ in the network in Fig. P6. 45.



Suggested Solution

for $t=0^-$

$$v_c = 12\left[\frac{R_2}{R_2+R_3}\right] = 6V$$

for $t=0^+$

by superposition

$$\mathrm{i_o} = \frac{-12}{R_1+R_2} + \frac{v_c(0^+)}{R_1+R_2} = -1+0.5 = -0.5mA$$

$$\mathrm{i_o}(0^+) = -0.5mA = k_1 + k_2$$

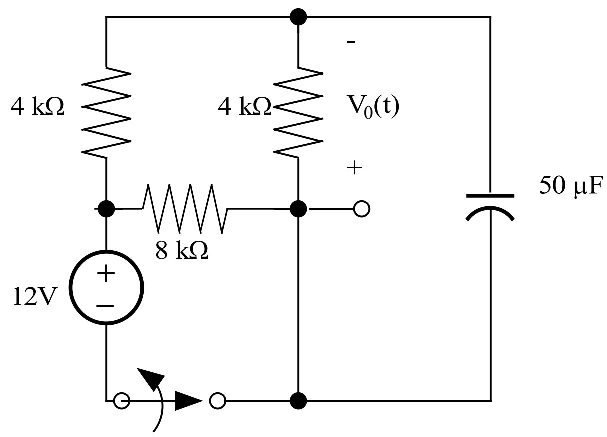
for $t \Rightarrow \infty$:

$$\mathrm{i_o}(\infty) = 0 = k_1 \Rightarrow k_2 = -0.5mA$$

$$\mathrm{i_o}(t) = -0.5e^{-5t}mA$$

Problem 6.46

Find $V_0(t)$ for $t > 0$ in the circuit in Fig. P6.46 using the step-by-step method.



Suggested Solution

for $t=0^-$

$$v_o = 12\left[\frac{R_3}{R_3 + R_1}\right] = 6V$$

$$v_c = -v_o = -6V = v_c(0^+)$$

for $t=0^+$

$$v_o = -v_c = -6V$$

$$v_o(0^+) = 6 = k_1 + k_2$$

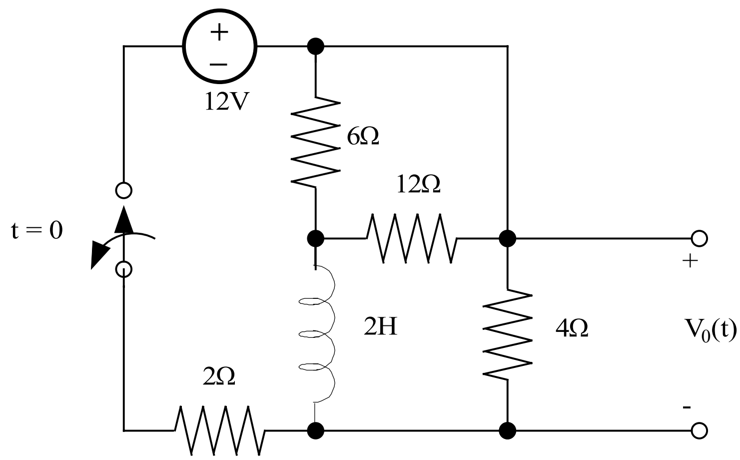
for $t \Rightarrow \infty$:

$$v_o(\infty) = 0 = k_1 \Rightarrow k_2 = 6$$

$$v_o(t) = 6e^{\frac{-t}{0.15}} v$$

Problem 6.47

Use the step-by-step technique to find $V_0(t)$ for $t > 0$ in the circuit in Fig. P6.47.



Suggested Solution

for t=0$^-$

$$i_s = \frac{12}{R_1 + (R_2 \parallel R)} = 3A$$

$$i_L = \frac{i_s R_2}{R_2 + R} = 1.5A = i_L(0^+)$$

$$\tau = \frac{L}{R_2 + R} = \frac{1}{4}s$$

for t=0$^+$

$$v_o = -i_L R_2 = -6 = k_1 + k_2$$

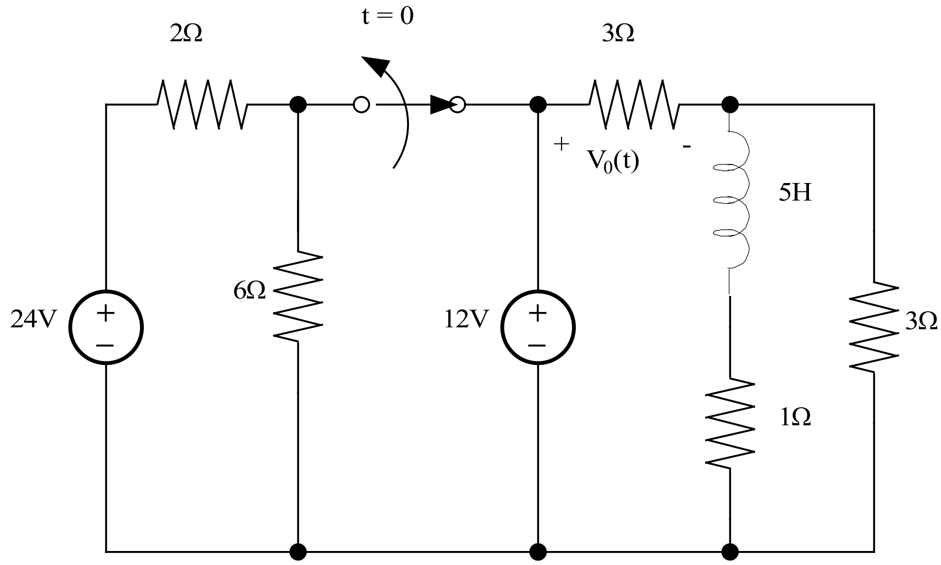
for t $\Rightarrow \infty$:

$$v_o = 0 = k_1 \Rightarrow k_2 = 6V$$

$$v_o(t) = -6e^{-4t}v$$

Problem 6.48

Find $V_0(t)$ for t > 0 in the circuit in Fig. P6.48 using the step-by-step method.



Suggested Solution

for t=0$^-$

$$i_o = \frac{12}{R_3 + (R_4 \parallel R_5)} = 3.2A$$

$$i_L = i_o\left(\frac{R_5}{R_4 + R_5}\right) = 2.4A$$

use superposition,

for t=0$^+$

$$v_o = 12\left(\frac{R_3}{R_3 + R_5}\right) + i_L(0^+)R_3\left(\frac{R_5}{R_4 + R_5}\right) = 9.6$$

$$v_o(0^+) = 9.6 = k_1 + k_2$$

for t $\Rightarrow \infty$:

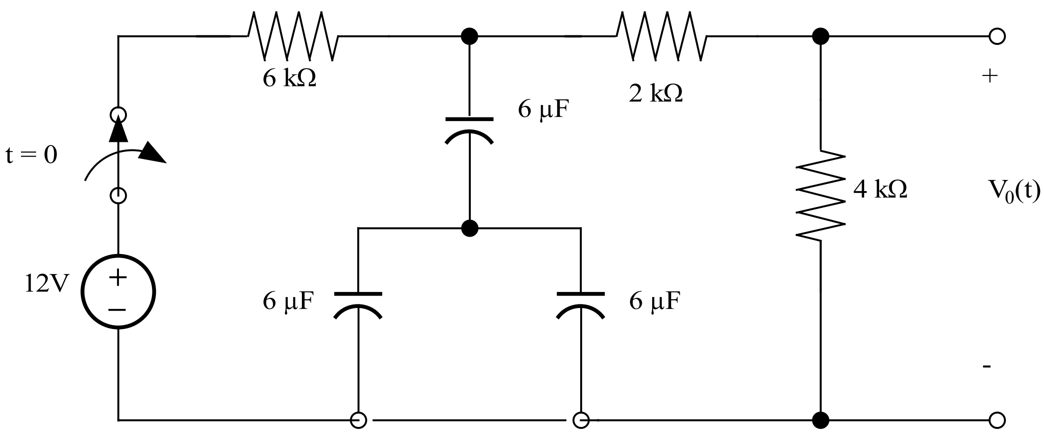
$$v_o = 12\left(\frac{R_3}{R_3 + (R_4 \parallel R_5)}\right) = 9.6 = k_1$$

$$k_2 = 0$$

$$v_o = 9.6V$$

Problem 6.49

Use the step-by-step method to find $V_0(t)$ for $t > 0$ in the network in Fig. P6.49.



Suggested Solution

$$v_o(t) = k_1 + k_2 e^{\frac{-t}{\tau}} \qquad v_o(0^+) = 6V$$

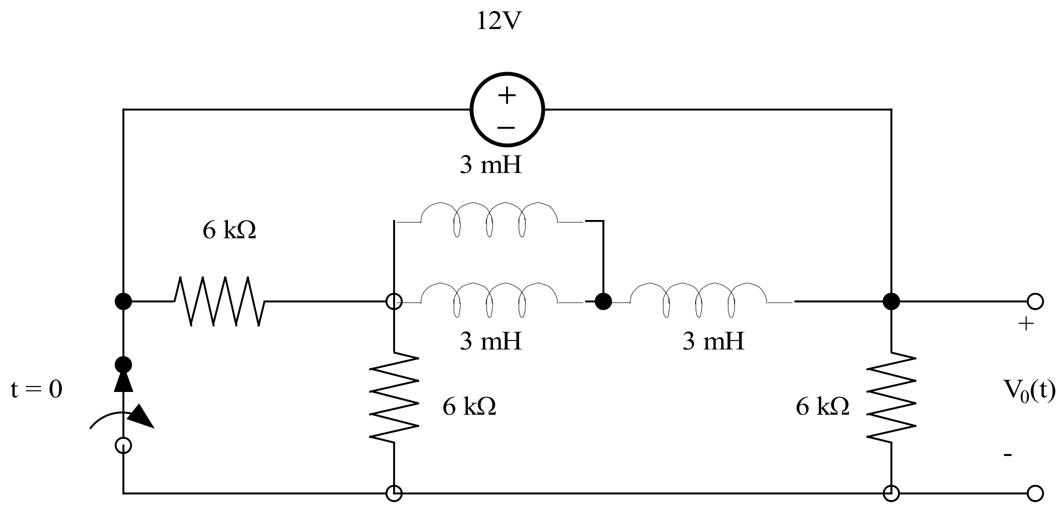
$$\tau = R_{AC} = 6k(\frac{4}{k}\Omega) = 0.0245$$

$$v_o(t) = 4e^{\frac{-t}{0.024}} v$$

$$t > 0$$

Problem 6.50

Find $V_0(t)$ for t > 0 in the network in Fig. P6.50 using the step-by-step method.



Suggested Solution

$$v_o(t) = k_1 + k_2 e^{\frac{-t}{\tau}} \qquad i_L(0^-) = \frac{12}{6k} + \frac{12}{6k} = \frac{4}{k} A$$

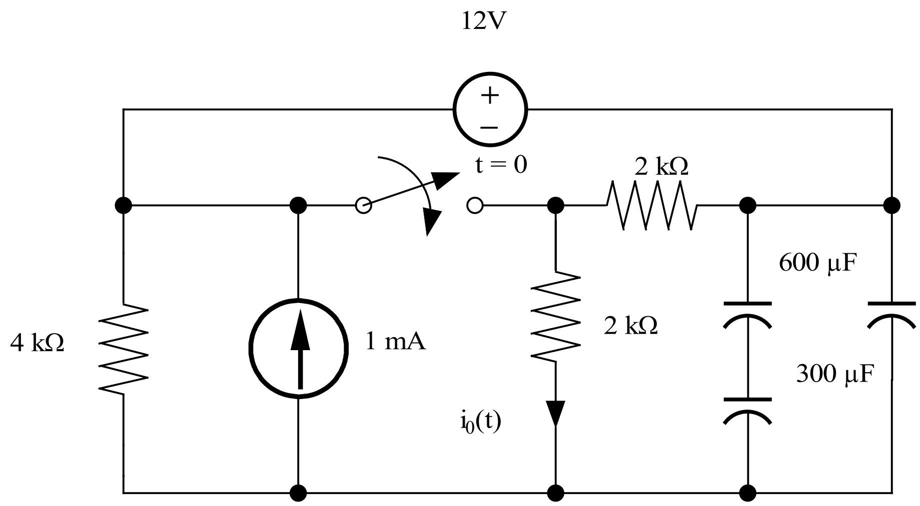
$$v_c(0^+) = \frac{-2}{k}\left(\frac{6k}{6k+12}\right)6k = -4V$$

$$\tau = \frac{L}{R_{th}} = \frac{4.5*10^{-6}}{4k}$$

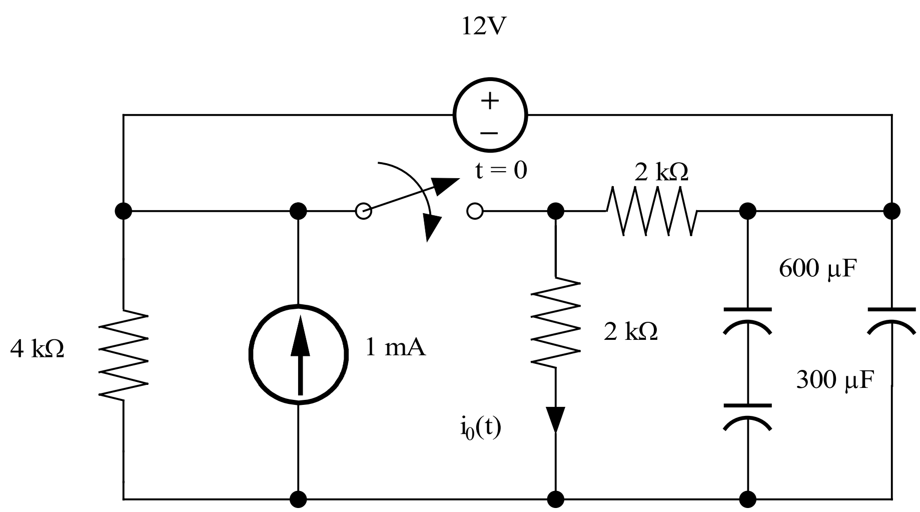
$$v_o(\text{t}) = -4e^{\frac{-t}{\tau}}$$

Problem 6.51

Use the step-by-step method to find $i_0(t)$ for $t > 0$ in the network in Fig. P6.51



Suggested Solution



$R1 = 4k\Omega, R2 = R3 = 2k\Omega$

$$c = 200\mu + \frac{1}{\frac{1}{600\mu} + \frac{1}{300\mu}} = 400\mu F$$

For

$t = 0^+$, Use Superposition

$$i_{012v} = \frac{-12}{R_2} = -6mA$$

$$i_{01mA} = 1m(0) = 0$$

$$i_{0v_c(0^+)} = \frac{8}{R_2} = 4mA$$

$$i_0(0^+) = \sum i_0 = -2mA = K_1 + K_2$$

$t = 0^-$, Use Superposition

$$v_c = 12\left(\frac{R_2 + R_3}{R_1 + R_2 + R_3}\right) + 1m\left(\frac{R_1}{R_1 + R_2 + R_3}\right)$$

$$v_c = 8v = v_c(0^+)$$

$$\tau = c[R_1 \parallel R_2] = \frac{8}{15}$$

for

$t \to \infty$

$$i_0 = 1m\left(\frac{R_1}{R_1 + R_2}\right) = \frac{2}{3}mA$$

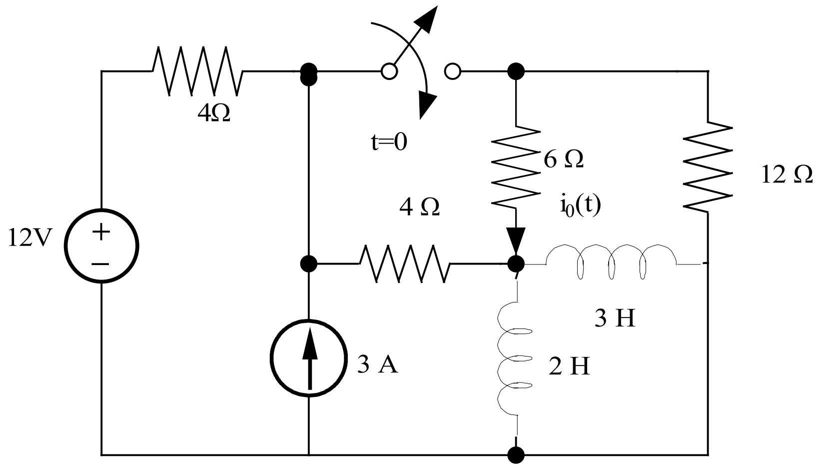
$$i_0(\infty) = \frac{2}{3}mA = K_1$$

$$K_2 = -\frac{8}{3}mA$$

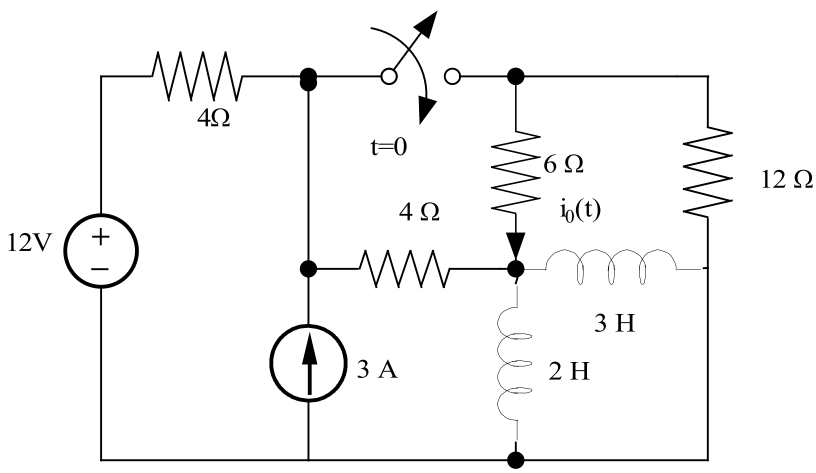
$$i_0(t) = \frac{2}{3} - \frac{8}{3}e^{-15t/8}mA$$

Problem 6.52

Use the step-by-step method to find $i_0(t)$ for $t > 0$ in the network in Fig. P6.52



Suggested Solution

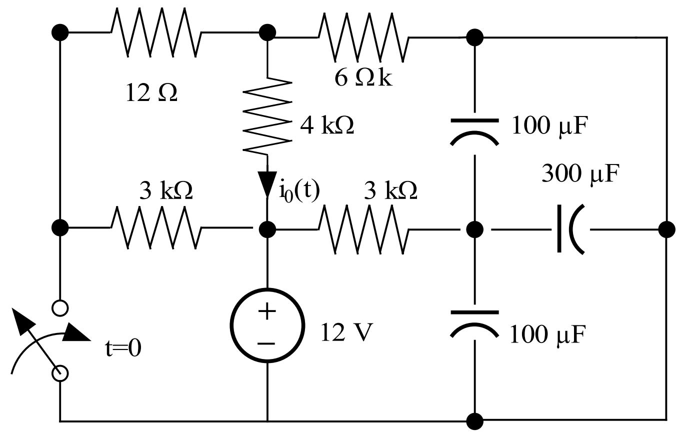


$$R_1 = 4 = R_2 = 4\Omega$$
$$R_3 = 6\Omega$$
$$R_4 = 12\Omega$$
$$L = \frac{1}{\frac{1}{2}+\frac{1}{3}} = 1.2H$$
$$\tau = \frac{L}{R_2 \parallel R_3 + R_1 \parallel R_3} = \frac{2}{9}$$

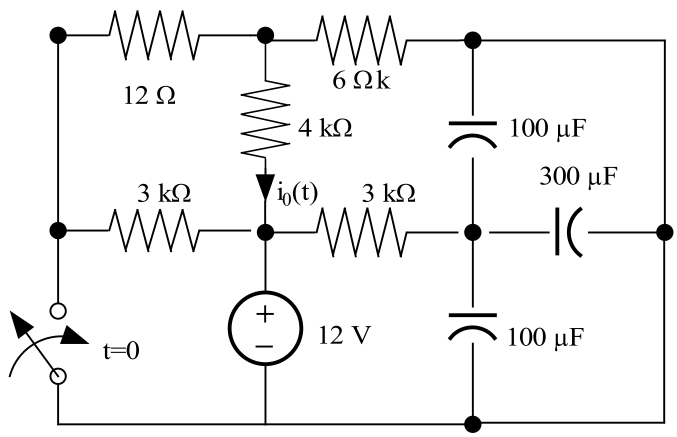
$$i_0 = \frac{4}{3} - \frac{2}{15}e^{-9t/2}A$$
For
$t = 0^-$
$$i_L = \frac{24}{R_1 + R_2} = 3A = i_L(0^+)$$
For
$t = 0^+$
$$i_0 = i_2(0^+)[\frac{R_2}{R_2 + R_3}] = 1.2A = K_1 + K_2$$
For
$t \rightarrow \infty$
$$i_0 = \frac{24}{R_1 + (R_2 \parallel R_3 \parallel R_4)}[\frac{R_2 \parallel R_4}{(R_2 \parallel R_4) + R_3}]$$
$$i_0 = \frac{4}{3} = K_1 \Rightarrow K_2 = \frac{-2}{15}$$

Problem 6.53

Find $i_0(t)$ for $t > 0$ in the circuit in Fig. P6.53 using the step-by-step method.



Suggested Solution



$R_1 = 12k\Omega$

$R2 = R4 = 3k\Omega$

$R3 = 4k\Omega$

$R5 = 6k\Omega$

$c = 300\mu + 100\mu + 100\mu = 500\mu F$

for

$t = 0^-$

$i_4 = 0$

$v_c = -12v$

for

$t = 0^+$

$v_c = -12v$

$i_4 = 0$

$$i_0 = \frac{-12}{R_3 + (R_1 \| R_5)} = -1.5mA$$

for

$t \to \infty$

$i_4 = 0$

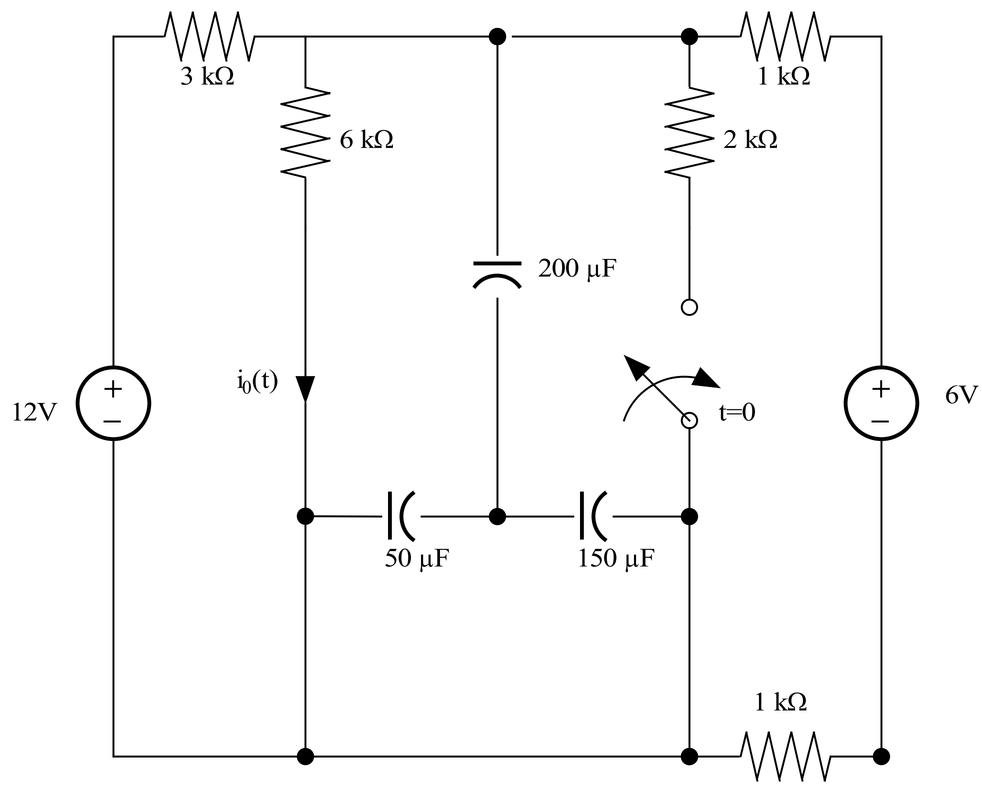
$v_c = -12v$

$$i_0 = \frac{-12}{R_3 + (R_1 \| R_5)} = -1.5mA$$

$i_0 = -1.5mA$

Problem 6.54

Find $i_0(t)$ for $t > 0$ in the circuit in Fig. P6.54 using the step-by-step technique.



Suggested Solution

$R_1 = 3k\Omega, R_2 = 6k\Omega, R_3 = 2k\Omega = R$

$c = 100\mu F$

for

$t \to \infty$

$$i0 = 12[\frac{1}{R_1 + (R_2 \| R_3 \| R)}](\frac{R_3 \| R}{R_3 \| R + R_2})$$

$$-6[\frac{1}{R_1 + (R_1 \| R \| R_2)}](\frac{R_1 \| R_3}{R_1 \| R_3 + R2})$$

$$i_0 = \frac{1}{9} mA = k_1$$

$$k_2 = \frac{1}{18} mA$$

for

$t = 0^-$

$$v_c = 12[\frac{R_1 \| R_2}{R \| R_2 + R_1}] - 6[\frac{R_1 \| R_2}{R_1 \| R_2 + R}]$$

$v_c = 1v = v_c(0^+)$

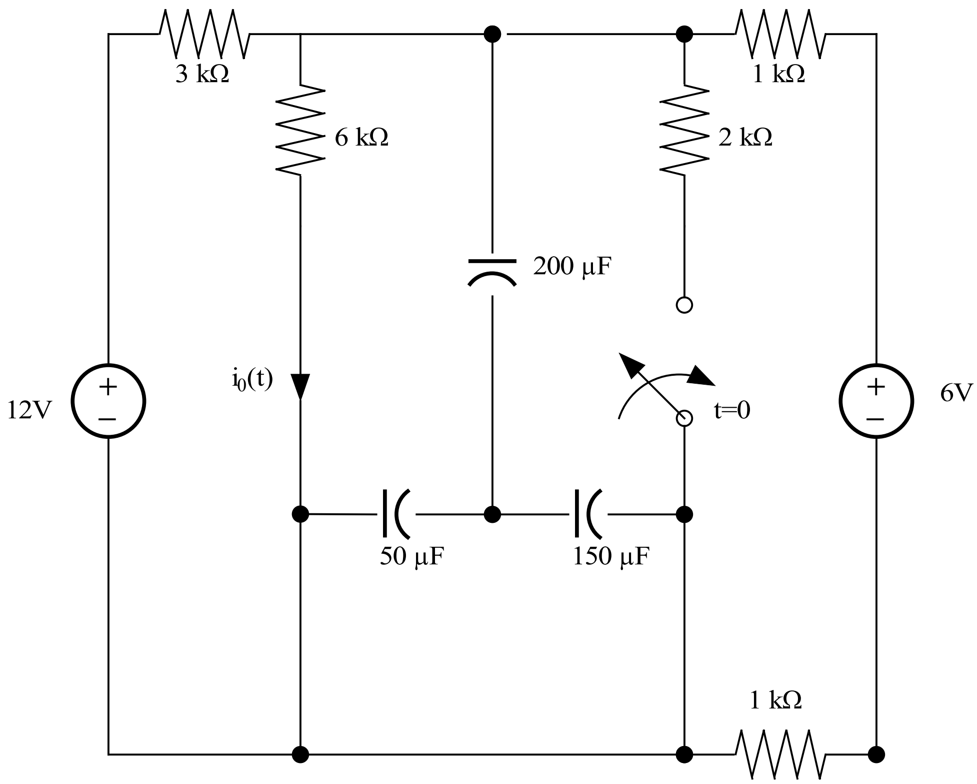
for

$t = 0^+$

$$i_0 = \frac{v_c}{R_2} = \frac{1}{6} mA$$

$$\tau = c[R_1 \| R_2 \| R_3 \| R] = \frac{1}{15} \sec$$

$$i_0 = \frac{1}{9} + \frac{1}{18} e^{-15t} mA$$



3 kΩ
6 kΩ
200 μF
2 kΩ
1 kΩ
12V
i0(t)
t=0
6V
50 μF
150 μF
1 kΩ

Problem 6.55

The differential equation that describes the current $i_0(t)$ in a network is

$$\frac{d^2 i_0(t)}{dt^2} + 6\left[\frac{di_0(t)}{dt}\right] + 8i_0(t) = 0$$

Find

(a) The characteristic equation of the network.
(b) The network's natural frequencies.
(c) The expression for $i_0(t)$

Suggested Solution

$$\frac{d^2 i_0(t)}{dt^2} + 6\left[\frac{di_0(t)}{dt}\right] + 8i_0(t) = 0$$

(a) The characteristic equation is $S^2+6S+8=0$
(b) The natural frequencies are
$S=-2$ and $S=-4$
(c) $i_0(t)=K_1e^{-2t}+K^2e^{-4t}$

Problem 6.56

The terminal current in a network is described by the equation

$$\frac{d^2 i_0(t)}{dt^2} + 10\left[\frac{di_0(t)}{dt}\right] + 25 i_0(t) = 0$$

Find

(a) The characteristic equation of the network.
(b) The network's natural frequency.
(c) The equation for $i_0(t)$.

Suggested Solution

$i_0(t)$: $$\frac{d^2 i_0(t)}{dt^2} + 10\left[\frac{di_0(t)}{dt}\right] + 25 i_0(t) = 0$$

(a) The characteristic equation is $S^2+10^S+25=0$
(b) The natural frequency is S=-5
(c) $i_0(t)=K_1e^{-5t}+K_2+e^{-5t}$

Problem 6.57

The voltage v_1 (t) in a network is defined by the equation

$$\frac{d^2 i_0(t)}{dt^2} + 10\left[\frac{di_0(t)}{dt}\right] + 25 i_0(t) = 0$$

Find
(a) The characteristic equation of the network.
(b) The circuit's natural frequencies.
(c) The expression for v_1 (t).

Suggested Solution

i_0(t): $$\frac{d^2 i_0(t)}{dt^2} + 10\left[\frac{di_0(t)}{dt}\right] + 25 i_0(t) = 0$$

(a) The characteristic equation is $S^2+2S+5=0$.
(b) The natural frequencies are
$S=-1+2\delta$, $S=-1-2\delta$
(c) $v_1(t)=K_1 e^{-t}\cos 2t+K_2 e^{-t}\sin 2t$.

Problem 6.58

The output voltage of a circuit is described by the differential equation

$$\frac{d^2 v_0(t)}{dt^2} + 6\left[\frac{dv_0(t)}{dt}\right] + 10v_0(t) = 0$$

Find

(a) The characteristic equation for the circuit.
(b) The networks natural frequencies.
(c) The equation for v_0 (t).

Suggested Solution

$$\frac{d^2 v_0(t)}{dt^2} + 6\left[\frac{dv_0(t)}{dt}\right] + 10v_0(t) = 0$$

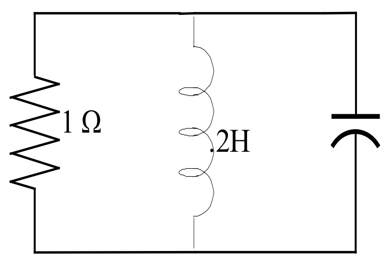
(a) The characteristic equation $S^2+6S+10=0$.
(b) Natural frequencies are S=-3 +δ. S=-3-8.
(c) v_0 (t)=K_1e^{-3t} cos t+K_2e^{-3t} sin t

Problem 6.59

The parameters for a parallel RLC circuit are R = 1Ω, L = 1/5 H, and C = ¼ F. Determine the type of damping exhibited by the circuit.

Suggested Solution

What type of damping occurs in this network?



Characteristic Equation:

$$S^2 + \frac{1S}{RC} + \frac{1}{LC} = 0 = S^2 + 4S + 20$$

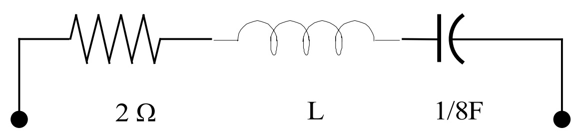
The Roots are S=-1±4δ which are complex conjugates. The network is underdamped.

Problem 6.60

A series RLC circuit contains a resistor R = 2Ω and a capacitor C = 1/x F.
Select the value of the inductor so that the circuit is critically damped.

Suggested Solution

What inductance cause critical damping?



If $S^2+\frac{R}{L}S+\frac{1}{L_C}=0$

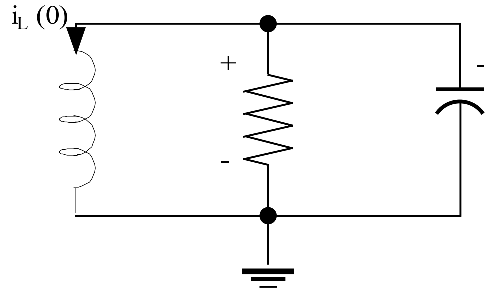
$S^2+\frac{2}{L}S+\frac{8}{L}=0$ with roots $S=\frac{\frac{-2}{L}\pm\sqrt{\frac{4}{L^2}-\frac{32}{L}}}{2}$

for critical damping $\frac{4}{L^2}-\frac{32}{L}=0.$

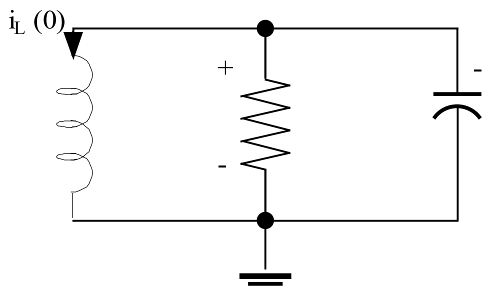
$$\boxed{L=\frac{1}{8}H}$$

Problem 6.61

For the underdamped circuit shown in Fig. 6.61 determine the voltage v(t) if the initial conditions on the storage elements are $i_L(0)$=1 A and v(0) = 10 V.



Suggested Solution



Find V(t) if $i_L(0)$=1A and $v_c(0)$=10v

$$\frac{d^2v(t)}{dt^2}+\frac{1}{RC}\frac{dv(t)}{dt}+\frac{1}{LC}v(t)=0$$

The characteristic equation is

$$s^2+8s+20=0$$

$$s=-4\pm 2j$$

so

$$v(t)=k_1e^{-4t}\cos 2t+k_2e^{-4t}\sin 2t$$

at

$$t=0, v_c=10$$

$$v(t)=k_1=10$$

then

$$\frac{dv(t)}{dt} = -2k_1e^{-4t}\sin 2t - 4k_1e^{-4t}\cos 2t + 2k_2e^{-4t}\cos 2t - 4k_2e^{-4t}\sin 2t$$

at

$t = 0$

$$\frac{dv(t)}{dt} = -4k_1 + 2k_2 = -40 + 2k_2$$

also

$$\frac{cdv(t)}{dt} + \frac{v(t)}{R} + i_L(t) = 0$$

at

$t = 0$

$$\frac{dv(t)}{dt} = \frac{1}{c}\left(\frac{-v(t)}{R} - i_L(t)\right) = \frac{1}{c}\left(\frac{-10}{R} - 1\right)$$

if

$-40 + 2k_2 = -120$

finally

$v(t) = 10e^{-4t}\cos 2t - 40e^{-4t}\sin 2t$

Problem 6.62

Given the circuit and the initial conditions of Problem 6.61, determine the current through the inductor.

Suggested Solution

Find $i_L(t)$ for the circuit

The network is Underdamped

$$i_L(t) = k_3 e^{-4t}\cos 2t + k_4 e^{-4t}\sin 2t$$

at

$$t = 0$$

$$i_L = k_3 = 1A$$

$$\frac{di_L(t)}{dt} = -2k_3 e^{-4t}\sin 2t - 4k_3 e^{-4t}\cos 2t + 2k_4 e^{-4t}\cos 2t - 4k_4 e^{-4t}\sin 2t$$

at

$$t = 0$$

$$\frac{di_L(t)}{dt} = -4k_3 + 2k_4 = -4 + 2k_4$$

$\sin ce$

$$v(t) = \frac{Ldi_L(t)}{dt}, \frac{di_L(t)}{dt} = \frac{v(0)}{L} = 5$$

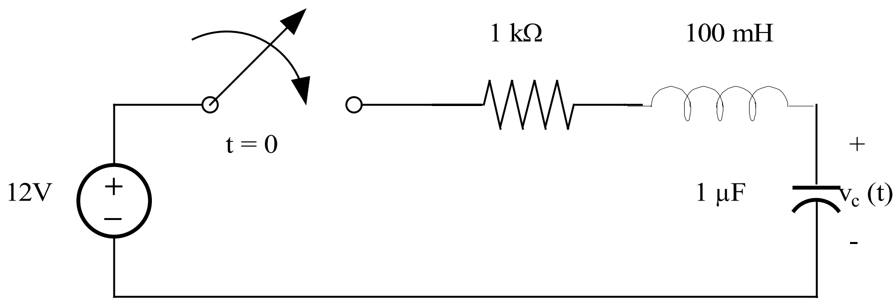
so

$$k_4 = \frac{1}{2}(5 + 4x3) = \frac{9}{2}$$

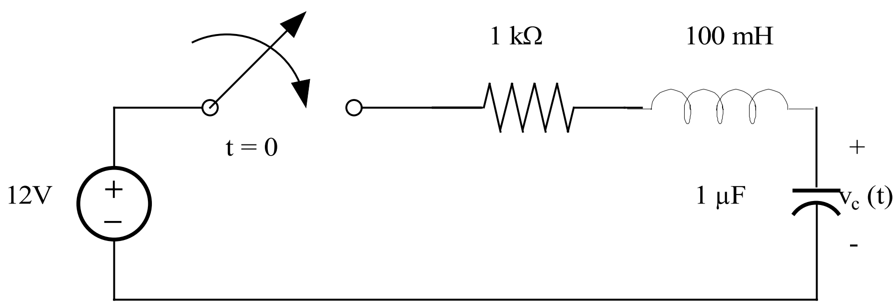
$$i_L(t) = e^{-4t}\cos 2t + \frac{9}{2}e^{-4t}\sin 2t$$

Problem 6.63

Find $v_C(t)$ for $t > 0$ in the circuit in Fig. P6.63 if $v_C(0) = 0$.



Suggested Solution



$R = 1k\Omega, c = 1\mu F, L = 100mH$

for

$t = 0^-$

$v_0 = 0$

$i_L = 0$

for

$t = 0^+$

$v_0 = 0, i_L = 0$

for

$t > 0$

series RLC with constant forcing function

$char\_eq$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$s^2 + 10^4 s + 10^7 = 0$$

$Roots\_are$

$$s_1 = -1127\frac{-r}{s}, s_2 = -8873\frac{r}{s} \Rightarrow overdamped$$

solution is

$$i_2(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t} + k_3$$

$$i_L(0^+) = 0 = k_1 + k_2 + k_3 = 0$$

$$i_L(\infty) = 0 = k_3$$

$$k_2 = -k_1$$

$$i_L(t) = k_1(e^{s_1 t} - e^{s_2 t})$$

KVL

$$12 = Ri_L(t) + L\frac{di_L(t)}{dt} + v_0(t)$$

at

$t = 0$

$$12 = R(0) + L[s_1 e^0 - s_2 e^0]k_1 + 0 \Rightarrow k_1 = 15.5mA$$

now

$$i_L(t) = 15.5(e^{s_1 t} - e^{s_2 t})mA$$

$$r_0(t) = \frac{1}{c}\int i_L(t) = \frac{15.5}{100}\frac{1}{10^{-6}}[\frac{e^{s_1 t}}{s_1} - \frac{e^{s_2 t}}{s_2}] + k$$

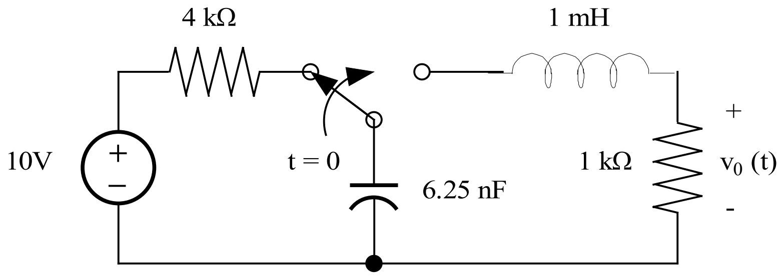
$$v_0(\infty) = 12 = k$$

$$v_0(t) = 12 - 13.75e^{-1127t} + 1.75e^{-8875t}v \quad t > 0$$

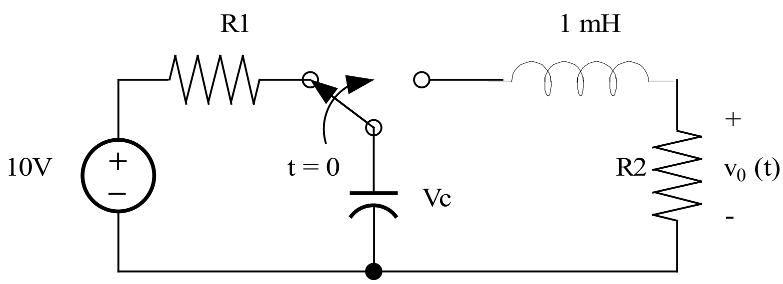
$$v_0(t) = 0v \quad t \le 0$$

Problem 6.64

Find $v_0(t)$ for $t > 0$ in the circuit in Fig. P6.64 and plot the response including the time interval just prior to moving the switch.



Suggested Solution

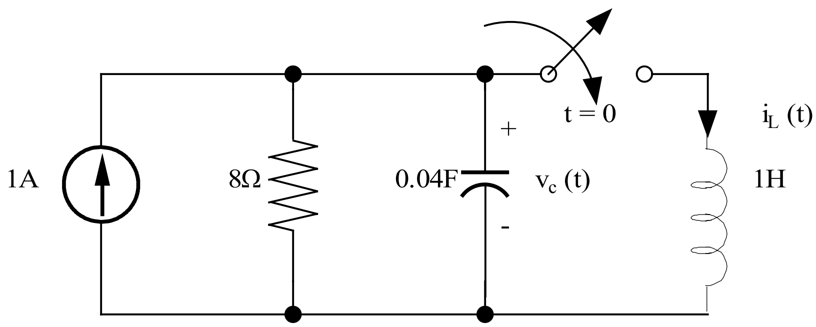


$R_1 = 4k\Omega, R_2 = 1k\Omega, L = 1mH$
$c = 6.25nF$

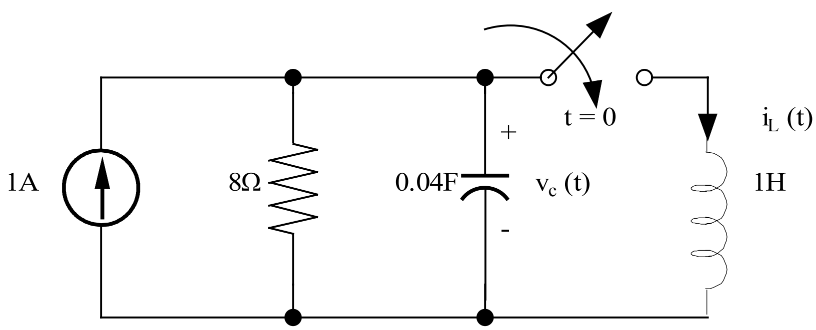
for
$t = 0^-$
$v_0 = 0$
$i_L = 0, v_c = 10v$
for
$t = 0^+$
$v_0 = 0, i_L = 0, v_c = 10v$
for
$t > 0$
series RLC with constant forcing function
$char\_eq$
$$s^2 + \frac{R_2}{L}s + \frac{1}{LC} = 0 \Rightarrow s^2 + 10^6 s + 1.6x10^{11} = 0$$
$roots\_are$
$s_1, s_2 = -2x10^5 and -8x10^5 r/s$
$overdamped$
$v_0(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t}$
at
$t = 0^+$
$v_0(0^+) = 0 = k_1 + k_2$
$k_2 = -k_2$
$also$
at
$t = 0^+$
$$v_c(0^+) = L\frac{dv_0/dt}{R_2}|_{t=0} + v_0(0^+) = 10v$$
or
$10^{-6}k_1[s_1 - s_2] = k_1(0.6) = 10 \Rightarrow k_1 = 16.67$
$v_0(t) = 0v$ t<0
$v_0(t) = 16.67[e^{-2x10^5 t} - e^{-8x10^5 t}]v$ $t \geq 0$

Problem 6.65

Find $v_C(t)$ for $t > 0$ in the circuit in Fig. P6.65.



Suggested Solution



$R = 8\Omega, c = 40mF, L = 1H$

for

$t = 0^-$

$i_L = 0, i_R = 1A, v_c = 8v, i_c = 0$

for

$t = 0^+$

$i_L = 0, i_R = 1A, v_c = 8v, i_c = 0$

for

$t = \infty$

$i_L = 1, v_c = 0$

Parallel RLC char eq is

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$roots$

$s_1, s_2 = 1.56 \pm j4.75$

system is underdamped and

$v_c(t) = e^{\sigma t}(A\cos\omega t + B\sin\omega t) + k$

$where$

$\sigma = -1.56$

$\omega = 4.75$

$v_c(0^+) = 8 = A + K$

and

$v_c(\infty) = 0 = k$

$A = 8$

by

KCL

at

$t = 0^+$

$$1 = \frac{v_c(0^+)}{R} + i_L(0^+) + c\frac{dv_c(t)}{dt}|_{t=0^+} = 1 + 0 + c\frac{dv_c(t)}{dt}|_{t=0}$$

so

$$\frac{dv_c(t)}{dt}|_{t=0} = 0 = B\omega + \sigma A \Rightarrow B = \frac{-\sigma A}{\omega} = 2.63$$

$v_c(t) = e^{-1.56t}[8\cos(4.75t) + 2.63\sin(4.75t)]v \quad \text{t>0}$

$v_c(t) = 8v \quad t \le 0$

Problem 6.66

Find iL(t) for t > o in the circuit in Fig. 6.65.

Suggested Solution

From problem 6.65 for t>0

$$v_c(t) = e^{-1.56t}[8\cos\omega t + 2.63\sin\omega t]$$

where

$$\omega = 4.75 r/s$$

KCL :

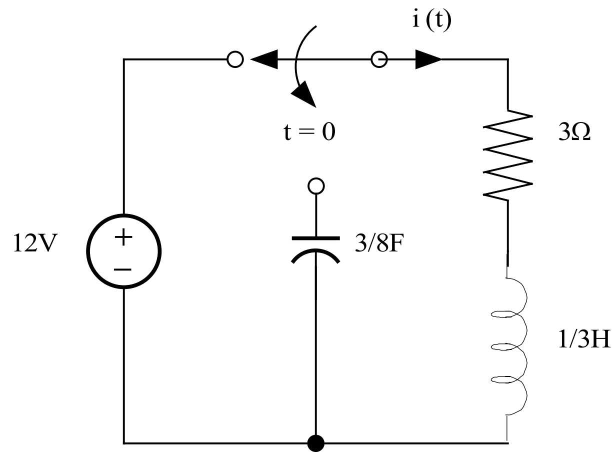
$$1 = \frac{v_c(t)}{R} + c\frac{dv_c(t)}{dt} + i_L(t)$$

$$i_L(t) = 1 - e^{-1.56t}[\cos\omega t + 0.329\sin\omega t] + [1.52\sin\omega t - 0.5\cos\omega t]e^{-1.56t} + 1.56e^{-1.56t}[8\cos\omega t + 2.63\sin\omega t]$$

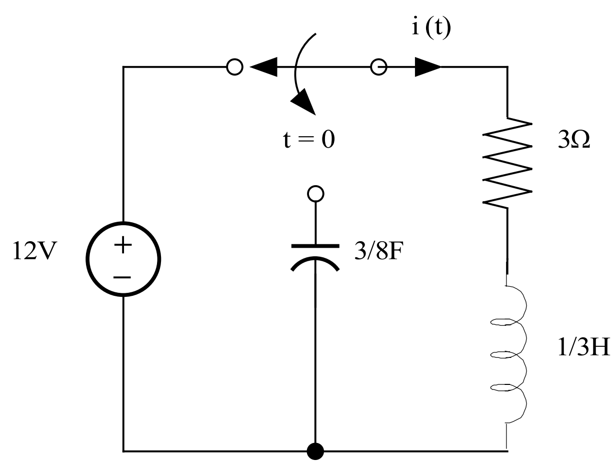
$$i_L(t) = 1 + e^{-1.56t}[10.98\cos(4.75t) + 5.29\sin(4.75t)]A$$

Problem 6.67

Given the circuit in Fig. 6.67, find the equation for i(t), t > 0.



Suggested Solution



for

$t < 0$

$i_L = 12/3 = 4A, v_c = 0v$

for

$t = 0^+$

$i_L = 4A, v_c = 0v$

for

$t > 0$

Series RLC circuit

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \Rightarrow s^2 + 9s + s = 0 \Rightarrow (s+8)(s+9) = 0$$

since roots are real and unequal, overdamped and

$i(t) = k_1 e^{-t} + k_2 e^{-8t}$

$i(0) = 4 = k_1 + k_2$

$KCL:$

$$v_c(0^+) = Ri(0^+) + L\frac{di(t)}{dt}|_{t=0} \Rightarrow 0 = 12 + \frac{1}{3}(-k_1 - 3k_2)$$

so

$k_1 + k_2 = 4$

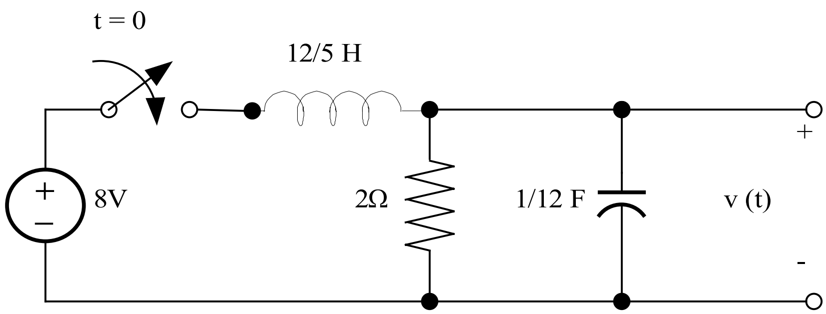
$k_1 + 8k_2 = 36$

$$k_1 = \frac{-4}{7}, k_2 = \frac{32}{7}$$

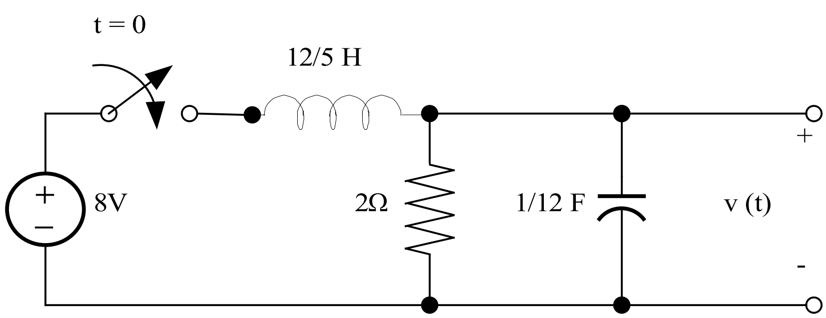
$$i(t) = \frac{32}{7}e^{-8t} - \frac{4}{7}e^{-t}A \quad \text{for t>0}$$

Problem 6.68

In the circuit shown in Fig. P6.68, find v(t), t > 0.



Suggested Solution



for

$t = 0^-$

$i_L = 0, v_c = v = 0v$

for

$t = 0^+$

$i_L = 0, v = 0$

for

$t > 0$

Parallel RLC circuit, char eq is

$$s^2 + \frac{s}{RC} + \frac{1}{LC} = s^2 + 6s + 5 = 0$$

$roots\_are$

$s_1 = -1, s_2 = -5 \Rightarrow overdamped$

$v(t) = k_1 e^{-t} + k_2 e^{-5t} + k_3$

$v(0^+) = 0 = k_1 + k_2 + k_3$

$v(\infty) = k_3 = 8v$

KCL

at

$t = 0^+$

$$i_L(0^+) = \frac{v(0^+)}{R} + c\frac{dv(t)}{dt}|_{t=0^+} \Rightarrow \frac{dv(t)}{dt}|_{t=0^+} = 0$$

or

$-k_1 - 5k_2 = 0$

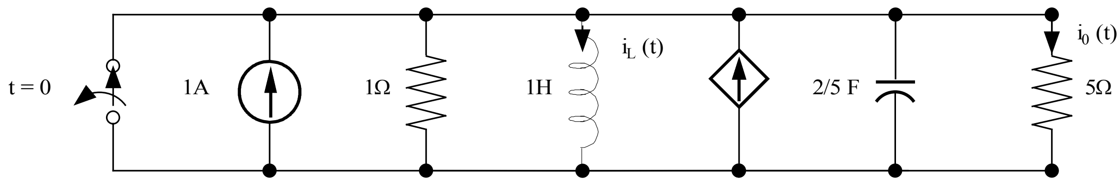
$k_1 + k_2 = -8$

$k_1 = -10, k_2 = 2$

$v(t) = 8 + 2e^{-5t} - 10e^{-t} v$

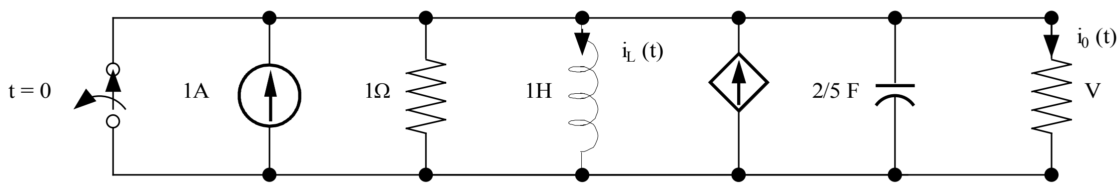
Problem 6.69

Find $i_0(t)$ for > 0 in the circuit in Fig. P6.69 and plot the response including the time interval just prior to opening the switch.



$$\frac{i_L(t)}{2}$$

Suggested Solution



$R_1 = 1\Omega, R_2 = 5\Omega, L = 1H, c = \frac{2}{3}F$

for

$t = 0^-$

$i_L = 0, v_c = 0, i_0 = 0, i_{R_1} = i_{R_2} = 0, i_c = 0$

for

$t = 0^+$

$i_L = 0, v_c = 0, i_0 = 0, i_{R_1} = 0, i_c = 1A$

for

$t \to \infty$

$v_c \to 0, i_0 \to 0$

for

$t > 0$

$KCL:$

$$1 = \frac{v}{R_1} + \frac{v}{R_2} + i_c + \frac{i_L}{2}$$

$$1 = v(G_1 + G_2) + c\frac{dv}{dt} + \frac{1}{2L}\int vdt$$

$$i_0 = \frac{v}{R_2}$$

$so,$

$$25 = 30i_0 + 10\frac{di_0}{dt} + 12.5\int i_0 dt \Rightarrow \frac{d^2 i_0}{dt^2} + \frac{3di_0}{dt} + 1.25 = 0$$

$char\_q$

$s^2 + 3s + 1.25 = 0$

$roots\_are$

$s_1 = -0.5, s_2 = -2.5 \Rightarrow overdamped$

$i_0(t) = k_1 e^{-t/2} + k_2 e^{-6t/2} + k_3$

$i_0(\infty) = 0 = k_3$

$i_0(0^+) = 0 = K_1 + k_2 \Rightarrow k_2 = -k_1$

$$i_c(0^+) = 1 = k_1(\frac{-1}{2} + \frac{5}{2}$$

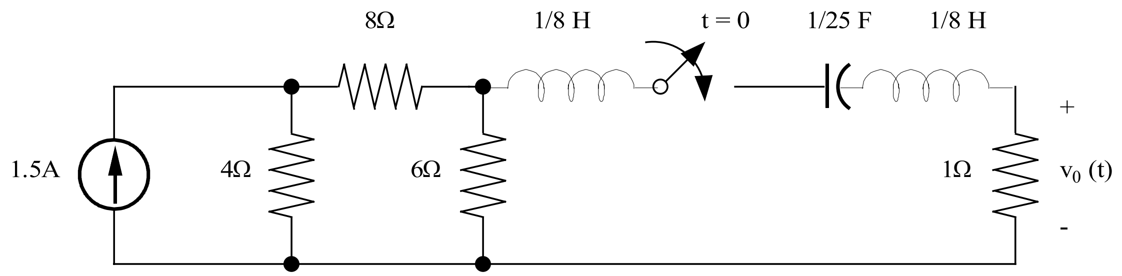
so

$$k_1 = \frac{1}{4}A$$

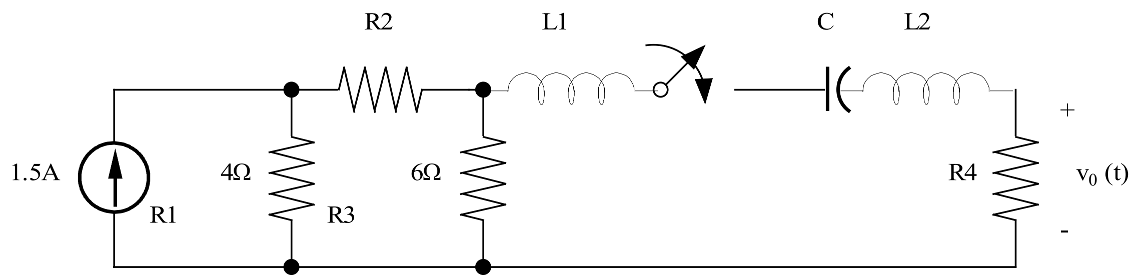
$$i_0(t) = \frac{1}{4}(e^{-t/2} - e^{-5t/2})A$$

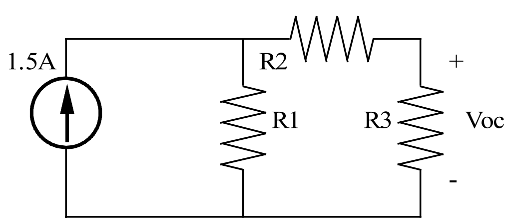
Problem 6.70

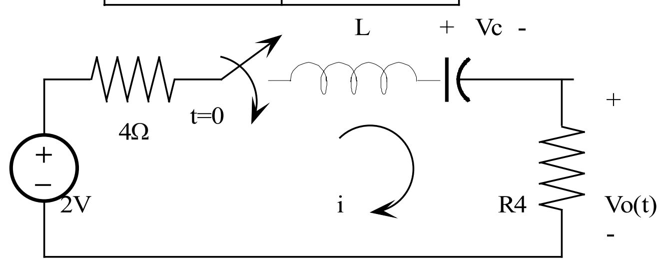
Find $v_0(t)$ for $t > 0$ in the circuit in Fig. P6.70 and plot the response including the time interval just prior to closing the switch.



Suggested Solution







$R_1 = 4\Omega, R_2 = 8\Omega, R_3 = 6\Omega, R_4 = 1\Omega, L_1 = L_2 = \frac{1}{8}H, c = \frac{1}{25}F$

$v_{oc} = 1.5[R_1 \parallel (R_2 + R_3)][\frac{R_3}{R_2 + R_3}] = 2.0v$

$R_{TH} = (R_1 + R_2) \parallel R_3 = 4\Omega$

For

$t = 0^-$

$i_L = 0, v_c = 0, v_0 = 0$

for

$t = 0^+$

$i_L = 0, v_c = 0, v_0 = 0, v_L = 2$

for

$t \to \infty$

$v_0 = 0$

$L = L_1 + L_2 = \frac{1}{4}H$

for

$t > 0$

series RLC

$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$

$s^2 + 20s + 100 = 0 \Rightarrow$

roots :

$s_1 = s_2 = -10r/s$

critically _ damped

$v_0(t) = k_1 e^{-10t} + k_2 t e^{-10t} + k_3$

$v_0(\infty) = 0 = k_3$

$v_0(0^+) = 0 = k_1$

so

$v_0 = k_2 t e^{-10t}$

$at, t = 0^+$

$v_L = 2 = L\frac{di(t)}{dt}|_{t=0}$

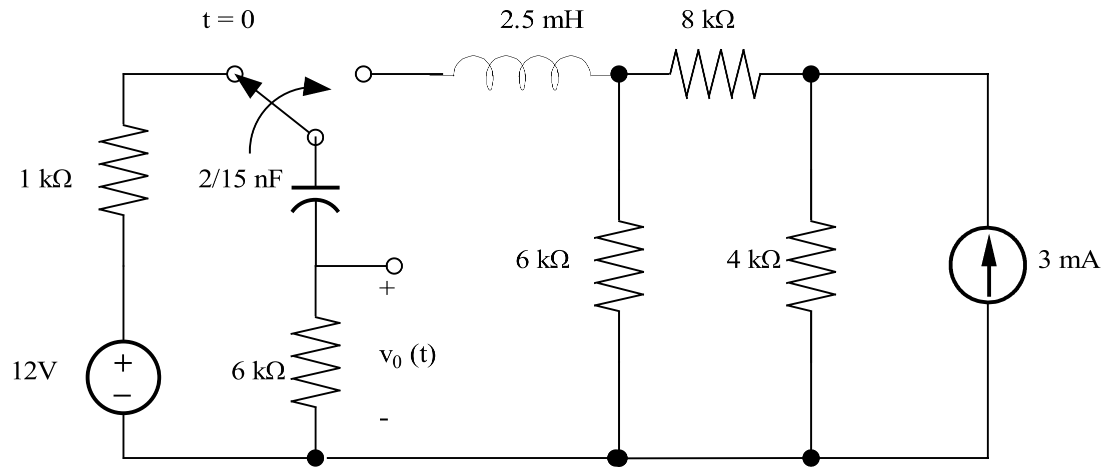
$i(t) = v_0(t)/R_4 = k_2 t e^{-10t}$

$2 = Lk_2 \Rightarrow k_2 = 8$

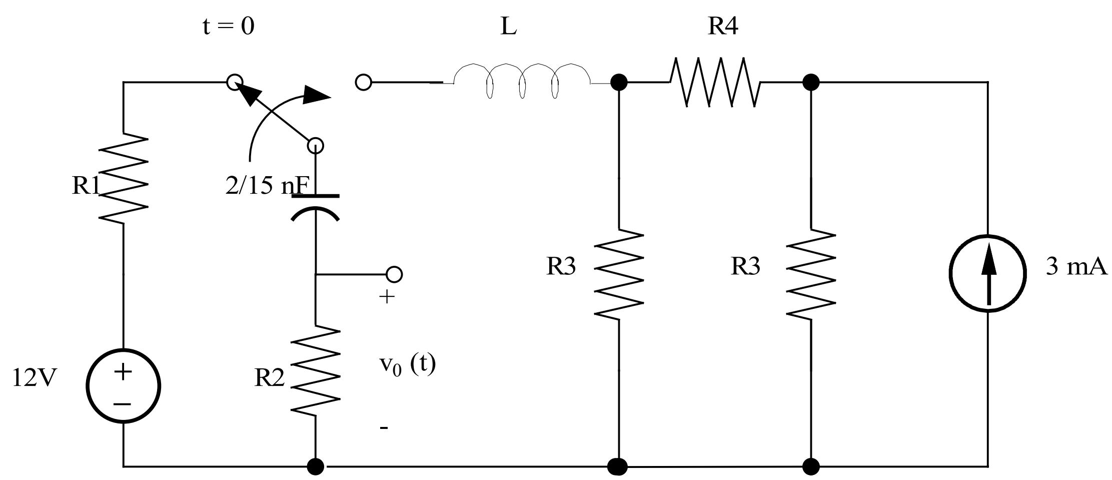
$v_0 = 8te^{-10t}v$

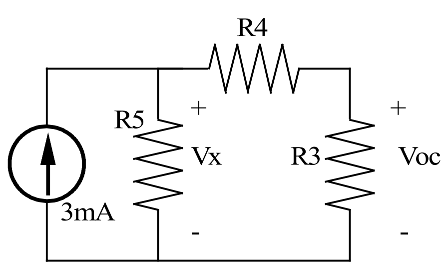
Problem 6.71

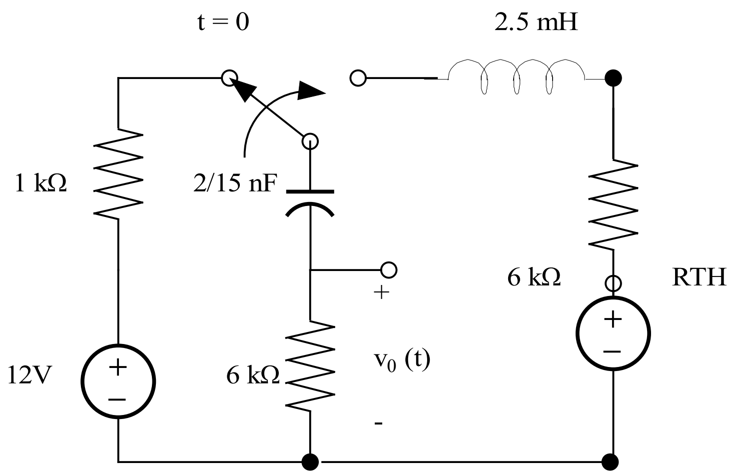
Find $v_0(t)$ for $t > 0$ in the circuit in Fig. P6.71 and plot the response including the time interval just prior to moving the switch.



Suggested Solution






t = 0
2.5 mH
1 kΩ
2/15 nF
6 kΩ
RTH
+
v0 (t)
-
12V
6 kΩ

$Theve\_eq$

$$v_{oc} = \frac{v_x R_3}{R_3 + R_4}$$

$$v_x = 3m[R_s \parallel (R_4 + R_3)]$$

$$R_{TH} = R_s \parallel (R_4 + R_3) = 4k\Omega, v_{oc} = 4v$$

for

$t = 0^-$

$$v_c = -12v, v_0 = 0, i_L = 0A$$

for

$t = 0^+$

$$v_c = -12v, i_L = 0, v_0 = 0, 4 + v_L = v_c + v_0 \Rightarrow v_L = -16v$$

for

$t \to \infty$

$v_0 \to 0$

for

$t > 0$

series RLC circuit

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$R = R_{TH} + R_2$$

$$s^2 + 4x10^6 s + 3x10^{12} = 0$$

$roots\_are$

$$s_1 = -1x10^6 \, and s_2 = -3x10^6 \Rightarrow overdamped$$

$$v_0(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t} + k_3$$

$$v_0(\infty) = 0 = k_3$$

$$v_0(0^+) = 0 = k_1 + k_2 \Rightarrow k_2 = -k_1$$

$$now, v_0(t) = k_1(s^{s_1 t} - e^{s_2 t})$$

$$v_L(0^+) = L\frac{di_L}{dt}\Big|_{t=0}$$

but

$$i_L(0^+) = \frac{-v_0}{R_L}$$

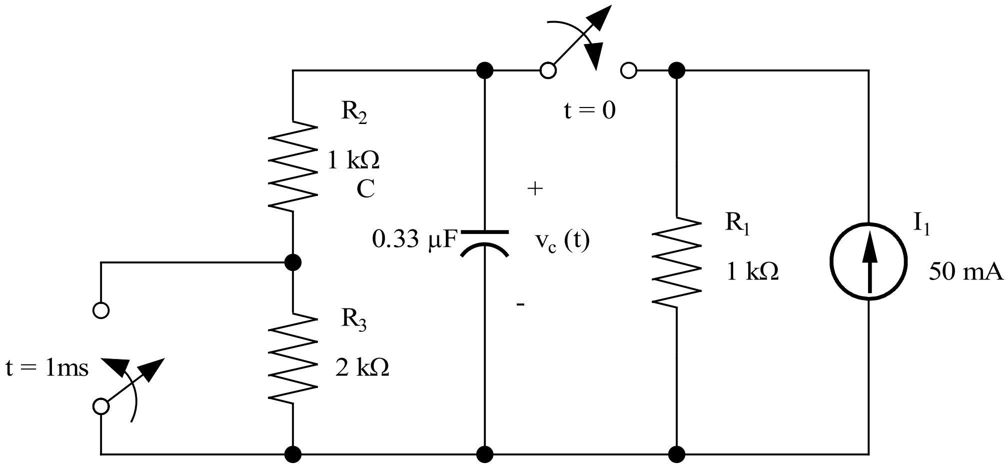
$$\frac{L}{R_2} k_1(s_1 - s_2) = 16 \Rightarrow k_1 = 19.2$$

$$v_0(t) = 19.2[e^{-10^6 t} - e^{-3x10^6 t}]V, t > 0$$

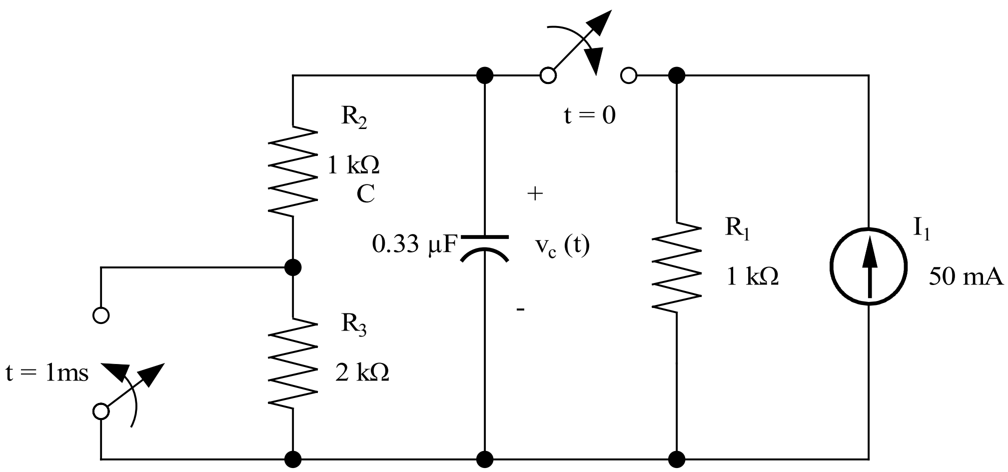
$$v_0(t) = 0v, t \leq 0$$

Problem 6.72

Using the PSPICE *Schematics* editor, draw the circuit in Figure P6.72, and use the PROBE utility to plot $v_C(t)$ and determine the time constants for $0 < t < 1$ ms and $1 \text{ ms} < t < \infty$. Also, find the maximum voltage on the capacitor.

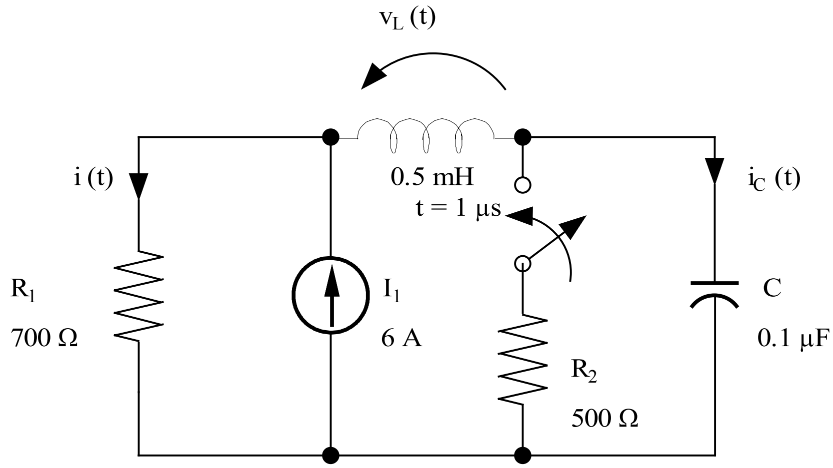


Suggested Solution

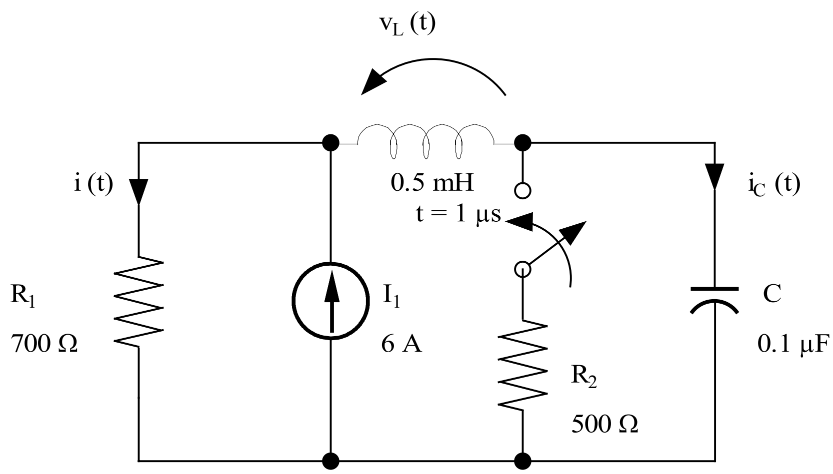


Problem 6.73

Using the PSPICE *Schematics* editor, draw the circuit in Figure P6.73, and use the PROBE utility to find the maximum values of $v_L(t)$, $i_C(t)$, and $i(t)$.



Suggested Solution



Problem 6.74

Design a series RCL circuit with R>= 1Kohm that has characteristic equation.

S^2 + 4*10^6S +4*10^14 = 0

Suggested Solution

Series RLC where R $\geq 1K\Omega$ *and* characteristic eq. is,

$$S^2 + \frac{R}{L}S + \frac{1}{LC} = 0 \Rightarrow LC = \frac{10^{-14}}{4}, L = \frac{R}{4*10^7}$$

choose R=40KΩ. Now, L=1mH & C=2.5ρF

R=40KΩ

L=1mH & C=2.5ρF

Problem 6.75

Design a parallel RLC circuit with R>=1kohm that has the characteristic equation

S^2 + 4*10^7S +3*10^14 = 0

Suggested Solution

Parallel RLC where R $\geq 1K\Omega$ *and* characteristic eq. is,

$$S^2 + \frac{S}{RL} + \frac{1}{LC} = 0 \Rightarrow LC = \frac{10^{-14}}{3} \;\&\; C = \frac{1}{4*10^7 R}$$

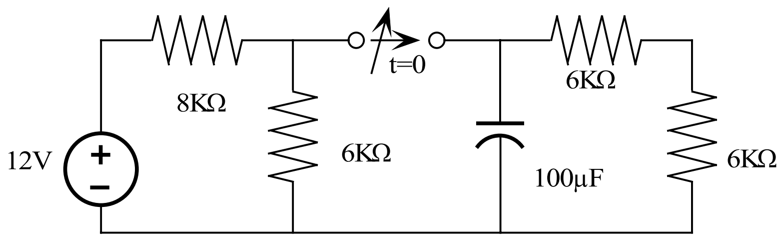
choose R=2.5KΩ. Now, L=333μH & C=10ρF

R=2.5KΩ

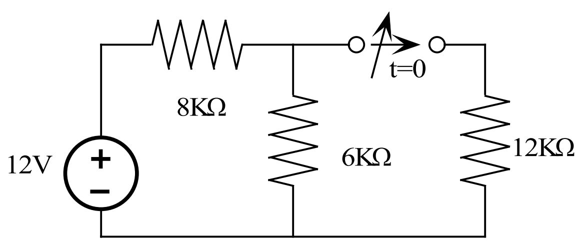
L=333μH & C=10ρF

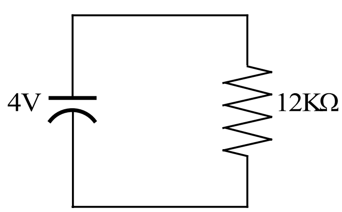
Problem 6FE-1

In the circuit in Fig the switch, which has been closed for a long time, opens at t=0. Find the value of the capacitor voltage Vc(t) at t=2s.



Suggested Solution





FOR t<0

6K || 12K=4K

$$V_C(0) = 12\left(\frac{4K}{4K+1K}\right) = 4V$$

$t > 0$ SWITCH OPENS

$$V_C(t) = 4e^{\frac{-t}{\tau}}V$$

$$\tau = 100*10^{-6}*12*10^{3} = 1.2SEC.$$

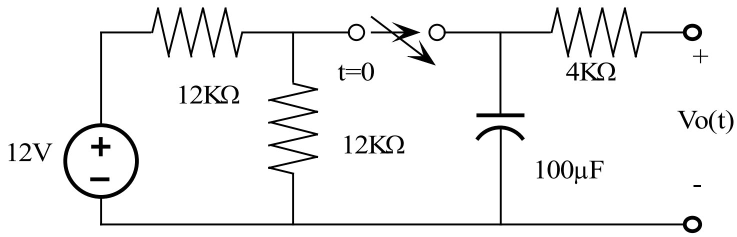
$$V_C(t) = 4e^{\frac{-t}{1.2}}j$$

AT t=2SEC

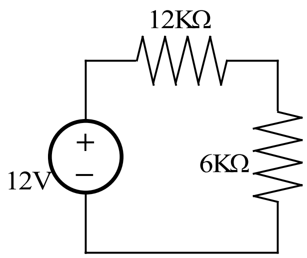
$$V_C(t) = 4e^{\frac{-t}{1.2}}V = 0.76V$$

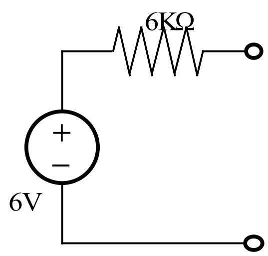
Problem 6FE-2

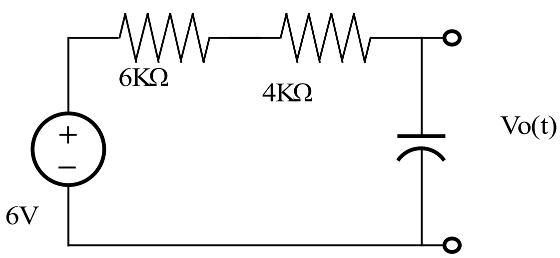
In the network in fig, the switch closes at t=0. Find Vo(t) at t=1s.



Suggested Solution







$$V_O(t) = 6(1 - e^{\frac{-t}{\tau}})$$

$$\tau = 10 * 10^3 * 100 * 10^{-6} = 1SEC.$$

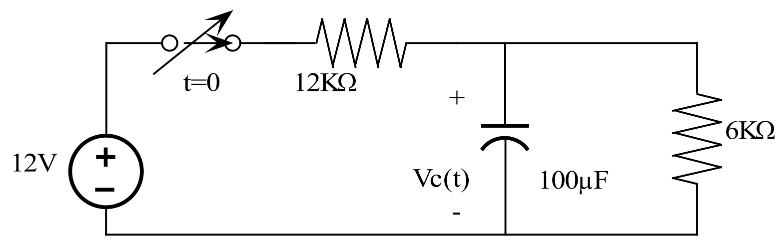
$$V_O(t) = 6(1 - e^{\frac{-t}{\tau}})V$$

AT t=1SEC,

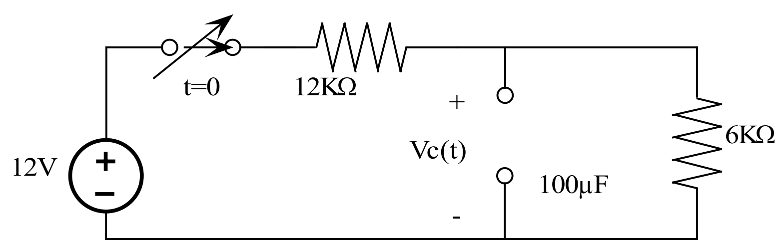
$$V_O(t) = 6(1 - e^{-1}) = 3.79V$$

Problem 6FE-3

Assume the switch, in the network in fig, has been closed for sometime. At t=0 the switch opens. Determine the time required for the capacitor voltage to decay to ½ of its initially charged value.



Suggested Solution



$$V_C(0^-) = 12\left(\frac{6K}{18K}\right) = 4V$$

$$V_C(t) = 4e^{\frac{-t}{\tau}}$$

$$\tau = 100*10^{-6}*6*10^3 = 0.6\,\text{sec}.$$

$$V_C(t) = 4e^{\frac{-t}{0.6}}V$$

1/2 OF INITIAL CHANGE VALUE = 2= $4e^{\frac{-t}{0.6}}V$

$$\therefore \frac{1}{2} = e^{\frac{-t}{0.6}} \Rightarrow t = 0.42\,\text{sec}.$$

Problem 7.1

Given i(t)=5 cos(400t –120°) A, determine the period of the current and the frequency in hertz.

Suggested Solution

$$f = \frac{\omega}{2\pi} = \frac{400}{2\pi} = 63.6Hz$$

$$T = \frac{1}{f} = .016s$$

Problem 7.2

Determine the relative position of the two sine waves

$V_1(t)=12 \sin(377t-45°)$
$V_2(t)=6 \sin(377t+675°)$

Suggested Solution

$V_2(t)$ can be rewritten as $V_2(t)=6 \sin(377t-45°)$, so $V_1(t)$ and $V_2(t)$ are in phase. PHASE=0.

Problem 7.3

Given the following currents:

$i_1(t)$=4 sin(377t – 10°) A
$i_2(t)$=-2 sin(377t – 195°) A
$i_3(t)$=-1 sin(377t – 250°) A

Compute the phase angle between each pair of currents.

Suggested Solution

$i_1(t)$ leads $i_2(t)$ by –85°. $i_2(t)$ leads $i_3(t)$ by 145°. $i_1(t)$ by $i_3(t)$ by 60°

Problem 7.4

Determine the phase angles by which v1(t) leads i1(t) and v1(t) leads i2(t), where:

$v_1(t)=4\sin(377t + 25°)$
$i_1(t)=0.05\cos(377t – 10°)$
$i_2(t)=-0.1\sin(377t + 75°)$

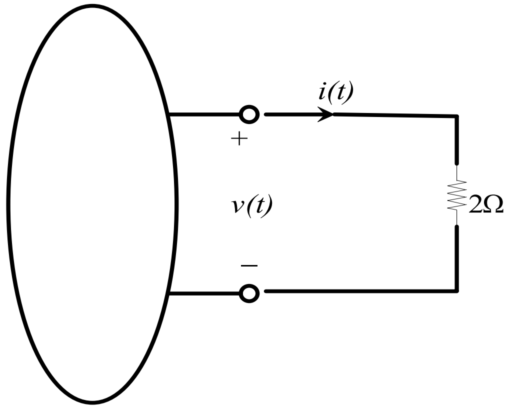
Suggested Solution

$v_1(t)$ leads $i_1(t)$ by $25°-80°=-55°$
$v_1(t)$ leads $i_2(t)$ by $25°-255°=-230°$

Problem 7.5

Calculate the current in the resistor if the voltage input is:

(a) v1(t)=10 cos(377t+180°)
(b) v2(t)=12 sin(377t+45°)



Give the answers in both the time and frequency domains.

Suggested Solution

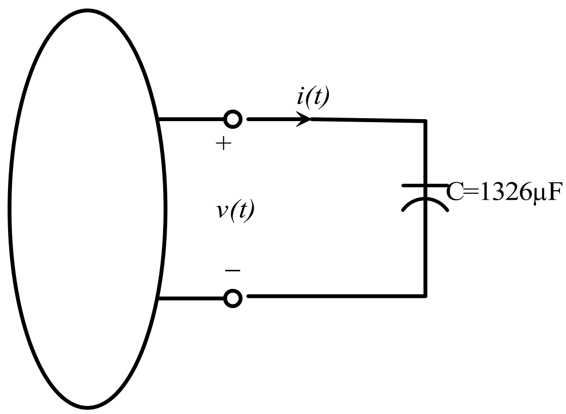
(a) $i(t) = \frac{v_1(t)}{R} = 5\cos(377t + 180°)A = 5\underline{|180°}\,A$

(b) $i(t) = \frac{v_2(t)}{R} = 6\sin(377t + 45°)A = 6\cos(377t - 45°)A = 6\underline{|-45°}\,A$

Problem 7.6

Calculate the current in the capacitor if the voltage input is:

(a) $v_1(t)=16\cos(377t-22°)$
(b) $v_2(t)=8\sin(377t+64°)$



Give the answers in both the time and frequency domains.

Suggested Solution

(a)

$$Z_c=\frac{1}{j\omega C}=\frac{1}{j(377)(1326\times10^{-6})}=\frac{1}{.5j}$$

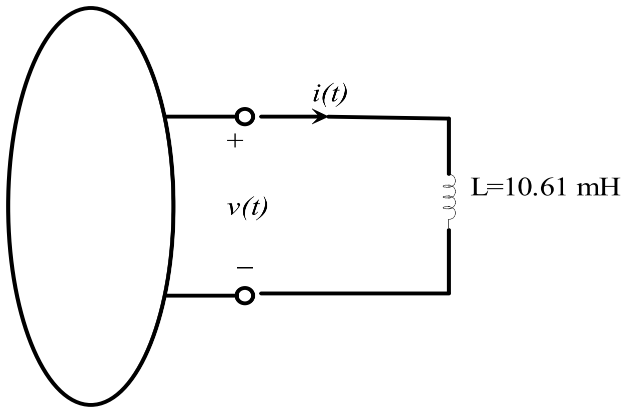
$$i(t)=\frac{v(t)}{Z_c}=\frac{16\angle -22°}{2\angle -90°}=8\angle 68°\,A=8\cos(377t+68°)A$$

(b) $$i(t)=\frac{v(t)}{Z_c}=\frac{8\angle 64°}{2\angle -90°}=4\angle 64°\,A=4\cos(377t+64°)A$$

Problem 7.7

Calculate the current in the inductor if the voltage input is:

(a) $v_1(t)=24\cos(377t+12°)$
(b) $v_2(t)=18\sin(377t-48°)$



Give the answers in both the time and frequency domains.

Suggested Solution

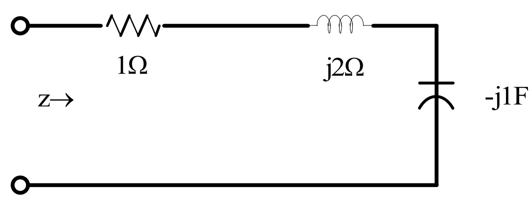
$$Z_L = j\omega L = j(377)(10.61\times10^{-3}) = 4j$$

(a)
$$i(t) = \frac{v(t)}{Z_L} = \frac{24\angle 12°}{4\angle 90°} = 6\angle -78°\,A$$

(b)
$$i(t) = \frac{v(t)}{Z_L} = \frac{18\angle 42°}{4\angle 90°} = 4.5\angle -48°\,A$$

Problem 7.8

Find the frequency domain impedance, **Z**, for the network shown.

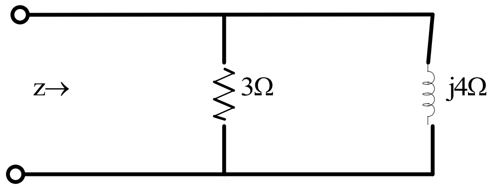


Suggested Solution

Z=1+j2-j1=1+j1Ω

Problem 7.9

Find the frequency domain impedance, **Z**, for the network shown.

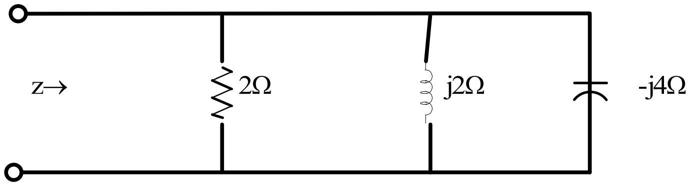


Suggested Solution

$$Z = 3 \parallel 4j = \frac{12j}{3+4j} = \frac{12\angle 90^\circ}{2.4\angle 53^\circ} = 2.4\angle 37^\circ = 1.92 + j1.44\Omega$$

Problem 7.10

Find the frequency domain impedance, **Z**, for the network shown.



Suggested Solution

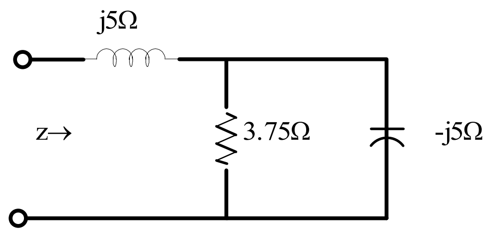
$$\frac{1}{Z} = \frac{1}{2} + \frac{1}{j2} - \frac{1}{j4} = \frac{1}{2} - j\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{1}{4}(2 - j)$$

$$Z = \frac{4}{2-j} = \frac{4(2+j)}{(2-j)(2+j)} = \frac{8+4j}{4+1} = 1.6 + j0.8$$

$$Z = 1.6 + j0.8\Omega$$

Problem 7.11

Find the frequency domain impedance, **Z**, for the network shown.

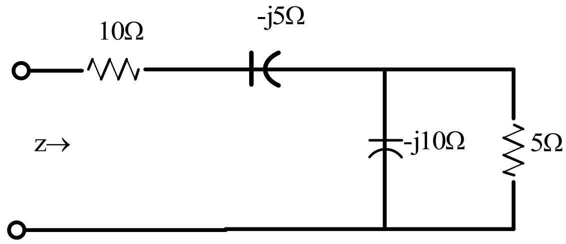


Suggested Solution

$$Z = 5j + (-5j \,||\, 3.75) = 5j + \frac{18.75j}{3.75 - 5j} = 5j + \frac{18.74\angle{-90°}}{6.25\angle{-53.1°}} = 5j + 3\angle{-36.9°} = 2.4 + j3.2 = 4\angle{53°}\Omega$$

Problem 7.12

Find the frequency domain impedance, **Z**, for the network shown.

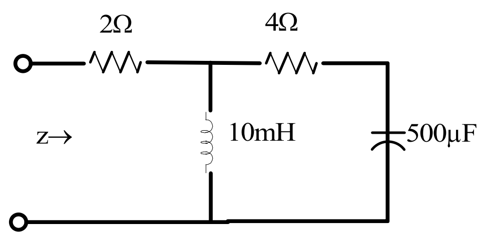


Suggested Solution

$$Z = 10 - 5j + \frac{5(-10j)}{5 - 10j} = 10 - 5j + \frac{5(-10j)}{11.2\angle -23.4^\circ} = 10 - 5j + 4 - 2j = 14 - 7j = 15.65\angle -26.6^\circ \Omega$$

Problem 7.13

Find **Z(jω)** at a frequency of 60 Hz for the network shown.



Suggested Solution

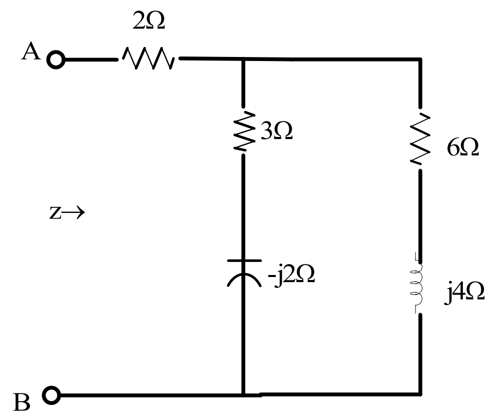
$$Z_L = j\omega L = j(377)(10\times10^{-3}) = j3.77$$

$$Z_c = \frac{1}{j\omega C} = \frac{10^6}{j377(5)} = -j5.3$$

$$Z = 2 + \frac{j3.77(4-j5.3)}{j3.77+4-j5.3} = 2 + \frac{j15.08+20}{4-j1.535} = 5.1 + j4.96\Omega$$

Problem 7.14

Calculate the equivalent impedance **Z** at the terminals A-B for the network shown.

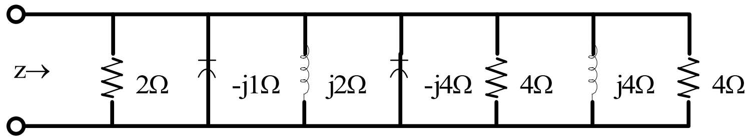


Suggested Solution

$$Z_{AB} = 2+(3-2j)\,||\,(6+4j) = 2+\frac{(3-2j)(6+4j)}{9+2j} = \frac{18+4j+26}{9+2j} = \frac{44.18\angle 5.19^\circ}{9.22\angle 12.53^\circ} = 4.8\angle -7.34^\circ\Omega$$

Problem 7.15

Find **Z** for the network shown.



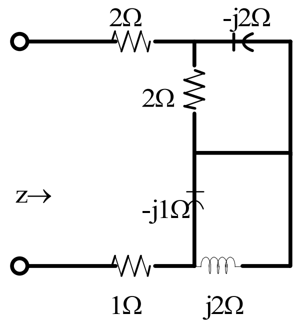
Suggested Solution

$$Y = 1 + \frac{1}{-j} + \frac{1}{-4j} + \frac{1}{2j} + \frac{1}{4j} = 1 + j + .25j - .5j - .25j = 1 + .5j = 1.12\angle 26.57^\circ$$

$$Z = \frac{1}{Y} = .89\angle -26.57^\circ \Omega$$

Problem 7.16

Find **Z** for the network shown.

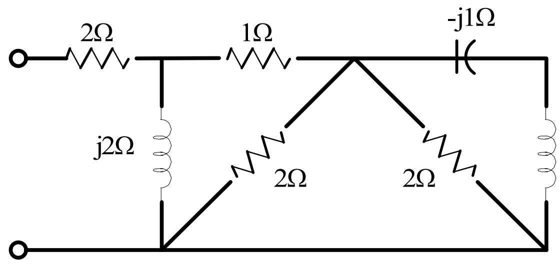


Suggested Solution

$$Z = 2+1+2\,||\,-2j+2j\,||\,-j = 3+\frac{-4j}{2-2j}+\frac{2}{j} = 4-3j = 5\underline{|-37°}\Omega$$

Problem 7.17

Find **Z** for the network shown.

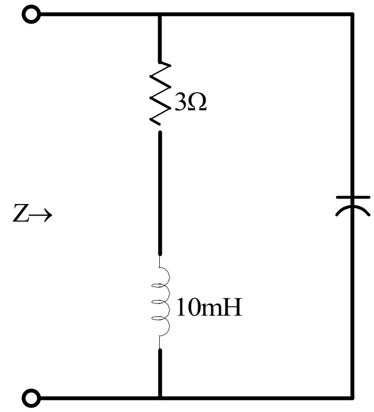


Suggested Solution

$$Z = 2 + 2j \,||\, (1 + 1 \,||\, j) = 2 + 2j \,||\, (1.5 + 0.5j) = 2 + \frac{2j - 4}{-1 + 4j} = 2.83\angle 16.92° \Omega$$

Problem 7.18

The impedance of the network shown is found to be purely real at f=60 Hz. What is the value of C?



Suggested Solution

$$Z = \frac{1}{j\omega C} \,||\, (R + j\omega L) = \frac{\frac{-j}{\omega C}(3+3.77j)}{\frac{-j}{\omega C}+3+3.77j} = \frac{\left(\frac{-3j}{\omega C}+\frac{3.77}{\omega C}\right)\left(3-j\left(3.77-\frac{1}{\omega C}\right)\right)}{3^2+\left(3.77-\frac{1}{\omega C}\right)^2}$$

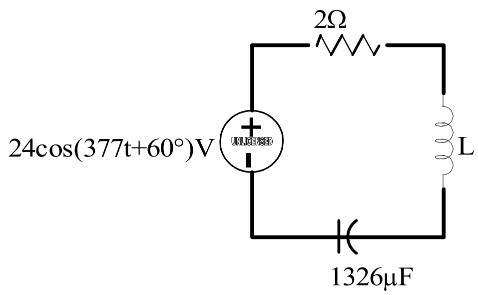
THE IMAGINARY COMPONENT IS ZERO

$$0 = \frac{-9j}{\omega C} - \frac{3.77j}{\omega C}\left(3.77-\frac{1}{\omega C}\right) = -9j - 3.77j\left(3.77-\frac{1}{\omega C}\right)^2$$

$$\omega = 2\pi 60 = 377 \rightarrow C = 431\mu F$$

Problem 7.19

In the circuit shown determine the value of the inductance such that the current is in phase with the source voltage.



Suggested Solution

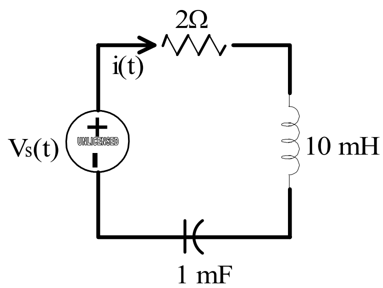
$\omega = 377$

$f = 60Hz$

$$\omega_L = \frac{1}{\omega C}, L = \frac{1}{\omega^2 C} = \frac{1}{\left(377^2\right)\left(1326\times10^{-6}\right)} = 5.3mH$$

Problem 7.20

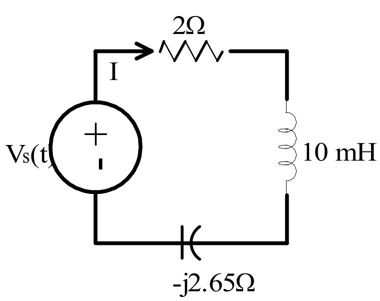
Draw the frequency domain circuit and calculate I(t) for the circuit shown if $v_s(t)=10\cos(377t+30°)$ V.



Suggested Solution

$$j\omega L = j(377)(10\times10^{-3}) = j3.77\Omega$$

$$\frac{1}{j\omega C} = \frac{-j}{377\times10^{-3}} = -j2.65\Omega$$

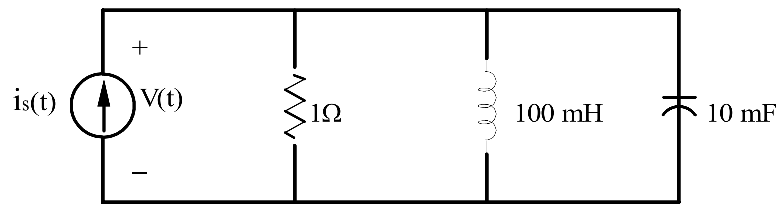


$$I = \frac{V_s}{2+j3.77-j265} = \frac{10\angle 30°}{2+j1.12} = \frac{10\angle 30°}{2.29\angle 29.25°} = 4.37\angle 0.75° A$$

$$i(t) = 4.37\cos(377t+0.75°)A$$

Problem 7.21

Draw the frequency domain circuit and calculate v(t) for the circuit shown if $I_s(t)$=20 cos(377t+120°) A.



Suggested Solution

$$Z = j\omega L = j37.7\Omega$$

$$Z_c = \frac{1}{j\omega C} = \frac{1}{j3.77}\Omega$$

$$V = I\left[\frac{1}{\frac{1}{R}+\frac{1}{Z_L}+\frac{1}{Z_C}}\right] = \frac{2\angle 120°}{1-\frac{j}{37.7}+j3.77} = \frac{2\angle 120°}{1+j3.74} = \frac{2\angle 120°}{3.84\angle 75.03°} = 0.52\angle 44.91° V$$

$$v(t) = 0.52\cos(377t + 44.97°)V$$

Problem 7.22

The voltages $v_R(t)$, $v_L(t)$, and $v_C(t)$ in the circuit shown can be drawn as phasors in a phasor diagram. Show that $v_R(t) + v_L(t) + v_C(t) = v_S(t)$.

Suggested Solution

$$V_R(t) = i(t)R = 2(4.37)\cos(377t + 0.75°) = 8.74\cos(377t + 0.75°)V$$

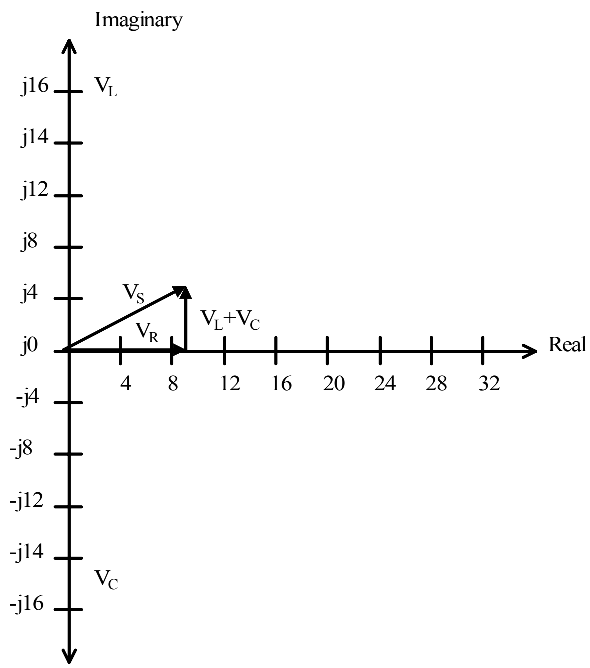
$$V_R = 8.74\underline{|0.75°}V$$

$$V_L(t) = L\frac{di}{dt} = (10^{-2})(4.37)(377)(-\sin(377t + 0.75°)) = 16.47\cos(377t + 90.75°)$$

$$V_L = 16.47\underline{|90.75°}V$$

$$V_C(t) = \frac{1}{C}\int idt = \left(\frac{1}{10^{-3}}\right)\left(\frac{1}{377}\right)(4.37)\sin(377t + 0.75°)$$

$$V_C(t) = 11.59\cos(377t - 89.25°)V$$



Problem 7.23

The currents $i_R(t)$, $i_L(t)$, and $i_C(t)$ in the circuit shown can be drawn as phasors in a phasor diagram. Show that $i_R(t) + i_L(t) + i_C(t) = i_S(t)$.

Suggested Solution

$$I_R(t) = \frac{V(t)}{R} = 0.52\cos(377t + 44.97°)A$$

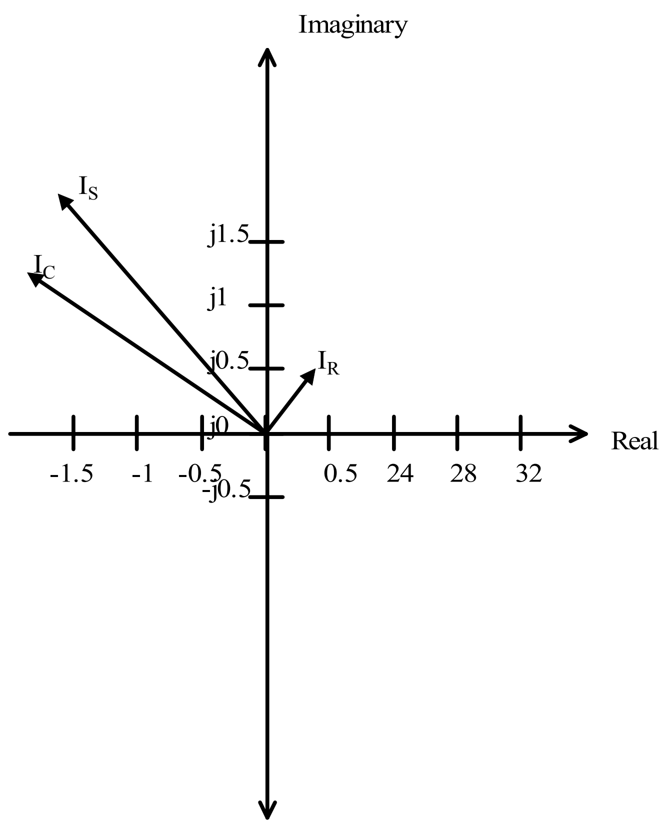
$$I_R = 0.52\underline{|44.97°}A$$

$$I_C(t) = C\frac{dv}{dt} = (10^{-2})(0.52)(377)(-\sin(377t + 44.97°)) = 1.96\cos(377t + 134.97°)$$

$$I_C = 1.96\underline{|134.97°}A$$

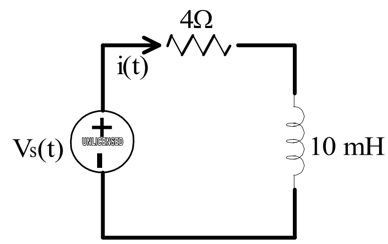
$$I_L(t) = \frac{1}{L}\int vdt = \left(\frac{0.52}{0.1}\right)\left(\frac{1}{377}\right)\sin(377t + 44.97°) = 13.79\cos(377t - 45.03°)mA$$

$$I_L(t) = 13.79\underline{|-45.03°}A$$



Problem 7.24

The voltages $v_R(t)$ and $v_L(t)$ in the circuit shown can be drawn as phasors in a phasor diagram. Use a phasor diagram to show that $v_R(t) + v_L(t) = v_S(t)$.

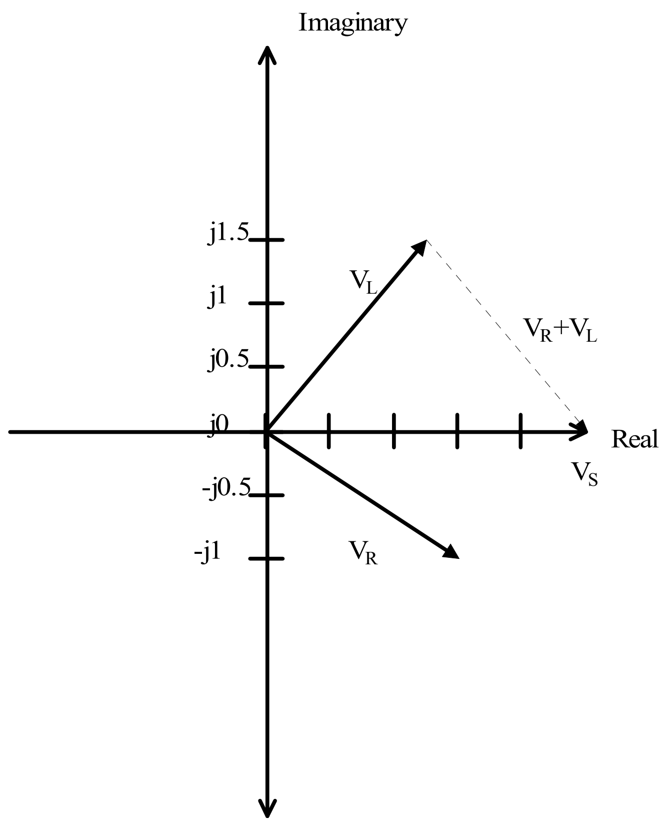


Suggested Solution

$$I = 0.36\angle{-43.38°}A$$

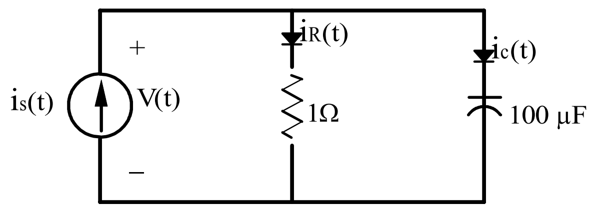
$$V_R = 4I = 1.44\angle{-43.38°}A$$

$$V_L = j3.77I = 1.36\angle{46.62°}V$$



Problem 7.25

The currents $i_R(t)$ and $i_C(t)$ in the circuit shown can be drawn as phasors in a phasor diagram. Use a phasor diagram to show that $i_R(t) + i_C(t) = i_S(t)$.



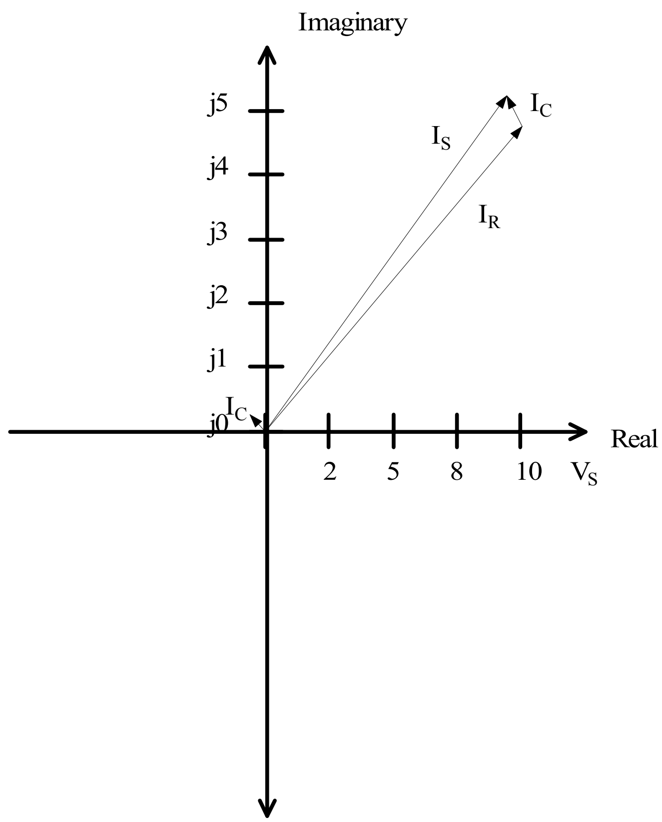
Suggested Solution

$$I_R(t) = \frac{V(t)}{R} = 9.99\cos(377t + 27.84°)A$$

$$I_R = 9.99\angle 27.84° A$$

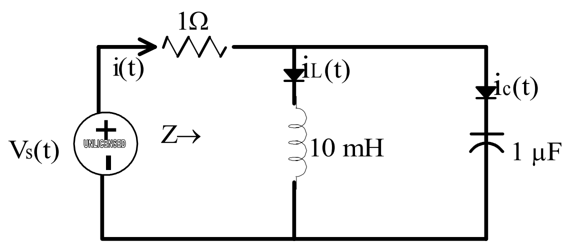
$$I_C(t) = C\frac{dv}{dt} = (10^{-4})(9.99)(377)(-\sin(377t + 27.84°)) = 0.38\cos(377t + 117.84°)$$

$$I_C = 0.38\angle 117.84° A$$



Problem 7.26

The currents $i_L(t)$ and $i_C(t)$ of the inductor and capacitor in the circuit shown can be drawn as phasors in a phasor diagram. Use a phasor diagram to show that $i_L(t) + i_C(t) = i_S(t)$.

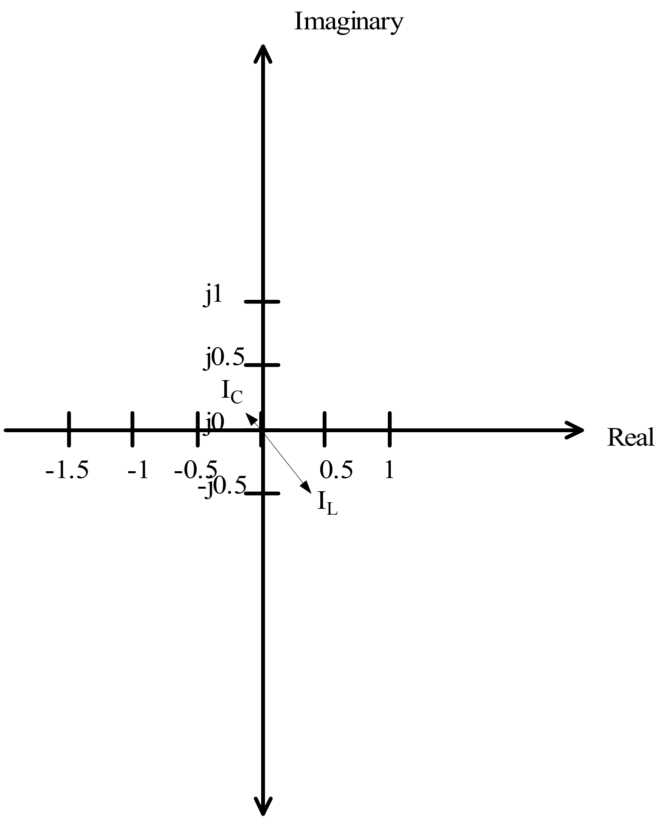


Suggested Solution

$$V = V_S - IR = 10\angle 30° - 0.99\angle -54.33° = 9.95\angle 35.67° V$$

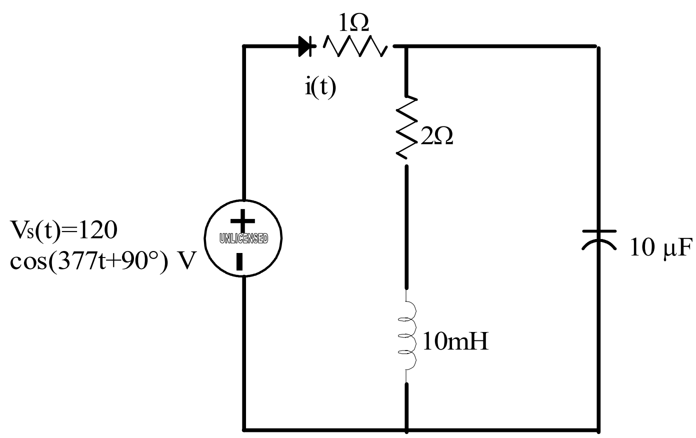
$$I_L = \frac{V}{j\omega L} = \frac{9.95\angle 35.67°}{10\angle 90°} = 1.00\angle 54.33° A$$

$$I_C = V(j\omega C) = [9.95\angle 35.67°]10^{-3}\angle 90° = 9.95\angle 125.67° mA$$



Problem 7.27

In the currents shown determine the frequency at which i(t) is in phase with $v_s(t)$.



Suggested Solution

$$Z = (2 + j\omega L) \parallel \frac{1}{j\omega C} = \frac{\frac{L}{C} - \frac{j2}{\omega C}}{2 + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{K_1 \left| \tan^{-1}\left(\frac{\frac{-2}{\omega C}}{\frac{L}{C}}\right)\right.}{K_1 \left| \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{2}\right)\right.}$$

For Z to be real

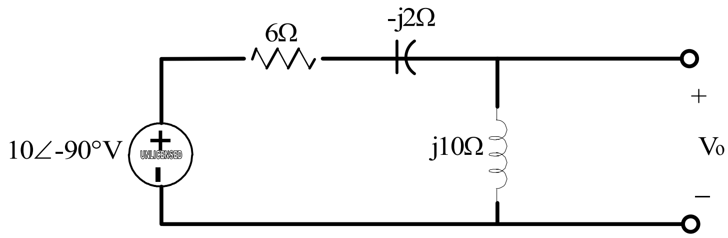
$$\frac{-2}{\omega C} = \frac{\omega L - \frac{1}{\omega C}}{2}$$

$$-4 = (\omega L)^2 - \frac{L}{C} \Rightarrow \omega = \frac{\sqrt{\frac{L}{C} - 4}}{L} = 3156$$

$$f = \frac{\omega}{2\pi} = 502.3 Hz$$

Problem 7.28

Find the frequency domain voltage $\mathbf{V}_0$ as shown.

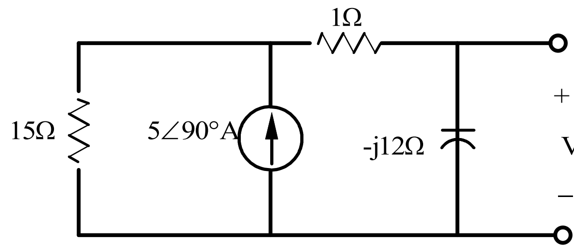


Suggested Solution

$$V_0 = 10\angle{-90°}\frac{10j}{6-3j+10j} = \frac{(10\angle{-90°})(10\angle{90°})}{10\angle{53°}} = 10\angle{-53.1°}V$$

Problem 7.29

Find the frequency domain voltage $\mathbf{V}_0$ as shown.



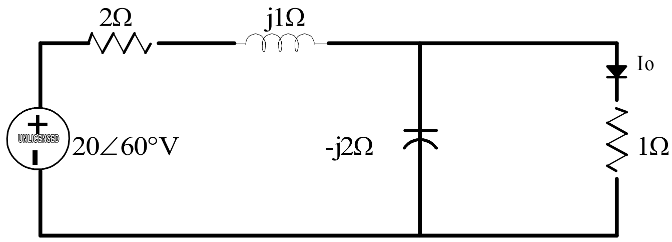
Suggested Solution

$$I_0 = 5\angle 30° \frac{15}{15+1-12j} = 3.75\angle 66.9° A$$

$$V_0 = -j12(3.75\angle 66.9°) = 45\angle -23.1° V$$

Problem 7.30

Find the frequency domain current $\mathbf{I}_0$ as shown.



Suggested Solution

$$V = 20\angle 60^\circ \left\{ \frac{Z}{Z + 2 + j1} \right\}$$

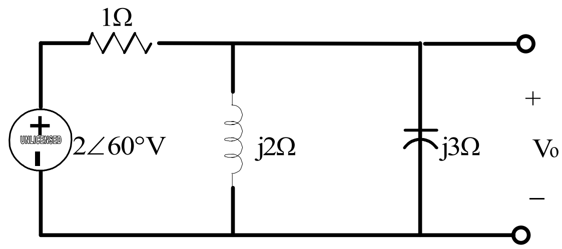
$$Z = 1 \,\|\, -j2 = \frac{-j2}{1 - j2} = \frac{2\angle -90^\circ}{\sqrt{5}\angle -64.43^\circ} = 0.89\angle -26.57^\circ \Omega$$

$$V_0 = 20\angle 60^\circ \left[\frac{0.89\angle -26.57^\circ}{0.89\angle -26.57^\circ + 2 + j1} \right] = 20\angle 60^\circ (0.31\angle -38.66^\circ) = 6.20\angle 21.34^\circ V$$

$$I_0 = \frac{V}{R} = 6.20\angle 21.34^\circ A$$

Problem 7.31

Find the frequency domain voltage $\mathbf{V}_0$ as shown.



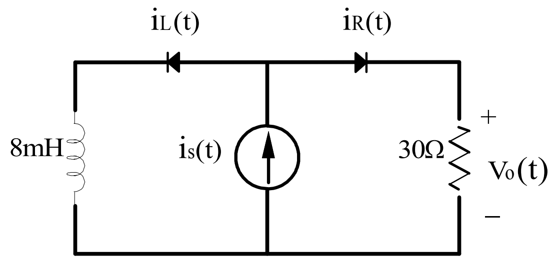
Suggested Solution

$$Z = \frac{j}{2} \parallel \frac{-j}{3} = \frac{\frac{1}{6}}{\frac{j}{2} - \frac{j}{3}} = -j$$

$$V_0 = 2\angle 60° \frac{-j}{1-j} = \frac{2\angle -30°}{\sqrt{2}\angle -45°} = \sqrt{2}\angle 15° V$$

Problem 7.32

Draw the frequency domain network and calculate $v_0(t)$ in the circuit shown if $i_S(t)=100\cos(5000t+8.13°)$mA. Also, using a phasor diagram, show that $i_L(t)+ i_R(t)= i_S(t)$.



Suggested Solution

$$i_S(t) = 100\angle 8.13°$$

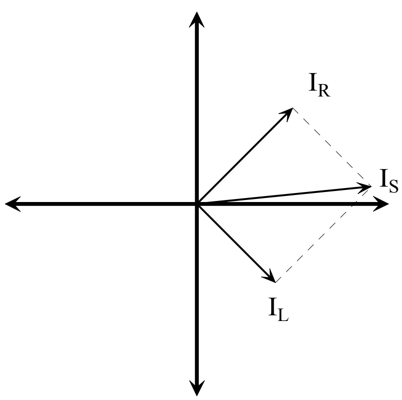
$$j\omega L = j0.008(5000) = 40j$$

$$V_0(t) = i_S(t)(30 \parallel 40j) = 100\angle 8.13° \left(\frac{30-40j}{30+40j}\right) = 2.4\angle 45°$$

$$V_0(t) = 2.4\cos(5000t + 45°)V$$

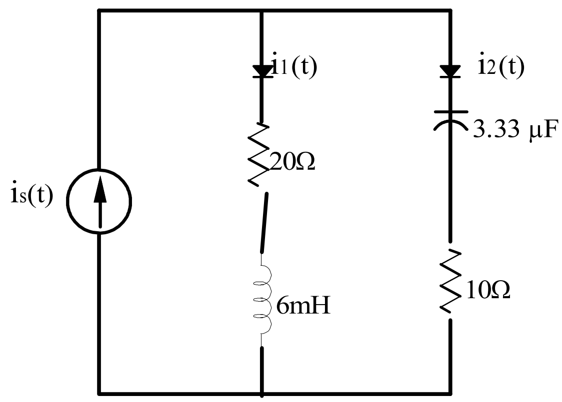
$$i_L = \frac{V_0(t)}{40j} = 60\angle -45° mA$$

$$i_R = \frac{V_0(t)}{30} = 80\angle 45° mA$$



Problem 7.33

Draw the frequency domain network and calculate $v_0(t)$ in the circuit shown if $i_S(t)$=100cos(5000t+8.13°)mA. Also, using a phasor diagram, show that $i_1(t)+ i_2(t)= i_S(t)$.



Suggested Solution

$$i_S(t) = 0.3\angle{-135°}A$$

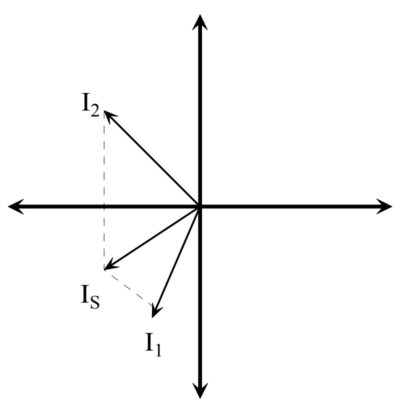
$$j\omega L = j0.006(10000) = 60j$$

$$V_0(t) = i_S(t)((20+60j)\parallel(10-30j)) = 14.14\angle{180°}$$

$$V_0(t) = 14.14\cos(10000t - 45°)V$$

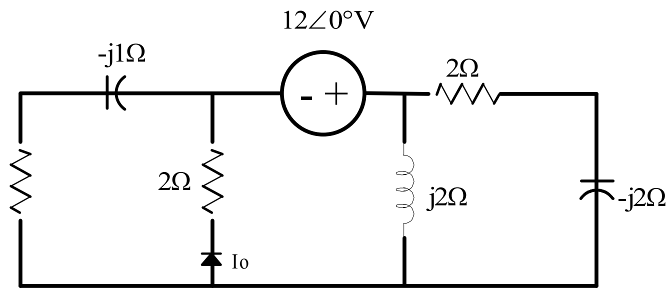
$$i_1 = \frac{V_0(t)}{20+60j} = .224\angle{251.6°}mA$$

$$i_2 = \frac{V_0(t)}{10-30} = .447\angle{-108.4°}mA$$



Problem 7.34

Find $\mathbf{I}_0$ in the network shown.



Suggested Solution

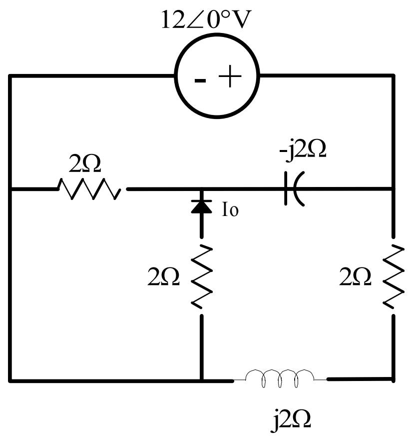
$$Z = 2j \parallel (2-2j) + 2 \parallel (1-j) = 2 + 2j + 0.8 - 0.4j = 2.8 + j1.6 = 3.22\angle 30^\circ$$

$$I_S = \frac{12\angle 0^\circ}{3.22\angle 30^\circ} = 3.73\angle -30^\circ$$

$$I_0 = 3.73\angle -30^\circ \left(\frac{1-j}{.8-.4j} \right) = 5.89\angle -48.4^\circ A$$

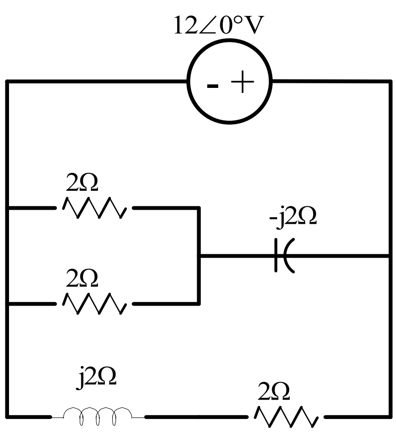
Problem 7.35

Find $\mathbf{I}_0$ in the network shown.



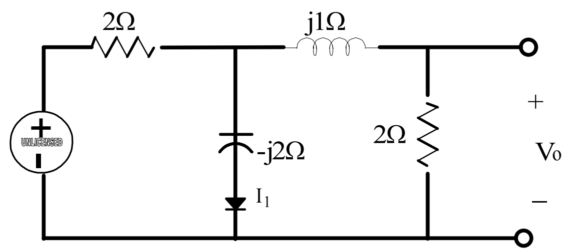
Suggested Solution

$$I_0 = \frac{1}{2}\left(\frac{12\angle 0^\circ}{1-2j}\right) = -2.68\angle 63.4^\circ A$$



Problem 7.36

In the circuit shown, if $V_0=4\angle 45°$ V, find I_1



Suggested Solution

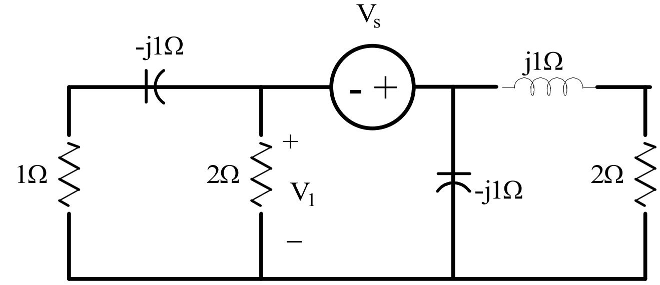
$$I_2 = \frac{4\angle 45°}{2} = 2\angle 45° A$$

$$V_1 = I_2(2+j) = 4.47\angle 71.6°$$

$$I_1 = \frac{V_1}{-j2} = \frac{4.47\angle 71.6°}{2\angle -90°} = 2.24\angle 162° A$$

Problem 7.37

Find V_S in the network shown if $V_0 = 4\angle 0°$ V.



Suggested Solution

$$Z_1 = \frac{2(1-j)}{3-j} = 0.8 - j0.4$$

$$Z_2 = \frac{(-j)(2+j)}{2} = 0.5 - j1$$

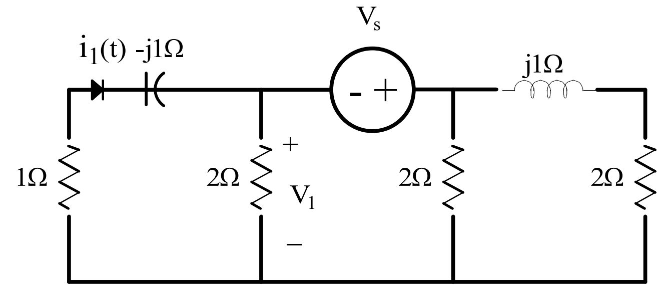
$$I = \frac{4\angle 0°}{0.8 - j0.4}$$

$$V_2 = IZ_2 = 5\angle -36.86° V$$

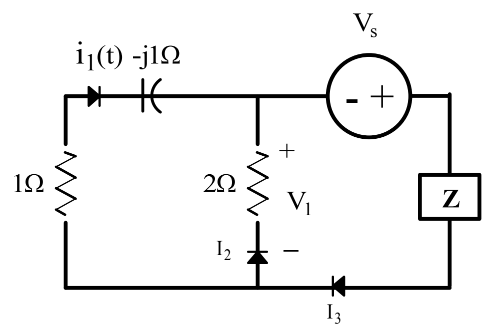
$$V_S = -(4\angle 0° + 5\angle -36.86° V) = -8 + j3 = -8.54\angle -20.56° V$$

Problem 7.38

Find V_S in the network shown if $I_0=2\angle 0°$ A.



Suggested Solution



$$Z = \frac{2(2+j)}{4+j}$$

$$V_1 = I_1(1-j) = 2 - j2$$

$$I_2 = \frac{V_1}{2} = 1 - j$$

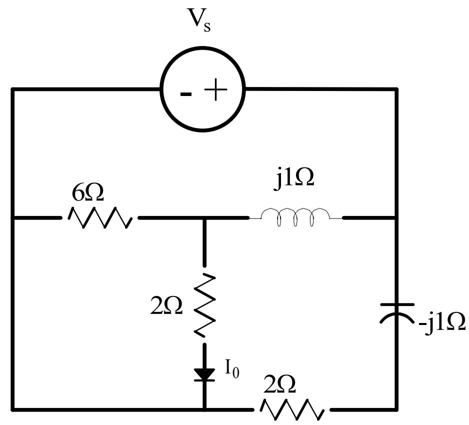
$$I_3 = I_1 + I_2 = 2 + 1 - j = 3 - j$$

$$V_2 = I_3 Z = (3-j)\left(\frac{4+2j}{4+j}\right) = \frac{14+j2}{4+j}$$

$$V_S = V_1 + V_2 = 2 - 2j + \frac{14+j2}{4+j} = 5.58\underline{\left|-14°\right.}V$$

Problem 7.39

Find V_S in the network shown if $I_0=2\angle 0°$ A.

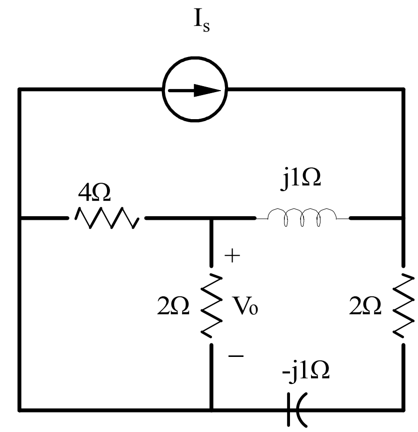


Suggested Solution

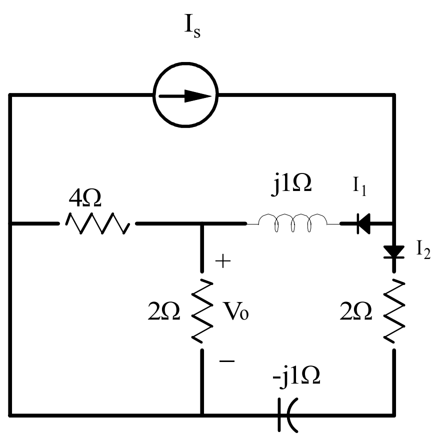
$$I_0 = 2\underline{|0°}$$
$$I_1 = 3\underline{|0°}$$
$$V_S = (3\underline{|0°})(2 + j) = 6 + 3j$$

Problem 7.40

Find I_S in the network shown if $V_0 = 2\angle 0°$ V.



Suggested Solution



$$V_1 = 8\angle 0° V$$

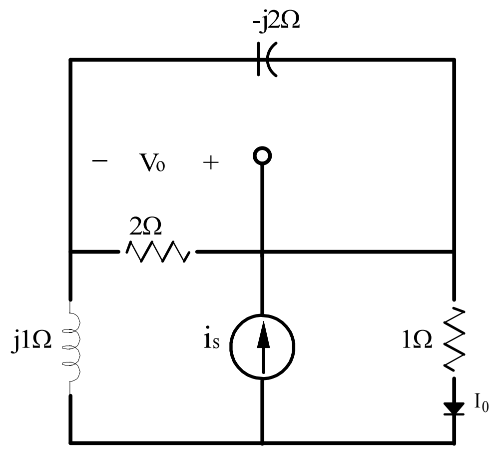
$$I_1 = \frac{8}{4/3} = 6A$$

$$I_2 = \frac{V_S}{2-j} = \frac{10\angle 37°}{2.24\angle -26.6°} = 4.46\angle 63.6°$$

$$I_S = I_1 + I_2 = 6 + 1.98 + 4j = 8 + 4j$$

Problem 7.41

Find I_0 in the network shown if $V_1=2\angle 0°$ V.



Suggested Solution

$$V_1 = V_0 - 4 = 4\angle 0° V$$

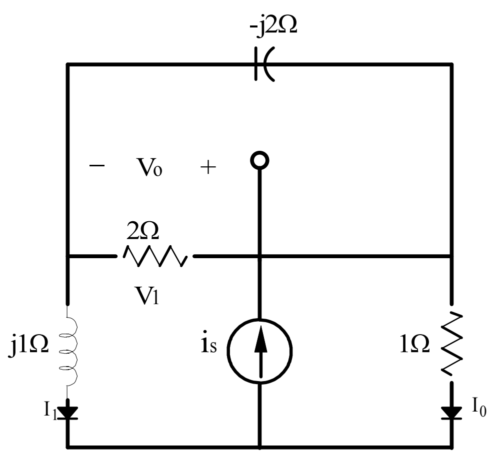
$$I_S = \frac{V_0}{1} + \frac{4}{2} + \frac{4}{-2j}$$

$$I_1 = 2 + 2j = I_S - V_0$$

$$V_0 = j(2+2j) = 2j - 2$$

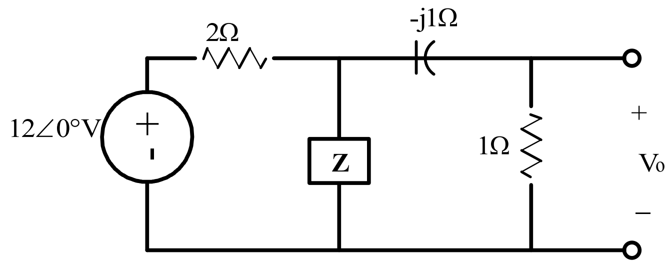
$$V_0 = V_0 + V_1 = 2j - 2 + 4 = 2 + 2j$$

$$I_0 = V_0 = 2 + 2j = 2.828\angle 45°$$



Problem 7.42

In the network shown $V_0=4\angle 45°$ V: find Z.



Suggested Solution

$$V_1 = 4\angle 45°(1-j) = 5.66\angle 0° V$$

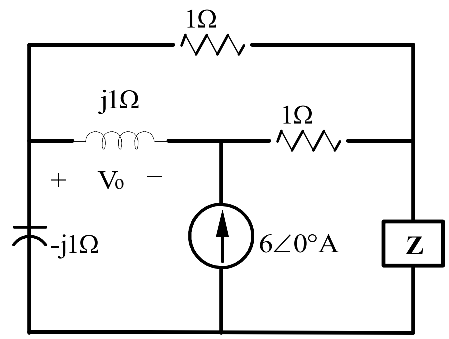
$$THEN, \frac{12-V_1}{2} = \frac{V_1}{Z} + 4\angle 45°$$

$$SO, \frac{12-5.66}{2} = \frac{5.66}{Z} + 2.83 + j2.83$$

$$Z = \frac{5.66}{2.85\angle -83°} = 2\angle 83°$$

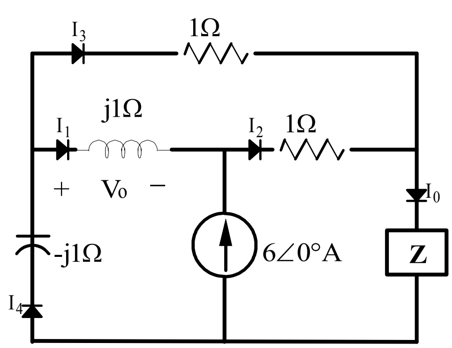
Problem 7.43

In the network shown $V_0 = 2\angle 45°$ V: find Z.



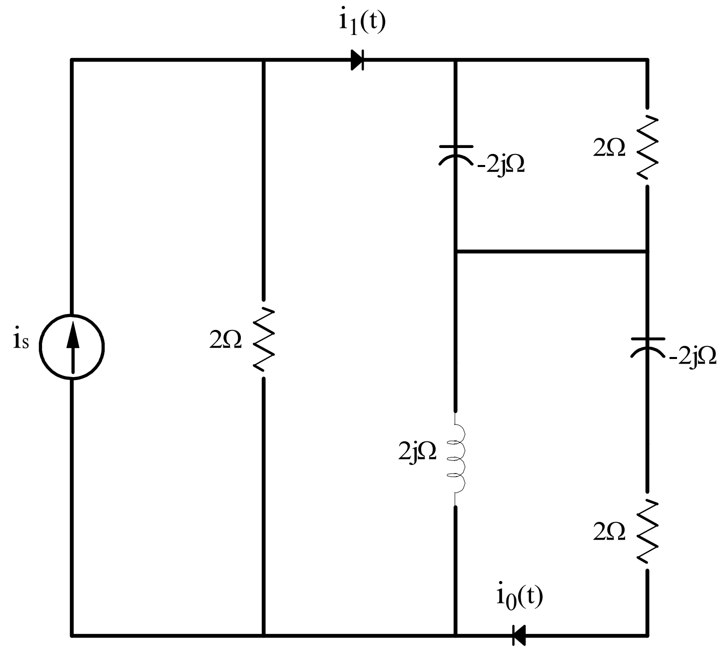
Suggested Solution

$$I_1 = \frac{2\angle 45°}{j} = 1.414 - j1.414$$
$$I_2 = 6 + I_1 = 7.414 - j1.414$$
$$V_2 = 1I_2 = 7.414 - j1.414$$
$$V_3 = V_1 + V_2 = 8.83$$
$$I_3 = \frac{V_3}{1} = 8.83$$
$$I_4 = I_1 + I_3 = 10.242 - j1.414$$
$$V_4 = -jI_4 = -1.414 - j10.242$$
$$V_4 + V_3 + V_0 = 0$$
$$V_0 = -(V_4 + V_3) = -(-1.414 - j10.242 + 8.83) = -7.416 + j10.242$$
$$I_0 = I_2 + I_3 = 7.414 - j1.414 + 8.83 = 16.25 - j1.414$$
$$Z = \frac{V_0}{I_0} = \frac{-(7.416 - j10.242)}{16.25 - j1.414} = 0.78\angle 130.87° \Omega$$



Problem 7.44

Find I_0 in the network shown if $I_S = 12\angle 0°$ A.



Suggested Solution

$$I_1 = I_S\left[\frac{2}{2+Z}\right]$$

$$I_0 = I_1\left[\frac{j2}{2-j2+j2}\right] = I_1(1\angle 90°)$$

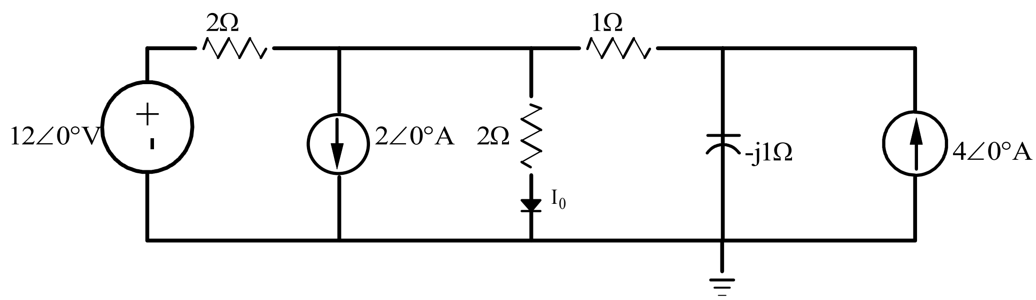
$$Z = (j2 \parallel (2-j2)) + (2 \parallel -j2) = \frac{j4+4}{2} - \frac{j4}{2-j2} = 2 + j2 - \frac{j2}{1-j} = \frac{4-j2}{1-j}$$

$$I_1 = 12\angle 0°\left[\frac{2}{2+\frac{4-j2}{1-j}}\right] = \frac{24(1-j)}{6-j4} = \frac{12(1-j)}{3-j2} = 4.69\angle -11.31° A$$

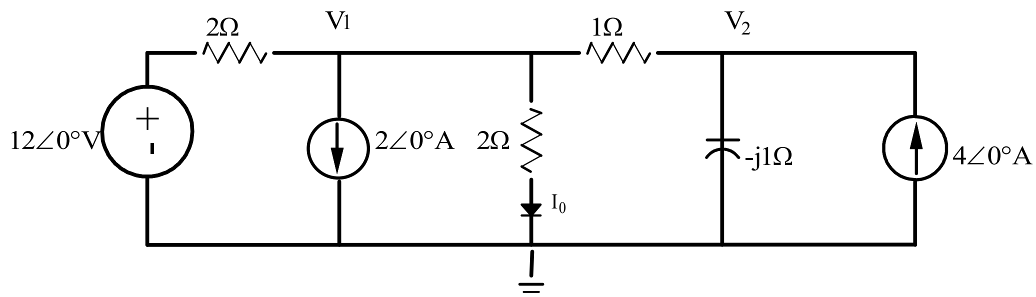
$$I_0 = (4.69\angle -11.31°)(1\angle 90°) = 4.69\angle 78.69° A$$

Problem 7.45

Use nodal analysis to find I_0 in the circuit shown.



Suggested Solution



$$V_1; \frac{12 - V_1}{2} = 2 + \frac{V_1 - V_2}{1} + \frac{V_1}{2}$$

$$8 = 4V_1 - 2V_2$$

$$V_2; \frac{V_1 - V_2}{1} + 4 = \frac{V_2}{-j1}$$

$$V_2 = \frac{4 + V_1}{1 + j1}$$

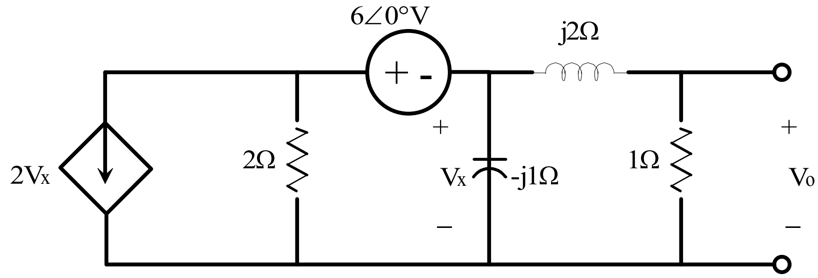
$$8 = 4V_1 - \frac{2(4 + V_1)}{1 + j1}$$

$$V_1 = \frac{4(2 + j)}{1 + j2}$$

$$I_0 = \frac{V_1}{2} = \frac{2(2 + j1)}{1 + j2} = 2\angle -36.87°$$

Problem 7.46

Use nodal analysis to find V_0 in the circuit shown.



Suggested Solution

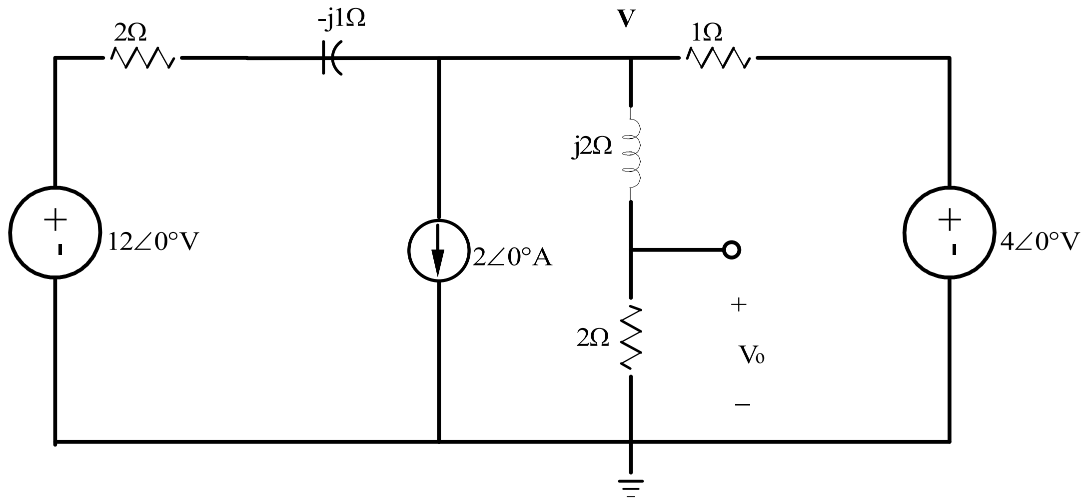
$$2V_x + \frac{V_x + 6}{2} + \frac{V_x}{-2} + \frac{V_x}{1+2j} = 0$$

$$V_x = \frac{-2(1+2j)}{1+4j} V$$

$$V_0 = V_x \left(\frac{1}{1+2j} \right) = -.49 \underline{|-76^\circ}$$

Problem 7.47

Find V_0 in the network shown using nodal analysis.



Suggested Solution

$$V_0 = V\left[\frac{2}{2+j2}\right] = \frac{V}{\sqrt{2}\angle 45^\circ} = \frac{V}{1+j1}$$

$$\frac{12-V}{2-j1} = 2 + \frac{V}{2+j2} + \frac{V-4}{1} = V\left[1+\frac{1}{2+j2}\right] - 2 = V\left[\frac{3+j2}{2+j2}\right] - 2$$

$$12 - V = V\left[\frac{(3+2j)(2-j1)}{2+j2}\right] - 4 + j2 = V\left[\frac{8+j1}{2+j2}\right] - 4 + j2$$

$$16 - j2 = V\left[1+\frac{8+j1}{2+j2}\right] = V\left[\frac{2+j2+8+j1}{2+j2}\right]$$

$$16 - j2 = V\left[\frac{10+j3}{2+j2}\right]$$

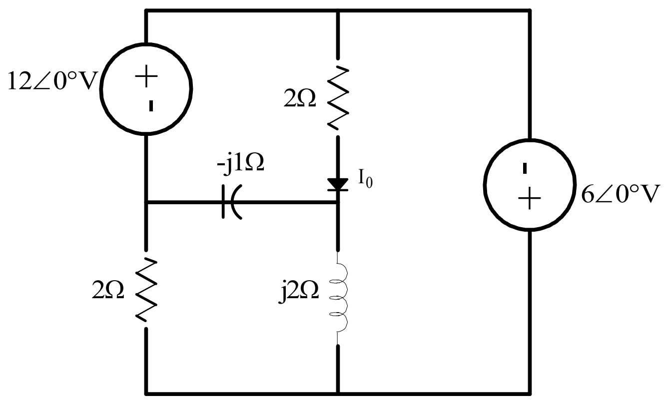
$$V = \frac{4(8-j1)(1+j1)}{(10+j3)}$$

$$V_0 = \frac{V}{1+j1} = \frac{4(8-j1)}{10+j3} = \frac{32.25\angle -7.13^\circ}{10.44\angle 16.70^\circ}$$

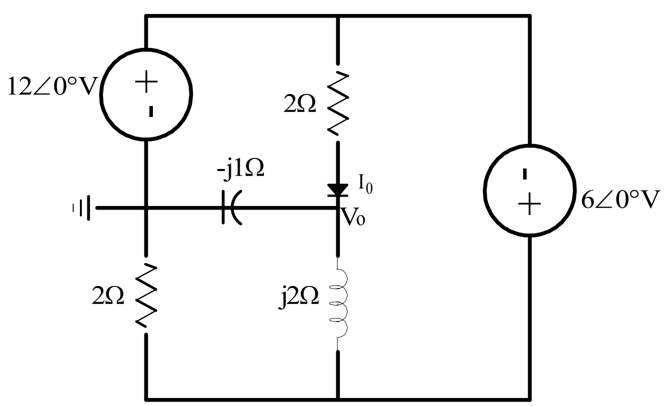
$$V_0 = 3.09\angle -23.83^\circ V$$

Problem 7.48

Use nodal analysis to determine I_0 in the network shown.
In addition, solve the problem using MATLAB.



Suggested Solution



$$V_0;\ \frac{12-V_0}{2}+\frac{18-V_0}{j2}=\frac{V_0}{-j1}=jV_0$$

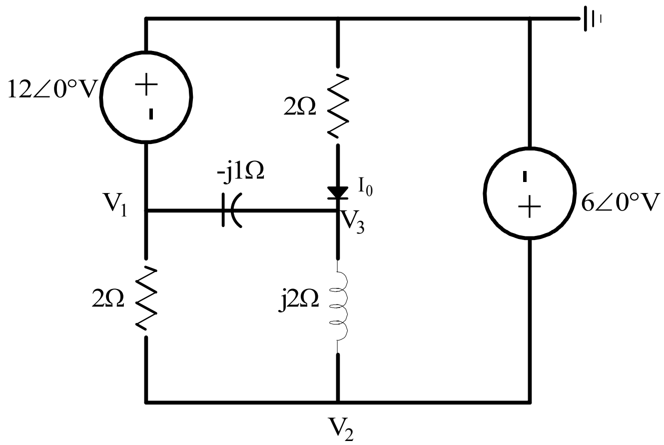
$$12-V_0-j18+jV_0=j2V_0$$

$$V_0(1+j1)=12-j18$$

$$V_0=\frac{12-j18}{1+j1}$$

$$I_0=\frac{12-V_0}{2}=\frac{12-\frac{12-j18}{1+j1}}{2}=\frac{12+j12-12+j18}{2(1+j1)}=\frac{j15}{1+j1}=10.64\underline{|45^\circ}A$$

MATLAB SOLUTION



$$V_1 = -12\underline{|0°} \qquad V_2 = 6\underline{|0°}$$

$$\frac{V_3}{2} + \frac{V_3 - V_1}{-j1} + \frac{V_3 - V_2}{j2} = 0$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & -1 & -1 + j1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -12 \\ 6 \\ 0 \end{bmatrix}$$

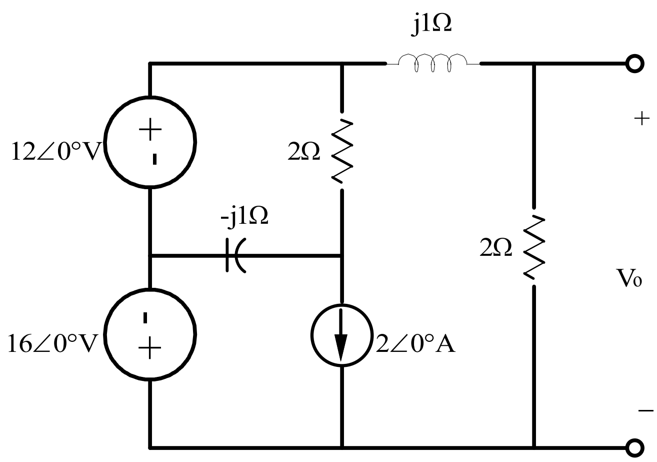
```
>>g=[2 -1 -1+1j;1 0 0;0 1 0]
>>i=[0;-12;6]
>>v=inv(g)*I

v=
      -12.0000
        6.0000
      -15.0000 -15.0000i
```

Since V_3=-15-j15

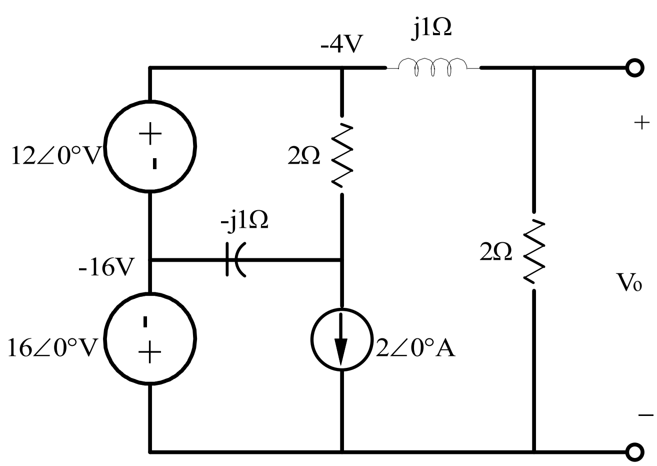
Then I_0=-V_3/2=7.5+j7.5 A=10.61/45° **A**

Problem 7.49

Find V_0 in the network shown.



Suggested Solution

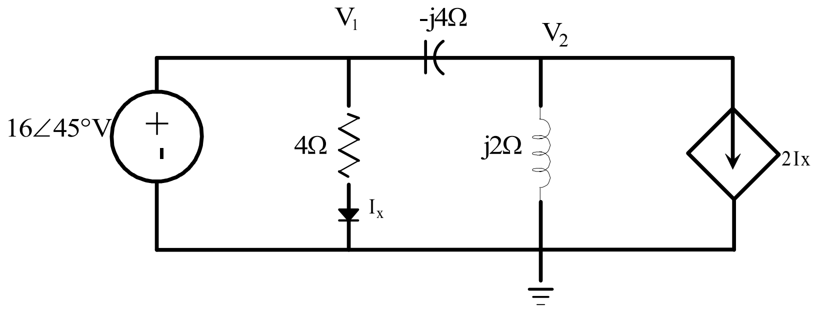


$$V_0 = -4\left[\frac{2}{2 + j1}\right]$$

$$V_0 = 3.58\underline{|153.43°}V$$

Problem 7.50

Find the voltage across the indicator in the circuit shown using nodal analysis.



Suggested Solution

$$KCL \ @ \ V_2; \quad \frac{V_1 - V_2}{-j4} = \frac{V_2}{2} + I_x$$

$$I_x = \frac{V_1}{4}; \qquad V_1 = 16\underline{|45^\circ}$$

$$V_1 - V_2 = -2V_2 - \frac{j8V_1}{4}$$

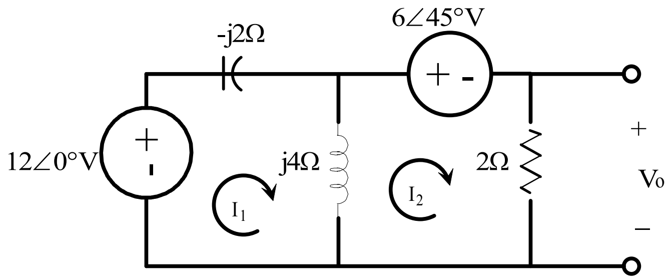
$$V_1(1 + j2) = -V_2$$

$$V_2 = -V_1(1 + j2) = (16\underline{|-135^\circ})(1 + j2)$$

$$V_2 = 35.78\underline{|-71.57^\circ} V$$

Problem 7.51

Use mesh analysis to find V_0 in the circuit shown.



Suggested Solution

$V_0 = 2I_2$

<u>*mesh equations*</u>

$$12 = I_1(j2) + I_2(-j4)$$

$$-6\angle 45° = -j4I_1 + I_2(2 + j4)$$

<u>*matrix form*</u>

$$\begin{bmatrix} j2 & -j4 \\ -j4 & 2+j4 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 6\angle -135° \end{bmatrix}$$

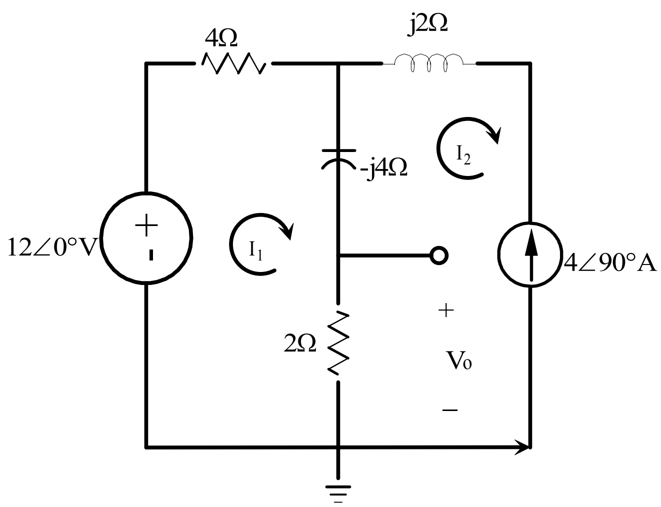
Solve for I_2

$$I_2 = \frac{\begin{bmatrix} j2 & 12 \\ -j4 & 6\angle 135° \end{bmatrix}}{\begin{bmatrix} j2 & -j4 \\ -j4 & 2+j4 \end{bmatrix}} = \frac{12\angle -45° + 48\angle 90°}{j4 - 8 + 16} = 4.52\angle 51.3° A$$

$$V_0 = 2I_2 = 9.04\angle 51.3° V$$

Problem 7.52

Use mesh analysis to find V_0 in the circuit shown.



Suggested Solution

mesh equations

$$12 = I_1(4 - j4 + 2) - I_2(2 - j4) \qquad (1)$$

$$I_2 = -4\angle 90^\circ = -j4A \qquad (2)$$

$$Substitute(2)into(1)$$

$$12 = I_1(6 - j4) + j4(2 - j4)$$

$$I_1(6 - j4) = 12 - j8 - 16 = -4 - j8$$

$$I_1 = \frac{2(-1 - j2)}{(3 - j2)}$$

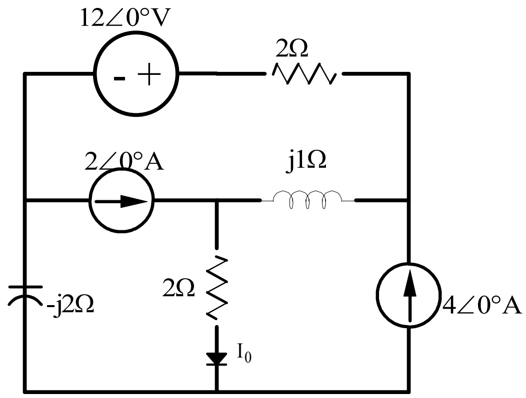
$$I_1 = 1.24\angle -82.87^\circ A = (0.15 - j1.23)A$$

$$V_0 = (I_1 - I_2)2 = (0.15 - j1.23 + j4)2$$

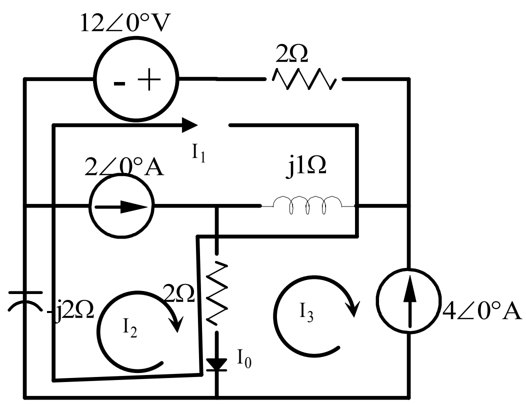
$$V_0 = 5.55\angle -86.9^\circ V$$

Problem 7.53

Using loop analysis, find I_0 in the network shown.



Suggested Solution



$I_3 = -4A \qquad I_2 = 2A$

Supermesh Equations

$$12 = I_1\left(2 + j1 + 2 - j2\right) - j1I_3 - 2I_3 + 2I_2 - j2I_2$$

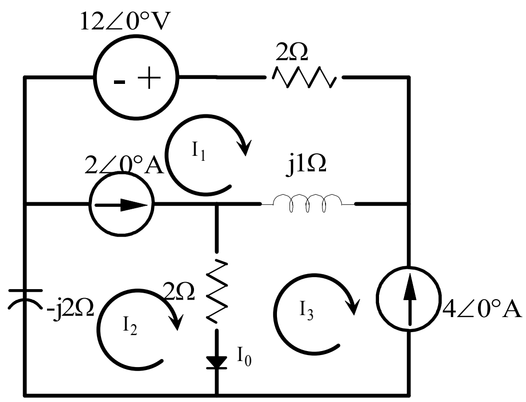
$$12 = I_1\left(4 - j1\right) - I_3\left(2 + j1\right) + I_2\left(2 - j2\right)$$

$$12 = I_1(4 - j1) + 8 + j4 + 4 - j4$$

$$I_1 = 0$$

$$I_0 = I_1 + I_2 - I_3 = 0 + 2 - (-4) = 6A$$

MATLAB SOLUTION



$$I_2 = -4\angle 0° \quad I_1 - I_3 = 2\angle 0°$$

$$12\angle 0° = 2I_3 + j1\left(I_3 - I_2\right) + 2\left(I_1 - I_2\right) - j2I_1$$

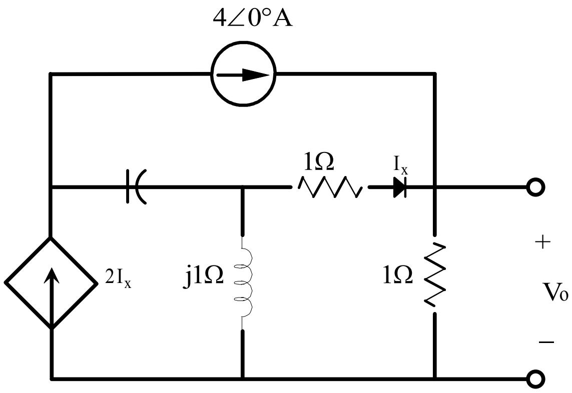
$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 2-j2 & -2-j1 & 2+j1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \\ 12 \end{bmatrix}$$

```
>> z=[2-2j -2-1j 2+1j; 0 1 0; 1 0 -1]
>> v=[12;-4;2]
>> i=inv(z)*v
i =
        2.0000 + 0.0000i
       -4.0000
       -0.0000 + 0.0000i
```

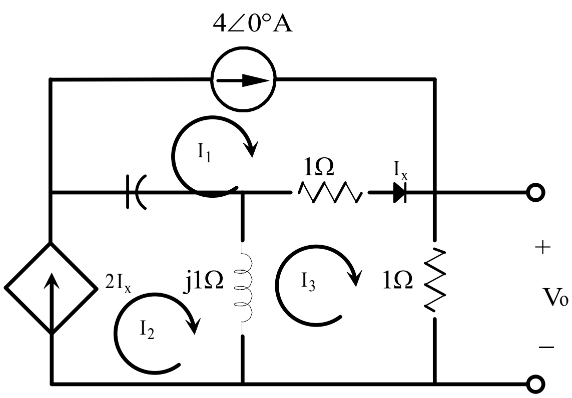
$I_0=I_1-I_2=2-(-4)=6A$

Problem 7.54

Find V_0 in the network shown.



Suggested Solution



$$I_1 = 4, \quad I_2 = 2I_x, \qquad I_x = I_3 - 4$$

$$j\left(I_3 - I_2\right) + 1\left(I_3 - I_1\right) + 1I_3 = 0$$

$$j\left(I_3 - 2I_x\right) + 1\left(I_3 - 4\right) + I_3 = 0$$

$$jI_3 - 2jI_x + I_3 - 4 + I_3 = 0$$

$$jI_3 - 2\left(I_3 + 4\right) + I_3 - 4 + I_3 = 0$$

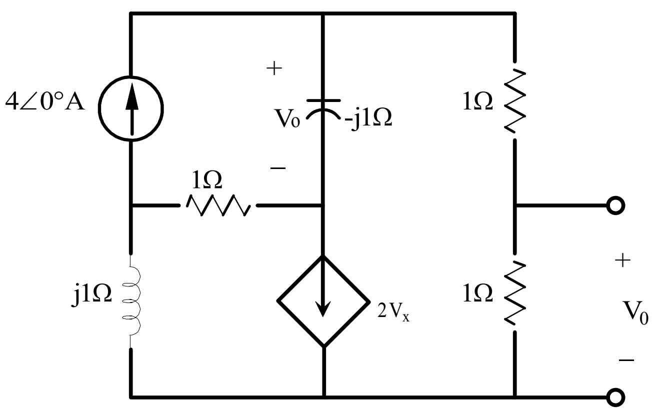
$$jI_3 - 2jI_3 + 8j + I_3 - 4 + I_3 = 0$$

$$I_3\left(j - 2j + 1 + 1\right) = 4 - 8j$$

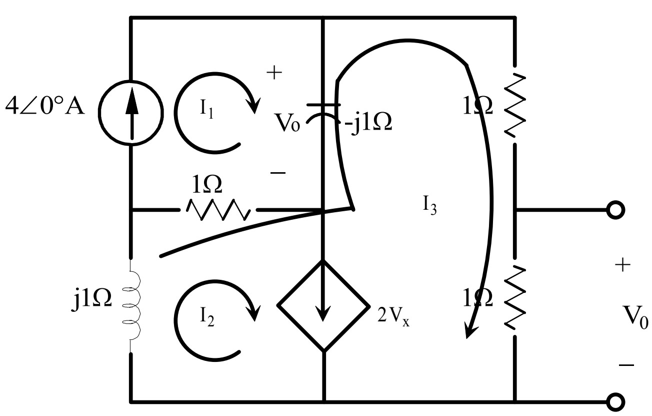
$$I_3 = \frac{4 - 8j}{2 - j} = 4\underline{|-36.86°}A$$

Problem 7.55

Find V_0 in the network shown.



Suggested Solution



$I_1 = 4\angle 0^\circ \qquad I_2 = 2V_x$

$V_x = -j(I_1 - I_3)$

$For\ I_3,$

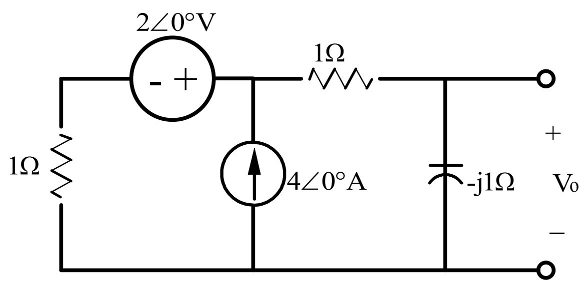
$j(I_2 - I_3) + 1(I_2 + I_3 - 4) - j(I_3 - 2V_x) + 2I_3 = 0$

$Solving\ I_3 = \dfrac{4+12j}{5}$

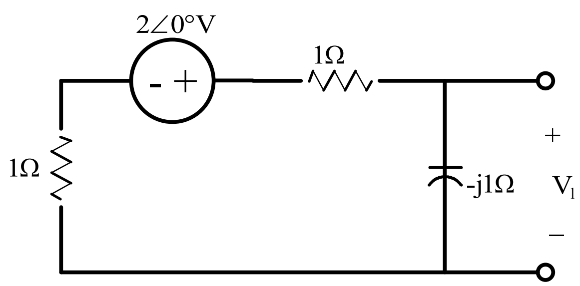
$V_0 = .8 + 2.4j$

Problem 7.56

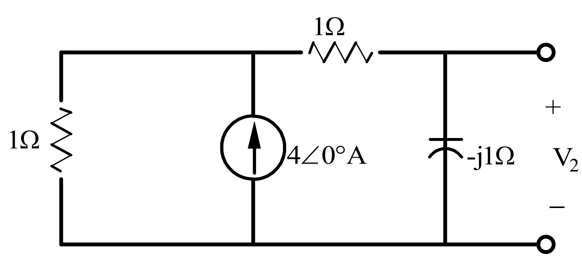
Use superposition to find V_0 in the network shown.



Suggested Solution



$$V_1 = -2\left(\frac{-j}{2-j}\right) = \frac{2j}{2-j} = \frac{2\angle 90^\circ}{2.236\angle -26.56^\circ} = 0.89\angle 116.56^\circ V$$

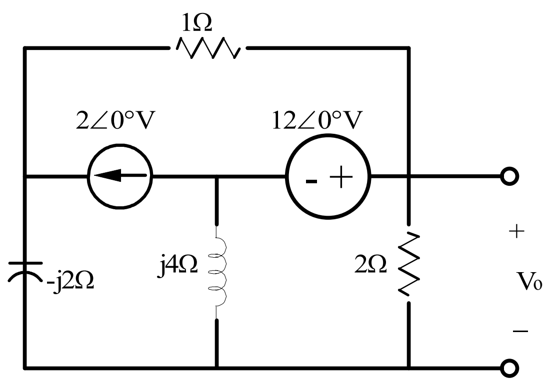


$$V_2 = \frac{4(1)}{2-j}(-j) = \frac{-4j}{2-j} = \frac{-4\angle 90^\circ}{2.236\angle -26.56^\circ} = 1.78\angle -63.4^\circ V$$

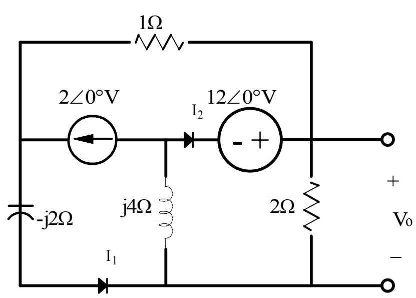
$$V_0 = V_1 + V_2 = -0.398 + j0.396 + 0.797 - j1.59 = 0.399 - j794 = 0.89\angle -63.6^\circ V$$

Problem 7.57

Find V_0 in the network shown using superposition.



Suggested Solution



For the 2A source,

$$I_1 = \frac{2+2}{2-2j+\dfrac{8j}{2+4j}} = \frac{4(2+4j)}{12+12j}$$

$$V_0 = I_1\left(\frac{8j}{2+4j}\right) = \frac{-32j}{12+12j}$$

For the 12V source,

$$I_2 = \frac{12}{4j+\dfrac{2(2-2j)}{2+2-2j}} = \frac{12(4-2j)}{12+12j}$$

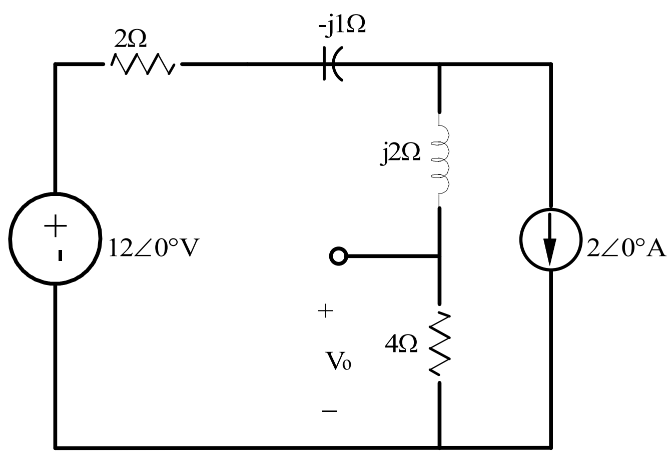
$$V_0 = I_2\frac{2(2-2j)}{2+2-2j} = \frac{12(4-4j)}{12+12j}$$

Summing

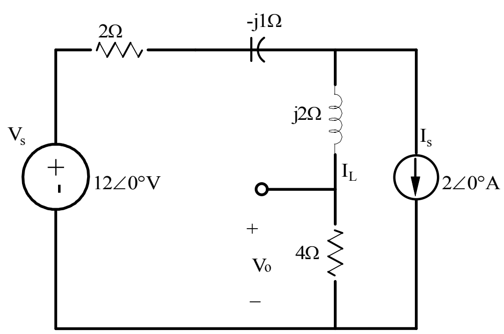
$$V_0 = \frac{-32j+48-48j}{12+12j} = 5.5\angle -104^\circ V$$

Problem 7.58

Using superposition, find V_0 in the circuit shown.



Suggested Solution



For the voltage source

$$V_1 = \frac{4}{4+2+j2-j1}\left(12\underline{|0^\circ}\right) = \frac{48}{6+j1}$$

For the current source

$$I_L = I_S\left[\frac{2-j1}{4+2+j2-j1}\right] = \frac{-4+j2}{6+j1}$$

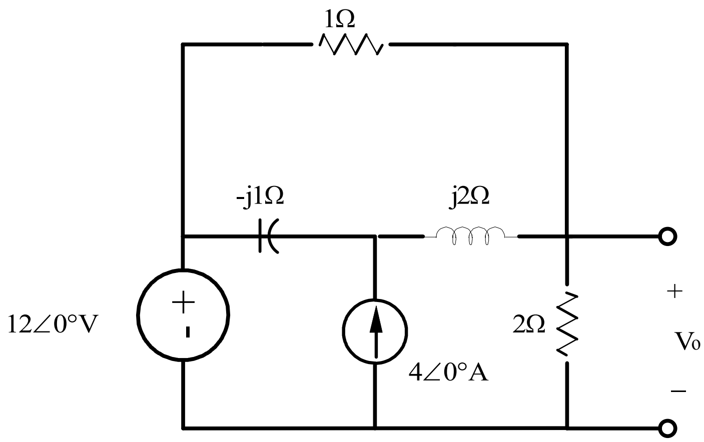
$$V_2 = 4I_L = \frac{-16+j8}{6+j1}$$

Summing

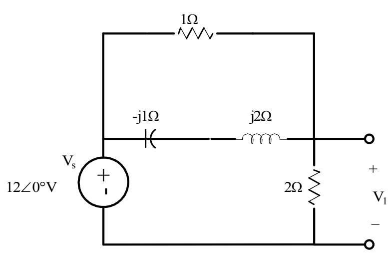
$$V_0 = V_1 + V_2 = \frac{32+j8}{6+j1} = 5.41\underline{|4.57^\circ}V$$

Problem 7.59

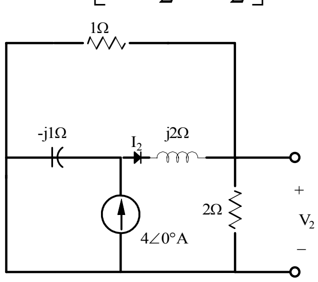
Use both superposition and MATLAB to determine V_0 in the circuit shown.



Suggested Solution



$$V_1 = V_0\left[\frac{2}{2+\frac{1}{2}+j\frac{1}{2}}\right] = 12\left(\frac{2}{\frac{5}{2}+j\frac{1}{2}}\right) = \frac{48}{5+j1} = 9.23 - j1.85V$$

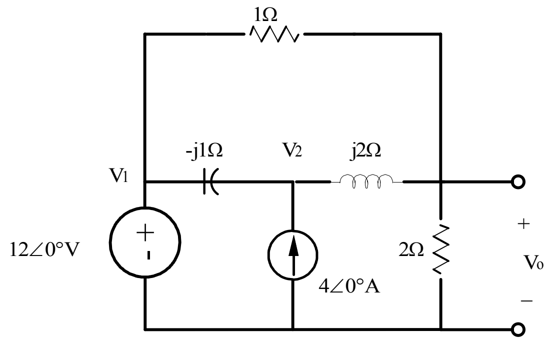


$$I_2 = 4\left[\frac{-j1}{-j1+j2+\tfrac{2}{3}}\right] = \frac{-j4}{j1+\tfrac{2}{3}} = \frac{-j12}{2+j3}$$

$$V_2 = \frac{2}{3}\left(I_2\right) = \frac{-j8}{2+j3} = -1.85 - j1.23V$$

$$V_0 = V_1 + V_2 = 7.38 - j3.08 = 8.00\underline{|-22.65°}V$$

MATLAB SOLUTION



$$V_1 = 12\underline{|0°}V$$

$$\frac{V_1 - V_2}{-j1} + 4 + \frac{V_0 - V_2}{j2} = 0$$

$$\frac{V_1 - V_0}{1} + \frac{V_2 - V_0}{j2} - \frac{V_0}{2} = 0$$

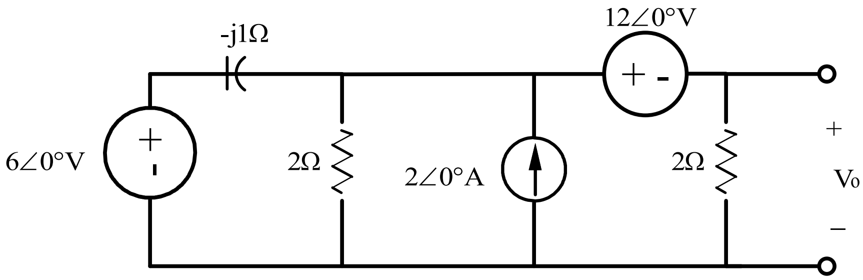
$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 1 \\ j2 & 1 & -1-j3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -j8 \\ 0 \end{bmatrix}$$

```
>> g=[0+2j 1 -1-3j;1 0 0;-2 1 1]
>> i=[0;12;-8j]
>> v=inv(g)*i

v =
      12.0000 - 0.0000i
      16.6154 - 4.9231i
       7.3846 - 3.0769i
```

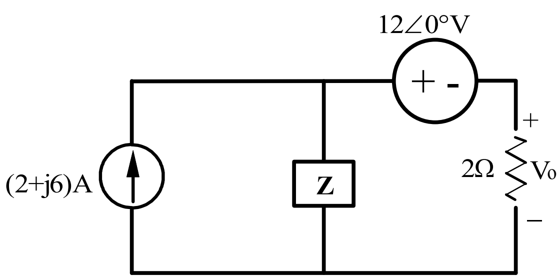
V_0=7.3846-j3.0769=8.00<u>/-22.62°</u> V

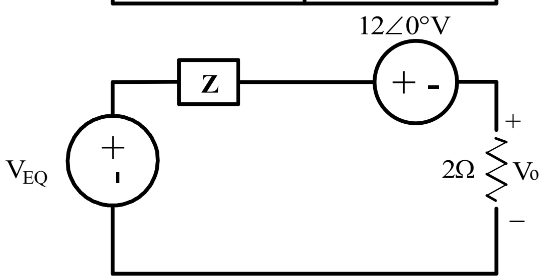
Problem 7.60

Use source exchange to determine V_0 in the network shown.



Suggested Solution





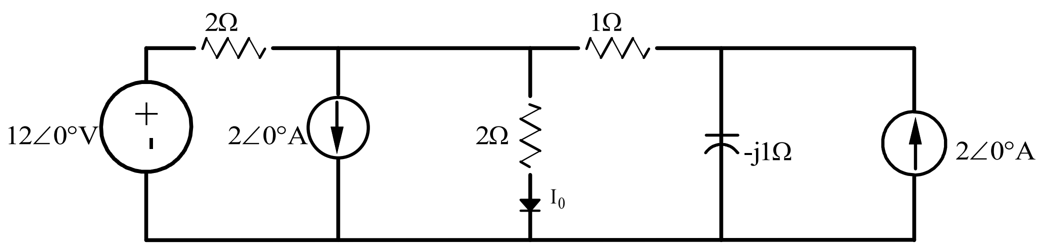
$$Z = 2 \parallel -j = \frac{-j2}{2-j1} = \frac{2}{1+j2}$$

$$V_{EQ} = (2+j6)(Z_{EQ}) = \frac{2(2+j6)}{1+j2} = 5.66\angle 8.13^\circ V = (5.60 + j0.80)V$$

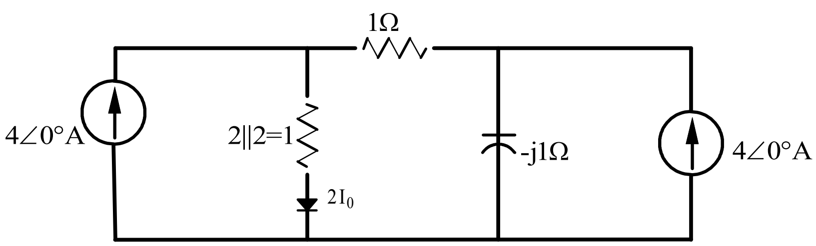
$$V_0 = \left(V_{EQ} - 12\right)\left[\frac{2}{2+Z_{EQ}}\right] = \left(-6.4 + j0.8\right)\left[\frac{4}{3+j2}\right] = 7.16\angle 139.18^\circ V$$

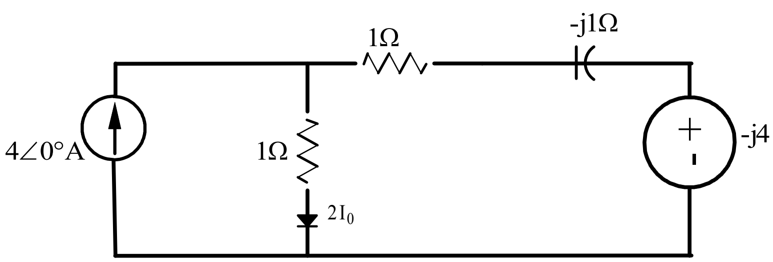
Problem 7.61

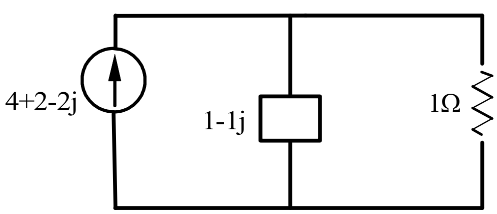
Use source exchange to find the current I_0 in the network shown.



Suggested Solution





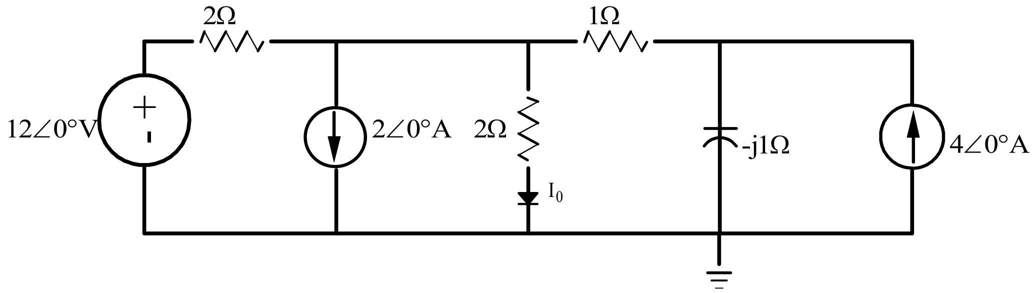


$$2I_0 = (6-2j)\frac{1-j}{2-j}$$

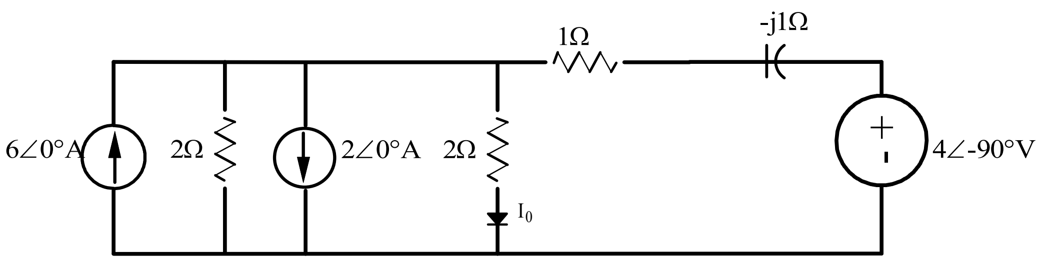
$$I_0 = 2\underline{|-37^\circ}$$

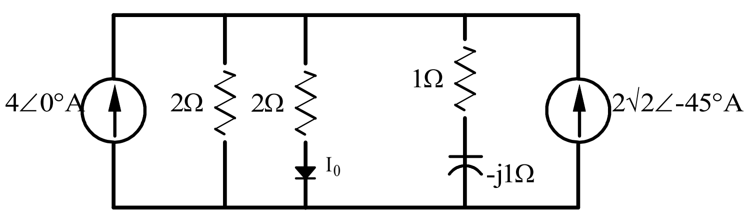
Problem 7.62

Use source transformation to determine I_0 in the circuit shown.

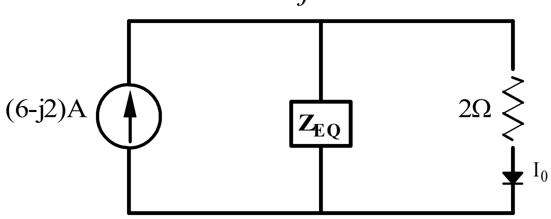


Suggested Solution





$$Z_{EQ} = 2 \parallel (1 - j1) = \frac{2 - j2}{3 - j1} \qquad I_{EQ} = 4 + 2\sqrt{2}\underline{|-45^\circ} = 6 - j2$$

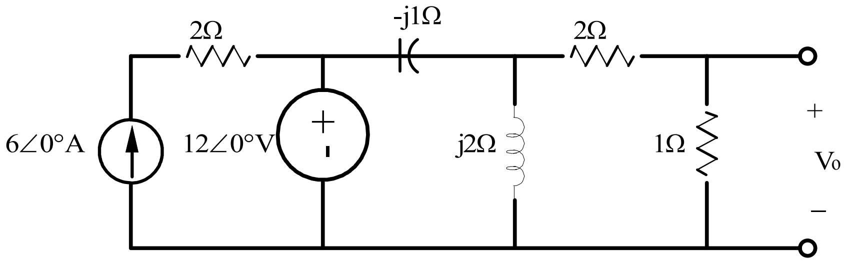


$$I_0 = (6 - j2)\left[\frac{Z_{EQ}}{Z_{EQ} + 2}\right] = (6 - j2)\left[\frac{2 - j2}{2 - j2 + 6 - j2}\right] = \frac{(6 - j2)(2 - j2)}{8 - j4}$$

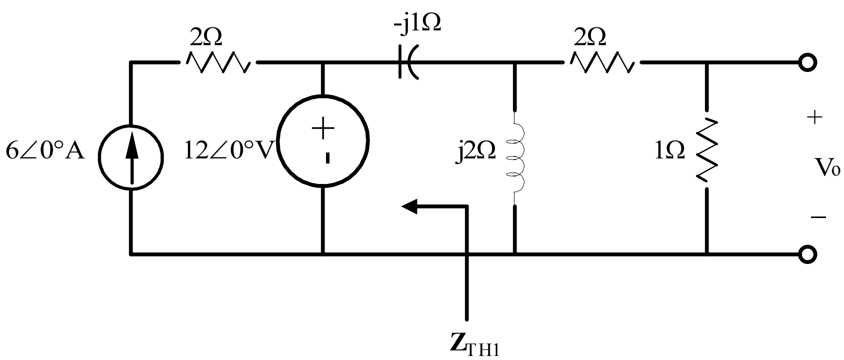
$$I_0 = \frac{(3 - j1)(1 - j1)}{2 - j1} = 2\underline{|-36.87^\circ} A$$

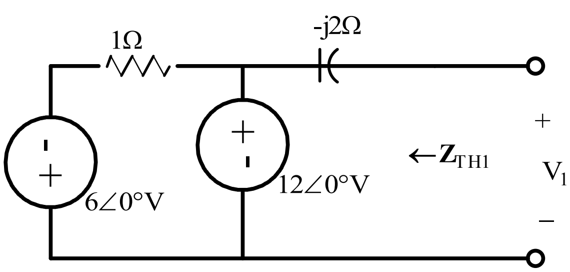
Problem 7.63

Use Thevenin's theorem to find V_0 in the circuit shown.



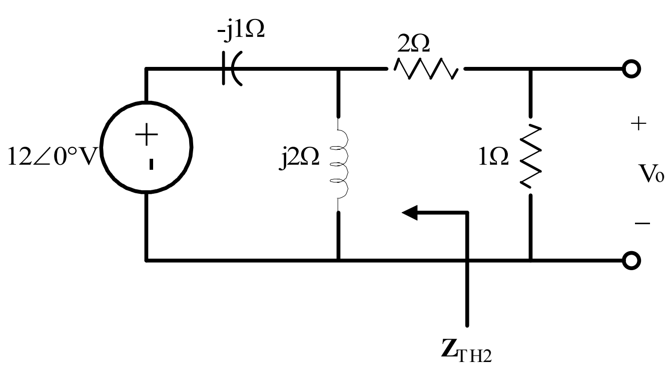
Suggested Solution

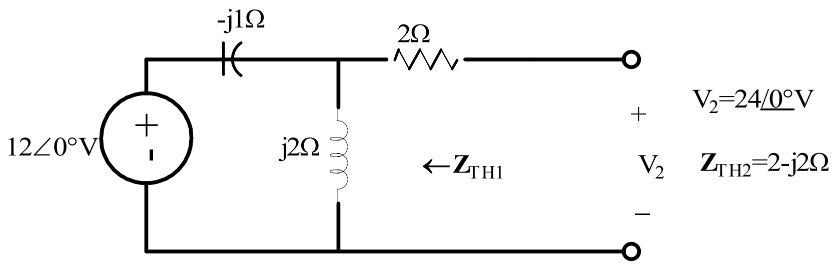


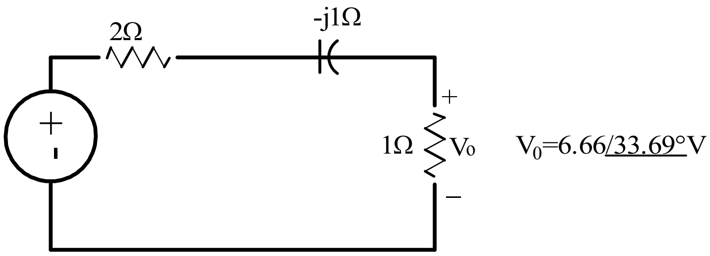


$V_1 = 12\underline{/0°}V$

$\mathbf{Z}_{TH1} = -j1\Omega$

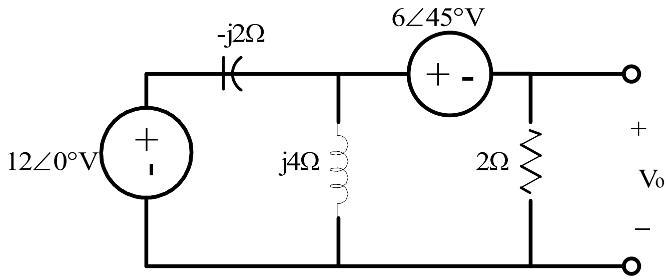



-j1Ω
2Ω
12∠0°V
j2Ω
←Z TH1
+
V2
−
V2=24/0°V
Z TH2=2-j2Ω


2Ω
-j1Ω
+
1Ω
V0
−
V0=6.66/33.69°V

Problem 7.64

Using Thevenin's theorem, find V_0 in the network shown.



Suggested Solution

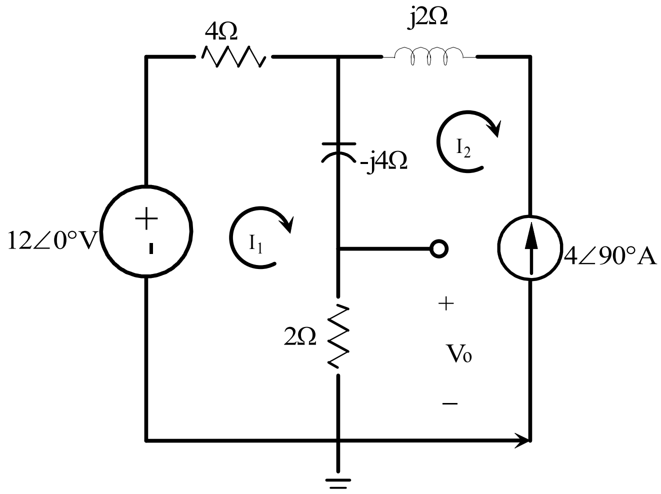
$$V_{oc} = \frac{12(j4)}{j2} - 6\underline{|45°} = 20.2\underline{|-12.1°}V$$

$$R_{TH} = \frac{(-j2)(j4)}{j2} = -j4\Omega$$

$$V_0 = \frac{(20.2\underline{|-12.1°})2}{2-j4} = 9.03\underline{|51.3°}V$$

Problem 7.65

Use Thevenin's theorem to find V_0 in the circuit shown.



Suggested Solution

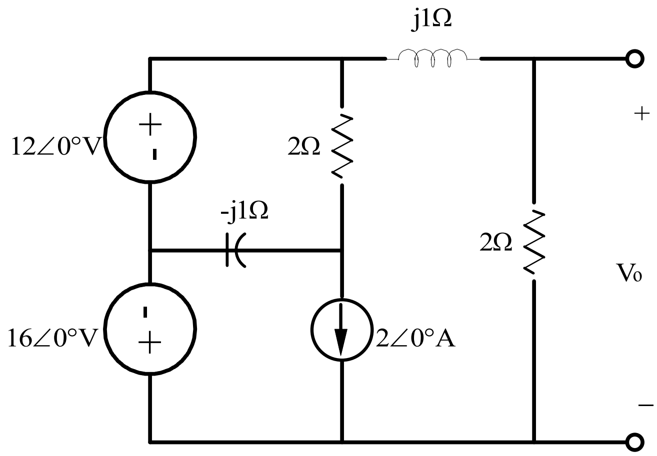
$$V_{oc} = 12 + 4\left(4\underline{|90°}\right) = 12 + j16V$$

$$R_{TH} = 4 - j4\Omega$$

$$V_0 = \frac{(12 + j16)2}{2 + 4 - j4} = 5.55\underline{|86.8°}V$$

Problem 7.66

Solve Problem 7.49 using Thevenin's theorem.



Suggested Solution

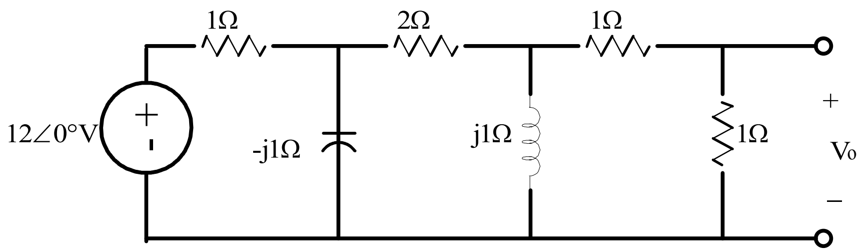
$$V_{TH} = 12\angle 0° - 16\angle 0° = 4\angle 180° V$$

$$Z_{TH} = 0\Omega$$

$$V_0 = 4\angle 180° \left[\frac{2}{2 + j1} \right] = 3.58\angle 153.43° V$$

Problem 7.67

Apply Thevenin's theorem twice to find V_0 in the circuit shown.



Suggested Solution

PART I

$$V_{OC} = \frac{12 - j}{1 - j} = \frac{-12j}{1 - j}$$

$$Z_{TH} = \frac{-j}{1 - j}\Omega$$

PART II

$$V_{OC} = \frac{\frac{-12}{1 - j}}{\frac{-j}{1 - j} + 2 + j} = (-j) = \frac{12}{3 - 2j}$$

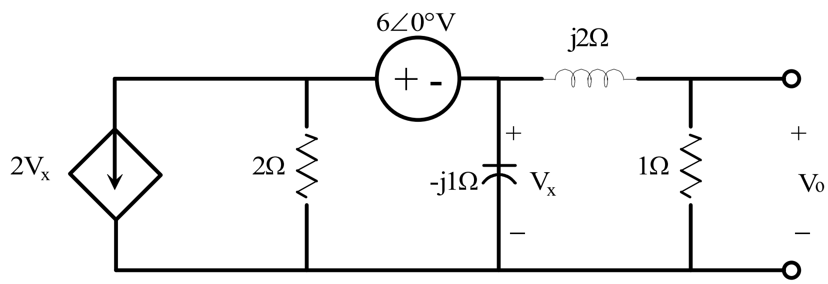
$$Z_{TH} = \frac{j\left(2 + \frac{-j}{1 - j}\right)}{j + 2 + \frac{-j}{1 - j}} = \frac{3 + 2j}{3 - 2j}\Omega$$

FINALLY

$$V_0 = \frac{\frac{12}{3 - 2j}}{2 + \frac{3 + 2j}{3 - 2j}}(1) = 1.3\underline{|12.5°}V$$

Problem 7.68

Find V_0 in the network shown using Thevenin's theorem.



Suggested Solution

$$V_{OC} = V_X$$

$$2V_X + \frac{V_{OC}+6}{2} + \frac{V_{OC}}{-j1} = 0 \Rightarrow V_{OC} = \frac{-6}{5+j2}$$

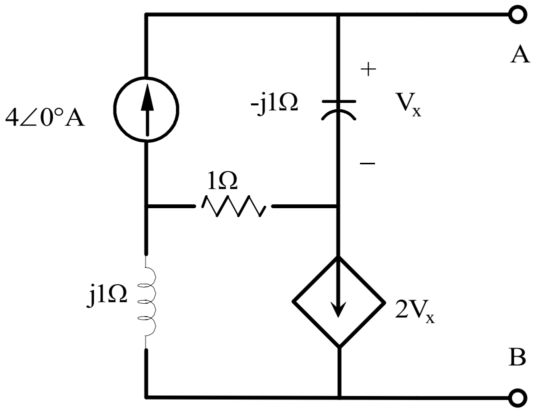
$$I_{SC} = \frac{-6}{2} = -3A$$

$$Z_{TH} = \frac{V_{OC}}{I_{SC}} = \frac{2}{5+2j}\Omega$$

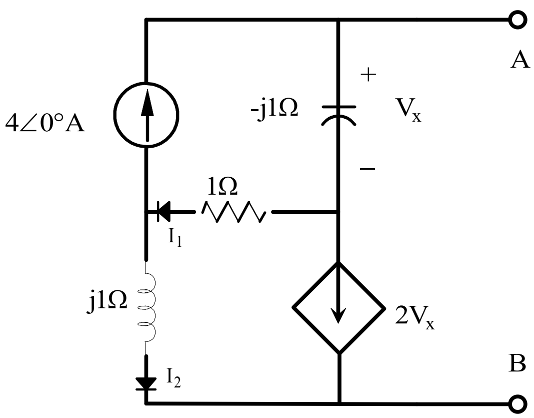
$$V_0 = V_{OC}\left[\frac{1}{1+j2+Z_{TH}}\right] = \left(\frac{-6}{5+j2}\right)\left(\frac{5+j2}{(1+j2)(5+j2)+2}\right) = \frac{-6(5+j2)}{3+j12} = 2.61\angle 125.8°V$$

Problem 7.69

Find the Thevenin equivalent for the network shown at the terminals A-B.



Suggested Solution



For V_{OC}

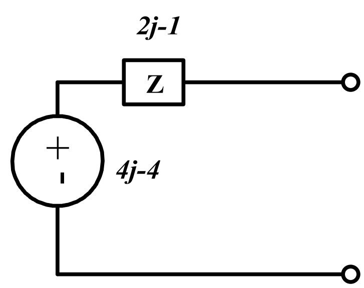
$V_X = 4\angle 0^\circ(-j) = -4j$

$2V_X = -8j$

$I_1 = 4 + 8j \qquad I_2 = 4 + 8j - 4 = 8j$

$V_1 = 8j - j = -8$

$V_{OC} = -8 + (4 + 8j) - 4j = -4 + 4j$



For I_{SC}

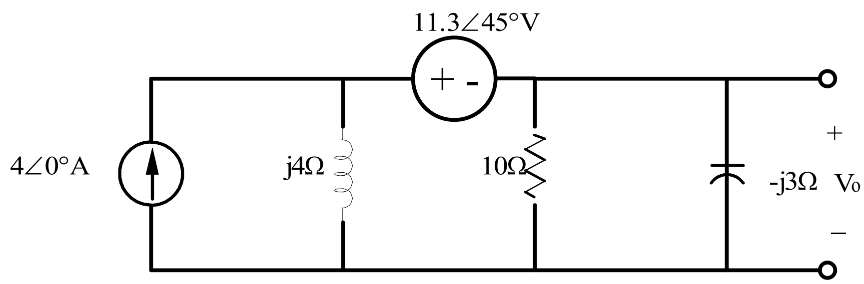
$j(I_{SC} + 2V_X) + 1(I_{SC} + 2V_X - 4) - j(I_{SC} - 4) = 0$

$V_X = -j(4 - I_{SC})$ *YIELDS* $I_{SC} = 2.4 + .8j$

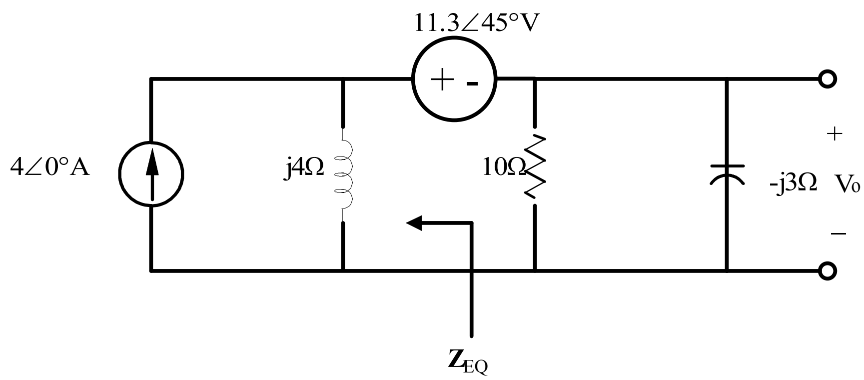
$$Z_{TH} = \frac{-4 + 4j}{2.4 + .8j} = -1 + 2j$$

Problem 7.70

Find V_x in the circuit shown using Norton's theorem.



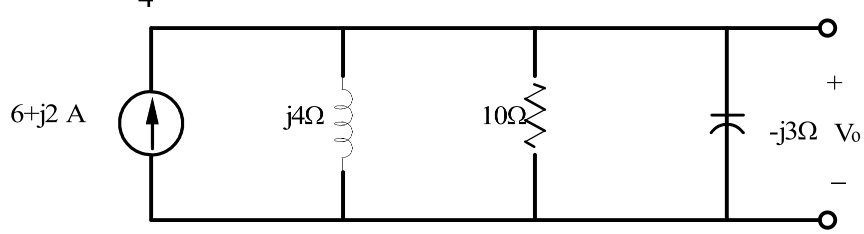
Suggested Solution



$$Z_{EQ} = j4\Omega$$

$$4 = \frac{V}{j4} + I_{SC} \quad where \ \ V = 11.3\angle 45°V$$

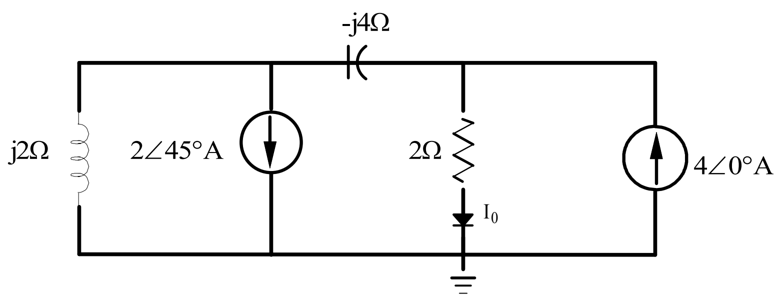
$$I_{SC} = 4 + \frac{jV}{4} = 6 + j2\,A$$



$$V_X = I_{SC}\left[10 \parallel j4 \parallel -j3\right] = I_{SC}\left[\frac{1}{\frac{1}{10}+\frac{1}{j4}+\frac{1}{-j3}}\right] = \frac{I_{SC}}{0.10 + j0.08} = 48.59\angle -21.37°V$$

Problem 7.71

Find I_0 in the network shown using Norton's theorem.



Suggested Solution

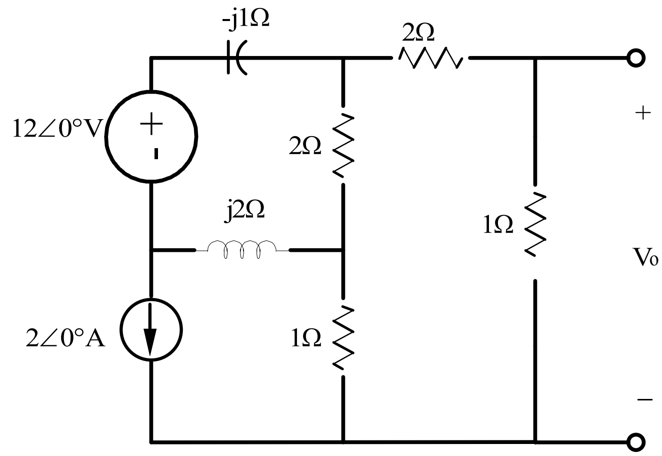
$$I_{SC} = -2\angle 45^\circ \frac{2j}{2j-4j} + 4\angle 0^\circ = 4 + 2\angle 45^\circ$$

$$Z_{TH} = -4j + 2j = -2j$$

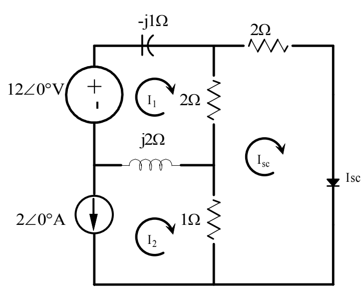
$$I_0 = I_{SC}\frac{Z_{TH}}{2+Z_{TH}} = (5.41 + 1.41j)\frac{-2j}{2-2j} = 4\angle -30.3^\circ A$$

Problem 7.72

Apply both Norton's theorem and MATLAB to find V_0 in the network shown.



Suggested Solution



mesh equations

$$12 = I_1(2 + j1) - 2I_{SC} - j2I_2 \quad (1)$$

$$-2 = I_2 \quad (2)$$

$$0 = I_{SC}(3) - 2I_1 - I_2 \quad (3)$$

substitute (2) *into* (1) & (3)

$$I_1(2 + j1) - 2I_{SC} = 12 - j4 \quad (4)$$

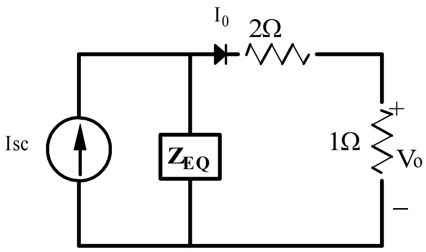
$$I_1 = 1 + \frac{3}{2}I_{SC} \quad (5)$$

substitute (5) *into* (4)

$$(2 + j1)\left(1 + \frac{3}{2}I_{SC}\right) - 2I_{SC} = 12 - j4 \Rightarrow I_{SC}\left(1 + j\frac{3}{2}\right) = 10 - j5$$

$$I_{SC} = \frac{10(2 - j1)}{2 + j3}$$

$$Z_{EQ} = 2 \parallel (j2 - j1) + 1 = 1 + \frac{j2}{2 + j1} = \frac{2 + j3}{2 + j1}$$

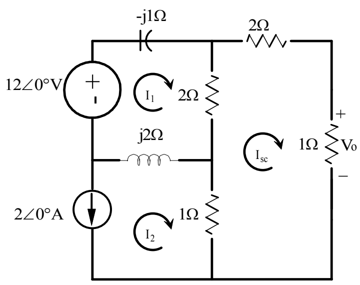


$$I_0 = I_{SC}\left[\frac{Z_{EQ}}{3+Z_{EQ}}\right] = \left(\frac{10(2-j1)}{2+j3}\right)\left[\frac{2+j3}{3(2+j1)+2+j3}\right]$$

$$V_0 = 1I_0 = \frac{10(2-j1)}{8+j6} = \frac{5(2-j1)}{4+j3}$$

$$V_0 = 2.24\angle -63.43° V$$

MATLAB SOLUTION



$$I_1 = 2\angle 0° A$$

$$12 = -j1I_2 + (I_2 - I_3) + j2(I_2 - I_1)$$

$$0 = 2I_3 + (1)I_3 + (I_3 - I_2)(1) + 2(I_3 - I_2)$$

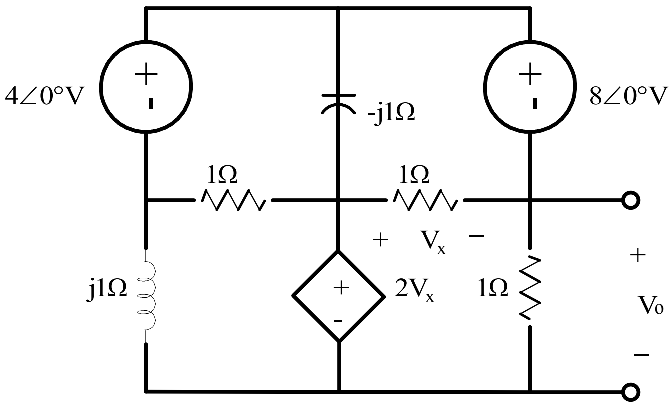
$$\begin{bmatrix} 1 & 0 & 0 \\ -j2 & 2+j1 & -2 \\ -1 & -2 & 6 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 12 \\ 0 \end{bmatrix}$$

```
>> z=[1 0 0;-2j 2+1j -2;-1 -2 6];
>> v=[-2;12;0];
>> i=inv(z)*v

i =
      -2.0000
        4.0000 - 6.0000i
        1.0000 - 2.0000i
```

$V_0 = I_3(1) = 1 - j2 = 2.24\angle -63.43°$ V

Problem 7.73

Find V_0 using Norton's theorem for the circuit shown.



Suggested Solution

$$V_X = 2V_X - V_{OC}$$

$$V_{OC} = V_1 - 4$$

$$V_X = V_1 - 4$$

$$\frac{V_1}{j1} + \frac{V_1 - 2V_X}{1} + \frac{V_1 + 4 - 2V_X}{-j1} + \frac{V_1 - 4 - 2V_X}{1} = 0$$

$$V_1 = 6V \qquad V_{OC} = 2V$$

$$V_X = 0$$

$$\frac{8}{-j1} + \frac{4}{1} + \frac{4}{j1} + I_{SC} = 0$$

$$I_{SC} = -4 - j4 \ A$$

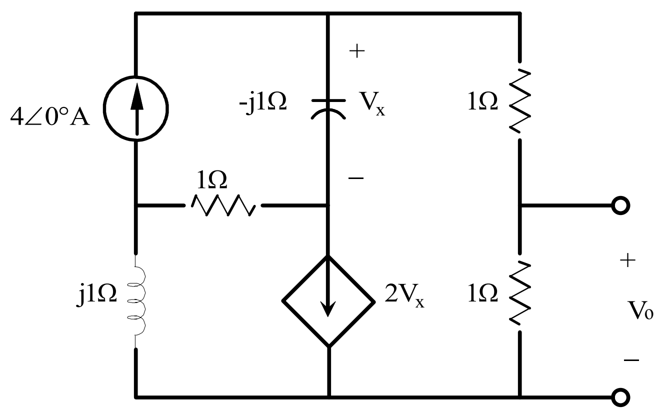
$$Z_{TH} = \frac{V_{OC}}{I_{SC}} = \frac{2}{-4 - j4} = -\frac{1}{4} + j\frac{1}{4}\ \Omega$$

$$I_0 = 2.53\angle -18.43\, A$$

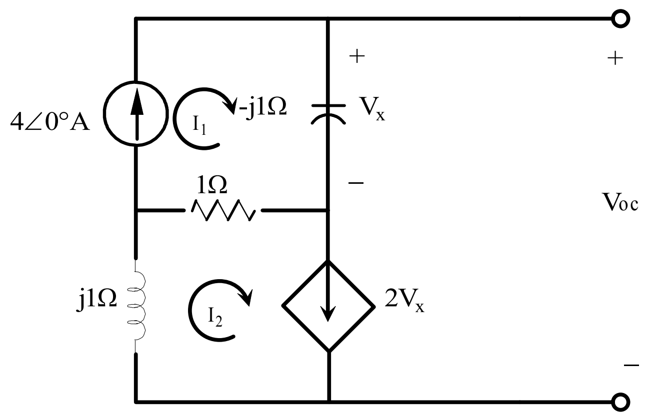
$$V_0 = I_0(1) = 2.53\angle -18.43\, V$$

Problem 7.74

Use Norton's theorem to find V_0 in the network shown.



Suggested Solution

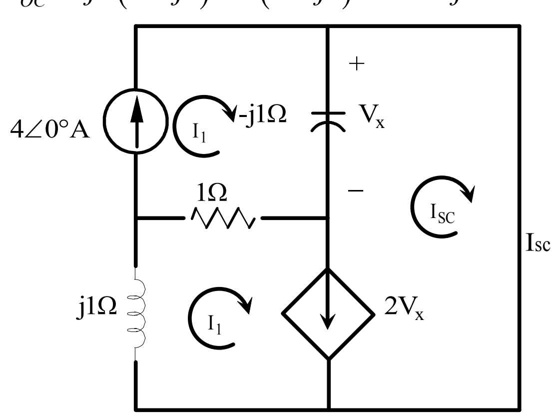


$I_1 = 4\angle 0°$ $\qquad I_2 = 2V_X$

$V_X = (-j1)I_1 = -j4V$ $\qquad I_2 = -j8\,A$

$I_2(1+j1) - I_1(1-j1) + V_{OC} = 0$

$V_{OC} = j8(1+j1) + 4(1-j1) = -4 + j4\ V$



$$I_1 = 4\angle 0^\circ \ A$$

$$I_2 - I_{SC} = 2V_X \qquad V_X = (-j1)(I_1 - I_{SC})$$

$$I_2(1+j1) - I_1(1-j1) + I_{SC}(-j1) = 0$$

$$\left[I_{SC}(1+j2) - j8\right](1+j1) - 4(1-j1) + I_{SC}(-j1) = 0$$

$$I_{SC} = \frac{-4+j4}{-1+j2} A$$

$$I_0 = I_{SC}\left[\frac{Z_{TH}}{Z_{TH}+2}\right]$$

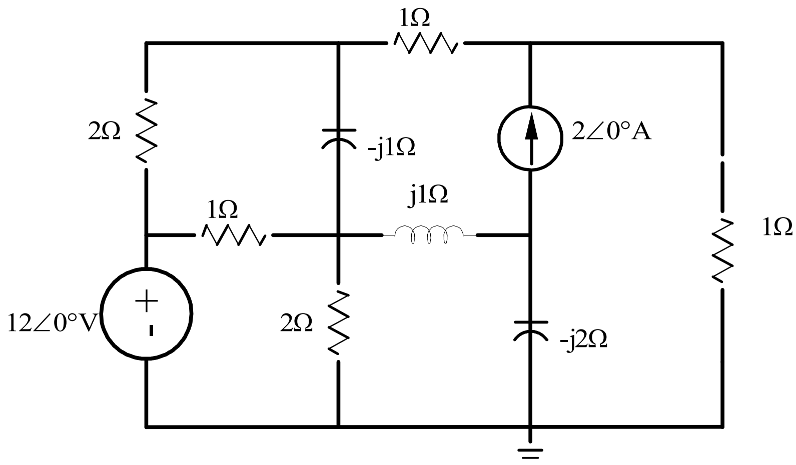
$$Z_{TH} = -1 + j2$$

$$I_0 = \frac{-4+j4}{1+j2} A$$

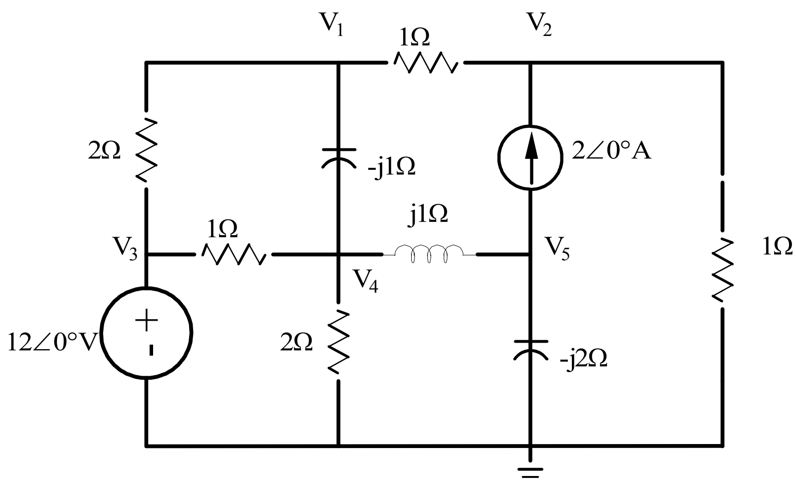
$$V_0 = (1) I_0 = 2.53\angle 71.57^\circ \ V$$

Problem 7.75

Use MATLAB to find the node voltages in the network shown.



Suggested Solution



$$\frac{V_1 - V_3}{2} + \frac{V_1 - V_4}{-j1} + \frac{V_1 - V_2}{1} = 0$$

$$\frac{V_2 - V_1}{1} + \frac{V_2}{1} = 2\underline{|0°}$$

$$V_3 = 12\underline{|0°}$$

$$\frac{V_4 - V_1}{-j1} + \frac{V_4 - V_3}{1} + \frac{V_4}{2} + \frac{V_4 - V_5}{j1} = 0$$

$$\frac{V_5 - V_4}{j1} + \frac{V_5}{-j2} = -2\underline{|0°}$$

ALSO

$$V_1(1.5 + j1) - V_2 - .5V_3 - j1V_4 = 0$$

$$-V_1 + 2V_2 = 2\underline{|0°}$$

$$V_3 = 12\underline{|0°}$$

$$-j1V_1 - V_3 + 1.5V_4 + j1V_5 = 0$$

$$j1V_4 - j0.5V_5 = -2\underline{|0°}$$

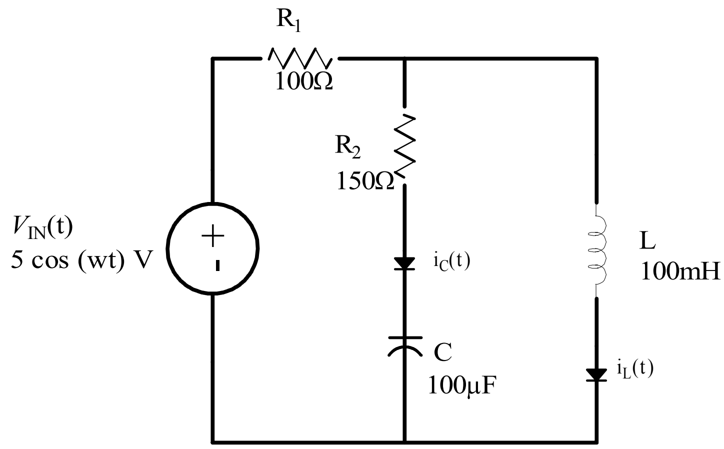
Matrix Form

$$\begin{bmatrix} 1.5 + j1 & -1 & -.5 & -j1 & 0 \\ -1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -j1 & 0 & -1 & 1.5 & j1 \\ 0 & 0 & 0 & j1 & -j0.5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 2\underline{|0°} \\ 12\underline{|0°} \\ 0 \\ -2\underline{|0°} \end{bmatrix}$$

```
>>Y = [ 1.5+j*1 -1 -.5 -j*1 0; -1 2 0 0 0; 0 0 1 0 0;-j*1 0 -1 1.5 j*1;
0 0 j*1 -j*0.5]
>>I = [0;2;12;0;-2]
>>V = inv(Y)*I
V=
       6.5800 - 2.0600i
       4.2900 - 1.0300i
      12.0000 - 0.0000i
       4.5200 - 1.6400i
       9.0400 - 7.2800i
```

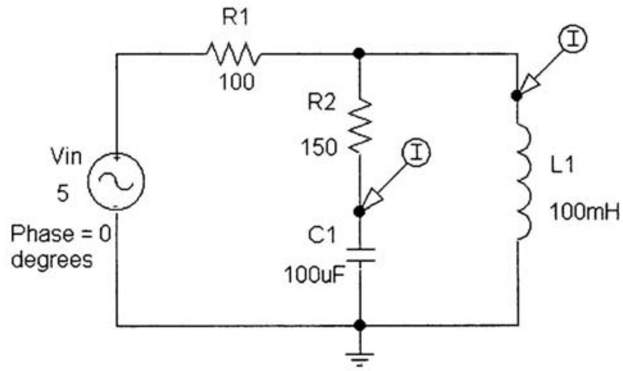
Problem 7.76

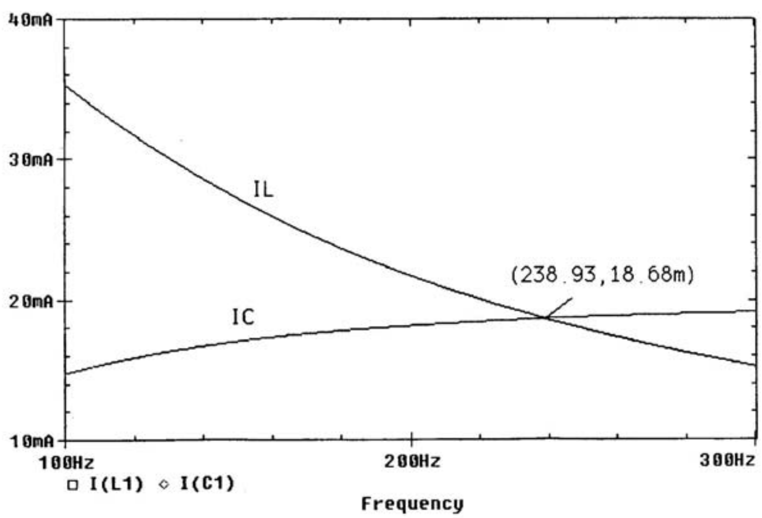
Using the PSPICE Schematics editor, draw the circuit shown. At what frequency are the magnitudes of $i_C(t)$ and $i_L(t)$ equal?



Suggested Solution

This *Schematics* circuit was simulated over the frequency range 100 Hz to 300 Hz. Since current into pin markers were placed in the circuit, PROBE will automatically plot the required current magnitudes.

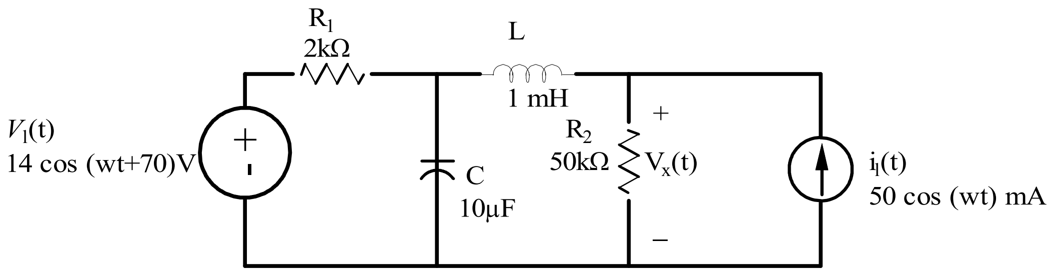




PROBE results show that the voltage and current phases are roughly equal at 238.9 Hz

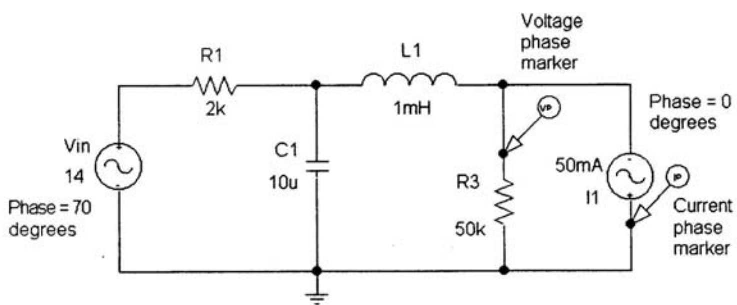
Problem 7.77

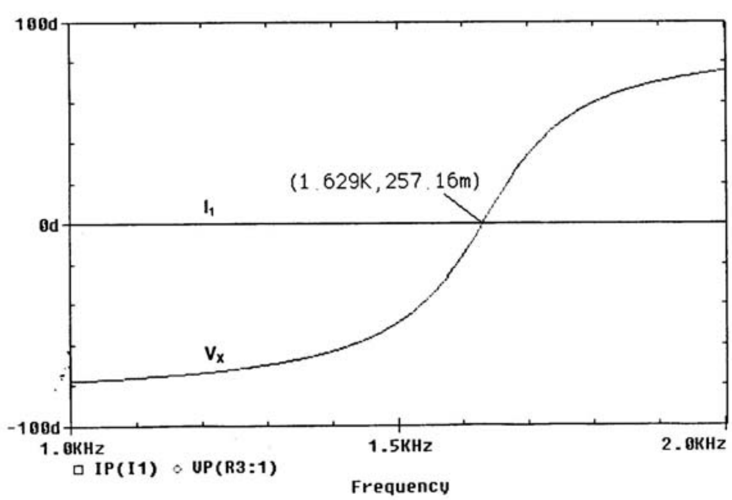
Using the PSPICE Schematics editor, draw the circuit shown. At what frequency are the phases of $i_l(t)$ and $v_x(t)$ equal?



Suggested Solution

This *Schematics* circuit was simulated over the frequency range 1 kHz to 2 kHz. Since vphase and iphase markers were placed in the circuit, PROBE will automatically plot the required voltage and current phase angles.

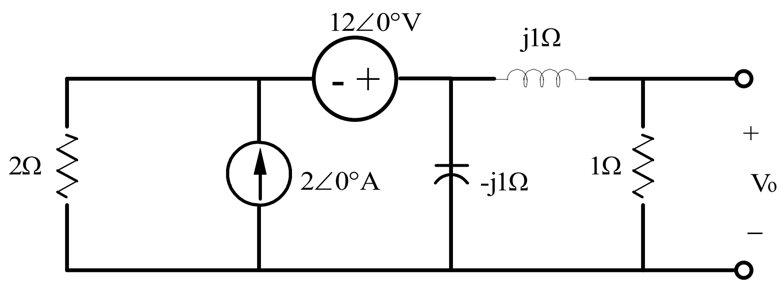




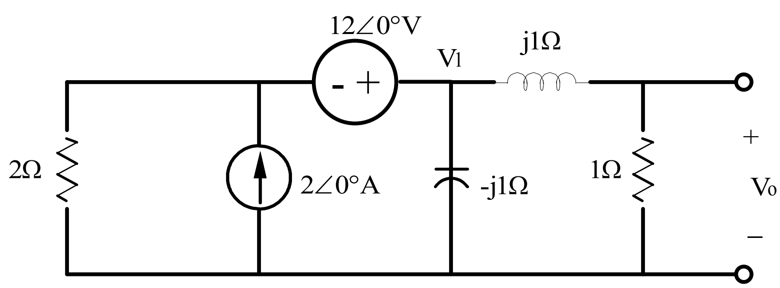
PROBE results show that the voltage and current phases are roughly equal at 1.63 kHz.

Problem 7FE-1

Find V_0 in the network shown.



Suggested Solution



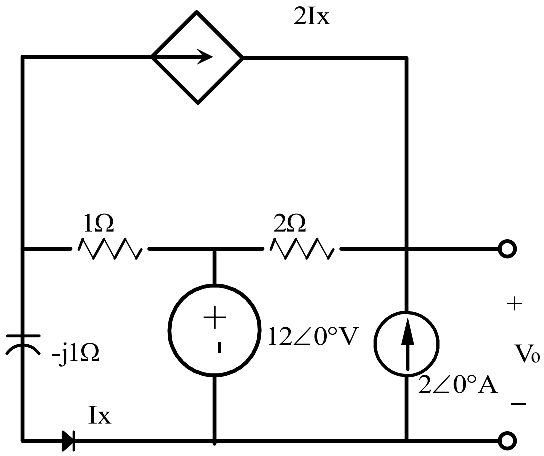
$$\frac{V_1 - 12}{2} - 2 + \frac{V_1}{-j1} + \frac{V_1}{1 + j1} = 0$$

$$V_1\left[\frac{1}{2} + \frac{1}{-j} + \frac{1}{1+j}\right] = 8 \qquad V_1 = \frac{32}{4 + j2} V$$

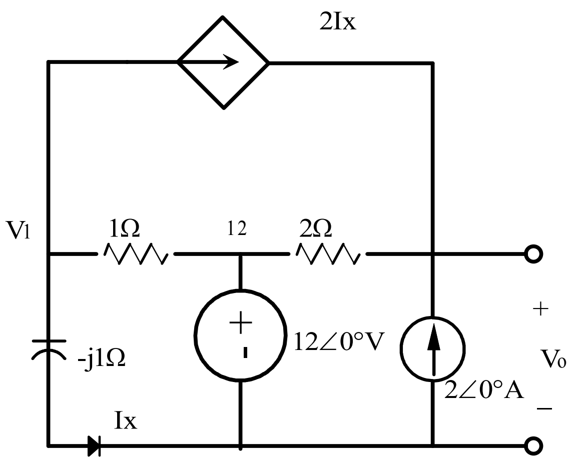
$$V_0 = V_1\left(\frac{1}{1+j}\right) = \frac{32}{2 + j6} = 5.06\underline{|-71.6}\ V$$

Problem 7FE-2

Find V_0 in the circuit shown.



Suggested Solution



$$\frac{V_1}{-j}+\frac{V_1-12}{1}+2I_X=0$$

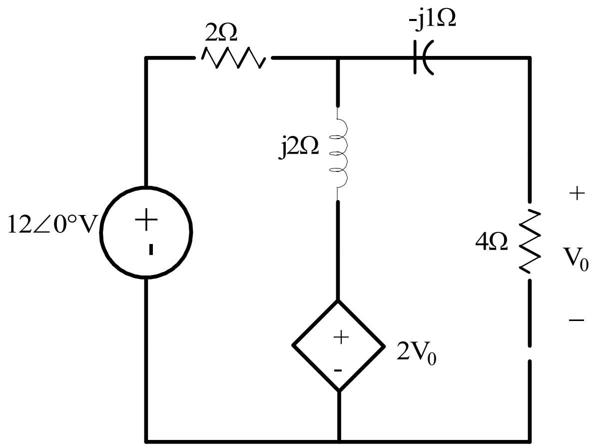
$$\frac{V_0-12}{2}-2-2I_X=0$$

$$I_X=\frac{V_1}{-j}$$

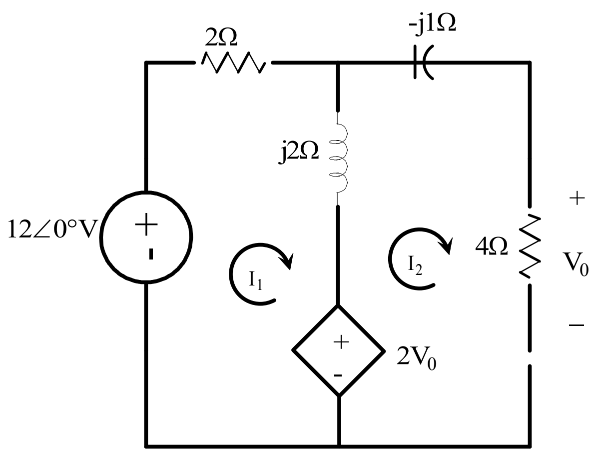
$$V_1=\frac{12}{1+3j}\Rightarrow\frac{V_0}{2}-2j\left(\frac{12}{1+3j}\right)=8\Rightarrow V_0=\frac{304+j48}{10}=30.78\underline{|8.97^{\circ}}V$$

Problem 7FE-3

Find the average power dissipated in the 4-Ohm load resistor.



Suggested Solution



$$-12 + 2I_1 + j2(I_1 - I_2) + 2V_0 = 0$$

$$-2V_0 + j2(I_2 - I_1) - jI_2 + 4I_2 = 0$$

$$V_0 = 4I_2$$

$$I_1(1 + j) + I_2(4 - j1) = 6$$

$$I_1(-j) + I_2\left(-2 + j\frac{1}{2}\right) = 0$$

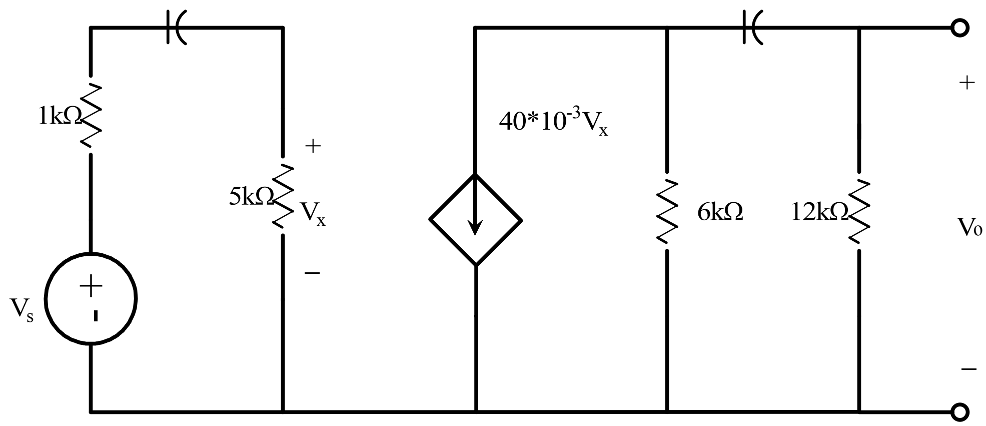
$$\Delta = (1 + j)\left(-2 + j\frac{1}{2}\right) - (-j)(4 - j) = \frac{-3 - j5}{2}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{2}{-3 + j5}\begin{bmatrix} -2 + j\frac{1}{2} & -4 + j \\ j & 1 + j \end{bmatrix}\begin{bmatrix} 6 \\ 0 \end{bmatrix} \Rightarrow I_2 = \frac{60 - j36}{34} = 2.05\underline{|-31.03^\circ}A$$

$$P_{4\Omega} = \frac{1}{2}(2.05)^2(4) = 8.41\,watts$$

Problem 7FE-4

Determine the mid-band (where the coupling capacitors can be ignored) gain of the single-stage amplifier shown.



Suggested Solution

$$V_X = \frac{V_S(5k)}{1k+5k} = \frac{5}{6}V_S \text{ and } V_0 = -40\times10^{-3}V_X(6k\,\|\,12k) = \frac{-40}{1000}\left(\frac{5}{6}\right)(4k)V_S = -133.33V_S$$

$$\frac{V_0}{V_S} = -133.33$$

Problem 8.1

Given the network in Figure P8.1

(a) Find the equations for $v_a(t)$ and $v_b(t)$

(b) Find the equations for $v_c(t)$ and $v_d(t)$

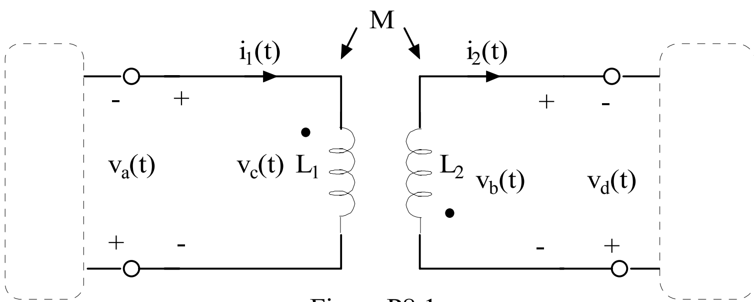


Figure P8.1

Suggested Solution

a)

$$v_a(t) = -L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$v_b(t) = -L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

b)

$$v_c(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_d(t) = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

Problem 8.2

Given the network in Figure P8.2 (a) write the equations for $v_a(t)$ and $v_b(t)$ and (b) write the equations for $v_c(t)$ and $v_d(t)$.

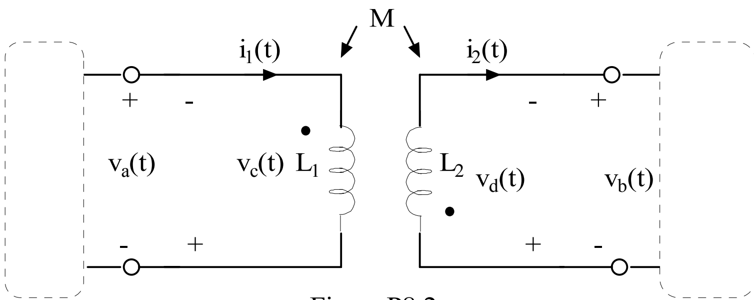


Figure P8.2

Suggested Solution

a)

$$v_a(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_b(t) = -L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

b)

$$v_c(t) = -v_a(t) = -L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_d(t) = -v_b(t) = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

Problem 8.3

Given the network in Figure P8.3 (a) write the equations for $v_a(t)$ and $v_b(t)$ and (b) write the equations for $v_c(t)$ and $v_d(t)$.

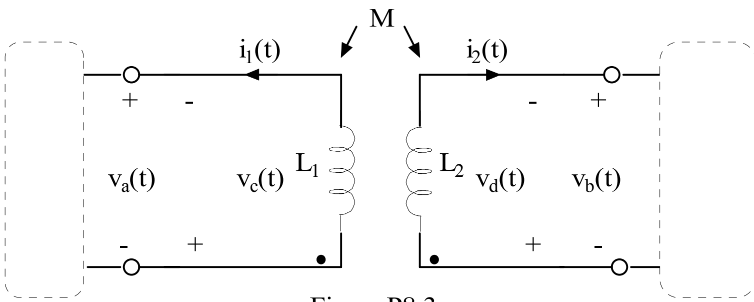


Figure P8.3

Suggested Solution

a)

$$v_a(t) = -L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_b(t) = -L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

b)

$$v_c(t) = -v_a(t) = -L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_d(t) = -v_b(t) = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

Problem 8.4

Given the network in Figure P8.4 (a) write the equations for $v_a(t)$ and $v_b(t)$ and (b) write the equations for $v_c(t)$ and $v_d(t)$.

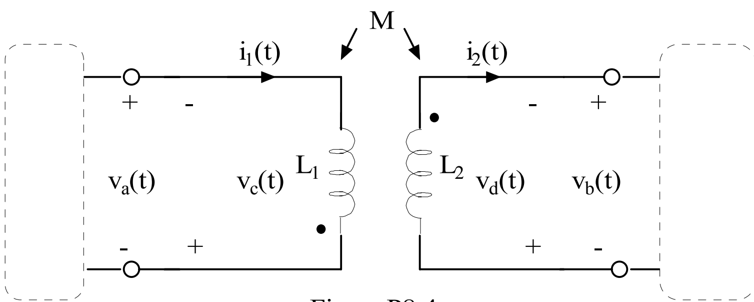


Figure P8.4

Suggested Solution

a)

$$v_a(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_b(t) = -L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

b)

$$v_c(t) = -v_a(t) = -L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$v_d(t) = -v_b(t) = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

Problem 8.5

Find the voltage gain Vo/Vs of the network shown in figure P8.5

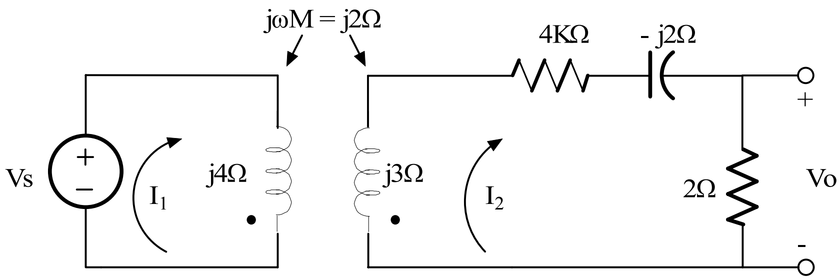
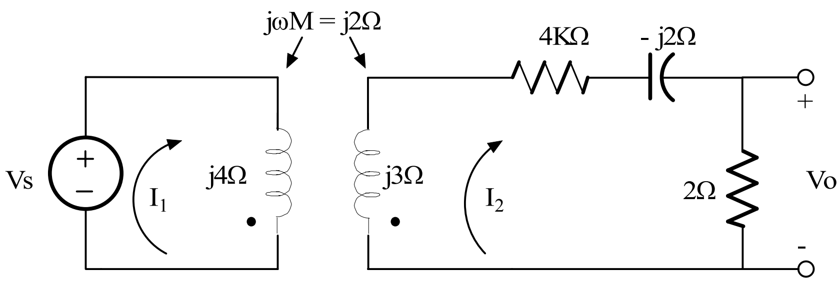


Figure P8.5

Suggested Solution



$$-Vs = 2(-I_1) + j4(-I_1) + j2I_2$$
$$Vs = (2 + j4)I_1 - j2I_2$$
$$0 = -j2I_1 + (6 + j1)I_2$$
$$Vo = 2I_2$$

$J2I1 = (6+1)I2$ or $I1 = \frac{6+1}{j2}(I2)$

then

$$Vs = (2 + j4)(\tfrac{6+1}{j2})I_1 - J2I_2 = (13 - j6)I_2$$

$I_2 = \frac{Vs}{3-j6}$ or $Vo = 2I_2 = \frac{2Vs}{13-j6} = \frac{Vo}{Vs} = \frac{2}{13-j6} = \boxed{0.140\angle 24.78°}$

Problem 8.6

Find the voltage gain Vo/Vs of the network shown in figure P8.6

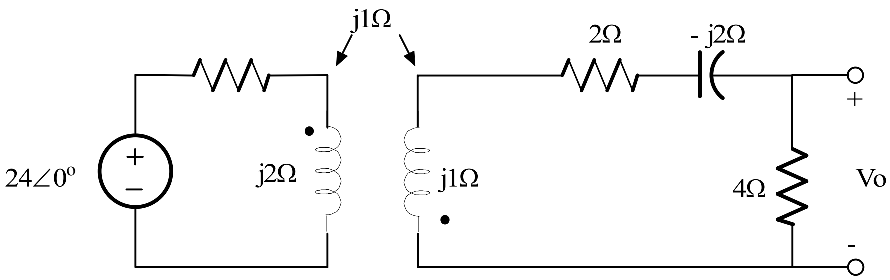
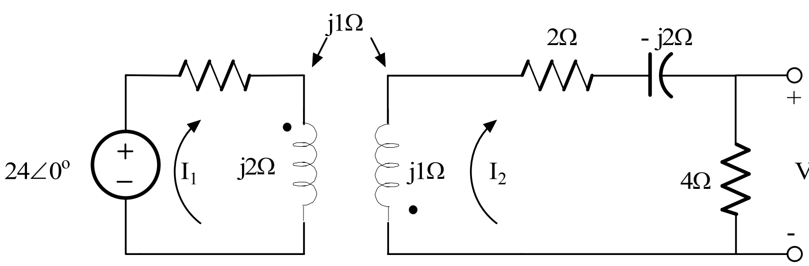


Figure P8.6

Suggested Solution



$24 = (2+j2)I_1 + J1\ I_2$
$0 = JI_1 + (6-j)I_2$
$Vo = 4I_2$

$-j\ I_1 = (6 - j)I_2$ or $I_1 = (1 + j6)I_2$
$24 = (2 + j2)(1 + j6)I2 + jI2$
$I_2 = 24 / (-10 + j5)$ $Vo = 4x24/(-10 + j15)$

$5.33\angle -123^{o}$ V

Problem 8.7

Find Vo in the network in Figure P8.7

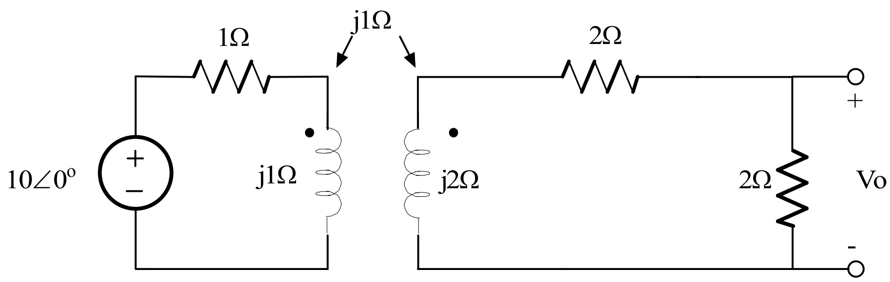
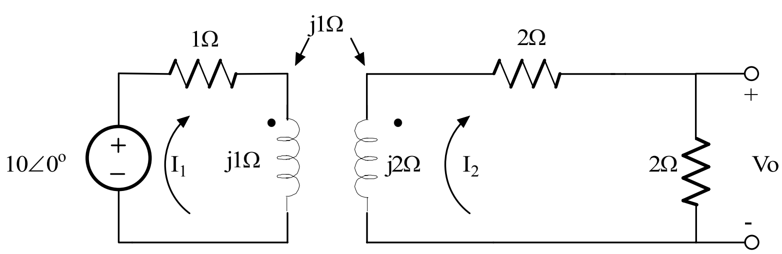


Figure P8.7

Suggested Solution



Loop equations

$10 = (1+j1)I_1 - j1\ I_2$ (1)

$0 = -jI_1 + (4 + 2j)I_2$ (2)

Solve (2) for I_1 and substitute into (1) and get I_2

$I_1 = I_2\ (4 + j2)\ /\ j1 = I_2\ (2 - j4)$

$10 = [(1 + j1)(2 - j4) - j1]\ I_2 = (2 + 4 + j2 - j4 - j1)I_2 = (6 - j3)\ I_2$

$I_2 = 10\ /\ (6 - j3)$ $\quad Vo = 2\ I_2 = 20\ /\ (6 - j3)$

$Vo = 2.98\angle 26.57^o$ V

Problem 8.8

Given the network in Figure P8.8 find Vo

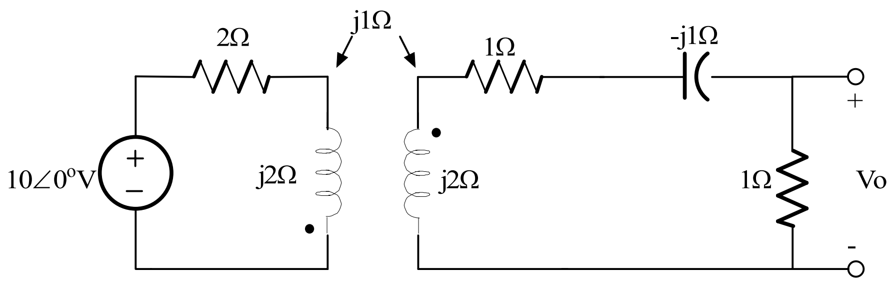
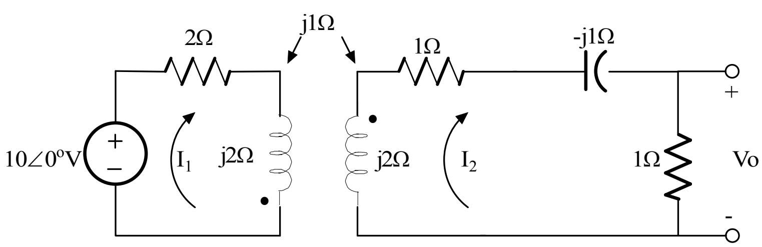


Figure P8.8

Suggested Solution



$10 = (2 + j2)I_1 + j1\ I_2$ (1)

$0 = j1I_1 + (2 + 2j)I_2$ (2)

Solve (2) for I_1 and substitute into (1) and get I_2

$I_1 = -\ I_2\ (2 + j1)\ /\ j1 = \ I_2\ (-1 + j2)$

$10 = [j1 - (-1 + j2)(2 + j2)]\ I_2 = (-2 - 4 - j2 + j4 + j1)I_2 = (-6 + j3)\ I_2$

$I_2 = 10\ /\ (-\ 6\ +\ j3) = 10\ /\ (-\ 6 + j3)$

$Vo = 1.49\angle{-153.43^o}$ V

Problem 8.9

Find Vo in the circuit in Figure P8.9

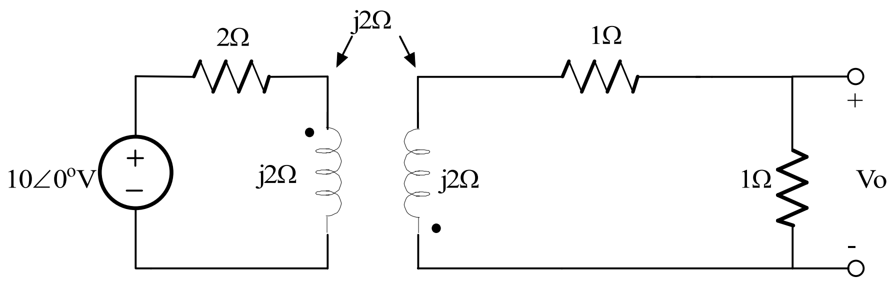
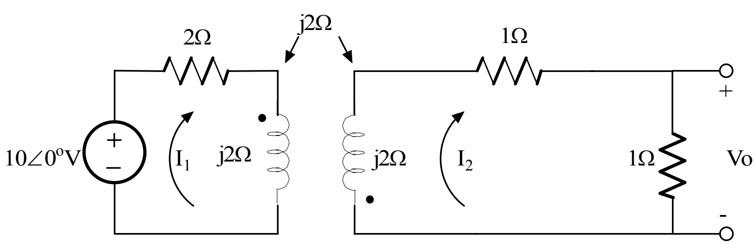


Figure 8.9

Suggested Solution



$10 = (2 + j2)I_1 + j2\ I_2$ (1)
$0 = j2I_1 + (2 + 2j)I_2$ (2)
Solve (2) for I_1 and substitute into (1) and get I_2

$I_1 = -\ I_2\ (2 + j2) / j2 = -\ I_2\ (2 - j1)$

$10 = [j2 - (1 - j1)(2 + j2)]\ I_2 = (j2 - 2 - 2 + j2 - j2)I_2 = (6 - j3)\ I_2$

$I_2 = 10 / (-4 + j2) = 5 / (-2 + j)$

$Vo = 2.24\angle{-153.43^o}$ V

Problem 8.10

Find Vo in the network in Figure P8.10

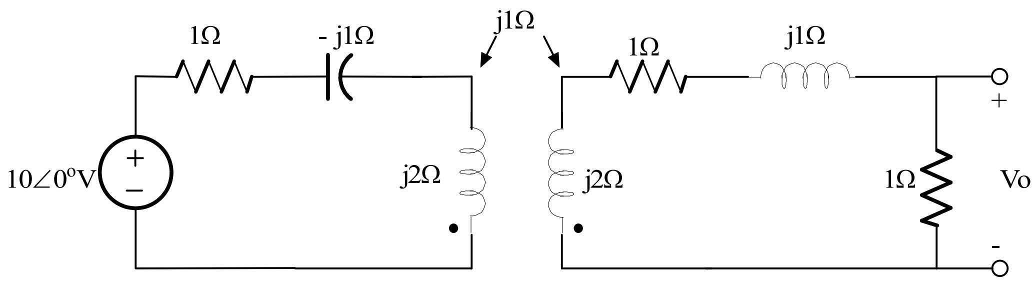
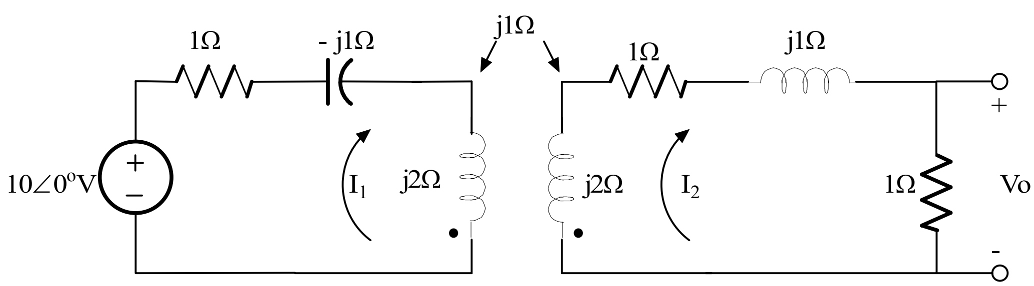


Figure P8.10

Suggested Solution



$10 = (1 + j1)I_1 - j1\ I_2$ (1)
$0 = - j1I_1 + (2 + j3)I_2$ (2)
Solve (2) for I_1 and substitute into (1) and get I_2

$I_1 = I_2\ (2 + j3) / j1 = I_2\ (3 - j2)$

$10 = [- j1 - (1 + j1)(3 - j2)]\ I_2 = (3 + 2 + j3 - j2 - j1)I_2 = (5)\ I_2$

$I_2 = 2A$

$Vo = (1)\ I_2 = 2\angle 0^o\ V$

Problem 8.11

Find Vo in the network in Figure P8.11

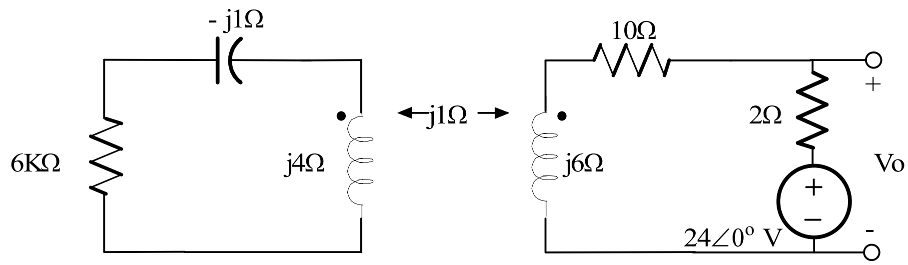
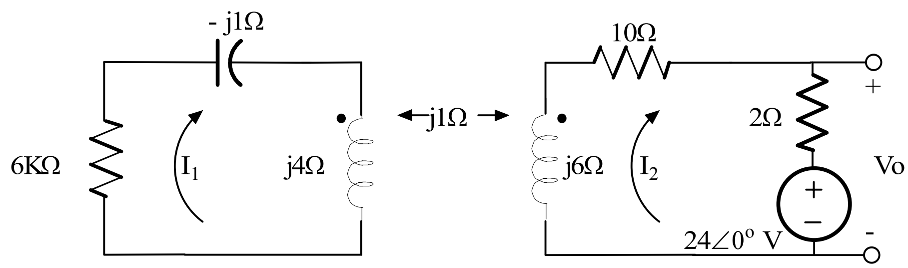


Figure 8.11

Suggested Solution



$(6 + j3)\ I_1 - jI_2 = 0$
$-jI_1 + (1_2 + j6)I_2 = -24$
$Vo = 2I_2 + 24$

$I_1 = jI_2/(6 + j3) \Rightarrow -j\ (\ j / (6 + j3)\ I_2 + (12 + j6)\ I_2 = -24$
$I_2 = -24(6 + j3) / (55 + j72) = -1.78\angle{-26.1^o}$

$Vo = 2\ I_2 + 24 = 20.86\angle 4.32^o$ V

Problem 8.12

Find Vo in the network in Figure P8.12

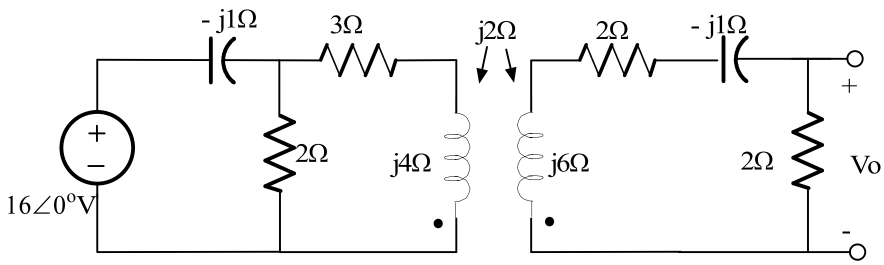
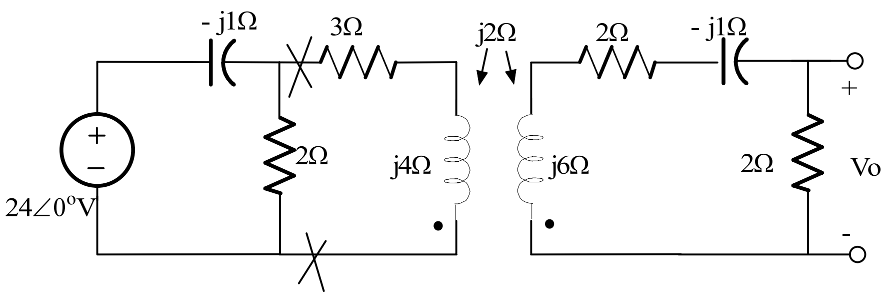


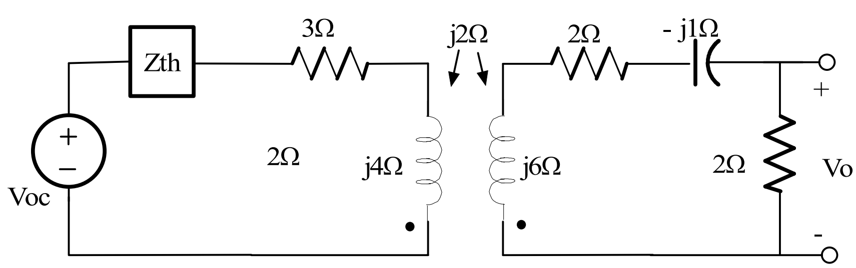
Figure P8.12

Suggested Solution



Forming a Thevenin equivalent at the input

$V_{oc} = 16(2)/(2 - j) = 32 / (2 - j)$ and $Z_{th} = -2j / (2 - j)$



$32 / (2 - j) = I_1 ((-2j) / (2 - j) + 3 + j4) - J2I_2$

$0 = -j2I_1 + (4 + 5j) I_2 \Rightarrow I_1 + (4 + 5j) I_2 \Rightarrow I_1 = [(4 + 5j) / J2] I_2$

$32 = [(30 + 3J)(4 + 5J) / J2 - (2 + j4)] I_2 \Rightarrow I_2 = 64j / (33 + j58)$

$V_o = 2I_2 = 1.92\angle 29.64^{o}V$

Problem 8.13

Find Vo in the circuit in Figure P8.13

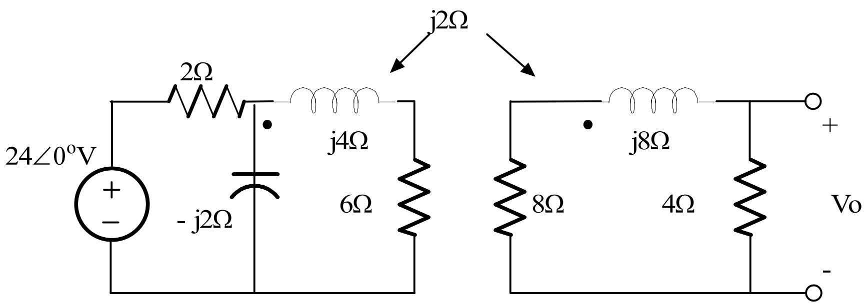
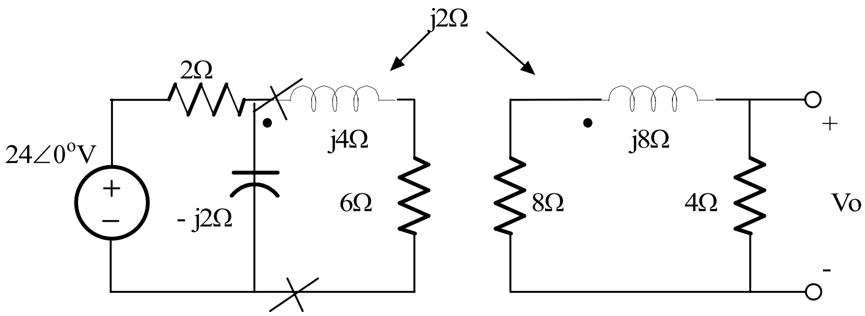


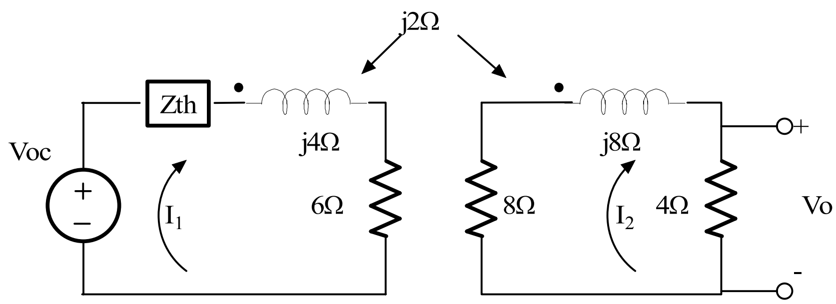
Figure P8.13

Suggested Solution



Thevenin equivalent at the input

Voc = 24(-j2) / (2 - j2) = - 24j (1 - j) and Zth = -4j / (2 - j2) = -2j / (1 - j)



$- 24j / (1 - j) = I_1 (-2j / (1 - j) + 6 + j4) + j2\ I_2$

$0 = j2\ I_1 + (12 + j8)\ I_2 \;\;=>\;\; I_1 = (-4 + j6)\ I_2$

$-24j = [(-4 + j6)(10-j4) + 2 + j2]\ I_2$

$I_2 = -24j / (-14 + j78) = 24j / (14 - j78)$

and

$Vo = 4I_2 = 1.21\angle 169.8^oV$

Problem 8.14

Find Io in the circuit in Figure 8.14

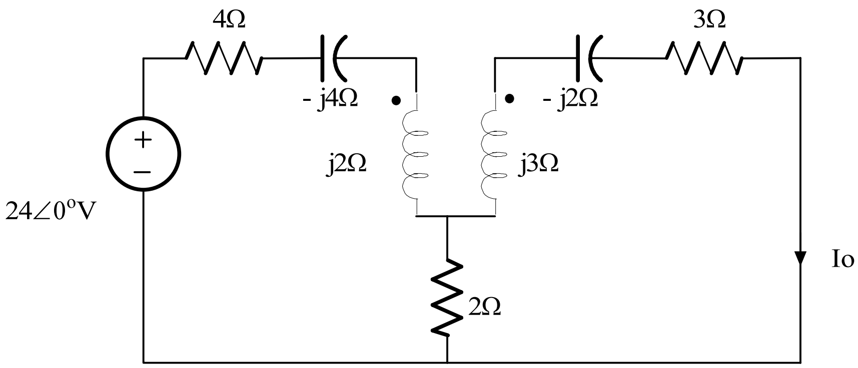


Figure P8.14

Suggested Solution

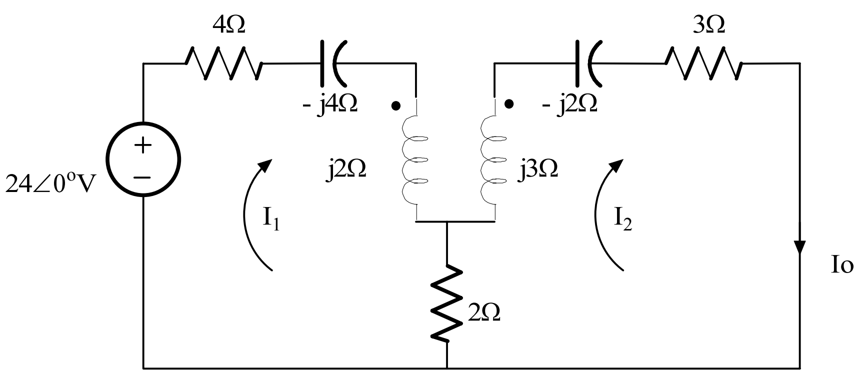


Figure P8.14

$24 = (6 - j2)\, I_1 - (2 + j1)\, I_2$

$0 = -(2 + j1)\, I_1 + (5 + j)\, I_2$

$I_1 = (5 + j) / (2 + j)\, I_2$

$24 = [\,(6 - j2)(5 + j) / (2 + j)\,]\, I_2 - (2 + j)\, I_2$

$I_2 = 1.78\angle 42^{o}A$

Problem 8.15

Write the mesh equations for the network in Figure P8.15

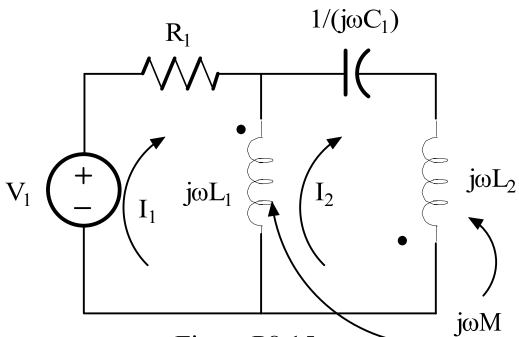
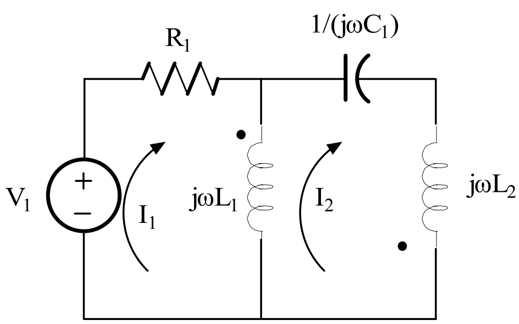


Figure P8.15

Suggested Solution

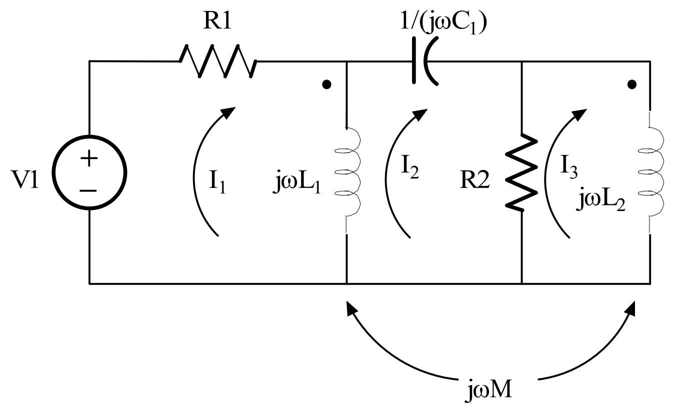


$V_1 = R_1\, I_1 + j\omega L_1(I_1 - I_2) - j\omega M I_2$

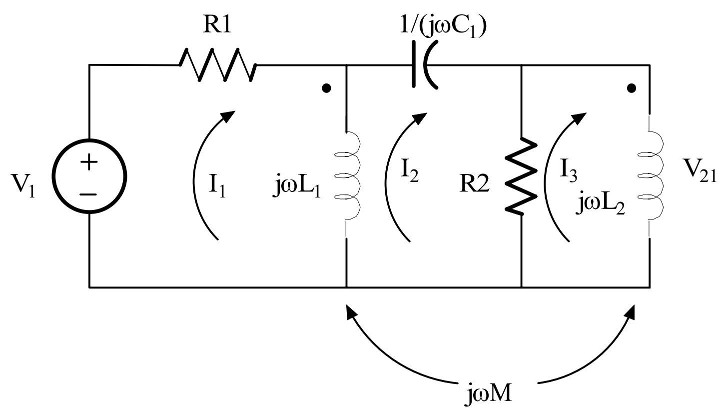
$0 = j\omega L_1(I_2 - I_1) + j\omega M I_2 + I_2(1/(j\omega C_1)) + j\omega C_2 I_2 + j\omega M(I_2 - I_1)$

Problem 8.16

Write the mesh equations for the network shown in Figure P8.16



Suggested Solution



Induced voltages are $V_{12} = +j\omega M I_3$ and $V_{21} = +j\omega M(I_1 - I_2)$

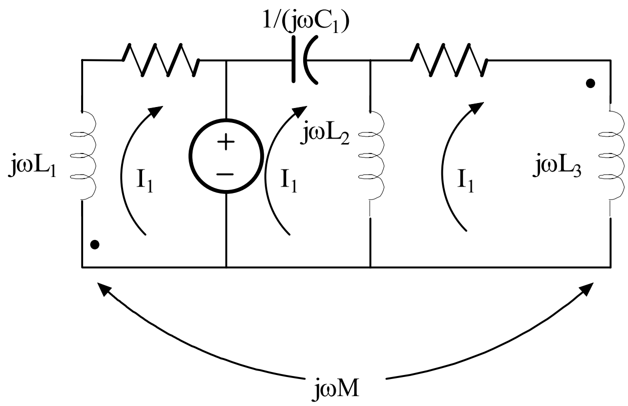
$V_1 = I_1(R_1 + j\omega L_1) - j\omega L_1 I_2 + I_3\, j\omega M$

$0 = -j\omega L_1 I_1 + I_2\,(R_2 + j\omega L_1 - j/(\omega C_1)) - I_3(R2 + j\omega M)$

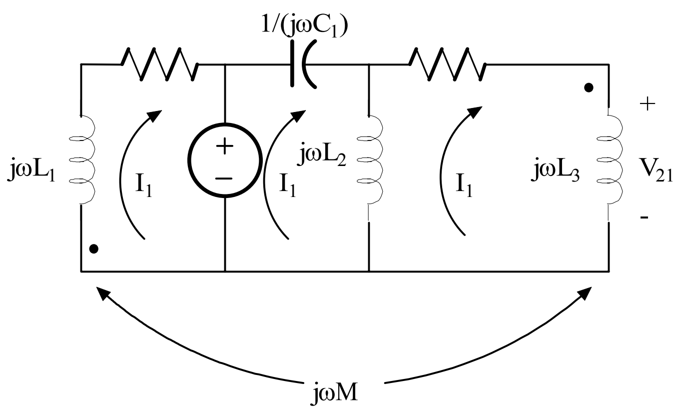
$0 = j\omega M I_1 - (R_2 + j\omega M)I_2 + I_3\,(R_2 + j\omega L_2)$

Problem 8.17

Write the mesh equations for the network shown in Figure P8.17



Suggested Solution



Induced voltages are $V_{12} = - j\omega MI_3$ and $V_{21} = + j\omega MI_1$

$- V_1 = I_1(R_1 + j\omega L_1) - j\omega MI_3$

$0 = I_2 (j\omega L_2 - j / (\omega C_1)) - j\omega L_2 I_3$

$0 = + j\omega MI_1 - j\omega L_2 I_2 + I_3 (R_2 + j\omega L_2 + j\omega L_3)$

Problem 8.18

Write the mesh equations for the network shown in Figure P8.18

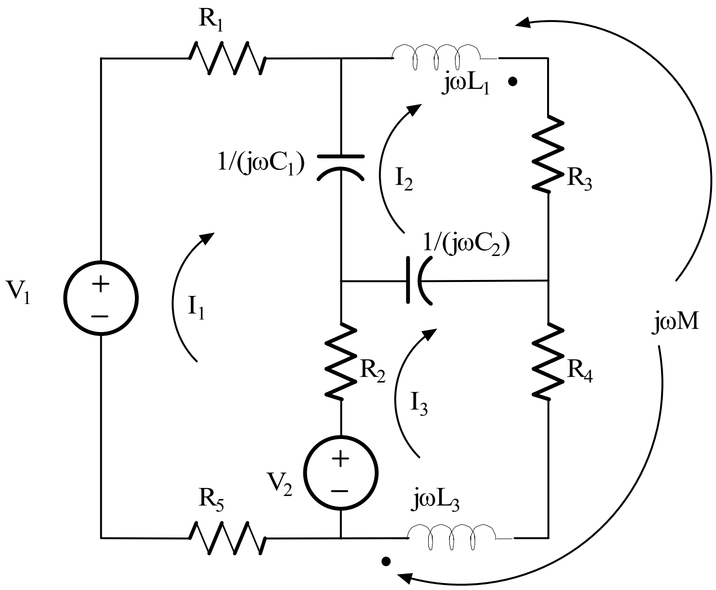
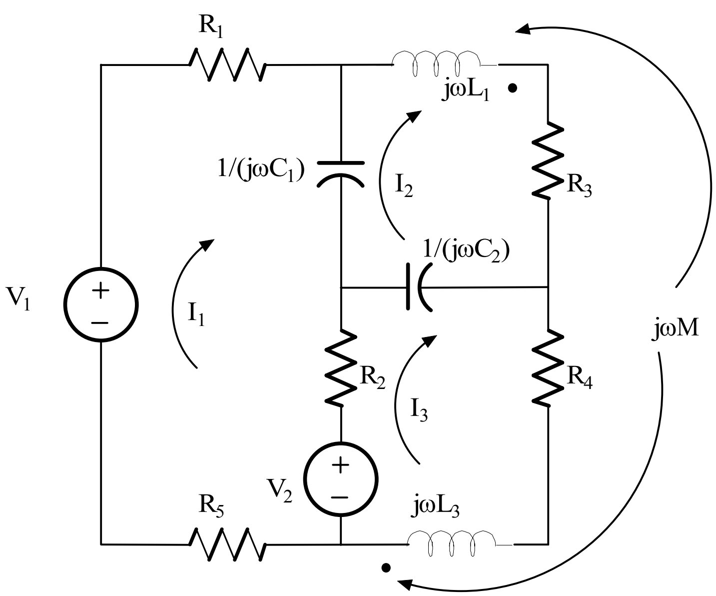


Figure P8.18

Suggested Solution



$V_1 - V_2 = I_1 (R_1 + R_2 + R_5 + 1/(j\omega C_1)) - 1/(j\omega C_1)I_2 - R_2 I_3$

$0 = - 1/(j\omega C_1) I_1 + I_2 (R_3 + j\omega L_1 + 1/(j\omega C_1) + 1/(j\omega C_2)) + I_3 (j\omega M - 1/(j\omega C_2))$

$V_2 = - R_2 I_1 + I_2 (j\omega M - 1/(j\omega C_2) + I_3 (R_2 + R_4 + j\omega L_3 + 1/(j\omega C_2))$

Problem 8.19

Write the mesh equations for the network shown in Figure P8.19

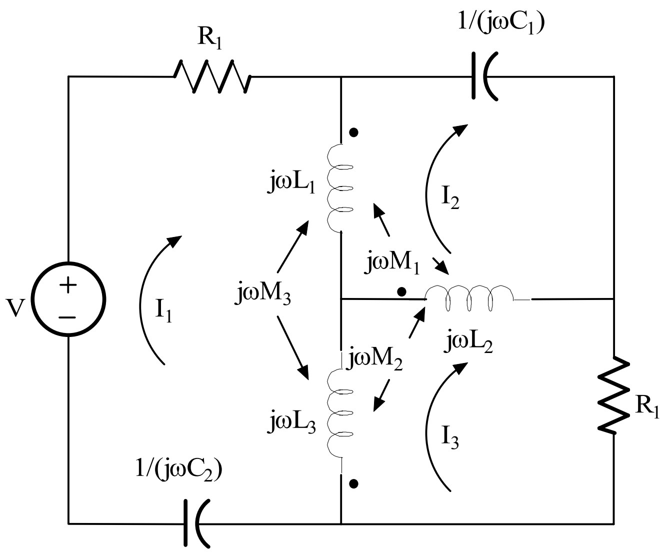
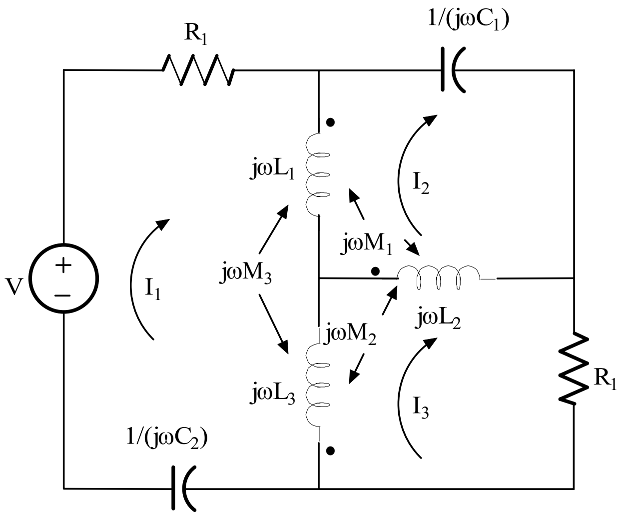


Figure P8.19

Suggested Solution



Induced voltages (positive @ dot)

IN	VIA	EXPRESION	NAME
L_1	M_1	$j\omega M_1(I_3 - I_2)$	V_{11}
L_1	M_3	$j\omega M_1(I_3 - I_1)$	V_{13}
L_2	M_1	$j\omega M_1(I_1 - I_2)$	V_{21}
L_2	M_2	$j\omega M_1(I_3 - I_1)$	V_{22}
L_3	M_2	$j\omega M_1(I_3 - I_2)$	V_{32}
L_3	M_3	$j\omega M_1(I_1 - I_2)$	V_{33}

Mesh equations

$V = I_1\ (R_1 + j\omega L_1 + j\omega L_3 + 1/(j\omega C_2)) - I2j\omega L_1 - I3j\omega L_3 + V_{11} + V_{13} - V_{32} - V_{33}$

$0 = - I_1 j\omega L_1 + I_2(j\omega L_1 + j\omega L_2 + 1/(j\omega C_1)) - I_3 j\omega L_2 + -V_{11} - V_{13} - V_{21} - V_{22}$

$0 = - I_1 j\omega L_1 + I_2(j\omega L_2 + I_3(R_2 + j\omega L_2 + j\omega L_3\ ) + V_{21} + V_{22} + V_{32} + V_{33}$

Let $Z_1 = R_1 + j\omega L_2 + j\omega L_3 + 1/(j\omega C_2)$; $Z2 = j\omega L_1 + j\omega L_2 + (1/j\omega C_1)$; $Z_3 = R_2 + j\omega L_2 + j\omega L_3$

Substituting from the table above:

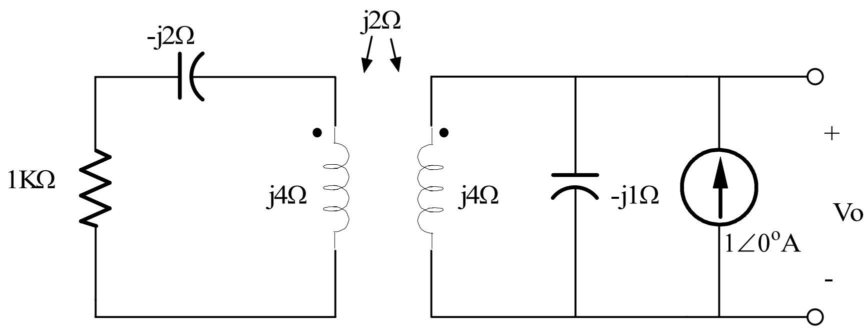
$V = I_1(Z_1 - j\omega ZM_3 + I_2[j\omega M(M_2 + M_3 - M_1 - L_1)] + I_3[j\omega(M_1 + - M_2 + M_3 - L_2)]$

$0 = I_1[\ j\omega(M_3 - M_1 + M_2 - L_1)] + I_2[Z_2 + j\omega ZM_1\ ) - I_3[j\omega(M_1 + M_2 + M_3 + L_2)]$

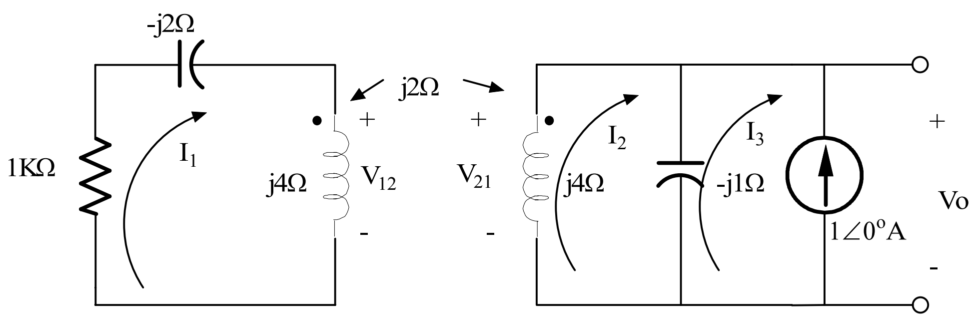
$0 = I_1[\ j\omega(M_1 - M_2 + M_3 - L_2)] - I_2[\ j\omega(M_1 + M_2 + M_3 + L_2)] + I_3(Z_3 + j\omega 2M_{2)}$

Problem 8.20

Find Vo in the network in Figure P8.20



Suggested Solution



Induced voltages: $V_{12} = -j2I_2 \quad V_{21} = +j2I_1$

$0 = I_1\,(4 + j2) - j2I_2$ and $0 = I_2(j3) + j1I_3 - j2I_1$ and $I_3 = -1\angle 0^o A$

From 1st equation: $I_1 = I_2[\,1 / (1 - j2)\,]$

Now: $= 0\ I_2[j3 - j2\,(1 - j2)\,] + j1I_3$ and $I_3 = -1$

or: $I_2\,[\,(6 + j3 - j2) / (1 - j2)\,] = j1$ or $I_2 = (2 + j1) / (6 + j1) = 0.37\angle 17.10^o A$

$Vo = (I_2 - I_3)\,(-j1) = [\,0.37\angle 17.10^o + 1\angle 0^o]\,(1\angle -90^o)$

$Vo = 1.36\angle -85.4^o V$

Problem 8.21

Find Vo in the network in Figure P8.21

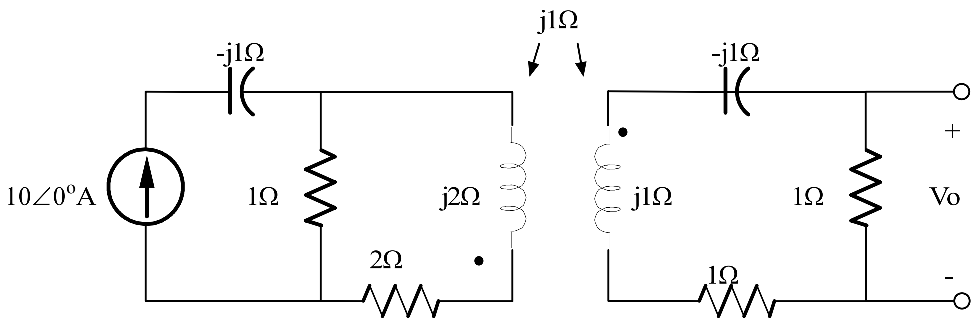
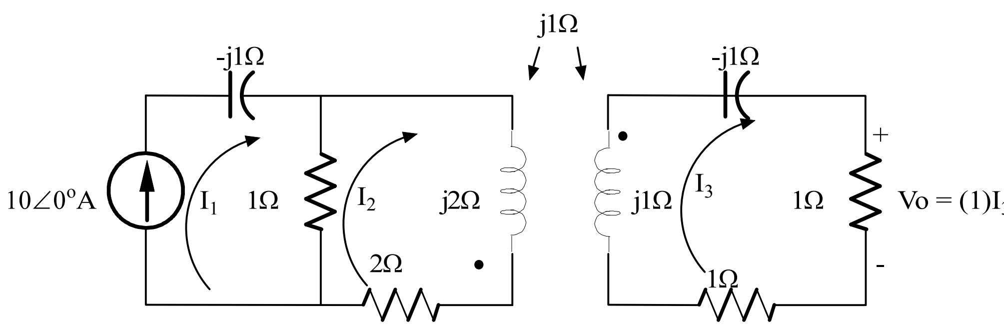


Figure P8.21

Suggested Solution



$I_1 = 10\angle 0^o A$ (1)

$0 = - I_1 + I_2(3 + j2) + J1I_3$ (2)

$0 - +j1I_2 + I_3(2)$ (3)

Solve (3) for I_2 and substitute into (2) along with (1).

I2 = (j2)I3

$0 = -10 + [(3 + j2)(j2) + j1]I_3 \Rightarrow I_3 = 10 / (-4 + j7) = 1.24\angle -119.74^o A$

$Vo = (1)\ I_3 = 1.24\angle -110.74^o V$

Problem 8.22

Find Vo in the network in Figure P8.22

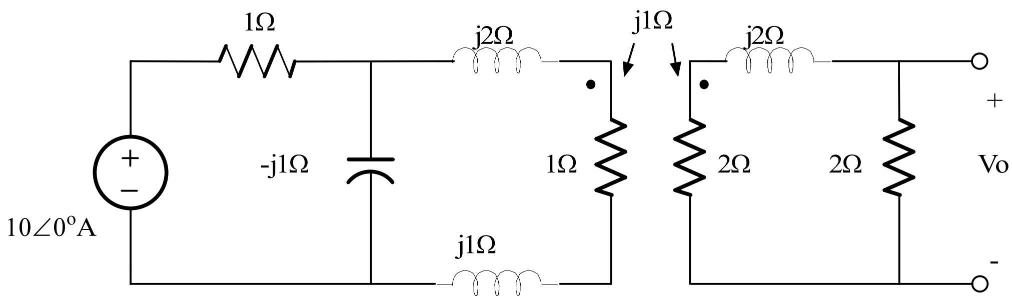
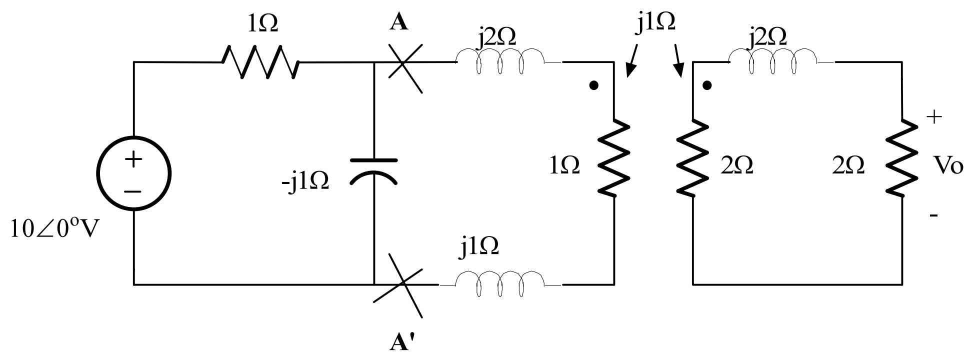
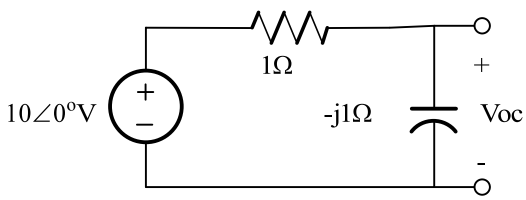


Figure P8.22

Suggested Solution

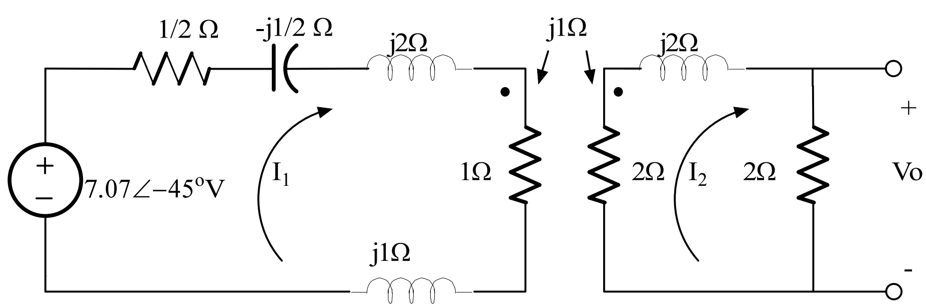


Thevenin Eq. at A - A'



$Voc = (- j1)/(1 - j1)\ (10) = 7.07\angle{-45^o}V$

$Zth = 1 \ || \ (-j1) = 1/2 - j1/2\ \Omega$



$7.07\angle{-45^o} = I_1\ (1.5 + j2.5) - J1\ I_2$ (1)

$0 = - j1\ I_1 + I_2\ (4 + j2)$ (2)

Solve (2) for I_1 and substitute into (1), solve for I_2

$7.07\angle{-45^o} = [\ (2 - j4)\ (1.5 + j2.5) - j1)]\ I_2 = [13 - j2]\ I_2$

$I_2 = (7.07\angle{-45^o}\ ) / (13 - j2) = 7.54\angle{-36.25^o}A$

$Vo = 2I2 = 1.08\angle{-36.25^o}V$

Problem 8.23

Find Vo in the network in Figure P8.23

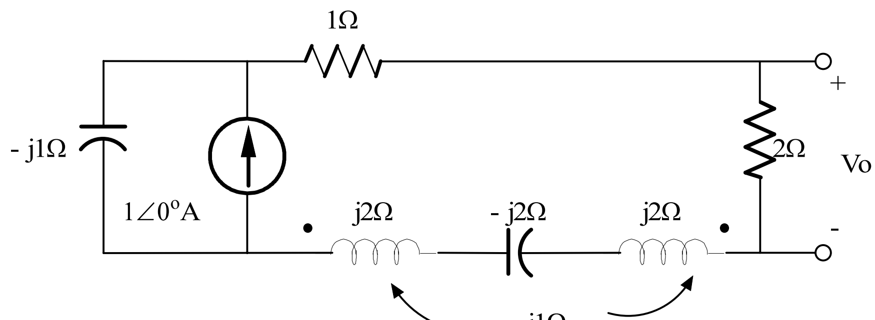
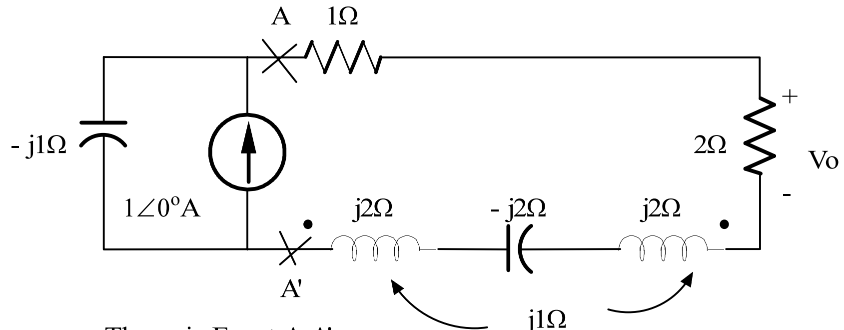
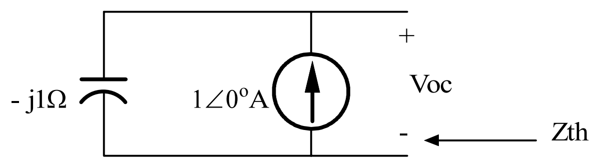


Figure P8.23

Suggested Solution



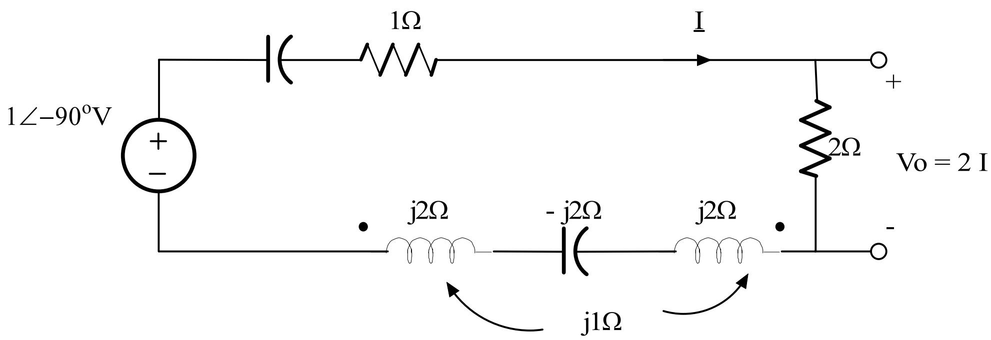
Thevenin Eq. at A-A'



$Voc = (-j1)\ (1) = 1\angle -90^o V$

$Zth = -j\ 1\Omega$

New Circuit



$1\angle -90^o V = I\ (3 - j1 + j2 - j2 + j2) - j1\ I - j1\ I$ (induced)

$I = 1\angle -90^o / (3 - j1) = 0.32\angle -71.57^o A$

$$Vo = 2I = 0.64\angle -71.57^o V$$

Problem 8.24

Find Vo in the network in Figure P8.24

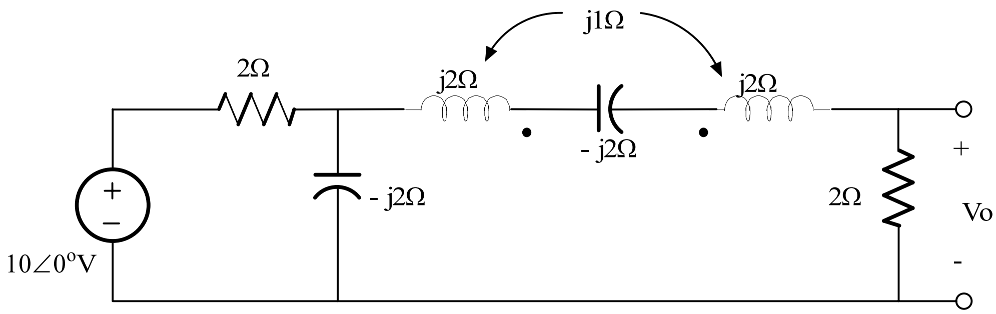
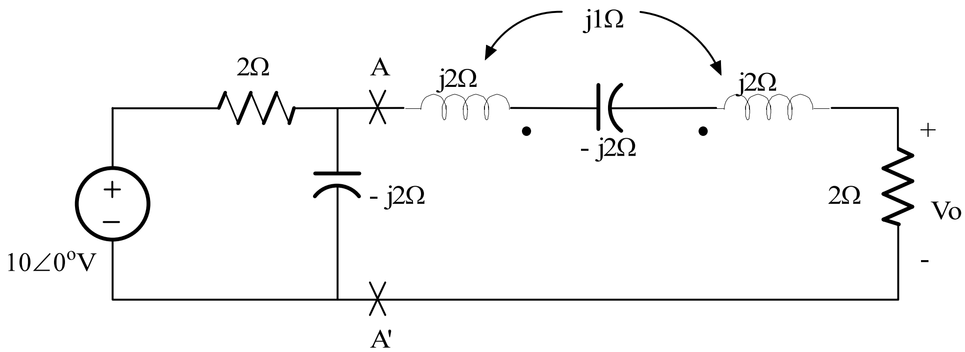
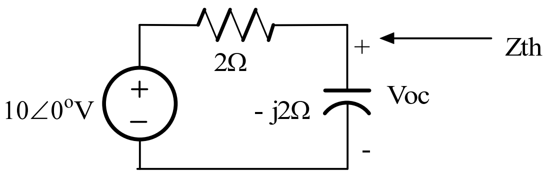


Figure P8.24

Suggested Solution



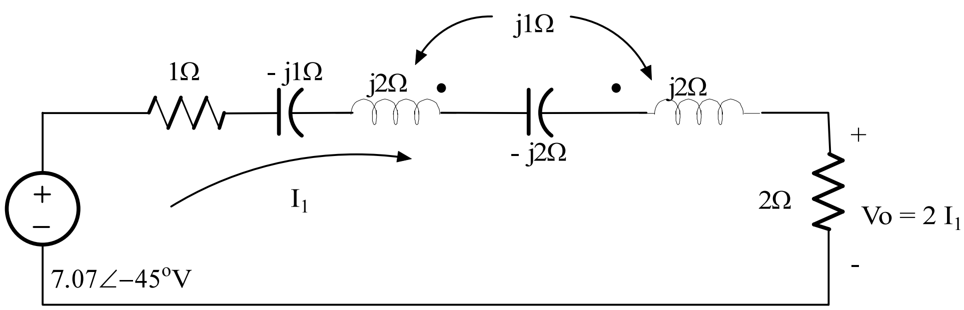
Thevenin Eq. at A - A'



$Voc = 10 (- j2) / (2 - j2) = 7.07\angle -45^o V$

$Zth = 2 \,||\, (- j2) = 1 - j1\Omega$

New Circuit



$7.07\angle -45^o = I_1 (1 - j1 + j2 - j2 + j2 + 2) - j1\ I_1 - j1\ I_1$
(induced voltages)

$I1 = (7.07\angle -45^o) / (3 - j1) = 2.24\angle -26.57^o A$

$$Vo = 2I_1 = 4.48\angle -26.57^o$$

Problem 8.25

Find Vo in the network in Figure P8.25

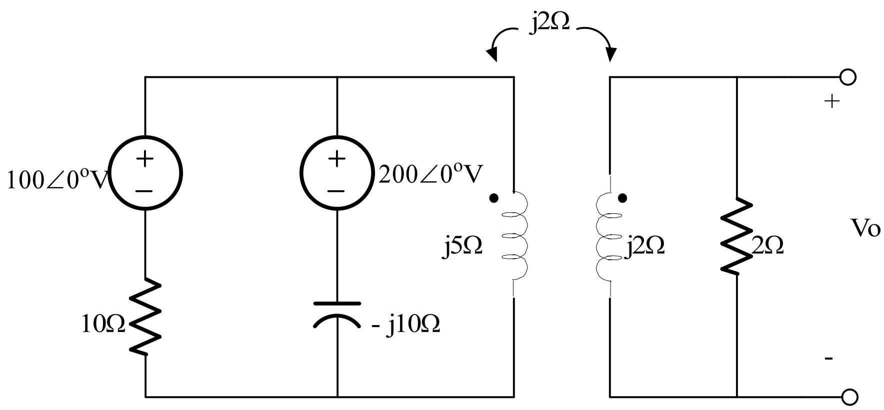
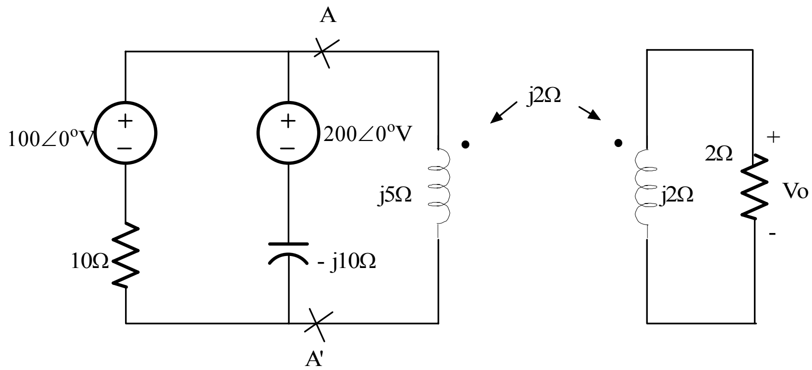
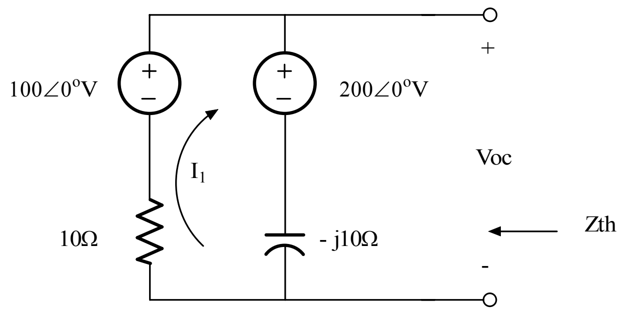


Figure P8.25

Suggested Solution



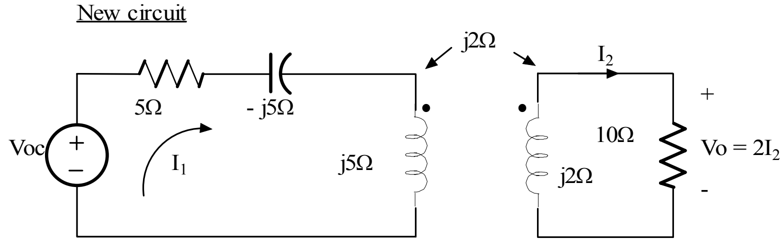


$I = (100 - 200) / (10 - j10) = 7.07\angle{-135^o}A$

$Voc = 200 - j10\,I = 150 + j50\ V$

$Zth = 10 \,||\, (-j10) = 5 - j5\ \Omega$

New circuit



$Voc = I_1\,(5 - j5 + j5) - j2\,I_2$ (1)

$0 = - j2I_1 + I_2\,(2 + j2)$ (2)

Solve (2) for I_1 and substitute into (1) to get I_2

$I_1 = I_2\,(1 - j1)$

$Voc = [(1 - j1)5 - j2]\ I2 = (5 - j7)\ I_2$

$I_2 = (150 + j50) / (5 - j7) = 18.38\angle 72.90^o A$

$Vo = 2\ I_2 = 36.76\angle 72.90^o V$

Problem 8.26

Find Vo in the network in Figure P8.26

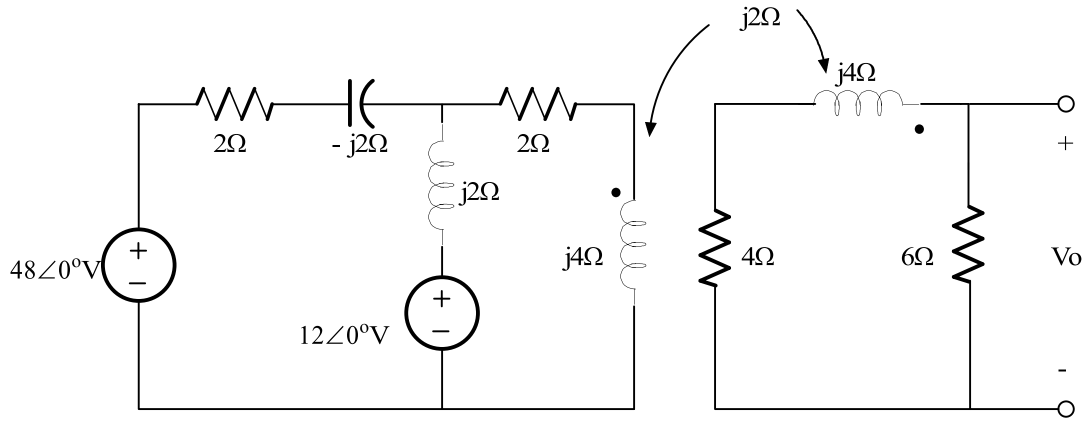
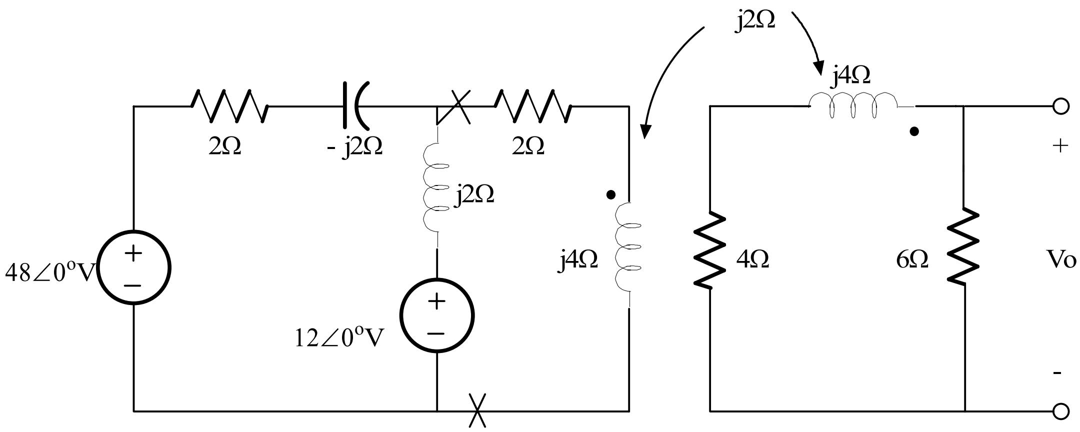


Figure P8.26

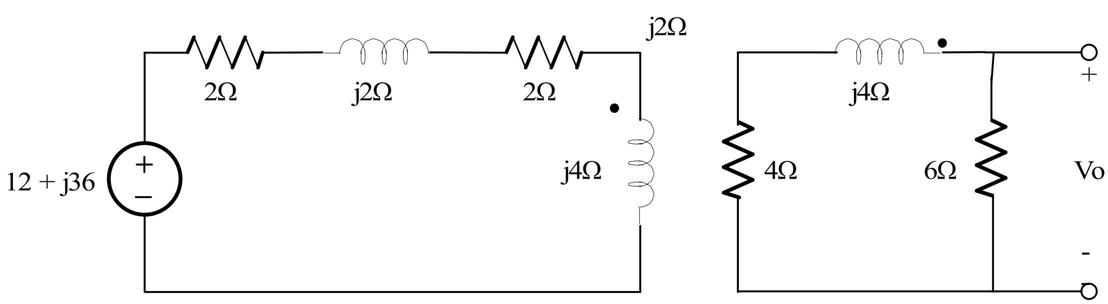
Suggested Solution



Thevenin equivalent at the input

Voc = [(48 - 12) / (2 - j2 + j2)] (j2 + 12) = 12 + j36

Zth = (2 - j2) (j2) /2 = 2 + j2



12 + j36 = (4 + j6)I1 - j2 I2
0 = -j2I1 + (10 + j4)I2
I1 = [(10 + j4) / j2] I2 = (2 - j5) I2
12 + j6 = [(4 + j6)(2 - j15) - j2 } I2
I2 = (12 + j36) / (38 - j10)

Vo = 6I2 = 6 (12 + j36) / (38 -j10)

V = 5.79∠86.31°V

Problem 8.27

Find Vo in the network in Figure P8.27

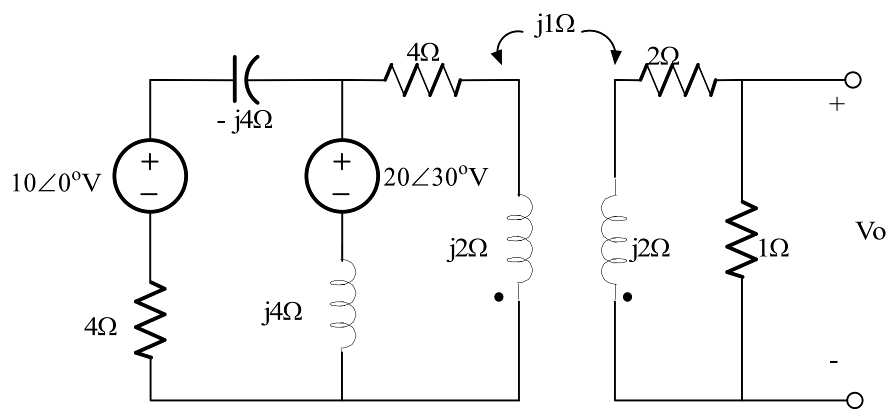
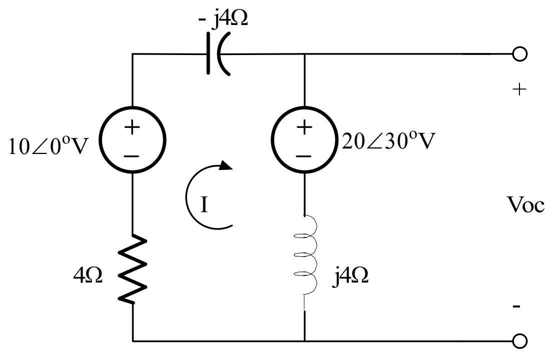


Figure P8.27

Suggested Solution

Thevenin Eq.

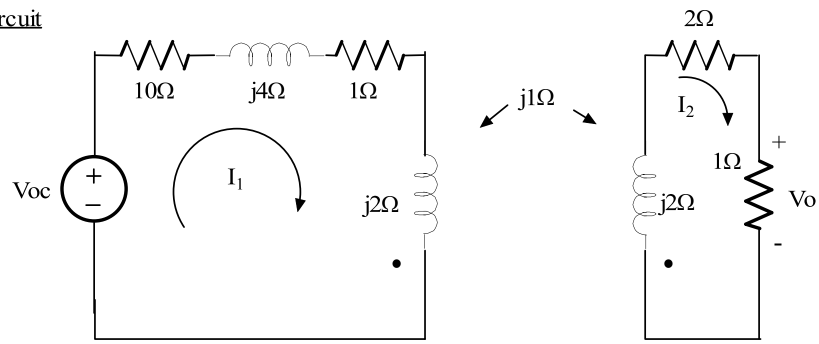


$I = (10\angle 0^o - 20\angle 30^o) / 4 = 12.39/4 \angle{-126.21^o}$

$Voc = 20\angle 30^o \;\; j4I = 27.45\angle 5.60^o$

$Zth = j4 \,||\, (4 - j4) = 4 + j4\ \Omega$

New circuit



$Voc = I_1\,(5 + j6) - j1\, I_2$ and $0 = (-j1)I_1 + I_2\,(3 + j2)$ (1) and (2)

Solve (2) for I_1, substitute into (1) to get I_2 and Vo

$I_1 = I_2\,(2 - j3) \;\; \Rightarrow \;\; Voc = [\,(2 - j3)(5 + j6) - j1]\, I_2$

$I_2 = Voc / (28 - j4) = 0.97\angle 13.73^o$

$\boxed{Vo = 0.97\angle 13.73^o V}$

Problem 8.28

Find Vo in the network in Figure P8.28

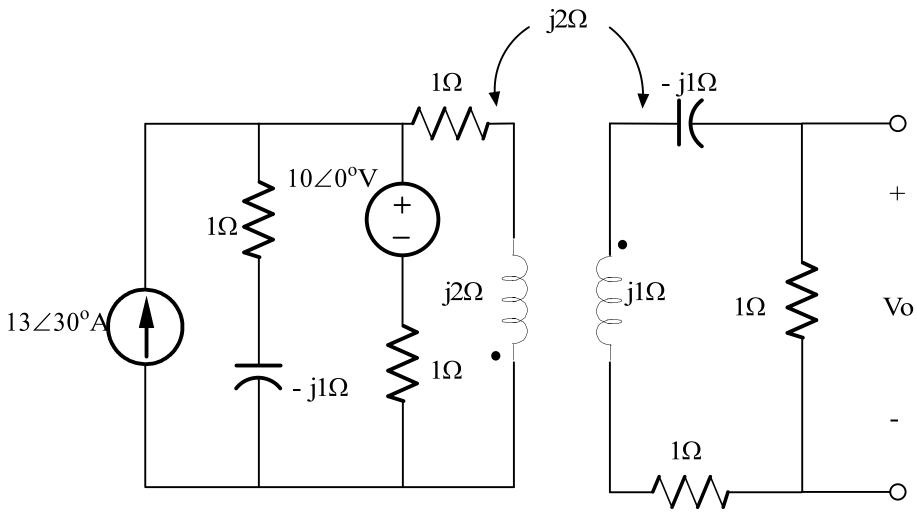
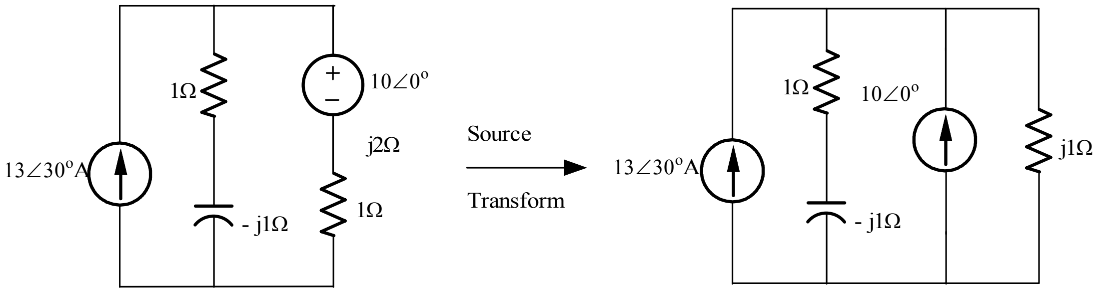
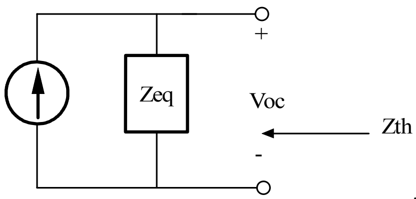


Figure P8.28

Suggested Solution



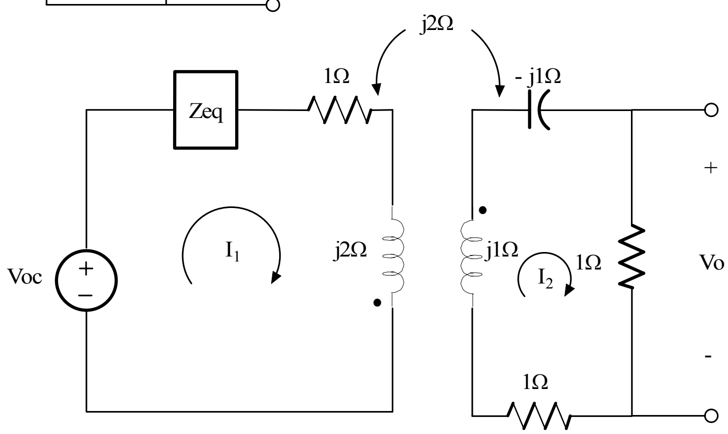


$Ieq = 10\angle 30^o + 10\angle 0^o = 19.32\angle 15^o$

$Ieq = (1 - j1) \parallel 1 = (1 - j1) / (2 - j1)\ \Omega$

$Voc = Ieq \quad Zeq = 12.22\angle -3.43^o V$

$Zth = Zeq$



$Voc = I_1 (Zth + 1 + j2) + j1\ I_2$ and $0 = jI_1 + 2I_2$

$I_1 = j2\ I_2$ and $Voc = [j2\ (Zth + 1 + j2) + j1]\ I_2$

$I_2 = [Voc / (-3 + j12)\ ](2 - j1)$ and $I_2 = 2.2\angle 134.1^o A$ and $Vo = 2.21\angle -134.1^o V$

$\boxed{Vo = 2.21\angle -134.1^o V}$

Problem 8.29

Find Vo in the network in Figure P8.29

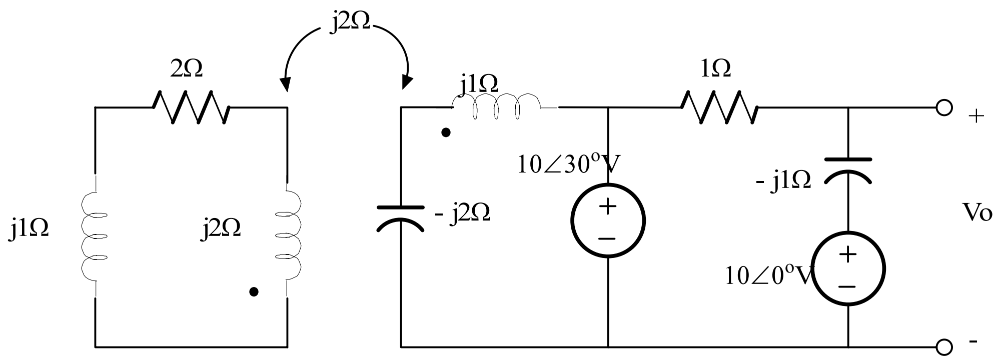
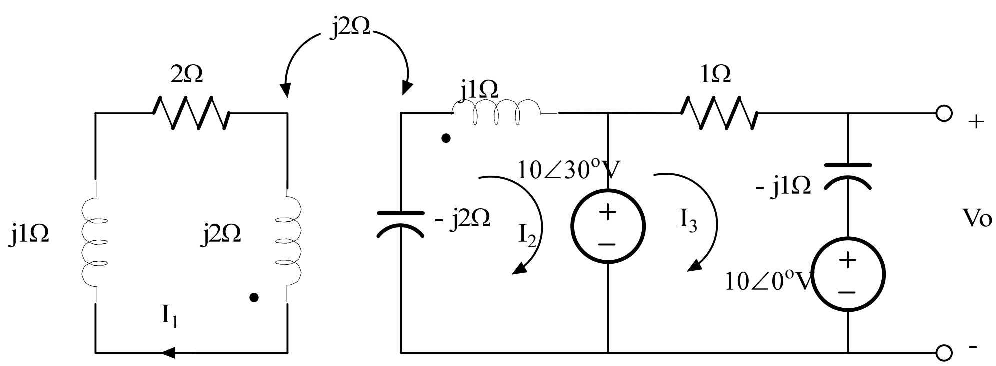


Figure P8.29

Suggested Solution



$0 = I_1 (2 + j3) - j1\ I_2$ (1)

$-10\angle 30^o = - j1\ I_1 + I_2(-j1)$ (2)

$10\angle 30^o - 10\angle 0^o = I_3\ (1 - j1)$ (3)

$Vo = (-j1)I_3 + 10\angle 0^o$ (4)

From (3) and (4)

we see that

Vo is independent of I1 and I2 !

From (3), $I_3 = (10\angle 30^o - 10\angle 0^o) / ((\sqrt{2})\angle -45^o) = 3.66\angle 150^o$ A

From (4), $Vo = 3.66\angle 60^o - 10\angle 0^o$

$Vo = 8.76\angle 158.80^o V$

Problem 8.30

Find Io in the circuit in Figure P8.30

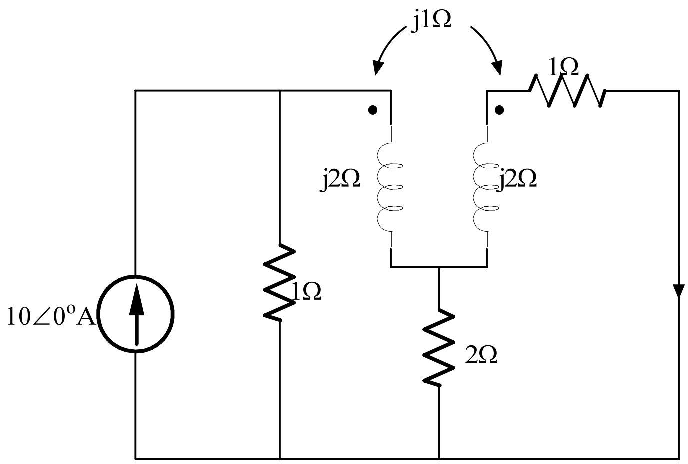
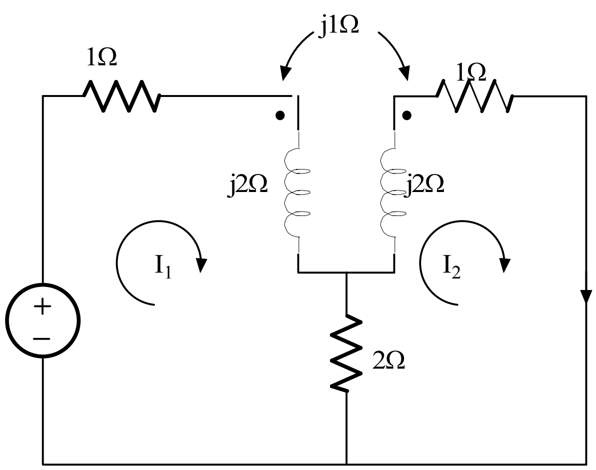


Figure P8.30

Suggested Solution



$10 = I_1(3 - j2) - J1I_2 - 2I_2$ (1)

$0 = I_2(3 + j2) - j\,1I_1 - 2I_1$ (2)

Solve (2) for I_1

$I_1 = I_2(3 + j2) / (2 + j1)$ (3)

Substitute (3) into (1) to find I_2

$10 = [\ (3 + j2)(3 + j2) / (2 + j1) - 2 - j1]\ I_2 = I_2\ [\ (9 - 4 + j12 - (2 + j1)(2 + j1)\ ) / (2 + j1)]$

$I_2 = Io = 10\ (2 + j1) / (2 + j8)$

$I_2 = Io = 2.71\angle -49.40^{o}$ A

Problem 8.31

Find Vo in the circuit in Figure P8.31

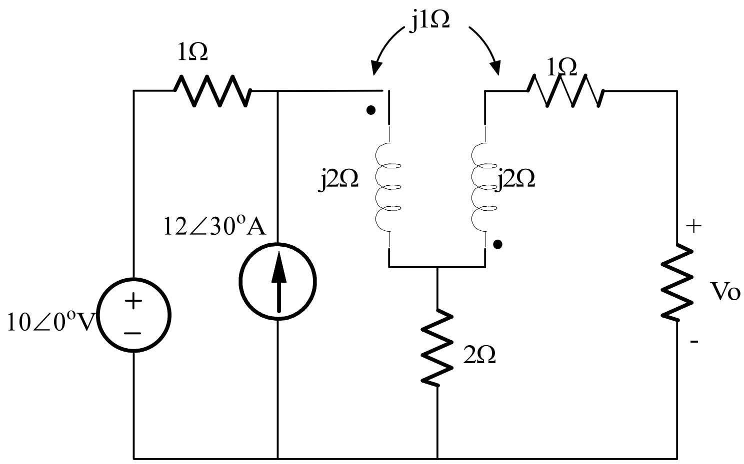
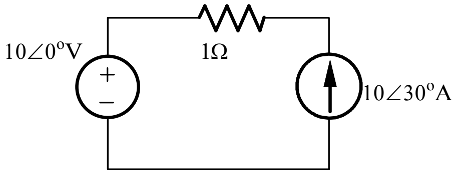


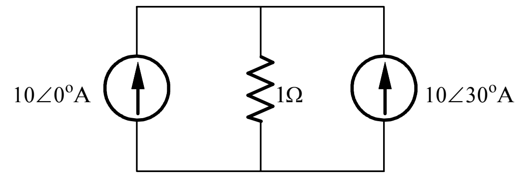
Figure P8.31

Suggested Solution

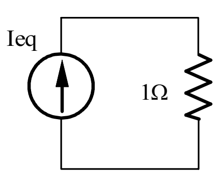
Try to simplify the left side of the circuit



Source transform



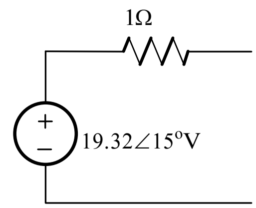
Add current sources



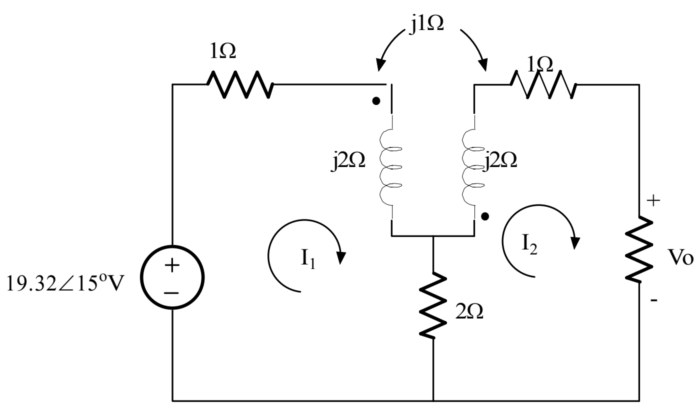
$Ieq = 10\angle 0^o + 10\angle 30^o$

$Ieq = 19.32\angle 15^o A$

Source transform



New Circuit



$Vs = I_1 (2 + j2) + j1\ I_2 - I_2$ (1)
$0 = j1\ I_1 - I_1 + I_2 (3 + 1)$ (2)

Solve (2) for I_1, substitute into (1) to get I_2 and Vo

$I_1 = I_2 (3 + j1) / (1 - j1) \Rightarrow Vs = [(2 + j2) (3 + j1) / (1 - j1) + j1 - 1] I_2$

$Vs = I_2 [j2(3 + j1) + j1 - 1] = I_2 [-3 + j7]$

$I_2 = 2.54\angle -98.20^o$ A

Problem 8.32

Find Io in the circuit in Figure P8.32

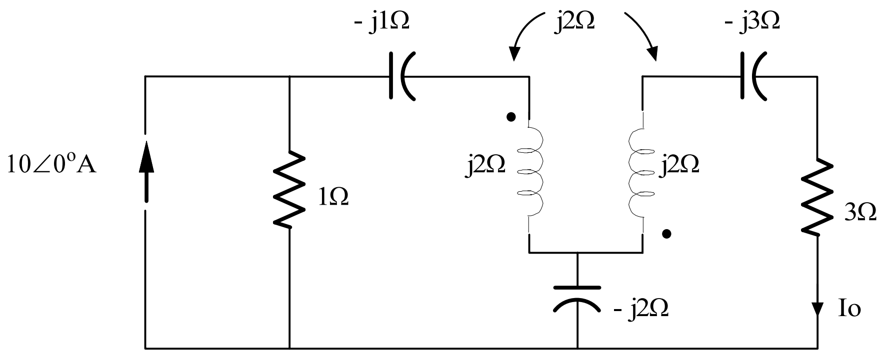
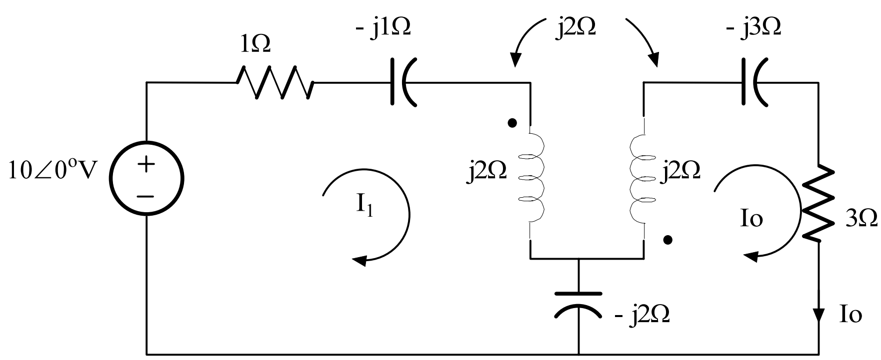


Figure P8.32

Suggested Solution



$10 = I_1\ (1 - j1) + Io\ (j2 + j1)$ (1)

$0 = I_1\ (j2 + j1) + Io\ (3 - j3)$ (2)

Solve (2) form Io substitute inti (1) to get Io

$I_1 = Io\ (1 + j1) \Rightarrow 10 = Io\ [\ (1 - j1)(1 + j1) + j3] = Io\ (2 + j3)$

$Io = 10 / (2 + j3)$

$Io = 2.78\angle{-56.31}^{o}A$

Problem 8.33

Find Vo in the network in Figure P8.33

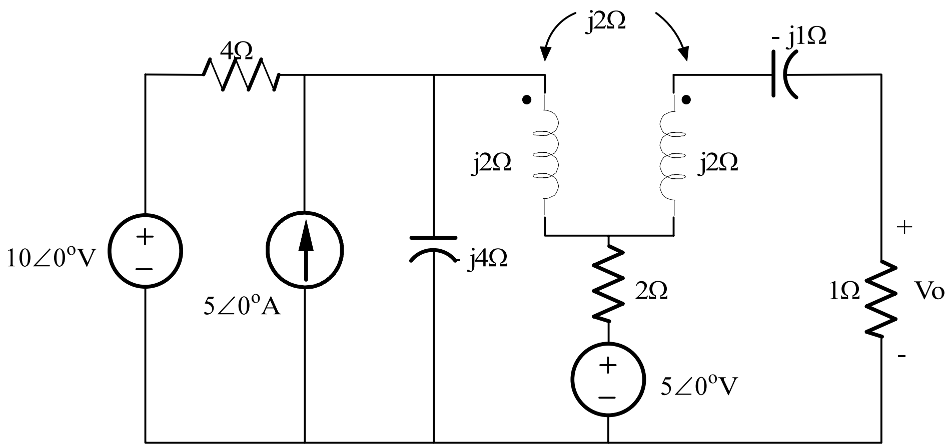
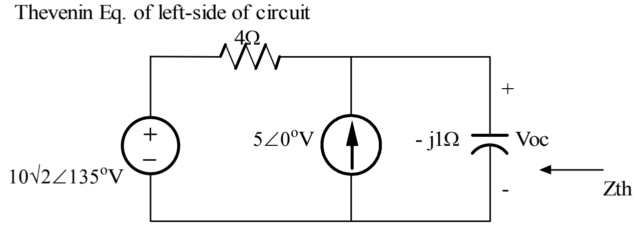


Figure P8.33

Suggested Solution

Thevenin Eq. of left-side of circuit



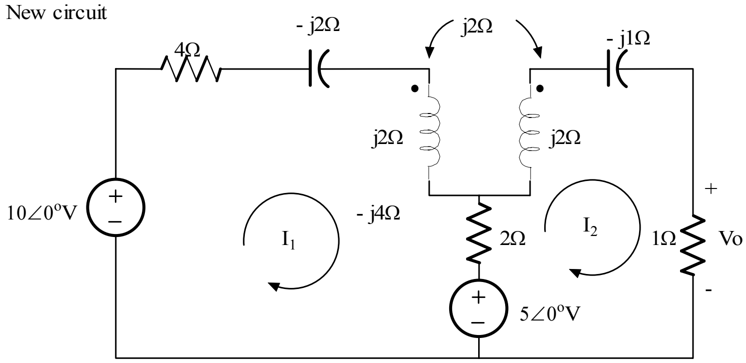
By superposition:

$Voc = 10\sqrt{2}\angle 135^o$ [-j4 / (4 - j4)] + 5 [4 || -j4]

$Voc = 10\angle 0^o V$

Zth = 4 || -j4 = 2 - j2Ω

New circuit



$5 = I_1 (4) - I_2 (j2 + 2)$ (1)

$5 = - I_1 (2 + j2) + I_2 (3 + j1)$ (2)

Solve (1) form I_1, substitute into (2) to get I_2 and Vo

$I_1 = [5 + I_2 (2 + j2)] / 4 \Rightarrow 5 = I_2 [(3 + j1) - (2 + j2)(2 + j2) / 4] - (2 + j2)(5/4)$

$I_2 (3 - j1) = 7.5 + j2.5 \Rightarrow I_2 = (7.5 + j2.5) / (3 - j1) = 2.5\angle 36.87^o A$

$Vo = (1) I_2 = 2.5\angle 36.87^o$ V

Problem 8.34

Find Vo in the network in Figure P8.34

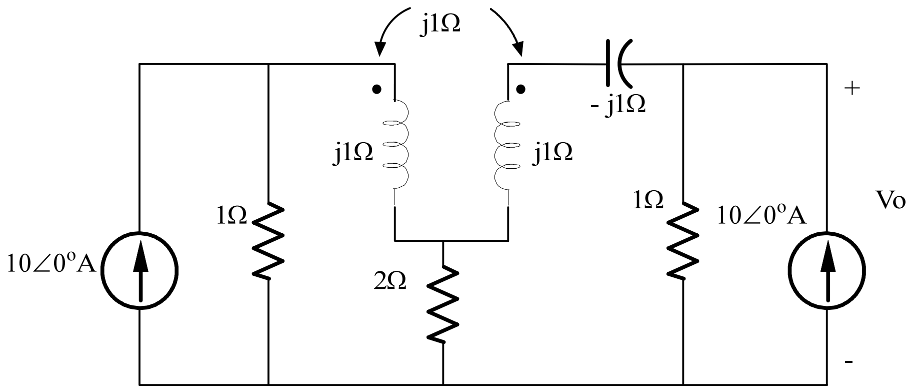
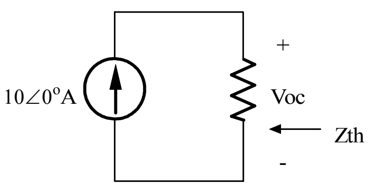


Figure P8.34

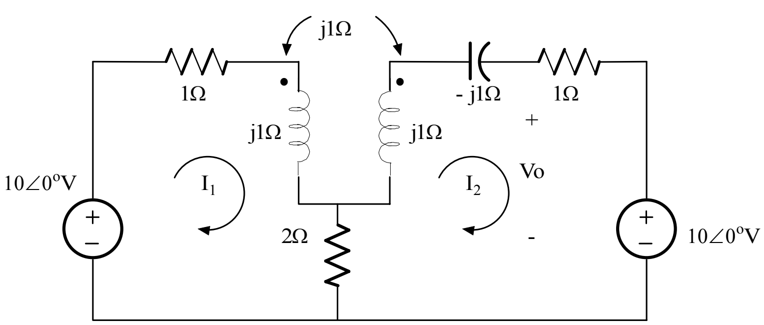
Suggested Solution

Thevenin eq. for left and right side of circuit



Voc = 10∠0°V Zth = 1Ω

New circuit



$10 = I_1(3 + j_1) - I2\ (2 + j_1)$ (1)

$-\ 10 = -\ I_1(2 + j1) + I_2(3)$ (2)

solve (2) for I_1

$I_1 = (10 + 3I_2) / (j1 + 2)$

$10 = (10 + 3I_2)[\ (3 + j1) / (2 + j1)\ ] - I_2\ (2 + j1) = 10\ [\ (3 + j1) / (2 + j1)\ ] + I_2\ (6 - j1) / (2 + j1)$

$I_2 = [\ 10(2 + j1) - 10(3 + j1)] / (6 - j1) = 1.64\angle{-170.54^o}\text{A}$

$\text{Vo} = (1)I_2 + 10 = 8.38\angle{-1.85^o}\ \text{V}$

Problem 8.35

Determine the impedance seen by the source in the network shown in Figure P8.35

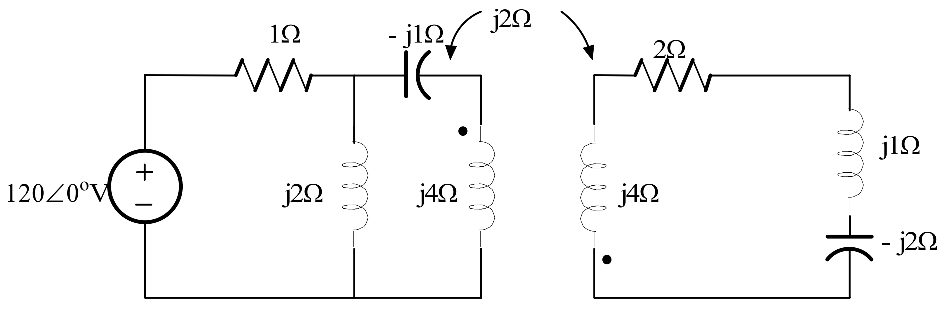
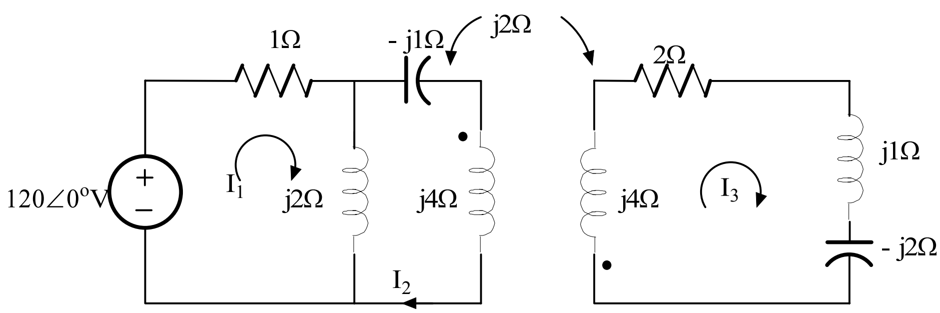


Figure P8.35

Suggested Solution



$120 = I_1\ (1 + j2) - j2\ I_2$; $0 = -j2I_1 + j5I_2 + j2\ I_3$; $0 = j2I_2 + I_3(2 + j3)$ (1) (2) and (3)

solve (3) for I_2 and usbstitute into (1) and (2)

$I_2 = I_3(-3/2 + j1)$ => $120 = I_1(1 + j2) + I_3(2 + j3)$; $0 = -j2I_1 + I_3(-5 - 11/2\ j)$ (4) and (5)

Solve (5) for I_3 and substitute into (4)

$I_3 = +j2I_1 / (-5 - j11/2) = -j4I_1 / (10 + j11)$

$120 = I_1\ [\ 1 + j2 + (2 + j3)(-j4) / (10 + j11)] = I_1[(10 + j11)(1 + j2) + 12 - j8] / (10 + j11)$

$120 = I_1\ [\ 1 + j2 + (2 + j3)(-J4) / (10 + J11)] = I_1\ [(10 + J11)(1 + J2) + 12 - J8] / (10 + J11)$

$Z_{source} = 120 / I_1 = (\ 10 - 22 + j11 + j20 + 12 - j8) / (10 + j11) = (j\ 23) / (10 + j11)$

$Z_{source} = 1.56\angle 42.27^o \Omega$

Problem 8.36

Determine the impedance seen by the source in the network shown in Figure P8.36

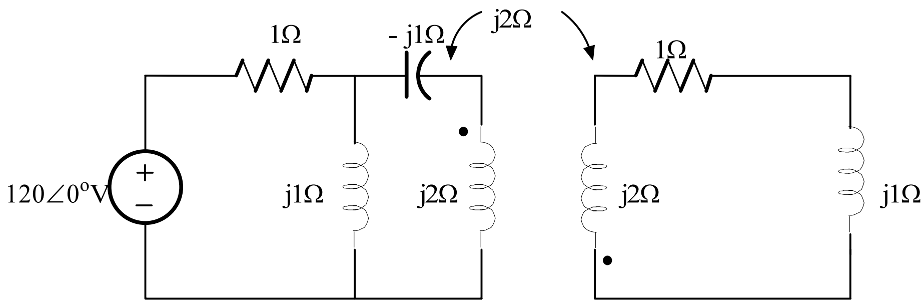
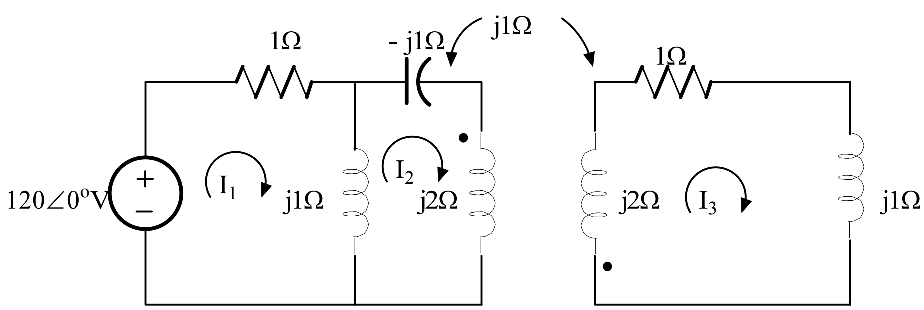


Figure P8.36

Suggested Solution



$120 = I_1\ (1 + j1) - j1\ I_2$; $0 = -j1I_1 + j2I_2 - j1\ I_3$; $0 = j1I_2 + I_3(1 + j3)$ (1) (2) and (3)

solve (3) for I_2 and usbstitute into (1) and (2)

$I_2 = I_3(-3 - j1)$ => $120 = I_1(1 + j1) + I_3(-1 - j3)$; $0 = -j1I_1 + I_3(2\ 5\ j)$ (4) and (5)

Solve (5) for I_3 and substitute into (4)

$I_3 = +j1I_1 / (2 + j5)$

$120 = I_1\ [\ 1 + j1 - (1 + j3)(j1) / (2 + j5)] = I_1[j6 / (2 + j5)$

$Z_{source} = 120 / I_1 =\ (j\ 6) / (2 + j5)$

$Z_{source} = 1.11\angle 21.8^o\Omega$

Problem 8.37

Determine the input impedance Zin of the circuit in Figure P8.37

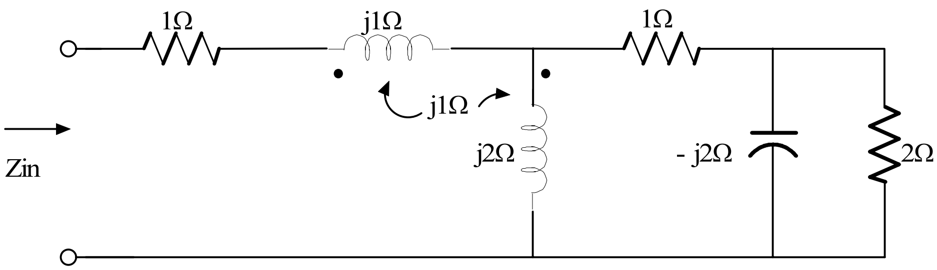
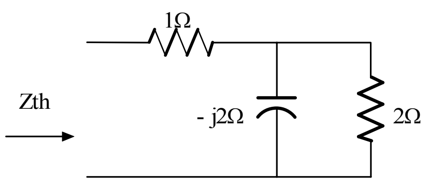


Figure P8.37

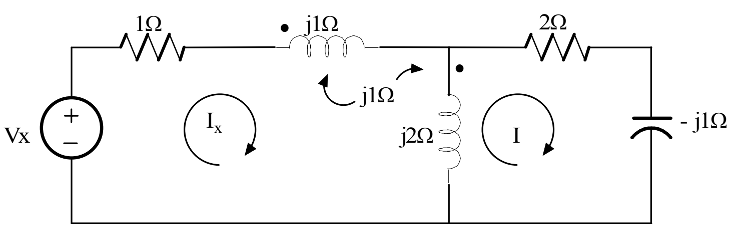
Suggested Solution

Thevenin's eq. at right side of circuit



$Zth = 1 + (2 \,||\, -j2) = 1 - (j4) / (2 - j2) = 1 + 1 - j1 = 2 - j1\ \Omega$

New circuit



$Vx = I_x\,(1 + j3) - j2I + j1I_x + j1(I_x - I)$

$0 = I\,(2 + j1) - j2I_x - j1I_x$

Solve (2) for I and substitute into (1)

$I = j3I_x / (2 + j1) \;\Rightarrow\; Vx = I_x\,[\,(1 + j3 + 2)\,(-j3)(j3) / (2 + j1)\,]$

$Vx = I_x\,[\,1 + j5 + (9) / (2 + j1)\,] = [\,(2 + j10 + j1 - 5 + 9) / (2 + j1)\,]\,I_x = (6 + j11) / (2 + j1)\,I_x$

$Zin = Vx / I_x = (6 + j11) / (2 + j1)$

Zιν = 5.60∠34.820° Ω

Problem 8.38

Determine the input impedance Zin in the network in Figure P8.38

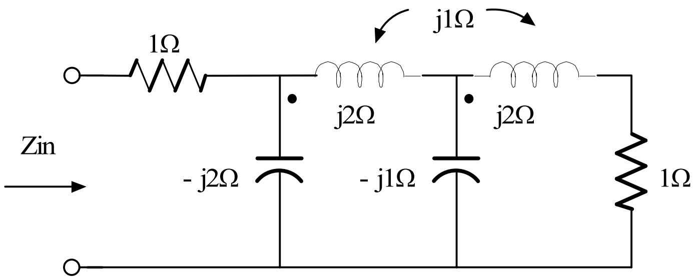
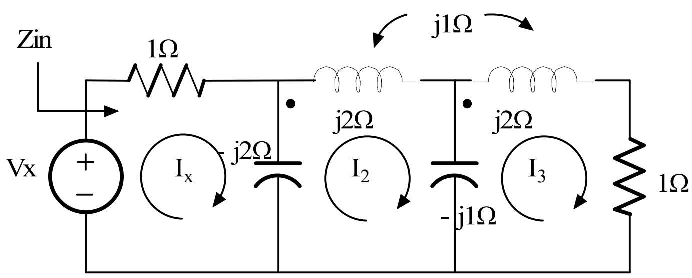


Figure P8.38

Suggested Solution



$Zin = Vx / I_x$

$Vx = I_x (1 - j2) + j2\ I_2$; $0 = j2I_x - j1I_2 + j2\ I_3$; $0 = j2I_2 + I_3(1 + j1)$ (1) (2) and (3)

solve (3) for I_2 and usbstitute into (1) and (2)

$I_2 = I_3(-1 + j1) / 2 \Rightarrow Vx = I_x (1 - j2) + I_3(-1 - j1)$; $0 = j2I_x + I_3(1/2 + 5/2\ j)$ (4) and (5)

Solve (5) for I_3 and substitute into (4)

$I_3 = +j4I_x / (1 + j5) \Rightarrow Vx = I_x [\ 1 - j2 + (1 + j1)(j4)/(1 + j5)\ ]$

$Zin = Vx / I_x = [(1 - j2)(1 + j5) - 4 + j4\ ] / (1 + j5) = (7 + j\ 7) / (1 + j5)$

$Zin = 1.94\angle{-33.69^{o}}\ \Omega$

Problem 8.39

Given the network shown in Figure P8.39 determine the value of the capacitor C that will cause the impedance seen by the 24∠0° V voltage source to be purely resistive. f = 60Hz.

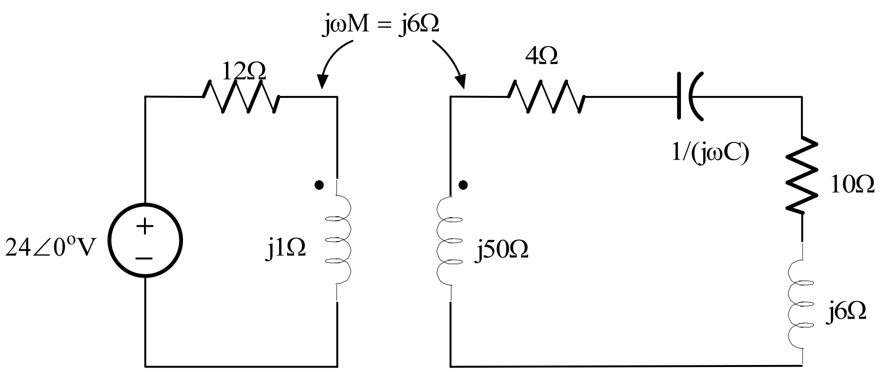
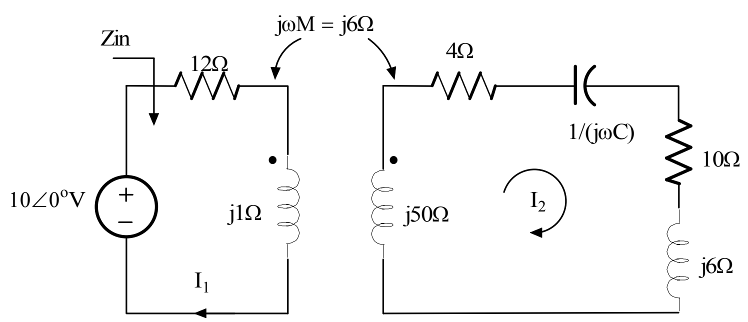


Figure P8.39

Suggested Solution



$Zin = 24 / I_1$

$24 = I_1 (12 + j1) - j6\, I_2$ and $0 = -j6I_1 + I_2 (14 + j(56 - Xc))$ (1) and (2)

Solve (2) for I_2 and substitute into (1)

$I_2 = j6I_1 / [14 + j(56-Xc)]$ => $24 = I_1 [\ 12 + j1 + 36 / (14 + j(56 - Xc)\ ]$

$Zin = 24 / I_1 = 12 + j1 + 36 / [14 + j(56 - Xc)] = Rin + j0$

Thus,

$36 / [14 + j(56 - Xc)] = R - J1$ where $Rin = 12 + R$

$36 = 14R + 56 - Xc$ and $0 = R(56 - Xc) - 14$ (3) and (4)

Solve (3) for Xc, substitute into (4) to find R

$Xc = 14R + 20$ => $0= R(56 - 14R - 20) - 14$ => $14R^2 - 36R + 14 = 0$

R = {2.094Ω, 0.478Ω} Xc = {49.316Ω, 26.692Ω} C = {53.79μF, 99.38μF} Rin = {14.09Ω, 14.48Ω}

Problem 8.40

Analyze the network in Figure P8.40 and determine if a value of Xc can be found such that the output voltage is equal to twice the input voltage.

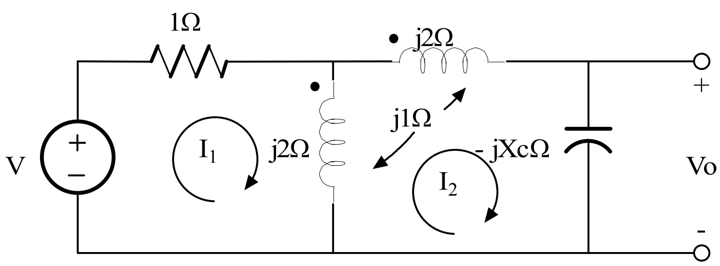
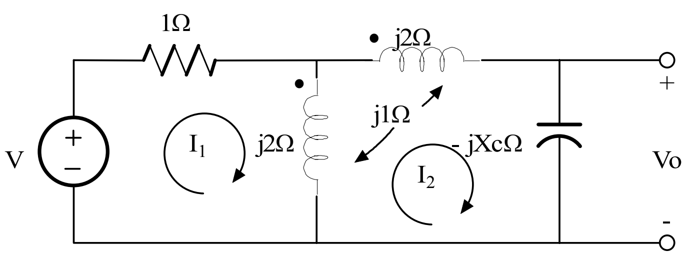


Figure P8.40

Suggested Solution



$Vo = I_2\ (-j\ Xc)$

$V = I_1\ (1 + j2) - J2\ I_2 + j1\ I_2$

$0 = - j2I_1 + I2(j4 - jXc) + j1\ (I_1 - I_2)$

$(j1)I_1 = I_2\ j(3 - Xc) \Rightarrow V = I_2[(3 - Xc)(1 + j2) - j1]$

But $I_2 - Vo / (-jXc)$ so $V = [\ Vo / (-jXc)\ ]\ (3 + j6 - Xc - j2Xc - j1)$

And $Vo/V = (-jXc) / (3 - Xc + j(5 - 2Xc)\ ) = Xc / [-5 + 2Xc + j(3 - Xc)\ ]$

For no phase shift between Vo and V1

Then

$3 - Xc = 0 \Rightarrow Xc = 3\Omega$

$Vo / V\ = Xc / (-5 + 2Xc) = 3 / (-5 + 6) = 3$

No value of Xc exists such that Vo = 2V

If we allow phase shift, then,

$|\ Vo/V\ | = 2 = Xc / \sqrt{(-5 + 2Xc)^2 + (3 - Xc)^2} \Rightarrow 4\ (34 - 26Xc + 5X^2c) = X^2c$

$19\ X^2c - 104Xc + 136 = 0$ and $Xc = 3.31\Omega$ and 2.14Ω

There are 2 values of Xc for $|Vo/V| = 2$

$Vo/V = (3.31) / (1.62 - 0.31j) = 2\angle 10.8^o$ and $Vo/V = (2.161) / (-0.678 + j0.839) = 2\angle 128.9^o$

Problem 8.41

Two coils in a network are positioned such that there is 100% coupling between them. If the inductance of one coil is 10 mH and the mutual inductance is 6mH, compute the inductance of the other coil

Suggested Solution

$k = \frac{M}{\sqrt{L_1 L_2}} \qquad k = 1, M = 6mH, L_1 = 10mH$

$L2 = \frac{1}{L_1}\left(\frac{M}{k}\right)^2 = \boxed{3.6mH}$

Problem 8.42

The currents in the magnetically coupled inductors shown in Figure P8.42 are known to be $i_1(t)$=8cos(377t-20)mA and $i_2(t)$=4cos(377t-50)mA. The inductor values are L_1 = 2H, L_2 = 1H, and k=0.6. Determine v_1 (t) and $v_2(t)$

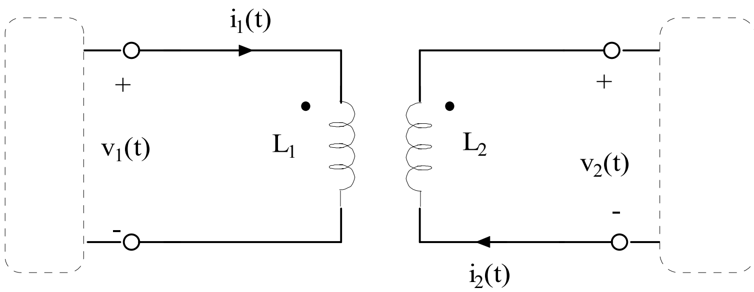


Figure P8.42

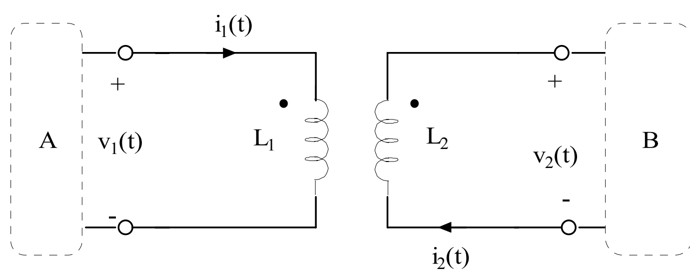
Suggested Solution

For the coupled inductors,

$i_1(t)$ = 8cos(377t - 20) mA, and

$i_2(t)$ = 4cos(377t - 50) mA, L_1 = 2

L_2 = 1,K = 0.6, and $v_1(t)$, $v_2(t)$.



Mutual coupling $M = 0.6\sqrt{2.1} = 0.85$

Then

$$v_1(t) = \frac{L_1 di_1(t)}{dt} - M\frac{di_2(t)}{dt}$$

$$v_1(t) = 2\frac{d}{dt}(8\cos(377t - 20°)) - 0.85\frac{d}{dt}(4\cos(377t - 50°))mA$$

$$\boxed{v_1(t) = -6\sin(377t - 20°) + 1.3\sin(377t - 50°)}$$

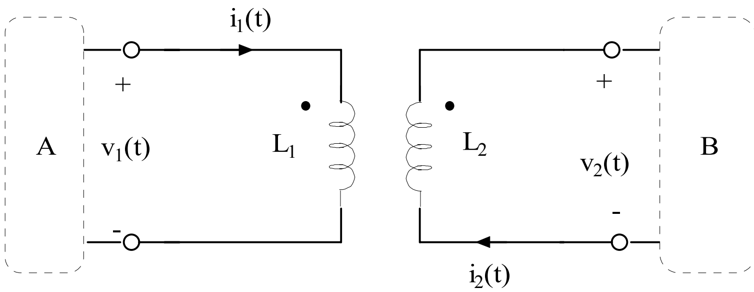
$$v_2(t) = \frac{-L_2 di_2(t)}{dt} + M\frac{di_1(t)}{dt}$$

$$\boxed{v_2(t) = 1.6\sin(377t - 50°) - 2.6\sin(377t - 20°)}$$

Problem 8.43

Determine the energy stored in the coupled inductors in Problem 8.42 at t=1ms

Suggested Solution



Find energy stored in inductors of P8.42 at t=1ms

At t = 1ms, $\omega t = (377)(0.001) = 0.337\text{rad} = 21.6°$

then, $i_1(t{=}0.001) = 8\cos(21.6° - 20°) = 8.0 \text{ mA}$

$i_2(t{=}0.001) = 4\cos(21.6° - 50°) = 3.52\text{mA}$

and $\omega(t{=}0.001) = 1/2\ (2)\ (2\text{mA})^2 + 1/2(1)(3.52\text{mA})^2 - 0.85(8)(3.53\text{mA})^2$

$\omega(t{=}0.001) = 46.3\ \mu\text{J}$

Problem 8.44

If L1 = L2 = 4H and k = 0.8, find i1(t) and I2(t) in the circuit in Figure P8.44

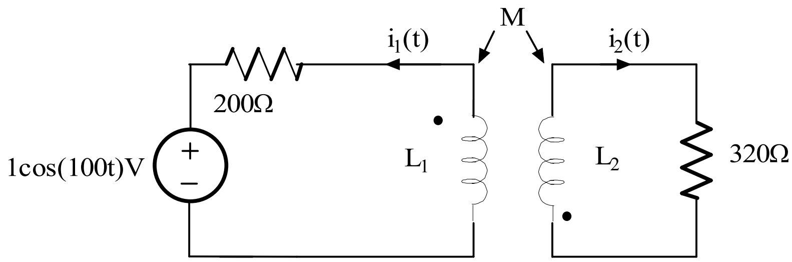
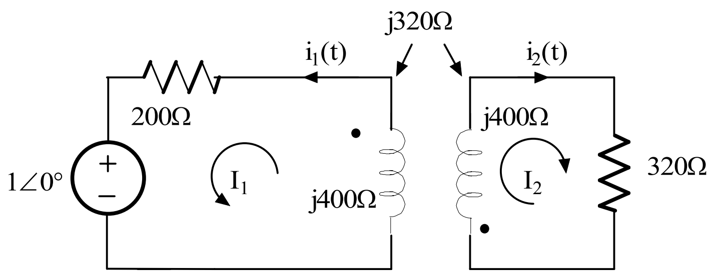


Figure P8.44

Suggested Solution

Convert to frequency domain!

$1\cos(100t) \rightarrow 1\angle 0°$; $i_1(t) \rightarrow I_1$; $i_2(t) \rightarrow I_2$ $M = k\,(L_1L_2)^{1/2} = 3.2\text{H}$



$1\angle 0° = -I_1\,(200 + j400) + j320\,I_2$ and $0 = I_2(320 + j400) - j320I_1$

$1\angle 0° = [(-1.25 + j1)(200 + j400) + j320]\,I_2$ $I_1 = I_2\,(1.25 - j1)$

$I_2 = 1 / (-650 + j20) = 1.54\angle -178.24°$ mA

$I_1 = I_2\,(1.25 - j1) = I_2\,(1.60\angle -38.66°) = 2.46\angle 143.10°$ mA

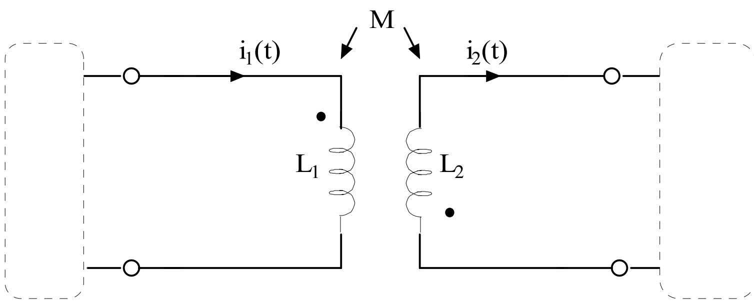
$i_2(t) = 1.54\cos(100t - 178.24°)$ mA

$i_1(t) = 2.46\cos(100t + 143.10°)$ mA

Problem 8.45

Determine the energy stored in the coupled inductors in the network in P8.44

Suggested Solution



L1 = L2 = 4H   k = 0.8

M = 3.2H

From problem 11.32, $i_1(t)$ and $i_2(t)$ as defined here are

$i_1(t) = -\ 2.46 \cos(100t + 143.10)$ mA

$i_2(t) = 1.54 \cos(100t - 178.24)$ mA

$\omega(t) = 1/2\ L1\ i_2 1(t) + 1/2\ L_2\ i^2{}_2(t) + M i_1(t) i_2(t)$

Evaluate at t = 2ms

$\omega(0.002) = 3.71\ \mu J$

Problem 8.46

Find all currents and voltages in the network in figure P8.46

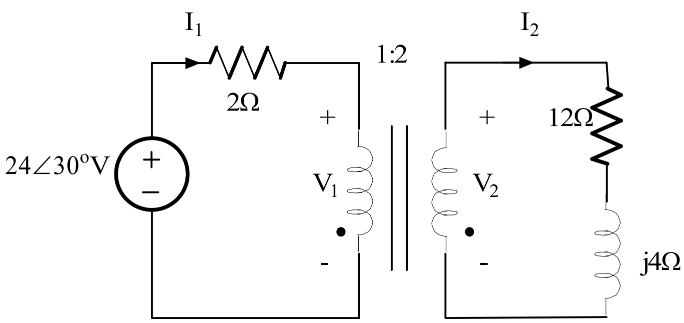


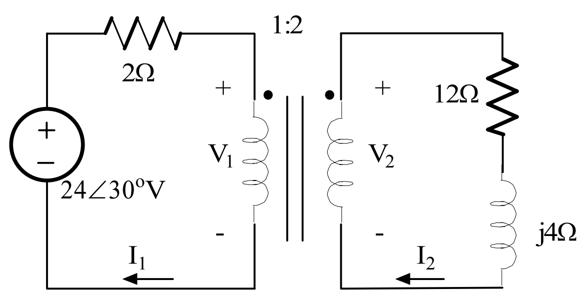
Figure P8.46

Suggested Solution

Find I_1, I_2, V_1, V_2

For the secondary

$Z = (12 + 4j) / (2^2) = 3 + j$



In the primary, $I_1 = (24\angle 30^o) / (2 + 3 + j) = \underline{4.7\angle 18.7^o A}$

Then $V_1 = I_1 Z_1 = (4.7\angle 18.7^o)(3.2\angle 18.4^o) = \underline{15\angle 37.1^o V}$

And $V_2 = 2V_1 = \underline{30\angle 32.1^o V}$, $I_2 = I_1/2 = \underline{2.35\angle 18.7^o A}$

Problem 8.47

Determine I1, I2, V1, and V2 in the network in Figure P8.47

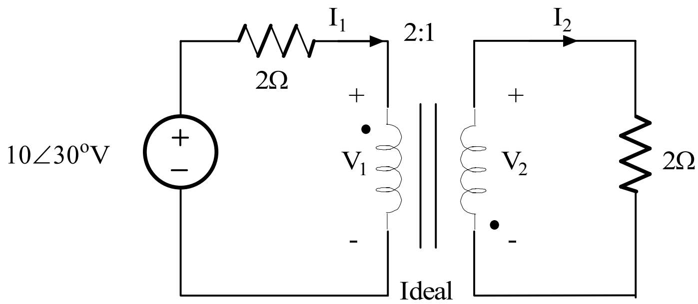
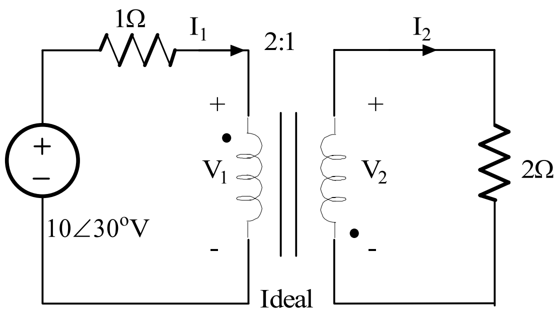


Figure P8.47

Suggested Solution



$Z_L = 2\Omega$, $n = L/2$, $Z_1 = Z_l/n_2 = 8\Omega$

$V_1 = 10\angle 30^o[Z_1 / (Z_1 + 1)] = 8.89\angle 30^oV$

$I_1 = (10\angle 30^o) / (1 + Z_1) = (10/9)\angle 30^oA = 1.11\angle 30^oA$

$I_2 = - I_1 / n = - 2.22\angle 30^oA = 2.22\angle -150^oA$

$V_2 = - n\ V_1 = 4.44\angle -150^oV$

Problem 8.48

Determine Vo in the circuit in Figure P8.48

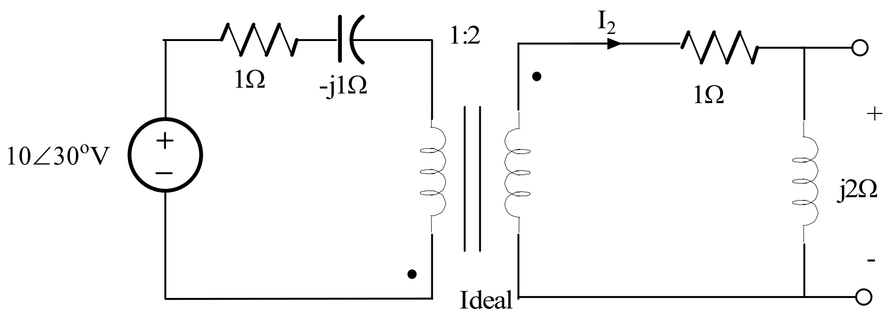
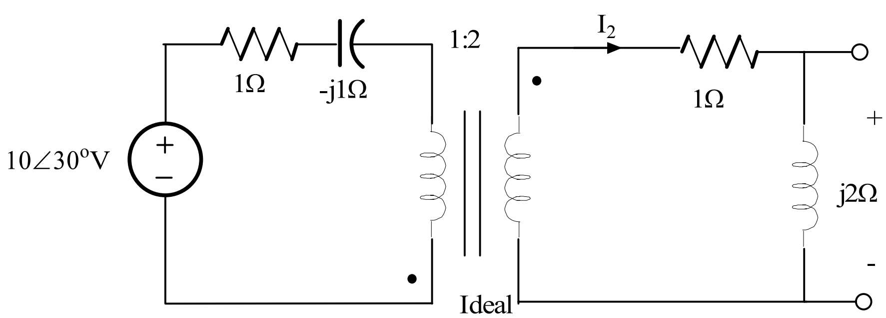
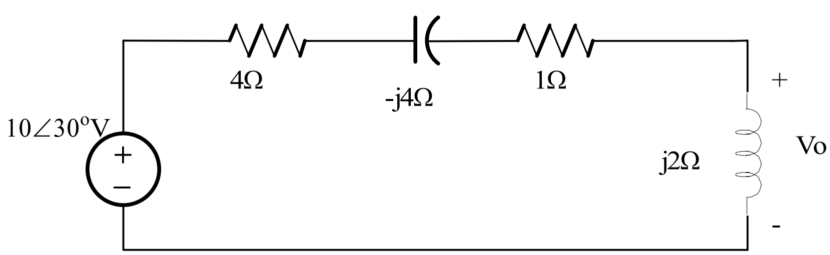


Figure P8.48

Suggested Solution



$n = 2$



$$V_o = -20\angle 30^{o}\,[j2 / (5 - j2)]$$

$$V_o = 7.43\angle -38.20^{o}V$$

Problem 8.49

Determine I1, I2, V1 in the network in Figure P8.49

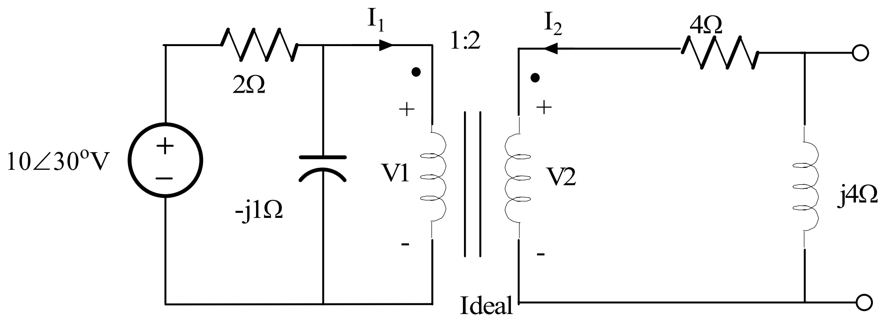
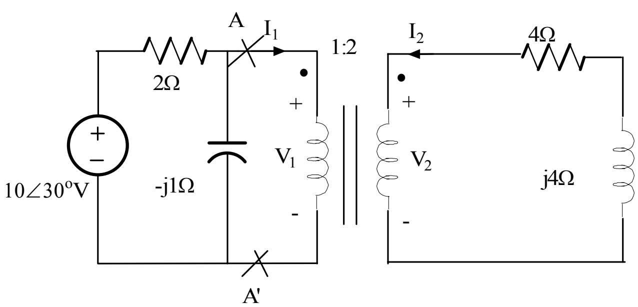


Figure P8.49

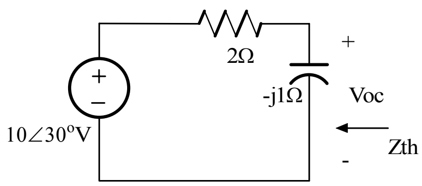
Suggested Solution



$Z_l = 4 + j4\Omega \quad n = 2$

$Z_1 = Z_l / n^2 = 1 + 1$

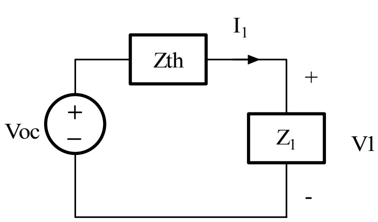
Thevenin Eq. at A-A'



$Voc = 10\angle 30^o[\ -j1/(2 - j1)\ ] = 4.47\angle -33.43^oV$

$Zth = 2 \parallel (-j1\Omega) = (-j2)/(2 - j1) = 0.89\angle -63.43^o\Omega$

New Circuit



$I_1 = Voc / (Zth + Z1) = Voc / (0.4 - j0.8 + 1 + j1) = Voc / (1.4 + j0.2)$

$I_1 = 3.16\angle -41.56^oA$

$\underline{V_1 = I_1 Z_1 = 4.47\angle 3.44^oV}$

$\underline{V_2 = n\, V_1 = 8.94\angle 3.44^oV}$ $\quad$ $\underline{I_2 = -\, I_1 / n = 1.58\angle 138.44^oA}$

Problem 8.50

Determine I_1, I_2, and V_1 in the network in Figure P8.50

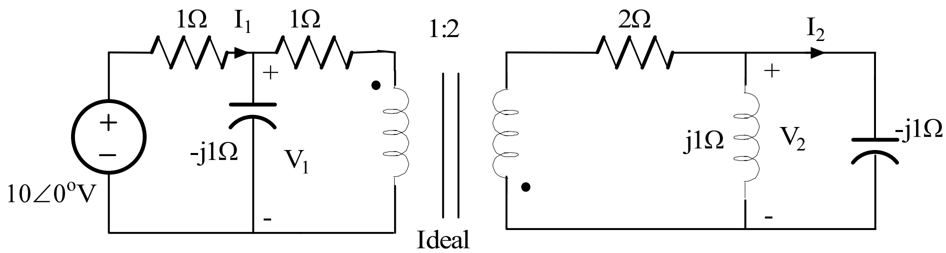
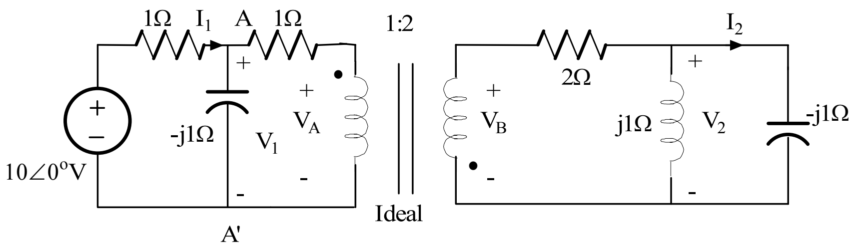


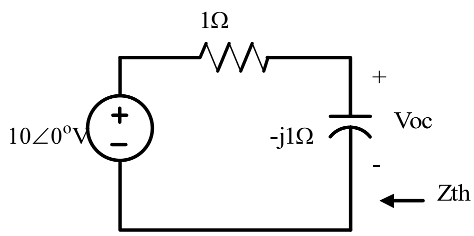
Figure P8.50

Suggested Solution



$ZL = 2 + (j1 \parallel (-j1) = 2\Omega \quad n=2 \quad Z_A = Z_L/n^2 = 1/2\Omega$

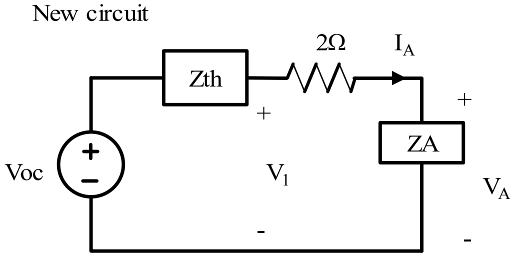
Thevenin equivalent at A - A'



$Voc = 10[\ (-j1) / (1 - j1) = 5 - j5 = 7.07\angle -45^oV$

$Zth = 1 \parallel (-j1) = (-j1) / (1 - j1) = 1/2 - j\ 1/2\ \Omega$

New circuit



$I_A = Voc / (Zth + 1 + Z_A) = 7.07\angle -45^o / (2 - 1/2j)$

$I_A = 3.43\angle -30.96^oA$

$V_1 = I_A(1 + Z_A) = 3/2\ I_A = 5.15\angle -30.96^oV$

$I_1 = V_1 / (-j1) + I_A = 5.15\angle 59.04^o + 3.43\angle -30.96^o = 6.19\angle 25.380^oA$

$V_B = -nV_A = -2\ (I_A\ Z_A) = 3.43\angle -149.04^oV$

$I_B = -I_A / n = 1.72\angle 149.040^oA$

In the secondary since $j1 \parallel (-j1) = \infty$, $V_2 = V_B = 3.43\angle 149.04^oA$

$I_2 = V_2 / (-j1) = 3.43\angle -120.96^oA$

Problem 8.51

Determine I_1, I_2, V_1, and V_2 in the network in Figure P8.51

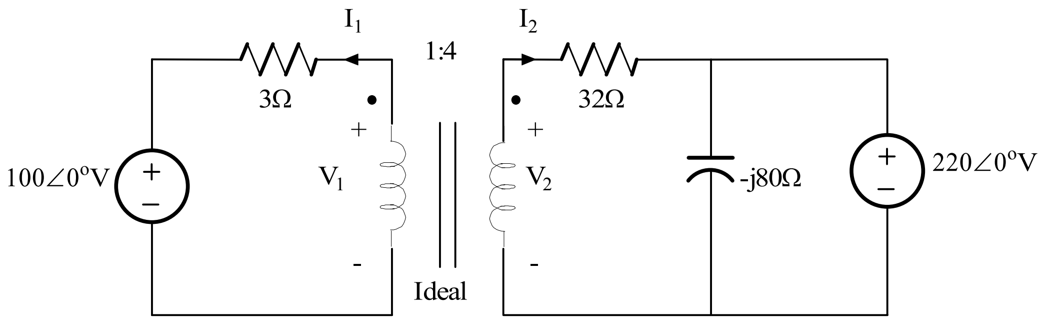
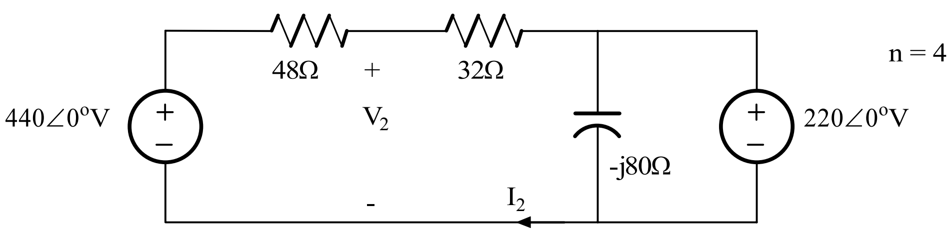


Figure P8.51

Suggested Solution



$I_2 = (440 - 220) / (80) = 2.75\angle 0^{o}A$ $V_2 = 220 + 32I_2 = 308\angle 0^{o}V$

$V_1 = V_2/n = 77\angle 0^{o}V$

$I_1 = -nI_2 = 11\angle 180^{o}A$

Problem 8.52

Determine the input impedance seen by the source in the circuit in Figure P8.52

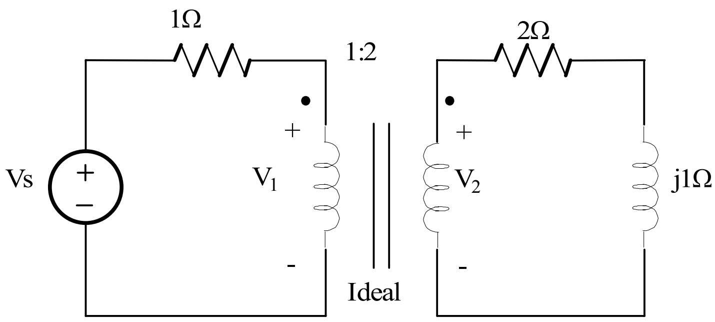
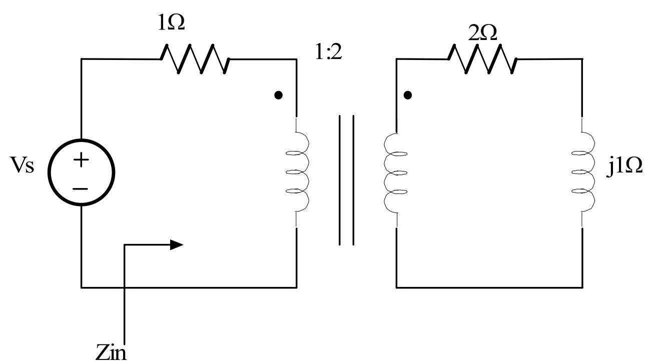


Figure P8.52

Suggested Solution



$Z_L = 2 + j1\Omega \quad n = 2$

$Z_1 = Z_L / n^2 = (2 + j1) / 4 = 0.5 + j0.25$

$Zin = 1 + Z_1 = 1.5 + j0.25\Omega$

Problem 8.53

Determine the input impedance seen by the source in the circuit in Figure P8.53

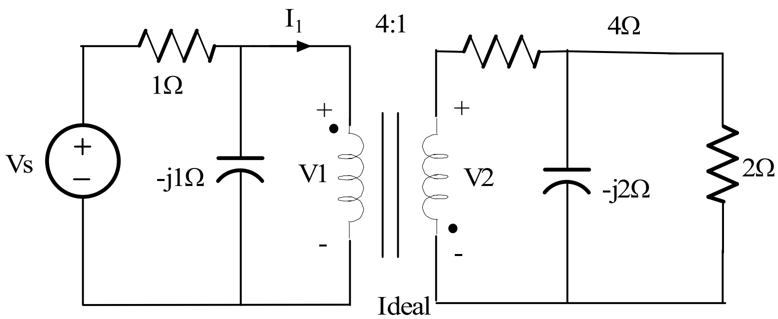
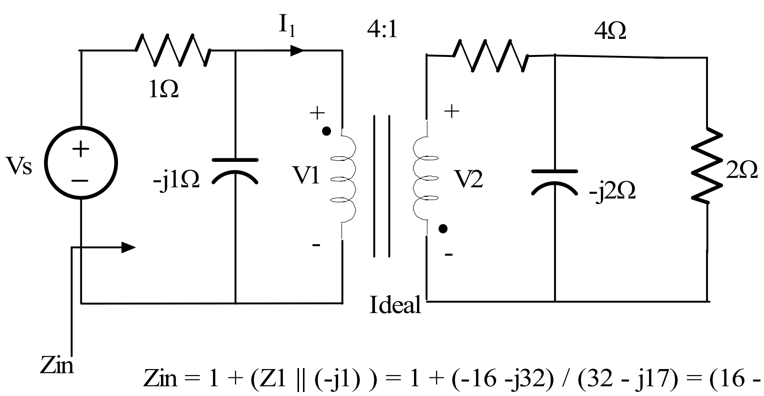


Figure P8.53

Suggested Solution



ZL = 1 + 2 || (-j2) = 2 - j1Ω

n = 1/4

Z1 = ZL/n2 = 32 - j16Ω

Zin = 1 + (Z1 || (-j1)) = 1 + (-16 -j32) / (32 - j17) = (16 - j49) / (32 - j17)

$Zin = 1.42\angle{-43.94}^{o}\Omega$

Problem 8.54

Determine the input impedance seen by the source in the network shown in Figure P8.54

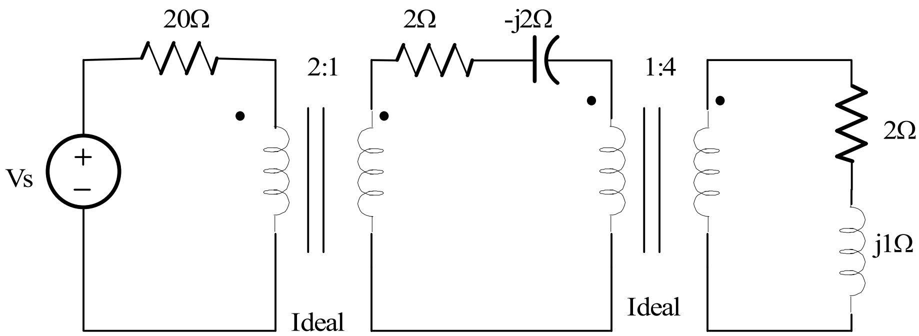
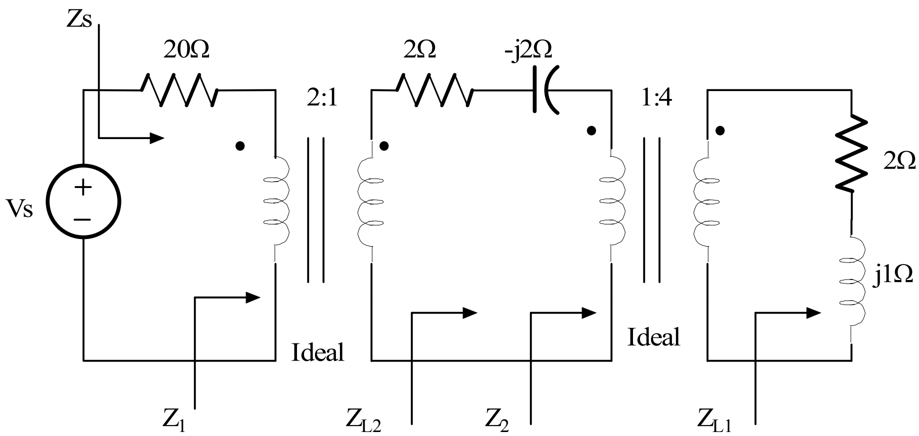


Figure P8.54

Suggested Solution



$Z_{L1} = 2 + j1\Omega \quad n_2 = 1/2 \quad Z_2 = Z_{L1}/n^2_2 = (2 + j1)/(1/4) = 8 + j4\Omega$

$Z_{L2} = 2 - j2 + Z_2 = 10 + j2\Omega \quad n_1 = 1/4 \quad Z_1 = Z_{L2} / n_1^2 = 160 + j32\Omega$

$Zs = 20 + Z_1 = 180 + j32\Omega$

Problem 8.55

Determine the input impedance seen by the source in the network shown in Figure P8.55

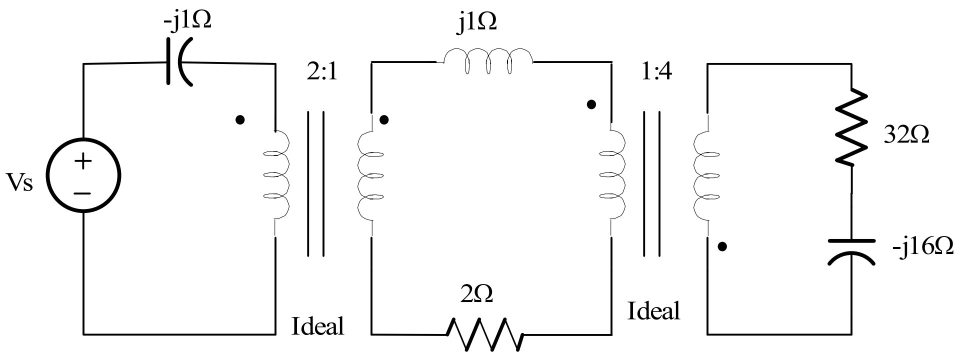
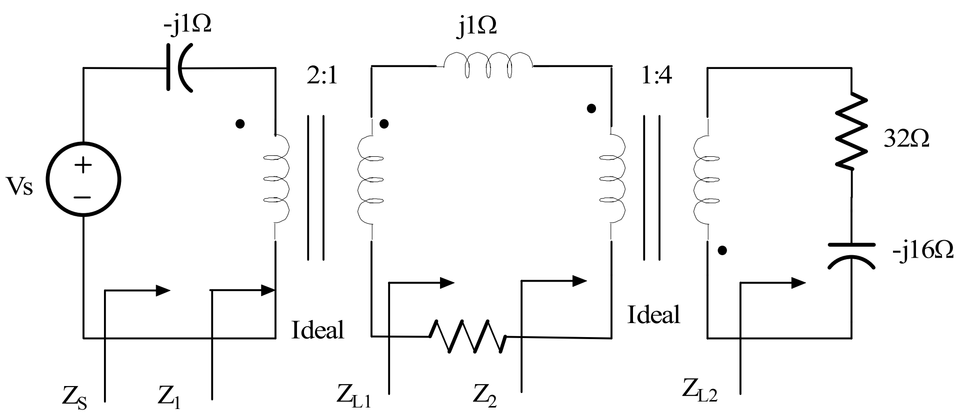


Figure P8.55

Suggested Solution



$n_2 = 4$

$n_1 = 1/2$

$Z_{L2} = 32 - j16\Omega \quad Z_2 = Z_{L2} / n_2^2 = 2 - j1\Omega$

$Z_{L1} = 2 + j1 + Z_2 = 4\Omega \quad Z_1 = Z_{L1}/n_1^2 = 16\Omega$

$Zs = 16 - j1\Omega$

Problem 8.56

The output stage on an amplifier in an old radio is to be matched to the impedance of a speaker as shown in Figure P8.56. If the impedance of the speaker is 8 ohms and the amplifier requires a load impedance of 3.2 K ohms determine the turns ratio of the ideal transformer.

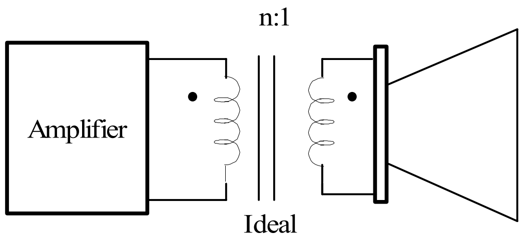
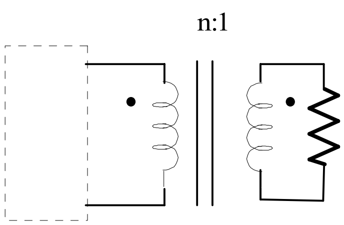


Figure P5.56

Suggested Solution



Speaker = 8Ω

$R_1 = (R_{Speaker})n^2 = 3200$

$n^2 = 400$

n= 20

Problem 8.57

Given that Vo = 48∠30° V in the circuit shown in Figure P8.57, determine Vs

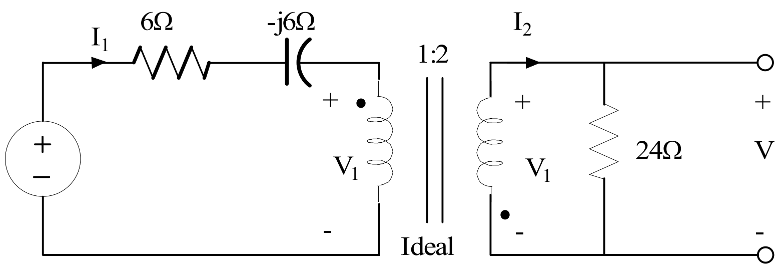
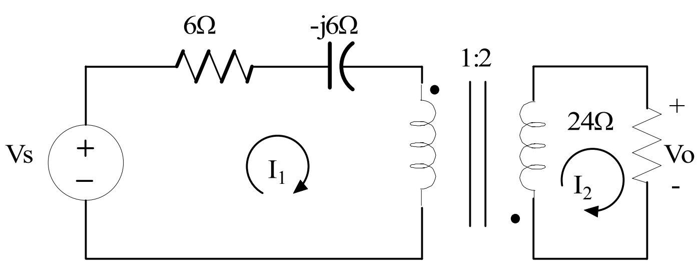


Figure P8.57

Suggested Solution



If $Vo = 48.30\angle 30^{o}V$, find Vs

If $Vo = 48.30\angle 30^{o}$, $I_2 = 2\angle 30^{o}$

$V_1 = -V_2 / 2 = -24\angle 30^{o}$, $I_1 = -2I_2 = -4\angle 30^{o}V$

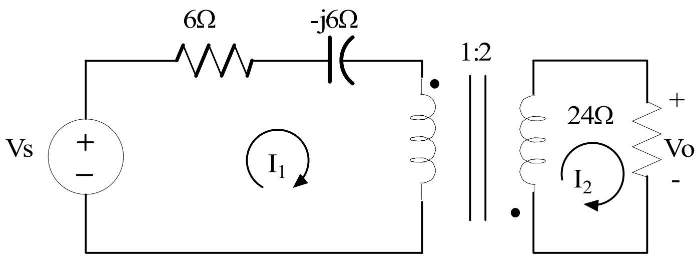
Then $Vs = V_1 + I_1 (6 - 6j)$

$53.66\angle -175.7^{o}V$

Problem 8.58

If the voltage source Vs in the circuit of Problem 8.57 is $50\angle 0°$V, determine Vo.

Suggested Solution



If Vs = $50\angle 0^o$V, find Vo

$I_1 = (50\angle 0^o) / (6 - 6j + 24/4) = 3.73\angle 26.6^o$

Then $I_2 = - I_1/2 = - 1.86\angle 26.6^o$

$Vo = 24\ I_2$

$44.72\angle -153^o V$

Problem 8.59

Determine Vs in the circuit in Figure P8.59

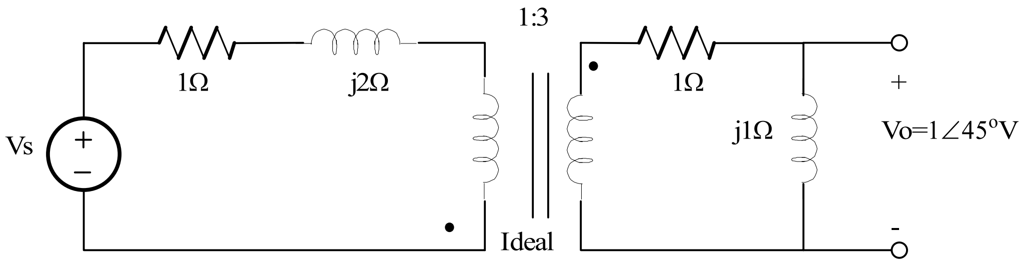
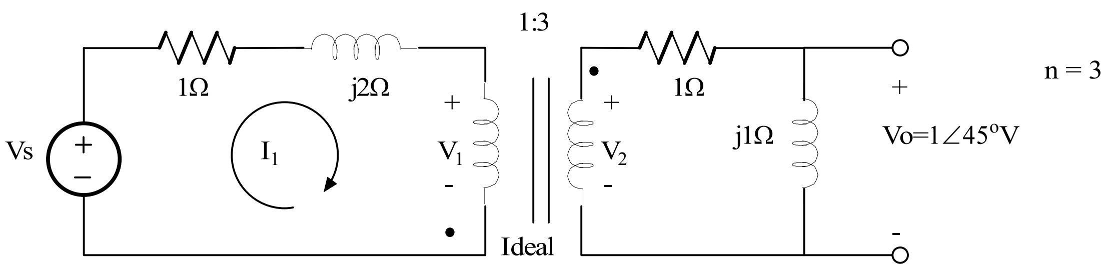


Figure P8.59

Suggested Solution



$I_2 = Vo / (j1) = 1\angle -45^o A \quad V_2 = I_2(1 + j1) = 1.41V$

$V_1 = - V_2/n = 0.47\angle 180^o V \quad I_1 = - nI_2 = 3\angle 135^o A$

$Vs = I_1 (1 + j2) + V_1 = 6.71\angle -161.57^o + 0.47\angle 180^o V$

$Vs = 7.16\angle -162.76^o V$

Problem 8.60

Determine Is in the circuit in Figure P8.60

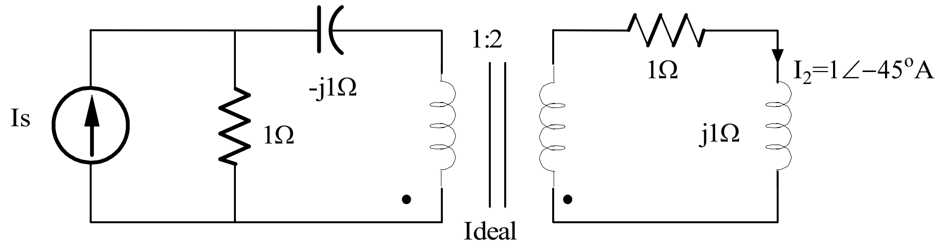
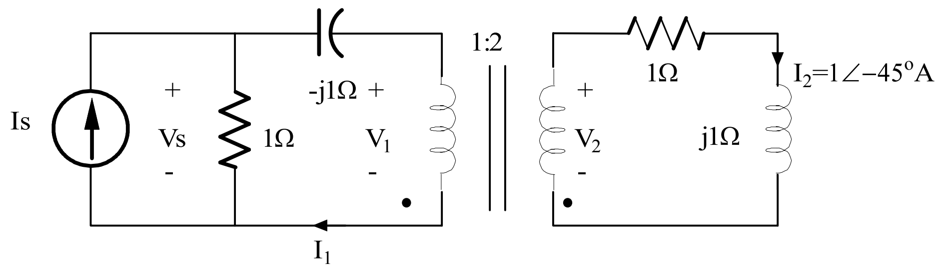


Figure P8.60

Suggested Solution



$V_2 = (1 + j1)I_2 = 1.41\angle 0^o V \quad V_1 = V_2/n = 0.707\angle 0^o V$

$I_1 = nI_2 = 2\angle -45^o A$

$Vs = V_1 - j1\ I_1 = 0.707\angle 0^o + 2\angle -135^o = 1.58\angle -116.57^o V$

$Is = Vs / 1 + I_1$

$Io = 2.91\angle -75.95^o A$

Problem 8.61

Find the current I in the network in Figure P8.61

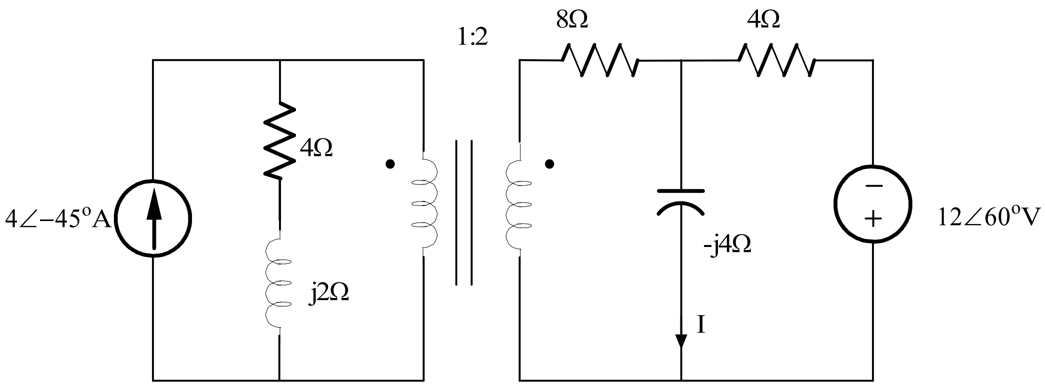
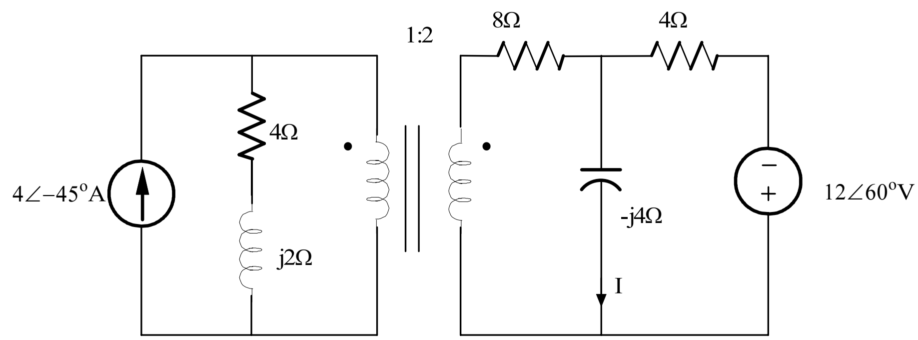


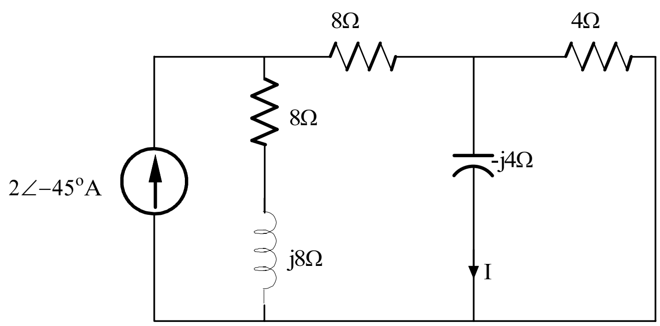
Figure P6.61

Suggested Solution

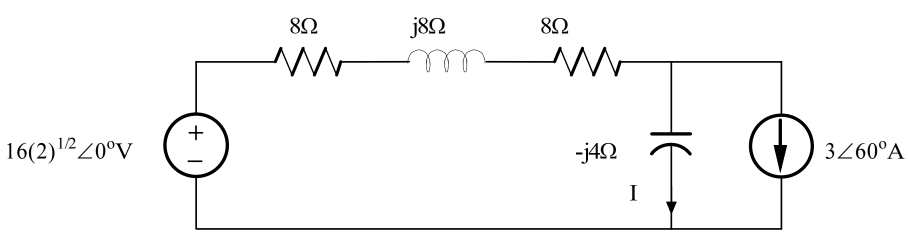


Refer primary to secondary

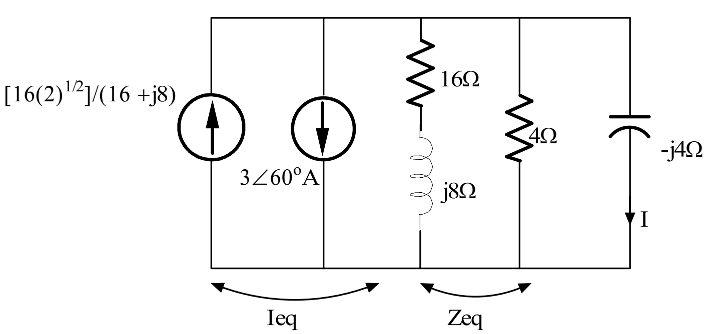
$I_{sec} = I_{prim} / n \quad Z_{sec} = Z_{prim}\, n^2$



Source transformations



Another source transformation



$I_{eq} = [16(2)^{1/2}]/(16 + j8) - 3\angle 60^\circ = 3.19\angle{-96.64^\circ}\text{A} \qquad Z_{eq} = 4 / (16 + j8) = 3.32\angle 4.76^\circ\ \Omega$

$I = I_{eq} [\ Z_{eq} / (Z_{eq} - j4\ ] = 2.13\angle{-43.50^\circ}$

$\boxed{I = 2.13\angle{-43.50^\circ}\text{A}}$

Problem 8.62

Find the voltage Vo in the network in Figure P8.62

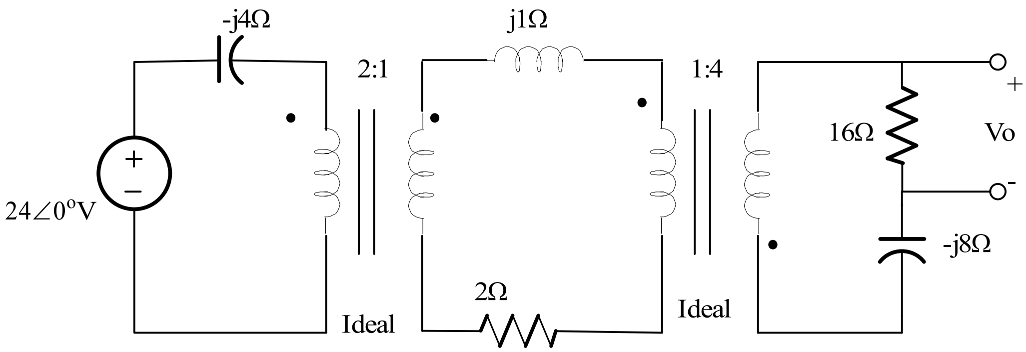
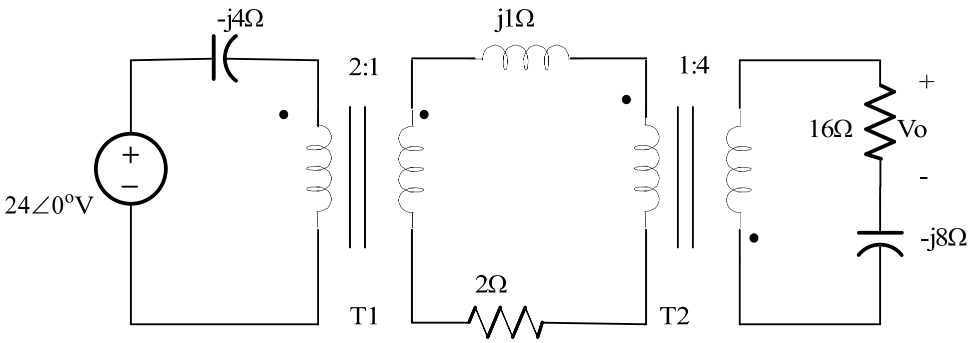


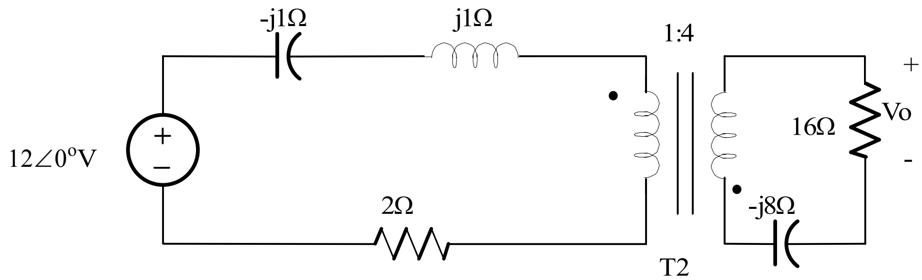
Figure 8.62

Suggested Solution

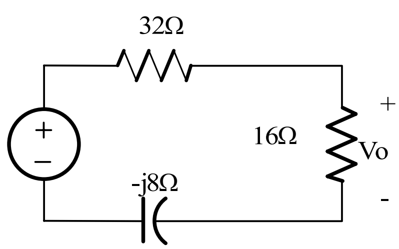


Refer primary circuit of T1 to secondary: $n_1 = 1/2$

$V_{sec} = V_{prim}\, n_1$ $Zsec = Zprim\, n_1^2$



Refer primary circuit of T2 to secondary: $n_2 = 4$ (Dots reversed!)



$Vo = -48\,[\,16 / (48 - j8)\,] = -96 / (6 - j1)$

$Vo = 15.78\angle 189.46^{o}V$

Problem 8.63

Find Vo in the network in Figure P8.63

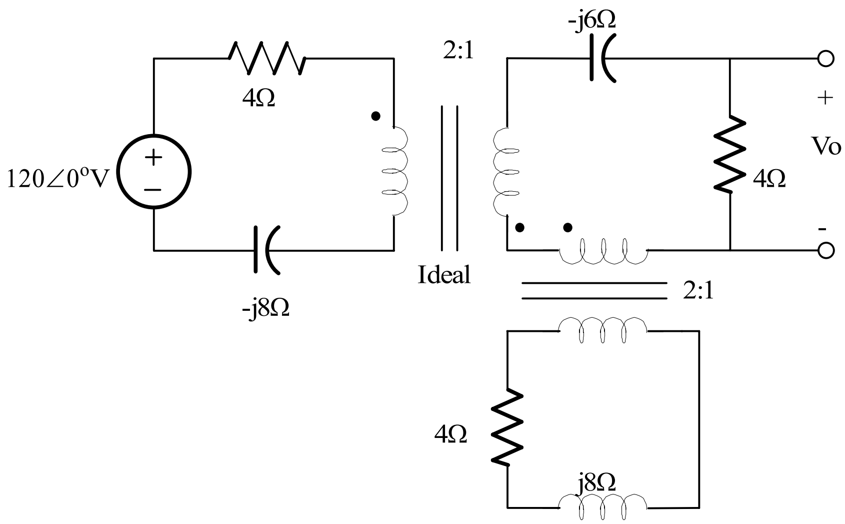
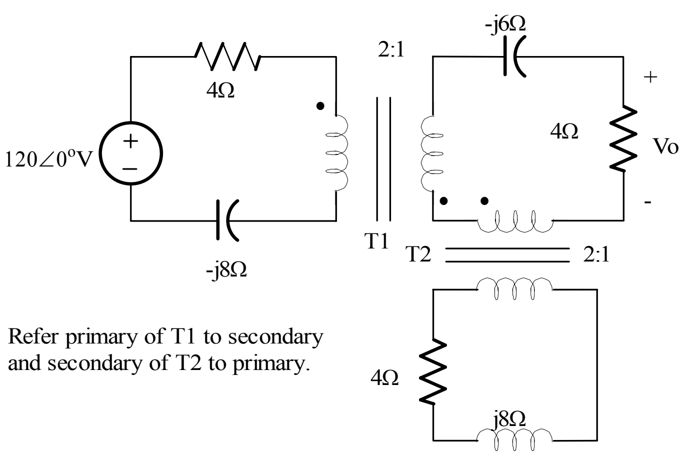


Figure P8.63

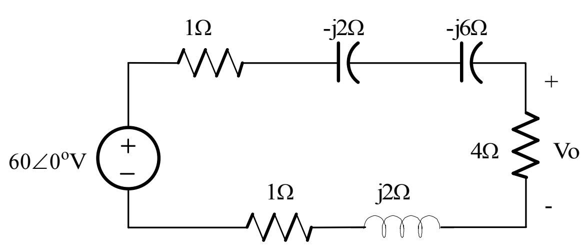
Suggested Solution



$n_1 = 1/2$

$n_2 = 1/2$

Refer primary of T1 to secondary and secondary of T2 to primary.



$$Vo = (-60)\,(4) / (6 - j2 + j2 - j6) = (-40) / (1 - j1)$$

$$\boxed{Vo = 28.28\angle{-135^{o}}\text{ V}}$$

Problem 8.64

Find Vo in the circuit in Figure P8.64

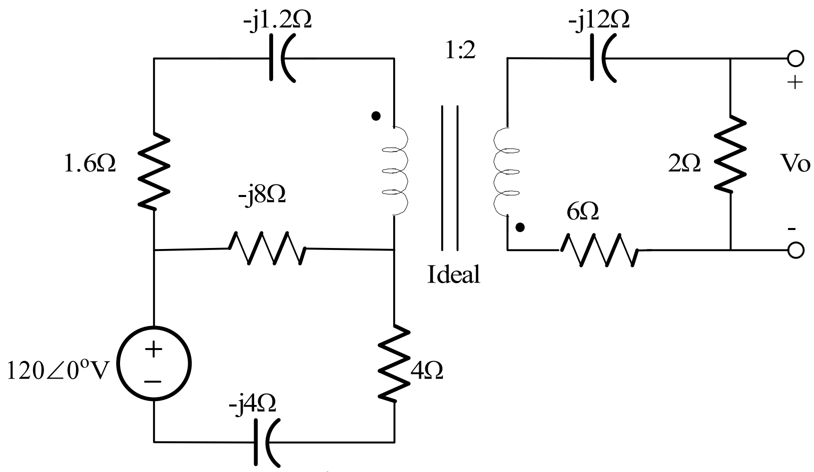
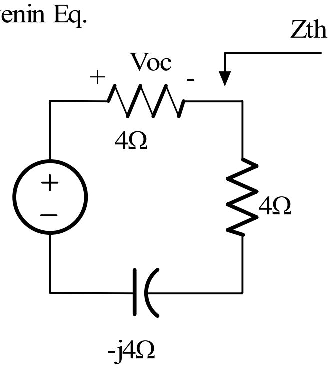


Figure P8.64

Suggested Solution

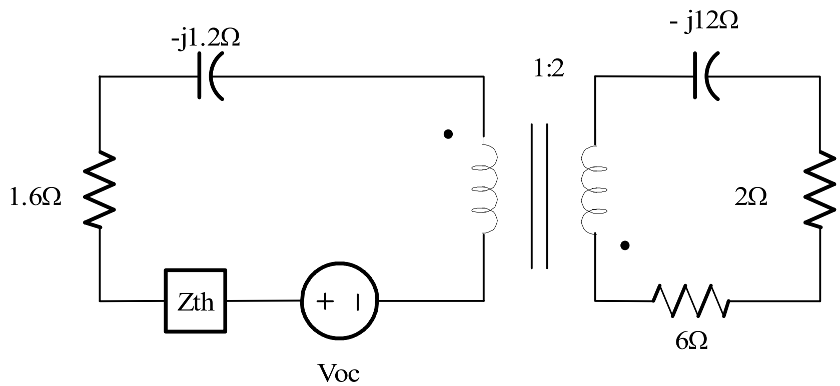
Thevenin Eq.



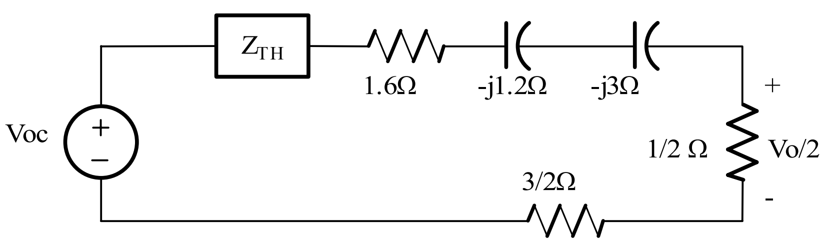
$$V_{oc} = 120\ [\ 4 / (8 - j4)\ ] = 120 / (2 - j1)$$

$$Z_{th} = 4 \parallel (4 - j4) = (4 - j4) / (2 - j1) = 2.4 - j0.8\ \Omega$$

New circuit



Refer secondary circuit to primary



$$V_o/2 = -V_{oc}\ [\ 0.5 / (3.6 - j4.2 + Z_{th})\ ]$$

$$V_o = 6.81\angle -113.630^\circ\ V$$

Problem 8.65

Find Vo in the circuit in Figure P8.65

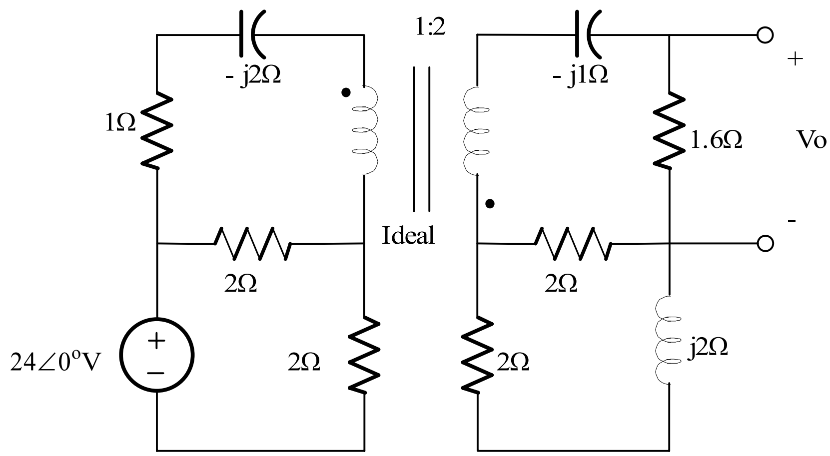
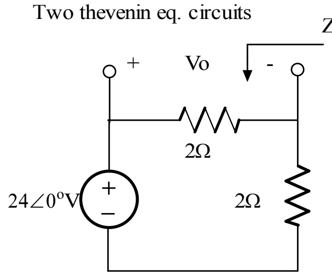


Figure P8.65

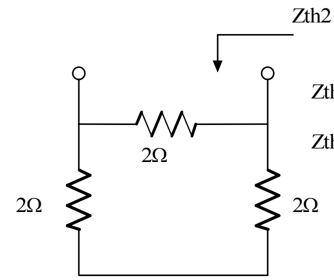
Suggested Solution

Two thevenin eq. circuits



$V_{oc} = 12\angle 0^{o}V$

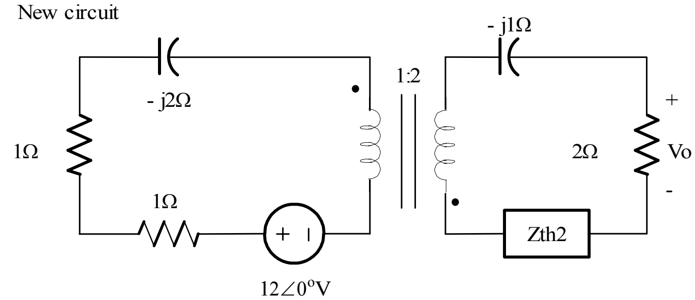
$Z_{th1} = 2 \,||\, 2 = 1\Omega$

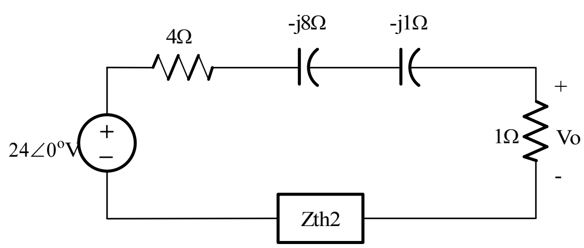


$Z_{th2} = 2 \,||\, (2 + j2)$

$Z_{th2} = (2 + j2) / (2 + j1)\ \Omega$

New circuit





$V_o = -24\,[\,1 / (9 - j9 + Z_{th2})\,] = (-24)(2 + j1) / [\,(9 - j9)(2 + j1) + 2 + 2\,] = (\,-24\,(\,2 + j1)\,) / (18 + 9 + 2 + j9 - j18 + j2)$

$V_o = (-24)\,(2 + j1) / (29 - j7)$

$V_o = 1.80\angle{-139.86}^{o}V$

Problem 8.66

In the circuit if Figure P8.66, if Ix = 6∠45°A, find Vo.

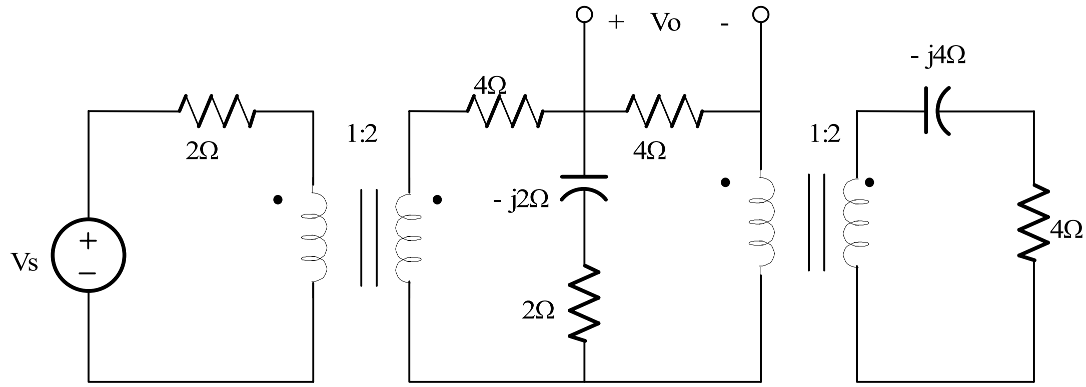
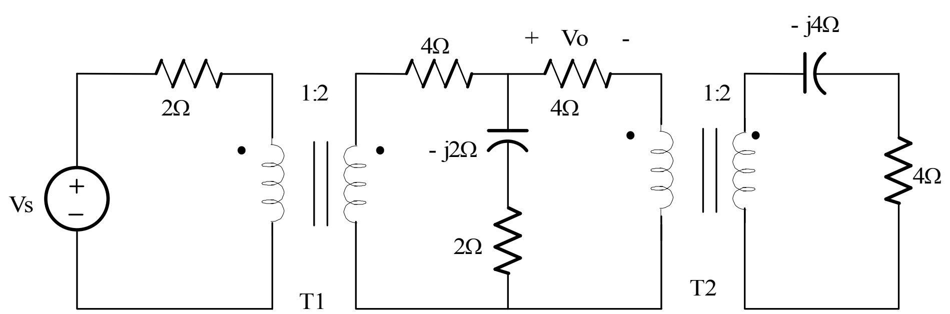


Figure P8.66

Suggested Solution

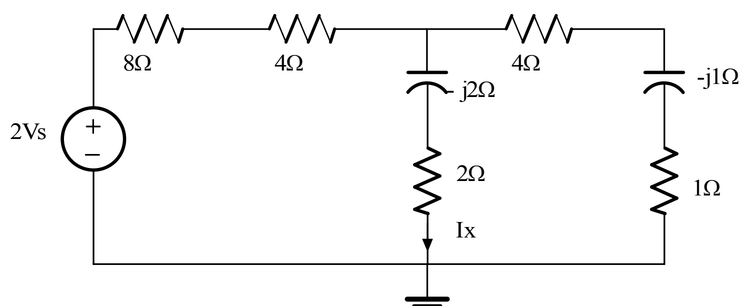


Ix = 6∠45°A

$N_2 = 2$

$N_1 = 2$

Refer primary of T1 to secondary and secondary of T2 to primary



Vx = Ix (2 - j2) = 6∠45°(2 – j2) = 16.97∠11.31° V

Vo = Vx[4 / (4 + 1 - j1)] = 4 Vx / (5 - j1)

Vo = 13.31∠11.31° V

Problem 8FE-1

In the network in Figure 8PFE-1 find the impedance seen by the source.

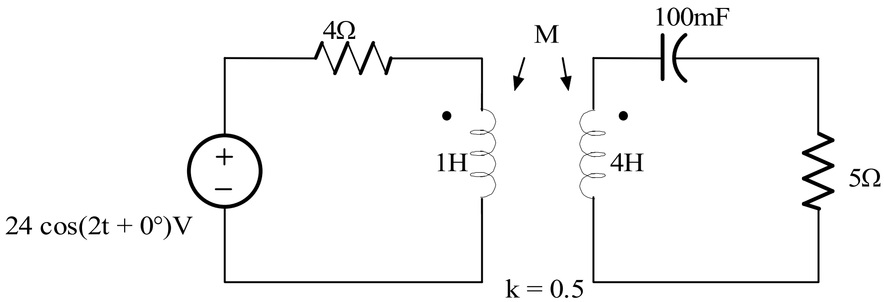
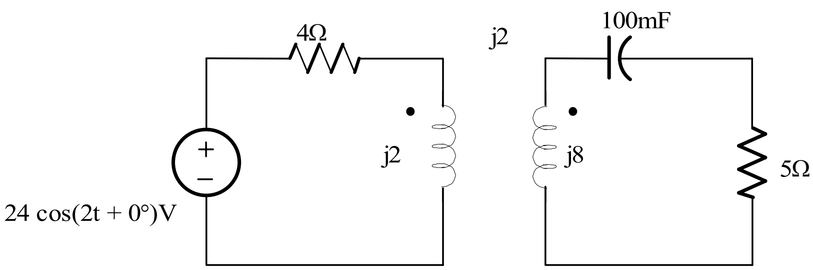


Figure 8PFE-1

Suggested Solution

Since $M = k(L_1L_2)^{1/2} = (0.5)(2) = 1H$



$V_1 = (4 + j2)I_1 - J2I_2$

$0 = -j2I_1 + (5 + j3)I_2$

$I_2 = (j2I_1)/(5 + j3)$

$V_1 = (4 + j2)I_1 - j2(j2) / (5 + j3)\ I_1 = [4 + j2 + 4/(5 + j3)]$

$Zs = V_1/I_1 = (18 + j22)/(5 + j3)$

$Zs = 4.88\angle 19°\ \Omega$

Problem 8FE-2

In the circuit in Figure 8PFE-2, select the value of the transformer's turns ratio $n = N_2/N_1$ to achieve impedance matching or maximum power transfer. Using this value of n calculate the power absorbed by the 3-Ohm resistor.

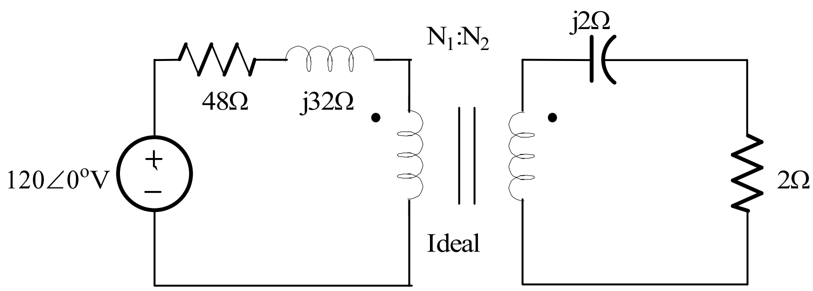


Figure P8PFE-2

Suggested Solution

The reflected impedance is $Z = Z_L/n^2$

The value of n that will make the reflected impedance the complex conjugate of the impedance in the primary is:

$\underline{n = 1/4}$ i.e. $(1/4)^2 (48 + j32) = (3 - j2)^*$ and $\underline{P_{3\Omega}} = (1/2)(120/96)^2(48) \underline{= 37.5W}$

Problem 8FE-3

In the circuit in Figure 8FE-3, select the turns ratio of the ideal transformer that will match the output of the transistor amplifier to the speaker represented by the 16-Ohm load.

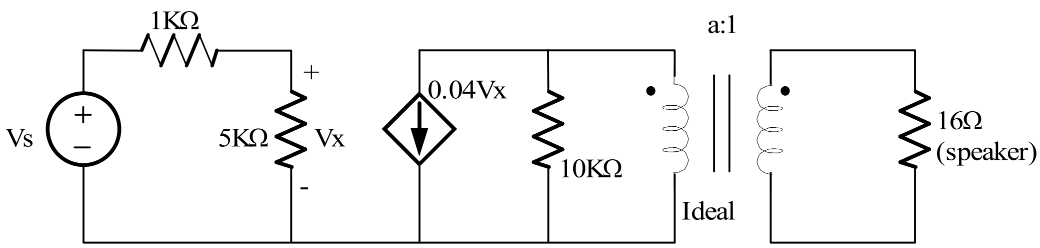


Figure P8PFE-3

Suggested Solution

$a^2(16) = 10K$ => $\underline{a = 25}$

Problem 8FE-4

Find the power absorbed in the 1-Ohm load in the network in Figure 8PFE-4

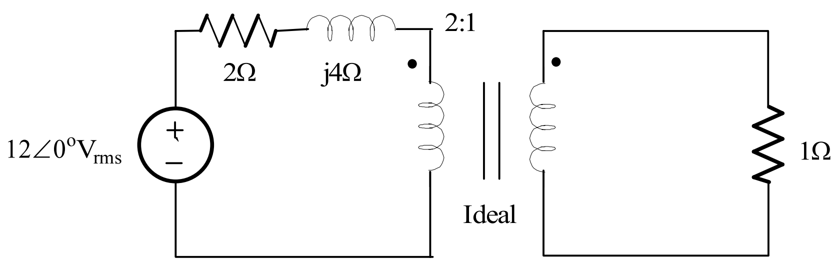
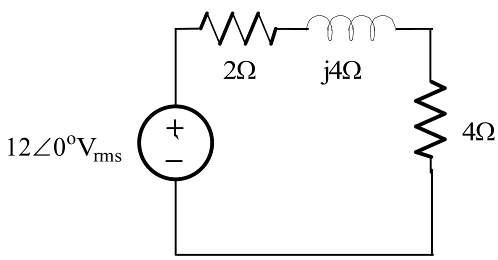


Figure PFE-4

Suggested Solution



$I = 12\angle 0° / (6 + j4) = 1.664\angle -33.69°$ Amps

$P = (1.664)^2(4) = 11.1W$

Problem 8FE-5

Determine the average power absorbed by the 1-Ohm resistor in the network in Figure 8PFE-5.

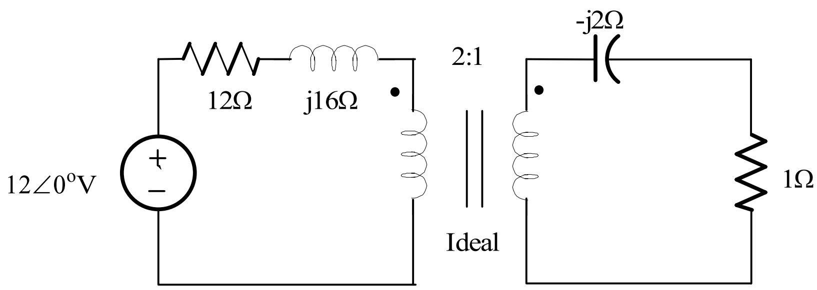
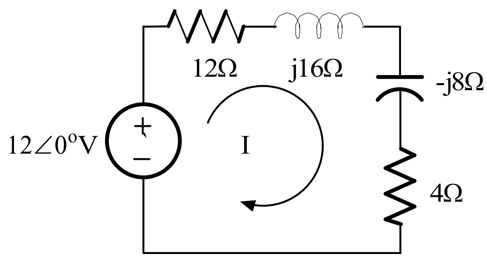


Figure PFE-5

Suggested Solution



$I = 12 / (16 + j8) = 0.07\angle{-26.57^{o}}$ Amps

$$P = 0.5(0.67)^2(4) = 0.9W$$

Problem 9.1

Determine the equations for the current and the instantaneous power in the network in Figure P 9.1

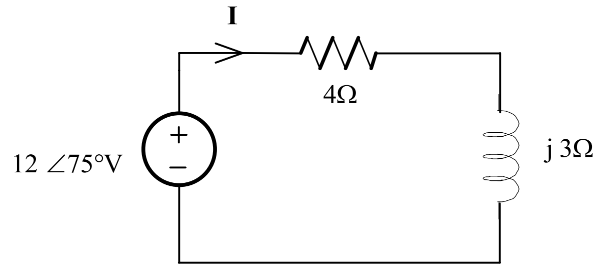


Figure P 9.1

Suggested Solution

$$I = \frac{12\angle 75^\circ}{4 + j3} = \frac{12\angle 75^\circ}{5\angle 36.90^\circ} = 2.4\angle 38.1^\circ A$$

$$L_{(t)} = 2.4\cos(\omega t + 38.1^\circ)A$$

$$P_{(t)} = \frac{12\times 2.4}{2}(\cos 36.9^\circ + \cos(2\omega t + 75^\circ + 38.1^\circ))$$

$$= 11.51 + 14.4\cos(2\omega t + 113.1^\circ)W$$

Problem 9.2

Determine the equations for the voltage and instantaneous Power in the network in Figure P 9.2

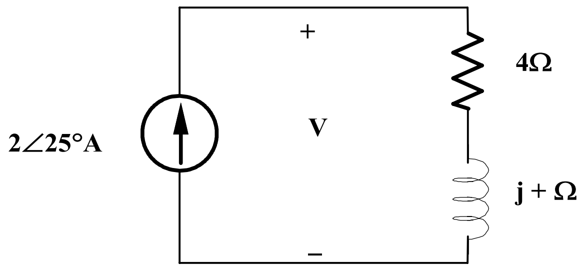


Figure P 9.2

Suggested Solution

$$V = IZ = (2\underline{|25^\circ})(4 + j4) = 11.3\underline{|70^\circ V}$$

$$L_{(t)} = 2\cos(\omega t + 25^\circ)A \,\&\, V_{(t)} = 11.3\cos(\omega t + 7025^\circ)(4 + j4) = 11.3\underline{|70^\circ})V$$

$$P_{(t)} = \frac{2\times 11.3}{2}[\cos 45^\circ + \cos(2\omega t + 95^\circ)]$$

$$= 8 + 11.3\cos(2\omega t + 95^\circ)W$$

Problem 9.3

Find the average power absorbed by the network shown in Figure P 9.3

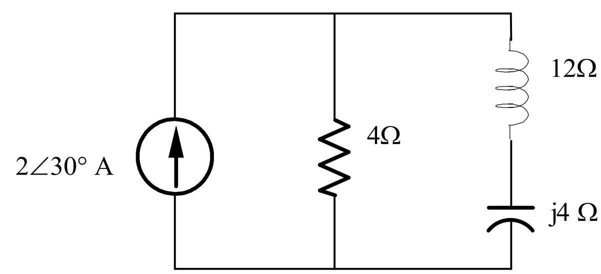


Figure P 9.3

Suggested Solution

Using Current division

$$I_R = \frac{(2\underline{|30^\circ})(-j4+j2)}{4-j4+j2} = .89\underline{|-33.43^\circ} A$$

Then

$$P = \frac{1}{2} I_M{}^2 R = \frac{1}{2}(.89^2)(4) = \boxed{1.58W}$$

Problem 9.4

Given the network in Fig. P 9.4, find the power supplied and the average power absorbed by each element.

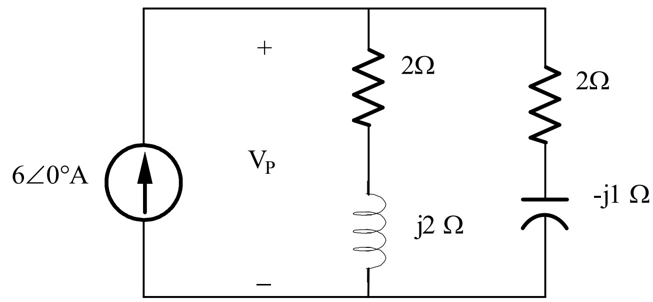


Figure 9.4

Suggested Solution

$$Z_T = \frac{(2+2j)(2-j)}{2+j2+2-j} = \frac{6+j2}{4+j}$$

$$V_T = I_T Z_T = (6\angle 0^\circ)\left(\frac{6.32\angle 18.43^\circ}{4.12\angle 14.04^\circ}\right) = 9.2\angle 4.39^\circ V$$

$$P_S = \frac{1}{2}(9.2)(6)\cos(4.39^\circ - 0^\circ) = 27.52W$$

$$I_1 = \frac{(6\angle 0^\circ)(2-j)}{4+j1} = 3.26\angle -40.61^\circ A$$

$$I_2 = \frac{(6\angle 0^\circ)(2+j2)}{4+j1} = 4.12\angle 30.96^\circ A$$

$$P_A = \frac{1}{2}I_1^2 R_1 + \frac{1}{2}I_2^2 R_2 = \frac{1}{2}[(3.26)^2(2) + (4.12)^2(2)]$$

$$P_A = \boxed{27.61W}$$

Problem 9.5

Given the circuit in Fig. 9.5, find the average power supplied and the average power absorbed by each element.

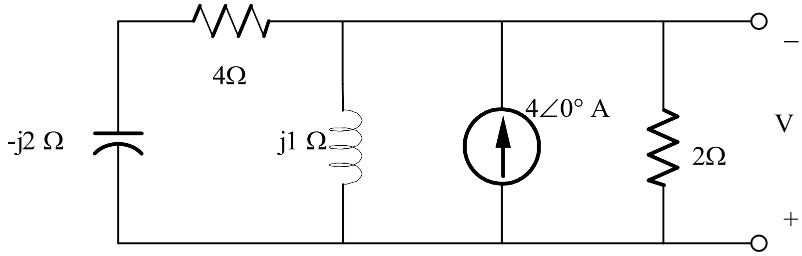
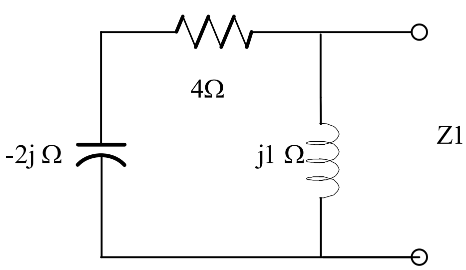


Figure P 9.5

Suggested Solution



$Z_1 = [(4 - j2)(j1)]/ (4-j2 +j1) = (2+ j4) / (4-j)$

$$Z_T = \frac{2(Z_1)}{2+Z_1} = \frac{4+j8}{10+j2}$$

$$V = (4\angle 0^\circ)(Z_T) = 3.51\angle 52.12^\circ V$$

$$P_S = \frac{1}{2}(4)(V)\cos(52.12^\circ - 0^\circ) = 4.31W$$

$$I_{2\Omega} = \frac{4\angle 0^\circ (Z_1)}{Z_1+2} = 1.75\angle 52.12^\circ A$$

$$I_{4\Omega} = \frac{4\angle 0^\circ (2)}{Z_1+2}\left(\frac{j1}{4-j}\right) = .78\angle 78.69^\circ A$$

$$P_{2\Omega} = \frac{1}{2}I_{2\Omega}^2(2) = \frac{1}{2}(1.75)^2(2) = \boxed{3.06W}$$

$$P_{4\Omega} = \frac{1}{2}I_{2\Omega}^2(4) = \frac{1}{2}(.78)^2(4) = \boxed{1.23W}$$

Problem 9.6

Compute the average power absorbed by the elements to the right of the dashed line in the network shown in Fig. P 9.6

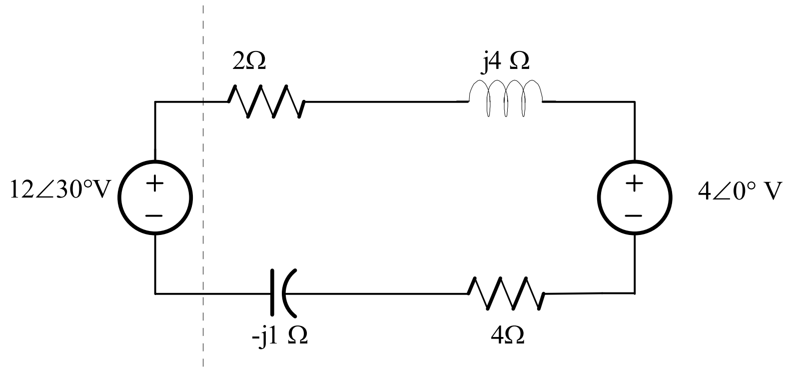
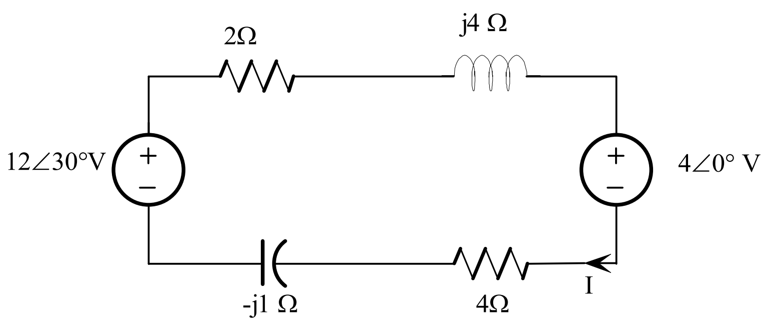


Figure P 9.6

Suggested Solution



$P_L = 0$ W $\quad P_C = 0$ W

$P_{R1} = (I^2R_1)/2 \quad P_{R1} = (I^2R_2)/2$

$$II = \frac{12\angle 30 - 4\angle 0}{2+4+j4-j1} = \frac{12\angle 30 - 4\angle 0}{6+j3} = \frac{8.77\angle 43.19^\circ}{6.71\angle 26.57^\circ} A$$

$$II = 1.31\angle 16.62^\circ A \Rightarrow I = 1.31A$$

$$P_{R1} = (\frac{1}{2})(1.31)^2(2) = 1.72W$$

$$P_{R1} = (\frac{1}{2})(1.31)^2(4) = 3.43W$$

$$\boxed{P_L = 0W}\ \boxed{P_C = 0W}\ \boxed{P_{R1} = 1.72W}\ \boxed{P_{R2} = 3.43W}$$

Problem 9.7

Given the network in Fig. 9.7, determine which elements are supplying power, which ones are absorbing power, and how much power is being supplied and absorbed.

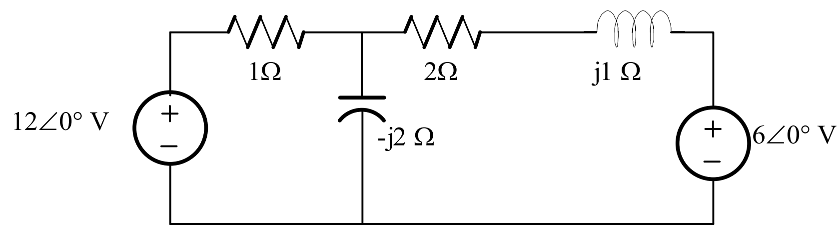


Figure P 9.7

Suggested Solution

The mesh currents are obtained from the equations:

$$(I - j2)I_1 + j2I_2 = 12\angle 0^\circ$$

$$j2I_1 + (2 - j)I_2 = -6\angle 0^\circ$$

$$or\ \begin{bmatrix} I1 \\ I2 \end{bmatrix} = \frac{1}{4 - j5}\begin{bmatrix} 2 - j & -j2 \\ -j2 & 1 - j2 \end{bmatrix}\begin{bmatrix} 12\angle 0^\circ \\ -6\angle 0^\circ \end{bmatrix}$$

$$= \begin{bmatrix} 3.75\angle 51.39^\circ \\ 2.1\angle -65.16^\circ \end{bmatrix}$$

$$P_{12\angle 0^\circ} = \frac{1}{2}(12)(3.75)\cos(0 - 51.39^\circ) = \boxed{14.04W \ \ \text{Sup}}$$

$$P_{1\Omega} = \frac{1}{2}(3.75)^2(1) = \boxed{7.03W \ \ \text{Abs}}$$

$$P_{2\Omega} = \frac{1}{2}(2.10)^2(2) = \boxed{4.41W \ \ \text{Abs}}$$

$$P_{6\angle 0^\circ} = \frac{1}{2}(6)(2.10)\cos(0 + 65.16^\circ) = \boxed{2.65W \ \ \text{Abs}}$$

Problem 9.8

Given the circuit in Fig P 9.8 find the average power absorbed by the network.

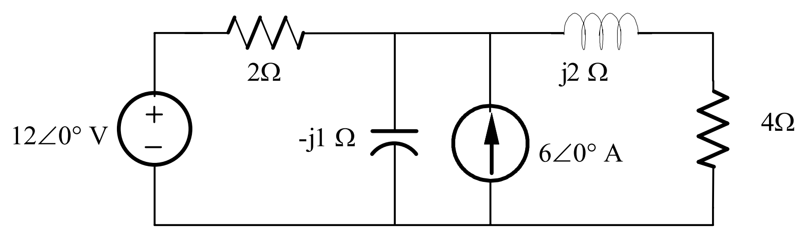
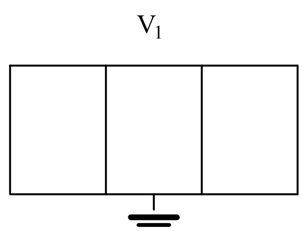


Figure P 9.8

Suggested Solution



The nodal equation is:

$(V_1 - 12)/2 + V_1/(-j) + V_1/(4 + j2) = 6\angle 0^\circ$

$$\text{Solving yields: } V_1 = 6.46 - j8.3V = 10.52\angle -52.12^\circ V$$

$$I_{\rightarrow 2\Omega} = \frac{12 - 6.46 + j8.3}{2} = 4.99\angle 56.28^\circ A$$

$$I_{4\Omega} = \frac{10.50\angle -52.12^\circ}{4 + j2} = 2.35\angle -78.69^\circ A$$

$$P_S = \frac{1}{2}(12)(4.99)\cos(0 - 56.28^\circ) + \frac{1}{2}(10.52)(6)\cos(-52.12^\circ - 0^\circ)$$

$$= 16.62 + 19.38 = \boxed{36W}$$

$$P_{abs} = \frac{1}{2}(4.99)^2(2) + \frac{1}{2}(2.35)^2(4)$$

$$= 24.9 + 11.05 = \boxed{35.95W}$$

Problem 9.9

Given the network in Fig 9.9, find the average power supplied to the circuit.

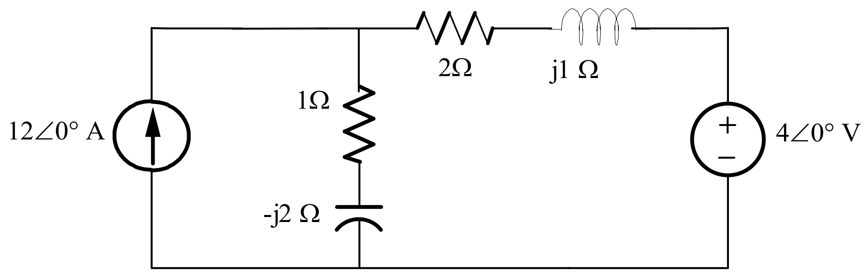
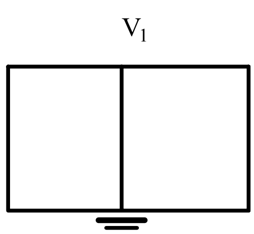


Figure P 9.9

Suggested Solution



$V_1/(1-j2) + (V_1 - 4)/(2 + j1) = 12$

$\therefore V_1 = 21.54\ \angle -21.81°\ V$

$$I_{1\Omega}\downarrow = \frac{21.54\angle -21.81^\circ}{1-j^2} = 9.63\angle 41.62^\circ A$$

$$I_{2\Omega}\rightarrow = \frac{21.54\angle -21.81^\circ - 4}{2+j1} = 8\angle -53.1^\circ A$$

$$P_S = \frac{1}{2}(12)(21.54)\cos(-21.81^\circ - 0^\circ) = 119.99W$$

$$P_{abs} = \frac{1}{2}(9.36)^2(1) + \frac{1}{2}(8)^2(2) + \frac{1}{2}(4)(8)\cos 53.13^\circ$$

$$= 46.37 + 64 + 9.6 = \boxed{119.97W}$$

Problem 9.10

Given the circuit in Fig. P 9.10, determine the amount of average power supplied to the network.

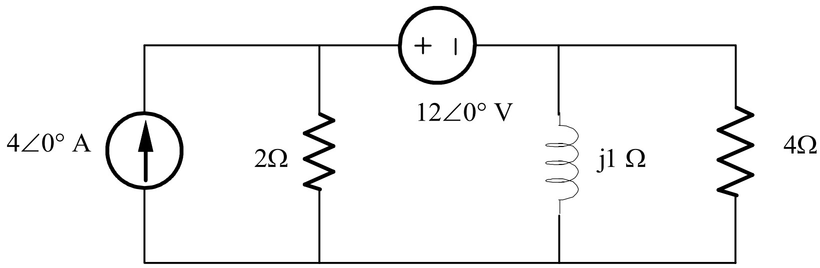
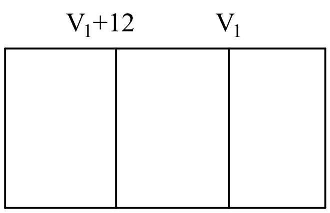


Figure P 9.10

Suggested Solution



$(V_1 + 12)/ 2 + V_1 / j + V_1/ 4 = 4$

$\therefore V_1 = (-8)/ (3 - 4j) = -1.6\angle 53.13° \text{ V}$

$$I_{2\Omega}\downarrow = \frac{V_1+12}{2} = 5.56\angle -6.61^\circ A$$

$$I_{4\Omega} = \frac{V_1}{2} = -.4\angle 53.13^\circ A$$

$$I_{12\angle 0^\circ} = I_{2\Omega} - 4\angle 0^\circ = 1.65\angle -22.83^\circ A$$

$$P_S = \frac{1}{2}(12)(1.65)\cos 22.83^\circ + \frac{1}{2}(11.11)(4)\cos(-6.61^\circ) = 31.19W$$

$$P_{abs} = \frac{1}{2}(5.56)^2(2) + \frac{1}{2}(.4)^2(4) = 30.91 + .32 = \boxed{31.23W}$$

Problem 9.11

Determine the average power absorbed by the 4-Ω resistor in the network shown in Fig. P 9.11

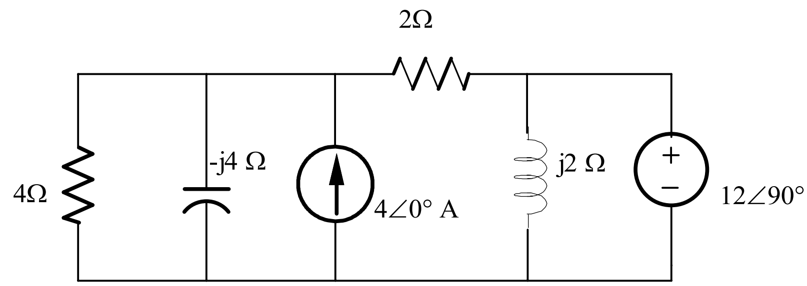


Figure P 9.11

Suggested Solution

Let V1 be the voltage across the 4Ω resistor.

$$\frac{V_1}{4}+\frac{V_1}{-j4}+\frac{V_1-12\angle 90^\circ}{2}=4$$

$$\therefore V_1 = 9.13\angle 37.88^\circ V$$

$$I_{4\Omega}=\frac{V_1}{4}=2.28\angle 37.88^\circ A$$

$$P_{4\Omega}=\frac{1}{2}(2.28)^2(4)=\boxed{10.4W}$$

Problem 9.12

Given the network in Fig. P 9.12, show that the power supplied by the source is equal to the power absorbed by the passive elements.

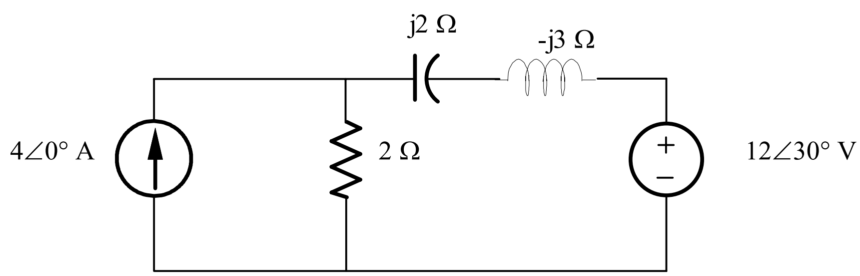
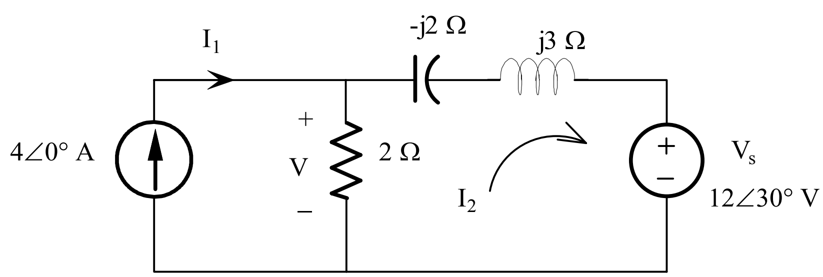


Figure P 9.12

Suggested Solution



Mesh Equations

$I_1 = 4\angle 0^\circ A$

$-12\angle 30^\circ = I_2(2 + j1) - 2I_1$

Find I_2

$$I_2 = \frac{2I_1 - 12\angle 30^\circ}{2 + j1} = \frac{8\angle 0^\circ - 12\angle 30^\circ}{2 + j1} = 2.89\angle -138.30^\circ A$$

Find V

$V = (I_1 - I_2)(2) = (4\angle 0^\circ - 2.89\angle -138.30^\circ)(2) = 12.90\angle 17.30^\circ V$

Power Delivered by current source

$$P_I = \frac{1}{2} V I_1 \cos\theta = \frac{1}{2}(12.90)(4)\cos(17.35 - 0) =$$

$\boxed{P_I = 24.63W}$

Power Delivered by voltage source

$$P_V = -\frac{1}{2} V_S I_2 \cos\theta = -\frac{1}{2}(12)(2.89)\cos(30 - (-138.3)) =$$

$\boxed{P_V = +16.98W}$

Power consumed by R

$P_L = 0W$ $\qquad\qquad$ $P_C = 0W$

$$P_V = \frac{1}{2}V^2 / R = \frac{1}{2}\frac{12.90^2}{2}$$

Note $P_R = P_I + P_V$

$$\boxed{P_R = 41.60W}$$

Problem 9.13

Calculate the average power absorbed by the 1-Ω resistor in the network shown in Fig. P 9.13

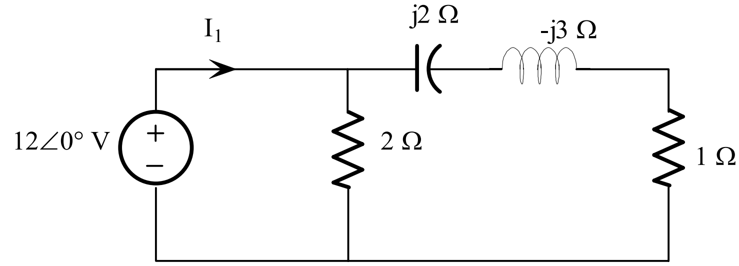
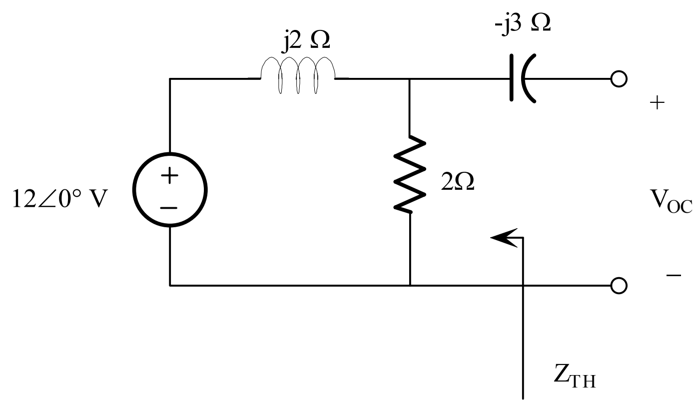


Figure P9.13

Suggested Solution

Therenin's Equation



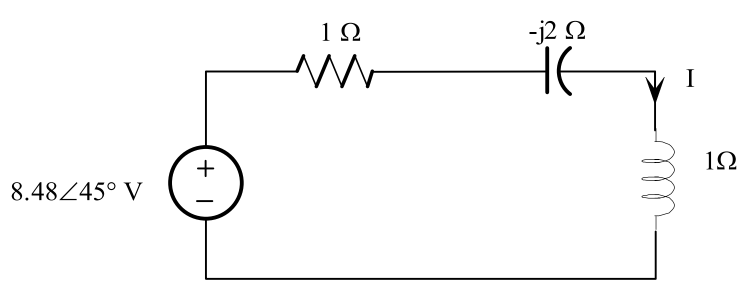
$$V_{OC} = 12\angle 0^\circ \left[\frac{2}{2+j2}\right]$$

$$V_{OC} = \frac{24\angle 0^\circ}{2\sqrt{2}\angle 45^\circ} \qquad V_{OC} = 8.49\angle -45^\circ V$$

$$Z_{TH} = -j3 + (2 \,//\, j2)$$

$$Z_{TH} = -j3 + \frac{j4}{2+j2} = -j3 + \frac{4\angle 90^\circ}{2\sqrt{2}\angle 45^\circ} = -j3 + \sqrt{2}\angle 45^\circ = -j3 + 1 - j1$$

$$Z_{TH} = (1 - j2)\Omega$$



$P_R = I^2R/2$

$$I = \frac{8.49\angle -45^\circ}{2 - j2} = \frac{8.49\angle -45^\circ}{2\sqrt{2}\angle -45^\circ} = 3\angle 0^\circ\, A$$

$$P_R = \frac{1}{2} I^2 R = (\frac{1}{2})(3)^2(1) = \boxed{4.5W}$$

Problem 9.14

Determine the average power absorbed by the 4-Ω resistor in the network shown in Fig 9.14

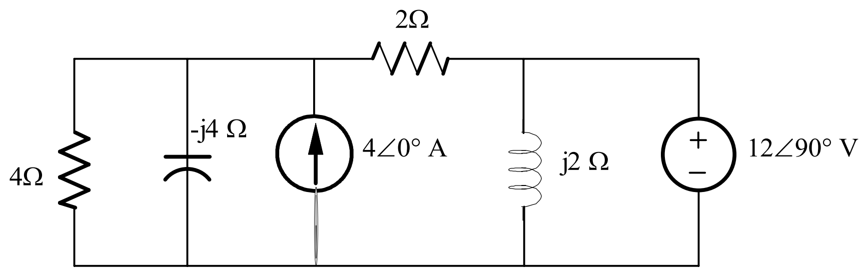
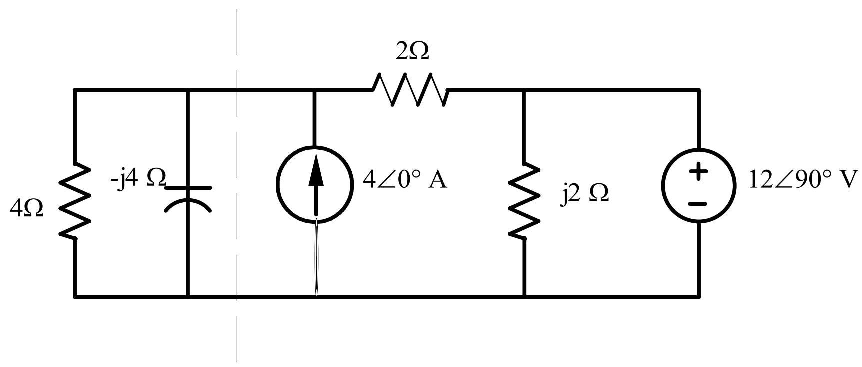
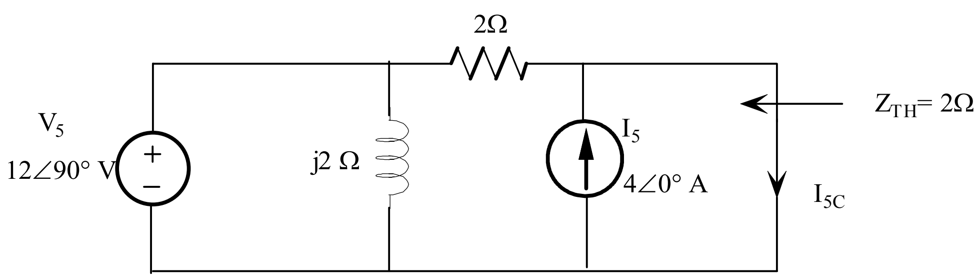


Figure P 9.14

Suggested Solution



Norton's Equivalent:



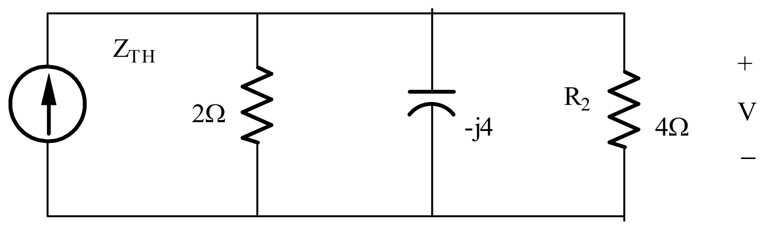
Find I_{5C} by superposition

I_{5C1} due to I_5: $I_{5C1} = I_5 = 4\angle 0^\circ A$

I_{5C2} due to V_5: $I_{5C2} = \frac{V_5}{2} = 6\angle 90^\circ A$

$I_{5C} = I_{5C1} + I_{5C2} = 4 + j6A = 7.21\angle 56.31^\circ$

Norton Equation of Circuit



I_{5C}=7.21∠56.31° A

$$V = I_{5C}[2 // 4 // -j4] = 9.12\angle 37.90^\circ V$$

$$P_{4\Omega} = \frac{1}{2}\frac{|V|^2}{R_2} = 10.40W$$

$$\boxed{P_{4\Omega} = 10.40W}$$

Problem 9.15

Find the average power absorbed by each element in the network shown in Fig. P 9.15

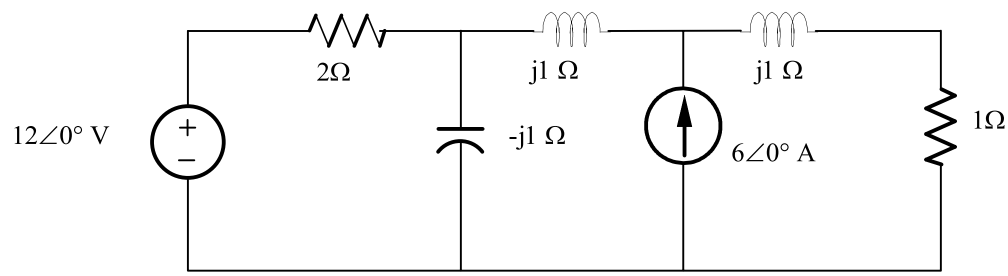
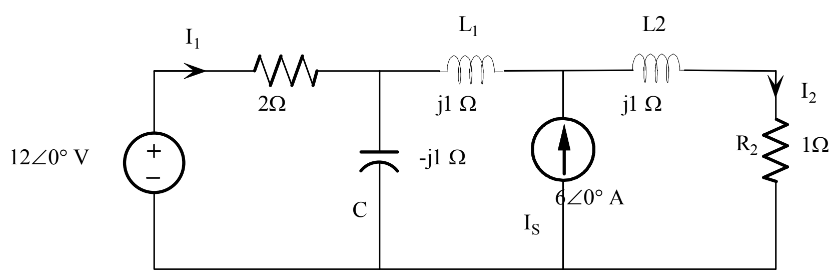


Figure P 9.15

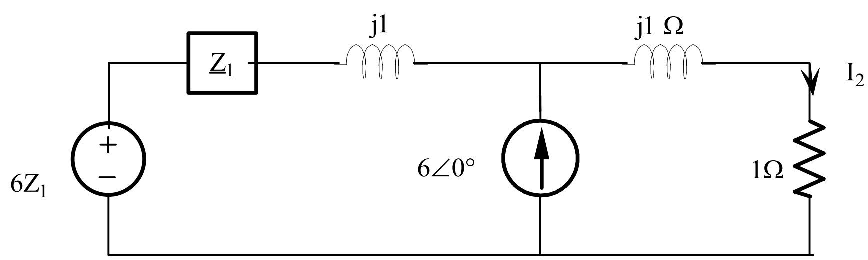
Suggested Solution

Find I_2 using source transformation

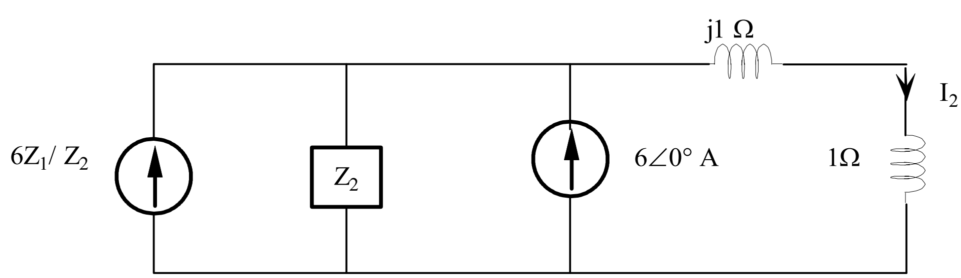


$P_C = 0$ W
$P_{L1} = P_{L2} = 0$ W

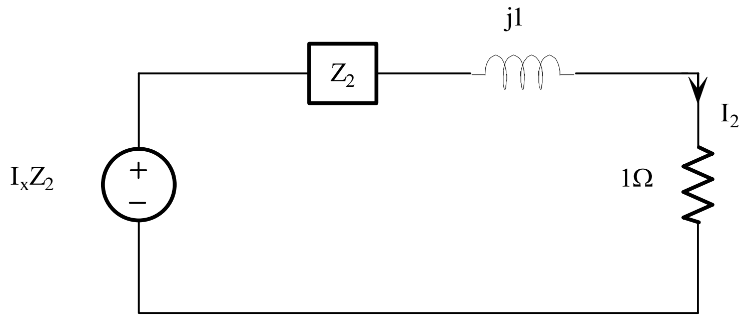
Let $Z_1 = (2 // -j1)$



Let $Z_2 = Z_1 + j1$



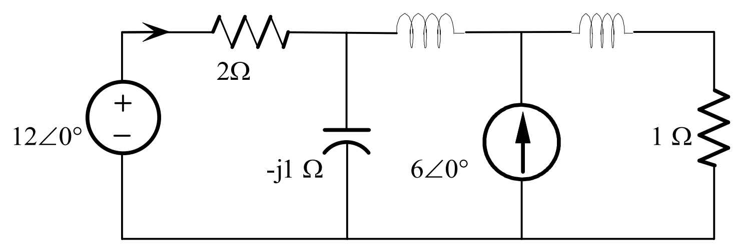
Let $I_x = 6Z_1/Z_2 + 6\angle 0°$



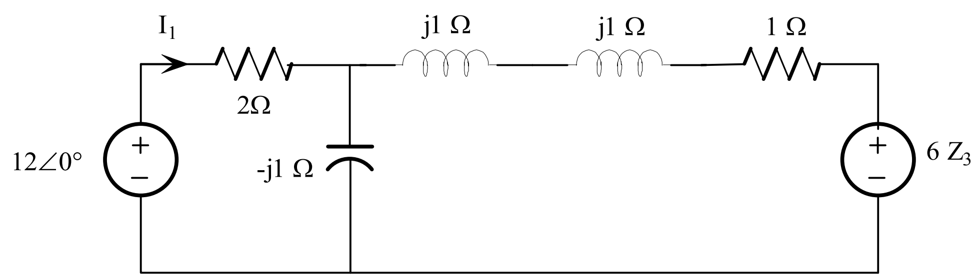
$$I_2 = \frac{I_x Z_2}{Z_2 + j1 + 1}$$

$$I_2 = 3.26\angle -77.4^\circ A$$

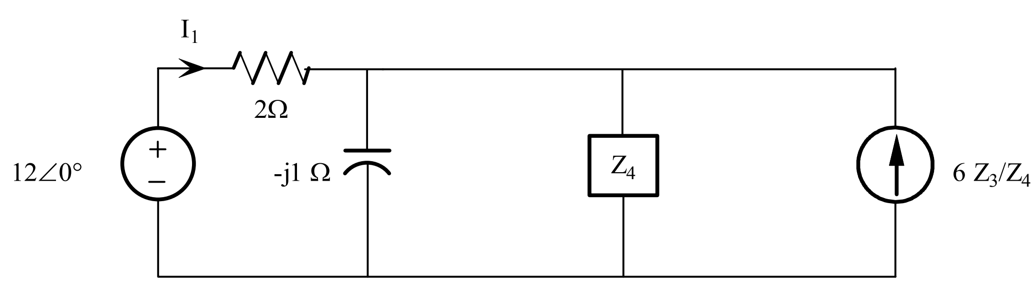
Finding I1 using source transformation:



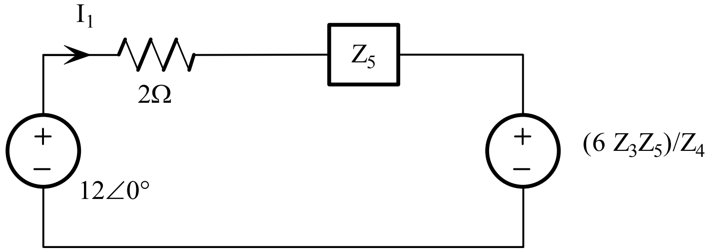
Let $Z_3 = 1 + j1\ \Omega$



Let $Z_4 = 1 + j2\ \Omega$



Let $Z_5 = Z_4//(-j1)$



$$I_1 = \frac{12 - \frac{6Z_3Z_5}{Z_4}}{2 + Z_5}$$

$$I_1 = 4.60\underline{|57.53^\circ} A$$

Power Absorbed by Each Element

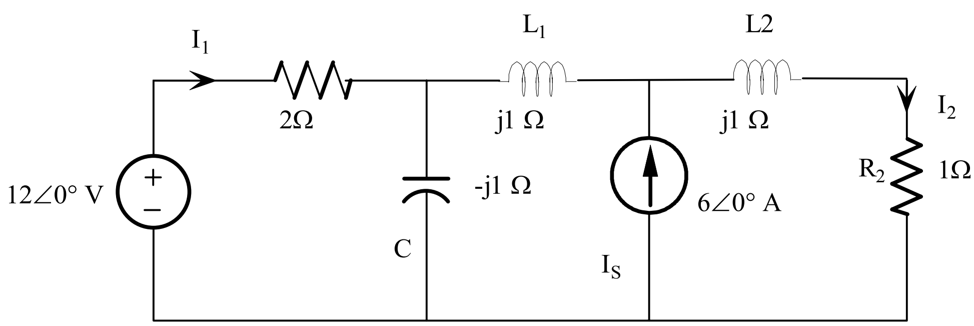
$$P_{R1} = \frac{1}{2}I_1^2R_1 = \frac{1}{2}(4.6)^2(2) = P_{R1} = 21.16W$$

$$P_{R2} = \frac{1}{2}I_2^2R_2 = \frac{1}{2}(3.26)^2(1) = P_{R2} = 5.31W$$

$$P_{VS} = \frac{1}{2}V_SI_1\cos(0 - 57.53^\circ) = \frac{1}{2}(12)(4.6)(.54)$$

$= 14.82W$ supplied

$P_{IS} = -P_{VS} + P_{R1} + P_{R2} = 11.65W$ supplied



$P_C = 0$ W
$P_{L1} = P_{L2} = 0$ W

$$P_{R1} = \frac{1}{2}I_1^2R_1 = \frac{1}{2}(4.6)^2(2) = P_{R1} = 21.16W$$

$$P_{R2} = \frac{1}{2}I_2^2R_2 = \frac{1}{2}(3.26)^2(1) = P_{R2} = 5.31W$$

$$P_{VS} = \frac{1}{2}V_SI_1\cos(0 - 57.53^\circ) = \frac{1}{2}(12)(4.6)(.54) = 14.82W \text{ supplied}$$

$$P_{IS} = -P_{VS} + P_{R1} + P_{R2} = 11.65W \text{ supplied}$$

Problem 9.16

Determine the average power supplied to the network shown in Fig. P 9.16

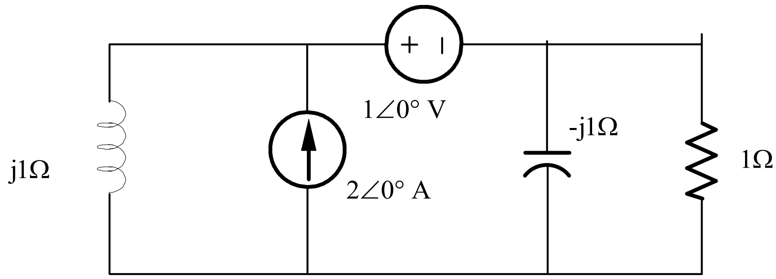
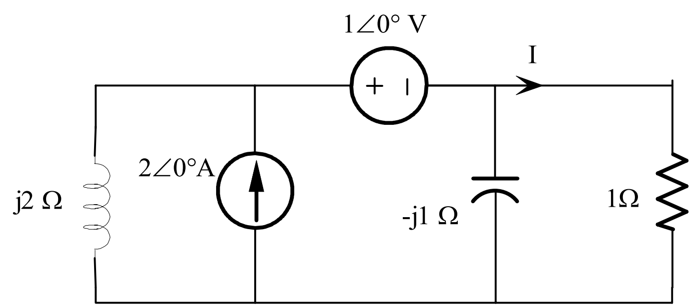
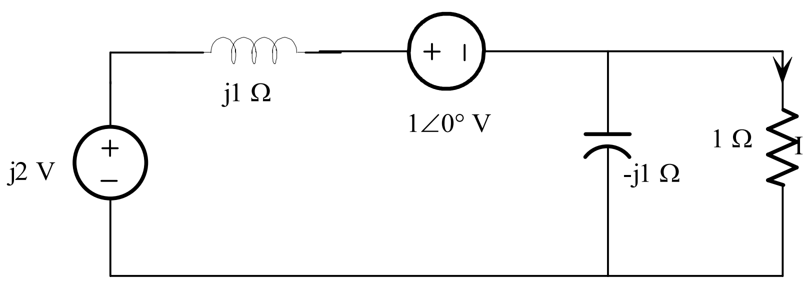


Figure P 9.16

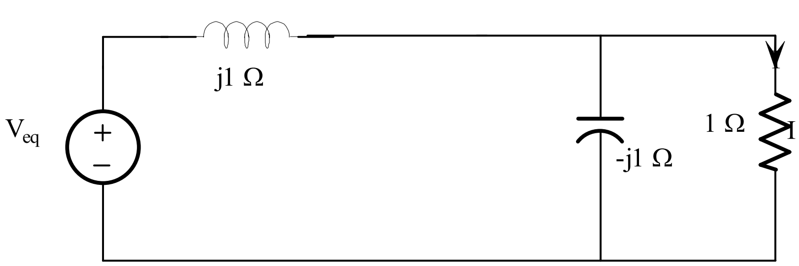
Suggested Solution



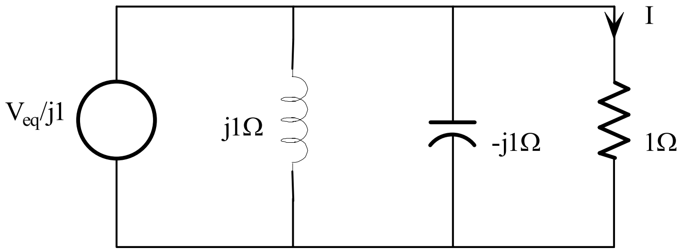
Find I using source transformation

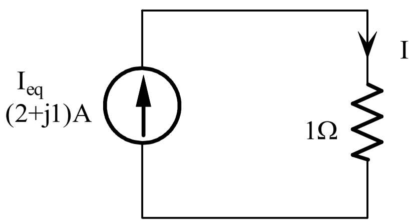


Let $V_{eq} = -1 + j2$ V



Another source transformation





$P_R = (1|I_{eq}|^2 R)/2$

$|I_{eq}| = \sqrt{5}$

$P_R = 2.5$ W

Problem 9.17

Determine the average power supplied to the network in Fig 9.17

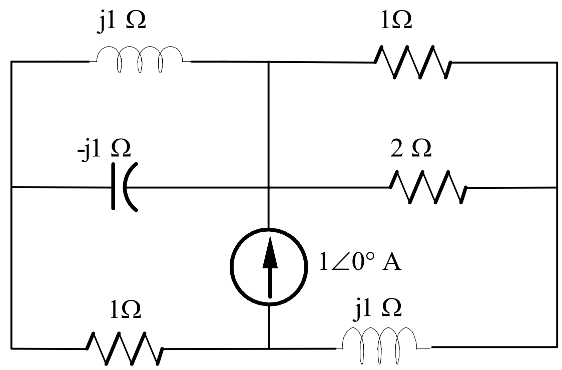
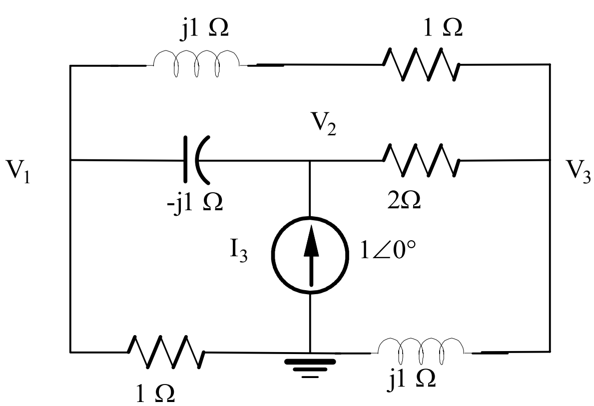


Figure P 9.17

Suggested Solution



Nodal Equations

$$@\ V_2:\ \ 1 = \frac{V_2 - V_1}{-j1} + \frac{V_3 - V_2}{2}$$

$$or\ \ 2 = V_2(2 + j2) - j2V_1 - V_3 \qquad (1)$$

$$@\ and:\ \ 1 = \frac{V_1}{1} + \frac{V_3}{j1}$$

-

$$or\ \ 1 = V_1 - j1V_3 \qquad (2)$$

$$V_1 = 1 + j1V_3$$

$$@V_1:\ \ \frac{V_3 - V_1}{1 + j1} = \frac{V_2 - V_1}{-j1} + \frac{V_1}{1}$$

Solve (2) for V_1 and substitute into (1) and (3)

$$2 = V_2(2+j2) - j2 + 2V_3 - V_3 \Rightarrow 2+j2 = V_2(2+j2) + V_3 \quad (4)$$

$$V_3 = 1+j2+V_3(-2+j1)+V_2(1-j1) \Rightarrow 1+j2 = -V_2(1-j1)+V_3(3-j1) \quad (5)$$

Solve (4) for V_3 and substitute into (5)

$$(1+j2) = -V_2(1-j1)+(3-j1)[2+j2-2V_2-j2V_2]$$

$$or \quad V_2 = \frac{7+j2}{9+j7} = 0.64\angle -21.93^\circ V$$

$$P_{IS} = \frac{1}{2}|I_S|\times|V_2|\cos(-21.93-0) = 0.297W$$

Power delivered = 0.297 W

Problem 9.18

Find the average power absorbed by the 2-Ω resistor in the circuit shown in Fig 9.18

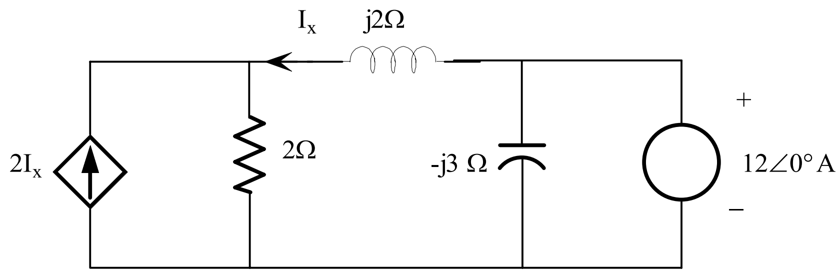
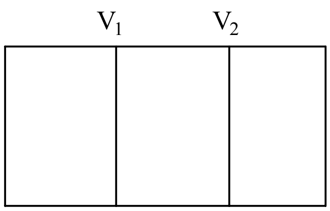


Figure P 9.18

Suggested Solution



$$\frac{V_1}{2}+\frac{V_1-V_2}{j2}-2I_x=0$$

$$I_x=\frac{-(V_1-V_2)}{j2};\quad V_2=12\angle 0^\circ V$$

Solving yields: $V_1=\dfrac{18\angle -90^\circ}{1.58\angle -71.57^\circ}=11.39\angle -18.43^\circ V$

$$I_R=\frac{V_1}{2}=5.7\angle -18.43^\circ A$$

$$P_R=\frac{1}{2}(5.7)^2(2)=\boxed{32.49W}$$

Problem 9.19

Given the network in Fig P 9.19 determine which elements are supplying power, which ones are absorbing power, and how much power is being supplied and absorbed.

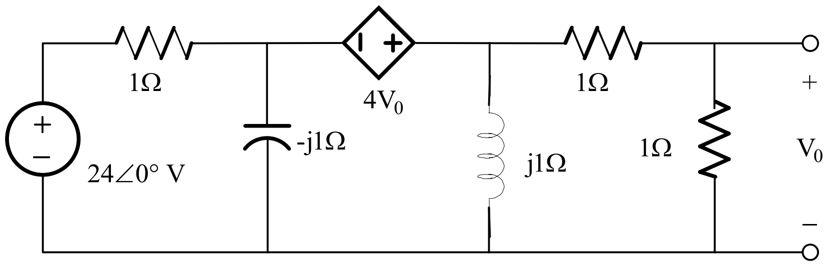
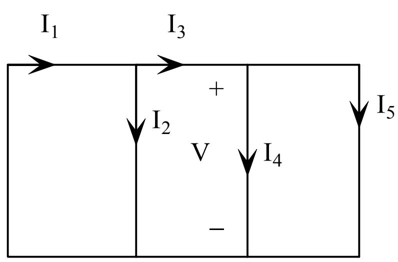


Figure P 9.19

Suggested Solution



$$\frac{V_1 - 4V_0 - 24\angle 0^\circ}{1} + \frac{V - 4V_0}{-3} + \frac{V}{j} + \frac{V}{2} = 0$$

Where $V_0 = \dfrac{V}{2}$

Solving for V yields: $V = 11.65\angle 104.04^\circ V$

$V - 4V_0 = -11.65\angle 104.04^\circ V$ SO

$$I_1 = \frac{24 + V}{1} = 24\angle 28.09^\circ A$$

$$I_2 = \frac{-V}{1\angle -90} = 11.3 + j2.83A$$

$$I_3 = I_1 - I_2 = 13.01\angle 40.63^\circ A$$

$$I_4 = \frac{V}{1\angle 90^\circ} = 11.3 + j2.83A$$

$$I_5 = \frac{V}{2} = -1.41 + j5.66A$$

Then $P_{abs} = \dfrac{1}{2}(24)^2(1) = 288W$

$$P_{2\Omega} = \frac{1}{2}(5.83)^2(2) = 34W$$

$$P_{\sup} = P_{24\angle 0^\circ} + P_{4V_0}$$

$$= \frac{1}{2}(24)(24)\cos(-28.09)$$

$$+\frac{1}{2}(2)(11.56)(13.01)\cos(104.04 - 40.63)$$

$$= 67.85 + 254 = \boxed{321.85W}$$

Problem 9.20

Determine the impedance Z_L for maximum average power transfer and the value of the maximum power transferred to the Z_L for the circuit shown in Fig P 9.20

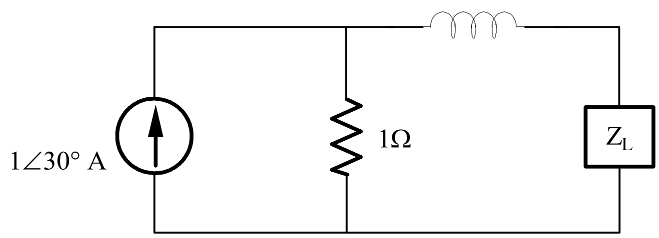
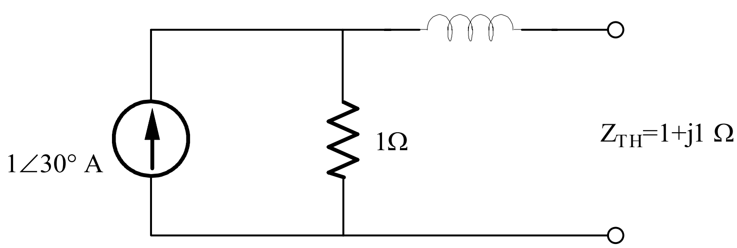


Figure P 9.20

Suggested Solution



$$Z_L = Z_{TH}^* = 1 - j1\Omega$$

$$\boxed{Z_L = 1 - j1\Omega}$$

$$V_{OC} = (1\angle 30^\circ)(1\angle 0^\circ) = 1\angle 30^\circ V$$

$$\boxed{P_{MAX} = \frac{1}{2}\left(\frac{V_{OC}}{2R_{TH}}\right)R_L = 0.125W}$$

Problem 9.21

Determine the impedance Z_L for maximum average power transfer and the value of the maximum average power transferred to Z_L for the circuit shown in Fig P 9.21

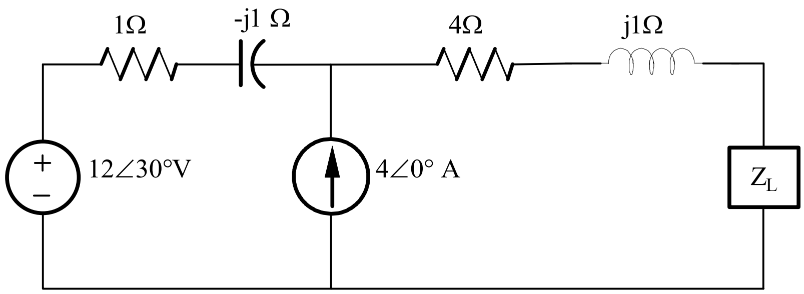
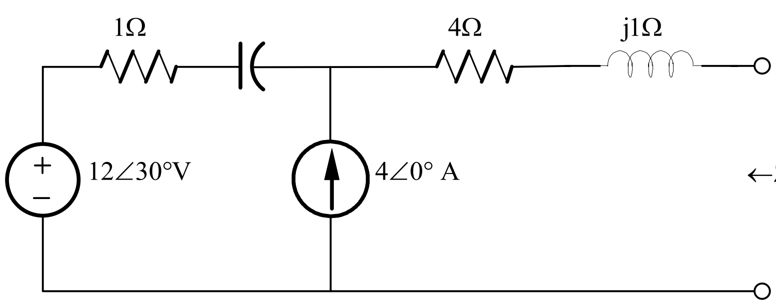
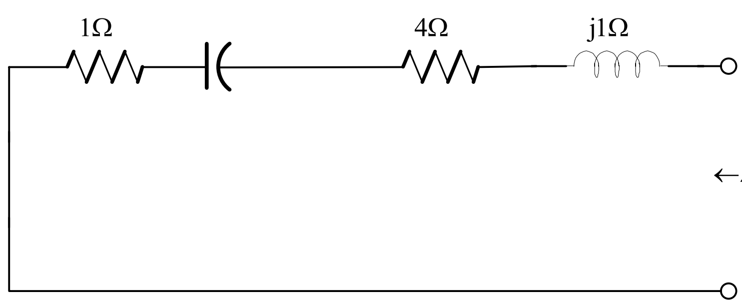


Figure P 9.21

Suggested Solution



$\leftarrow Z_{TH} = 12\angle 30° + 4\angle 0°(1\text{-}j)$
$= 14.53$



$\leftarrow Z_{TH} = 5\Omega$
$\therefore Z_L = 5\Omega$

$$P_L = \frac{1}{2}\left(\frac{14.53}{10}\right)^2(5) = 5.28W$$

Problem 9.22

Determine the impedance Z_L for maximum average power transfer and the value of the maximum average power absorbed by the load in the network shown in Fig P 9.22

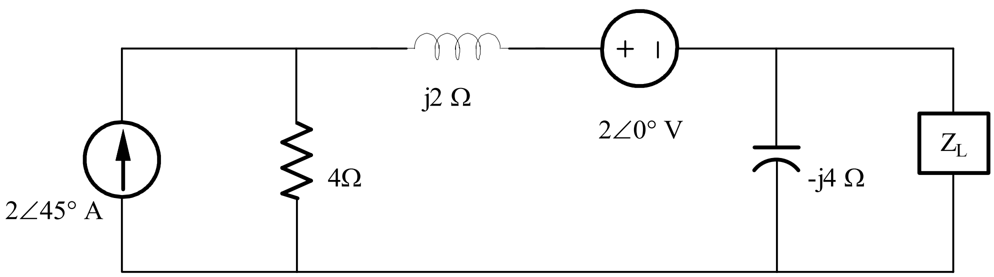
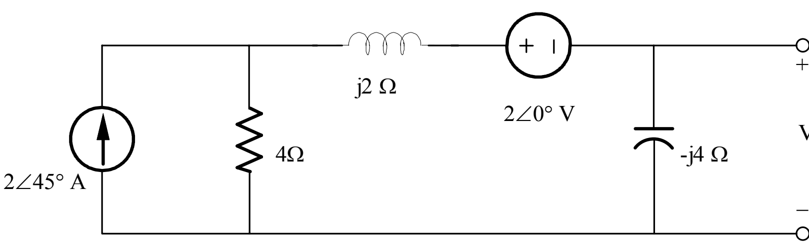
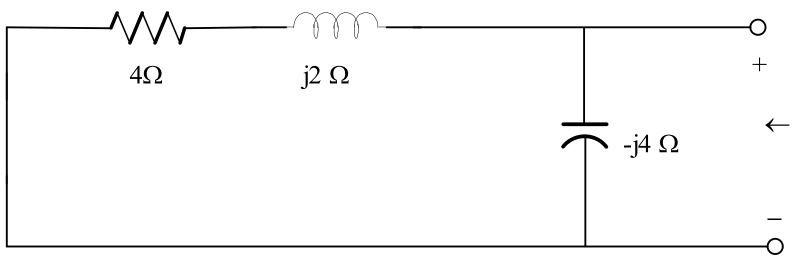


Figure P 9.22

Suggested Solution



V_{OC}= [(8∠45°- 2∠0°)/ (4- j^2)](-j4)
=6.03 ∠6.32° V



← Z_{TH}= (4+ j^2)(-j4) / (4- j^2)
=3.2 - j2.4Ω

$\therefore Z_L = 3.2 + j2.4$ and $P_L = \frac{1}{2}(\frac{6.03}{3.2+3.2})^2(3.2) = 1.42W$

Problem 9.23

Determine the impedance Z_L for maximum average power transfer and the value of the maximum average power absorbed by the load in the network shown in Fig P 9.23

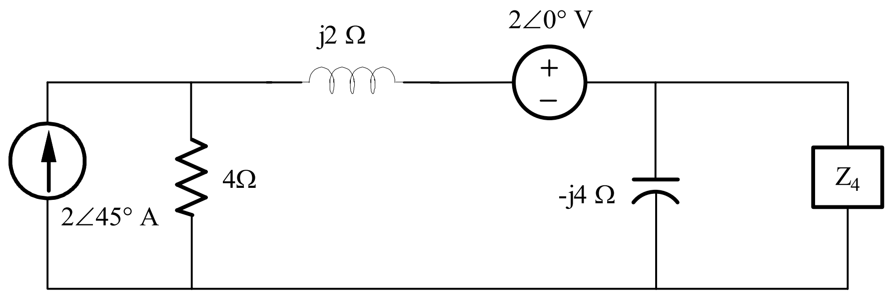
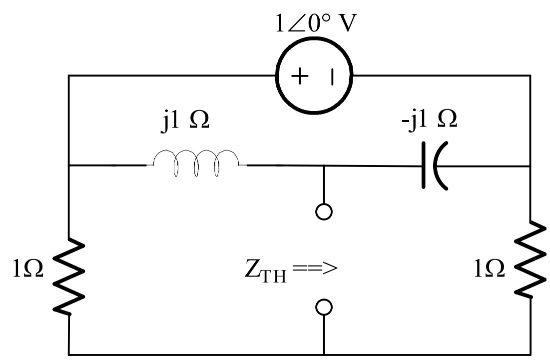
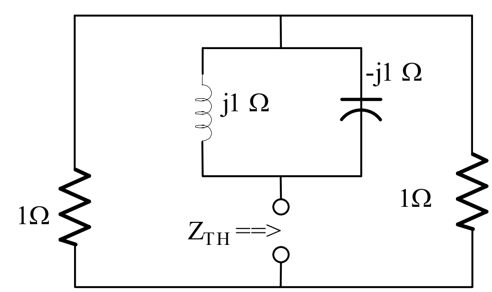


Figure P 9.23

Suggested Solution



==>



$$Z_{TH} = [j1//(-j1)] + (1//1) = \frac{1}{j1 - j1} + \frac{1}{2} = \infty$$

$\boxed{Z_{TH} = \infty}$

$\boxed{P_{Max} = 0}$

Problem 9.24

Determine the impedance Z_L for maximum average power transfer and the value of the maximum average power absorbed by the load in the network shown in Fig P 9.24

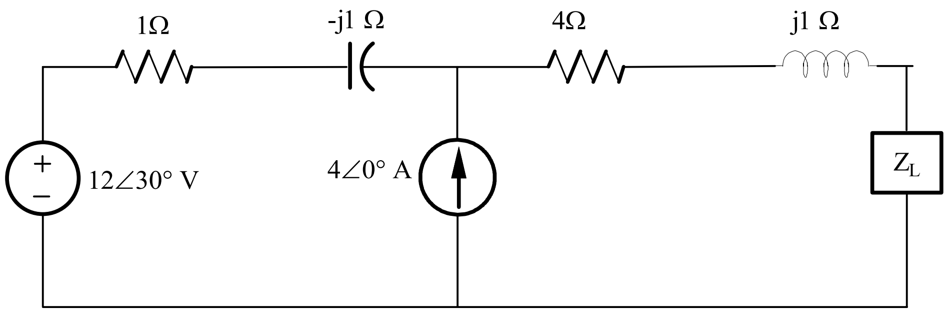
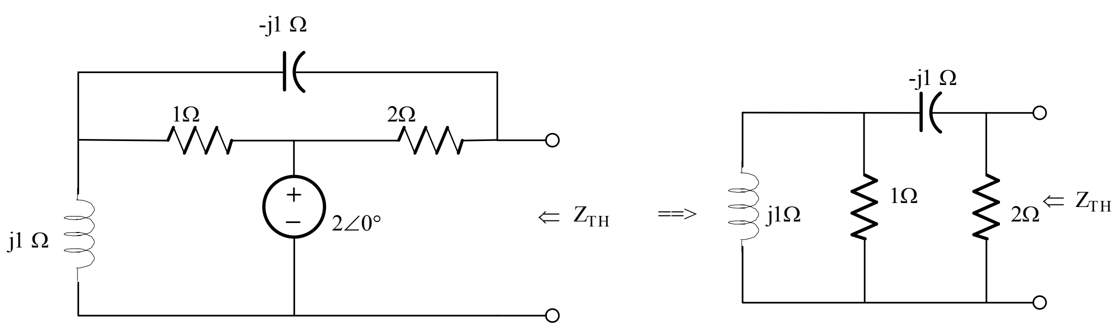


Figure P 9.24

Suggested Solution



$$Z_{TH} = [(1//j1) - j1]//2$$

$$= [\frac{j1}{1+j1} - j1]//2 = (\frac{1}{1+j1})//2 = \frac{2/(1+j1)}{2+1/(1+j1)} = \frac{2}{1+2+j2}$$

$$Z_{TH} = \frac{2}{3+j2} = 0.55\angle{-33.69^\circ}\Omega$$

$$\boxed{Z_{TH} = 0.55\angle{-33.69^\circ}\Omega}$$

Find V_{OC}

$$Z_S = j1 + 1//(2-j1) = 1.14\angle{52.1^\circ}\Omega$$

$$I_S = \frac{2\angle{0^\circ}}{1.14\angle{52.1^\circ}} = 1.75\angle{-52.1^\circ} A$$

$$V_{OC} = V_S - I_{2\Omega}(2\angle 0^\circ)$$

$$= 2(1 - 0.56\angle -33.7^\circ)$$

$$= 1.24\angle 29.7^\circ V$$

$$\therefore \boxed{P_{Max} = \frac{(1.24)^2}{(8)(0.46)} = 0.42W}$$

Problem 9.25

Determine the impedance Z_L for maximum average power transfer and the value of the maximum average power absorbed by the load in the network shown in Fig P 9.25

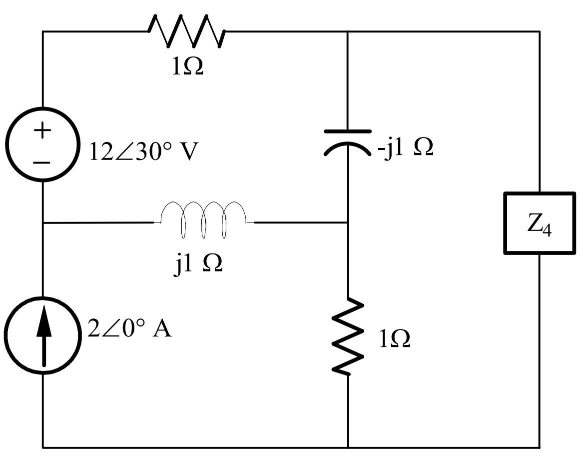
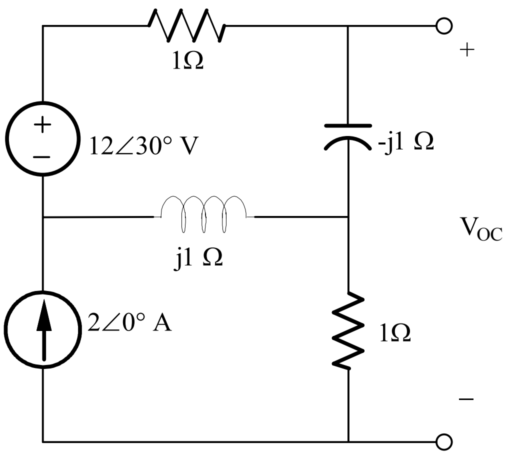


Figure P 9.25

Suggested Solution



$$12\angle 30^\circ - (1-j)I_1 - j(I_1 - I_2) = 0$$

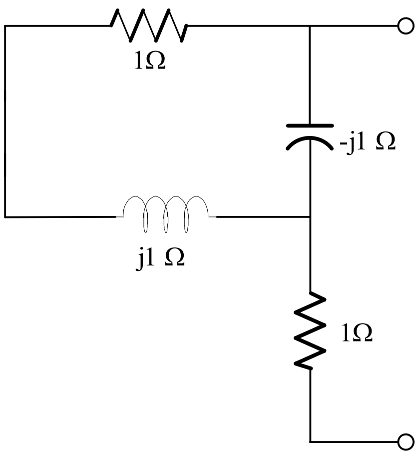
$$I_2 = 2\angle 0^\circ A$$

$$\therefore I_1 = 13.11\angle 37.6^\circ A$$

$$V_{OC} = -jI_1 + I_2$$

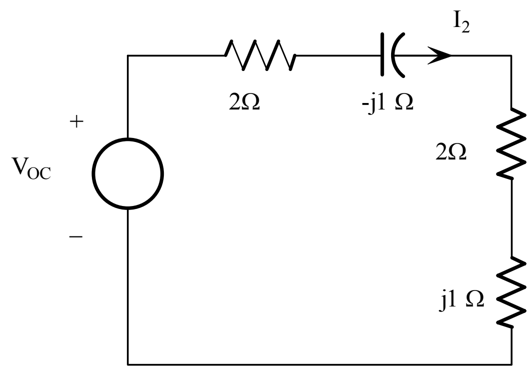
$$= 14.42\angle -46.09^\circ V$$

Z_{TH} is determined from the network below



$\leftarrow Z_{TH} = \{[(1+j)(-j)] / (1+j-j)\} - 1 = 2-j$

∴



$I_2 = (14.42\angle -46.09°)/4$

$= 3.6\angle -46.09°$

$P_L = [1(3.6)^2(2)]/2 = 13W$

Problem 9.26

Repeat problem 9.24 for the network in Fig P 9.27

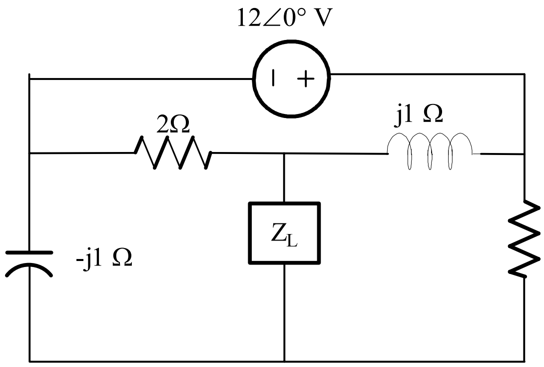
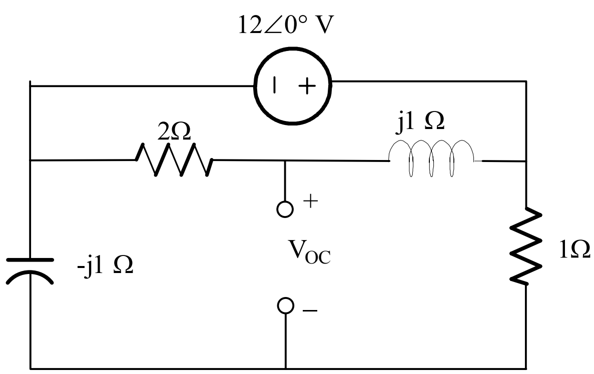
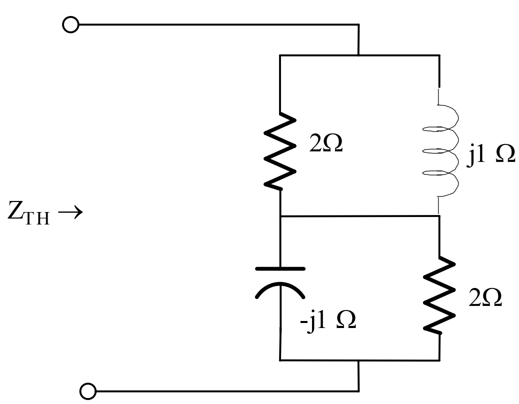


Figure 9.26

Suggested Solution



$$V_{OC} = \frac{12\angle 0^\circ}{2+j}(2) - \frac{12\angle 0^\circ}{1-j}(-j) = 3.79\angle 18.43^\circ V$$



$$Z_{TH} = \frac{2j}{2+j} + \frac{-j}{1-j}$$

$$= \frac{3}{3-j} = .9 + j.3\Omega$$

$$\therefore Z_L = .9 - j.3\Omega$$

$$\boxed{P_L = \frac{1}{2}\left(\frac{3.79}{1.8}\right)^2 (.9) = 2W}$$

Problem 9.27

Repeat problem 9.24 for the network in Fig P 9.27

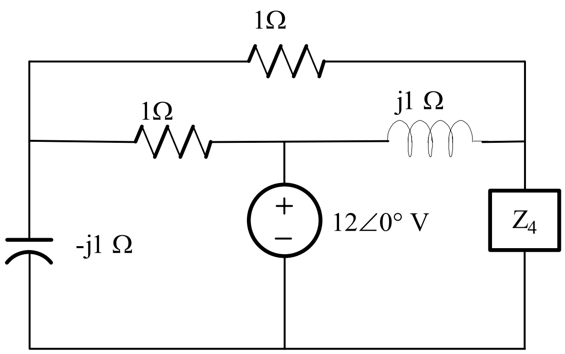


Figure 9.27

Suggested Solution

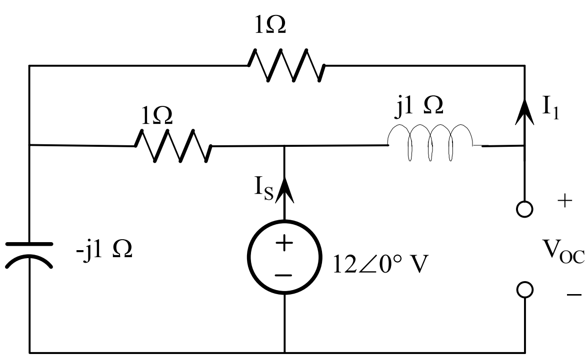
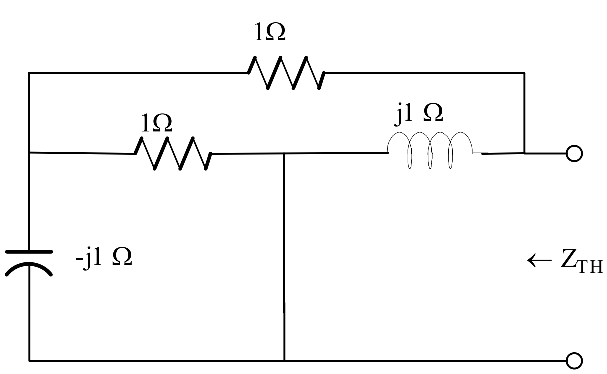


Figure 9.27

$$I_S = \frac{12\angle 0^\circ}{\frac{(1+j)(1)}{1+j+1} - j} = \frac{12(2+j)}{2-j}\,A$$

$$I_1 = \frac{I_3(1)}{2+j} = \frac{12}{2-j}\,A$$

$$V_{OC} = 12\angle 0^\circ - I_1(j) = 15.15\angle -18.43^\circ V$$



$$Z_{TH} = (\frac{-j}{1-j} + 1) // (j)$$

$$= \frac{2+j}{2-j} \Omega$$

$$\boxed{P_L = \frac{1}{2}(\frac{15.15}{1.2})^2 (.6) = 47.82W}$$

Problem 9.28

Repeat problem 9.24 for the network in Fig P 9.28

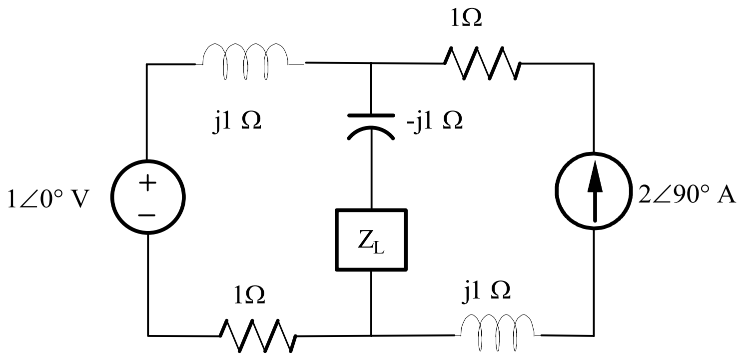
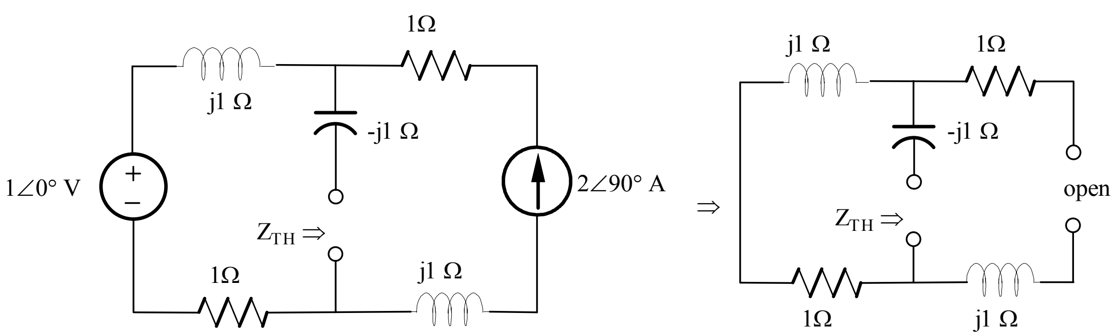


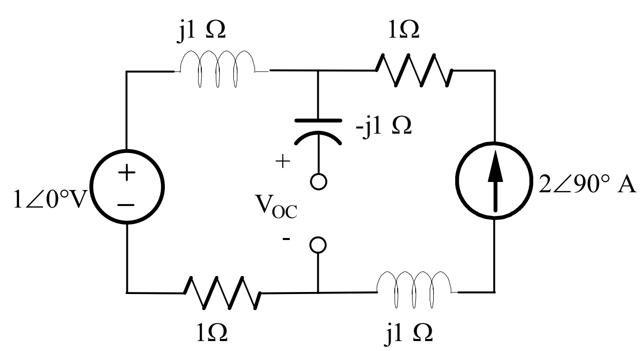
Figure P 9.28

Suggested Solution



Z_{TH} = -j1+j1+1=1Ω

$Z_L = Z_{TH}^* = 1\Omega$



$$V_{OC} = (1+j)(2\angle 90^\circ) + 1\angle 0^\circ$$

$$= -1 + j2 = 2.23\angle 116.57^\circ V$$ g

$$\therefore \boxed{P_L = \frac{1}{2}\left(\frac{2.23}{2}\right)^2 (1) = 0.622W}$$

Problem 9.29

Repeat problem 9.24 for the network in Fig P 9.29

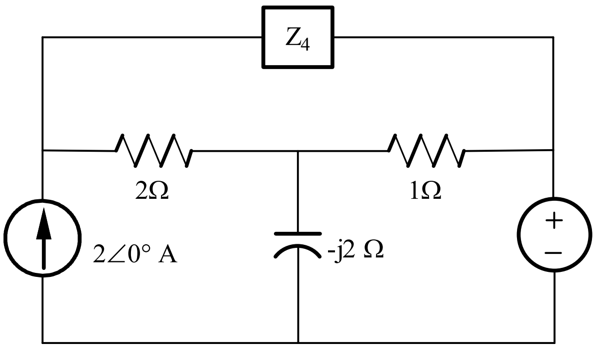
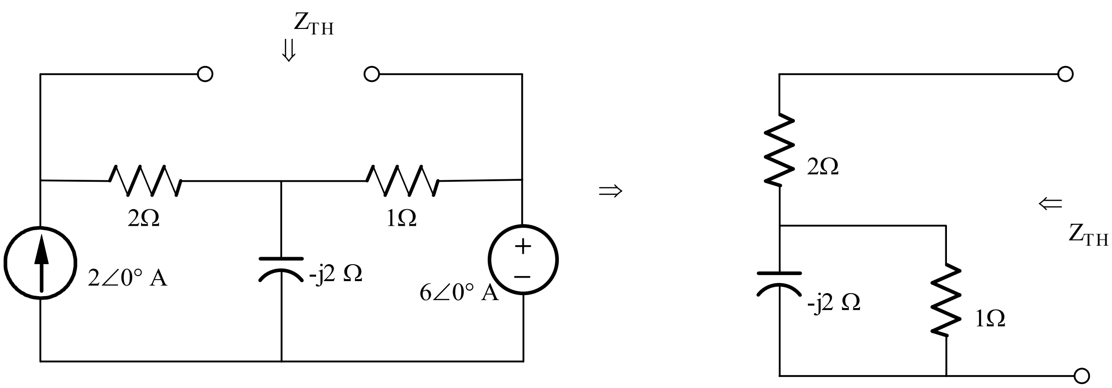


Figure P 9.29

Suggested Solution



$$Z_{TH} = 2 + (1 // -j2) = 2 - \frac{j2}{1-j2} = \frac{2-j6}{1-j2} = 2.83\angle{-8.13^\circ}\Omega$$

$$\boxed{Z_L = Z_{TH}^{\;*} = 2.83\angle{-8.13^\circ}\Omega}$$

$$I_{1\Omega} = \frac{-6-j4}{1-j2} = 3.22\angle{-82.9^\circ}A$$

$$V_{OC} = (2\angle{0^\circ})(2) + (3.22\angle{-82.9^\circ})(1) = 5.44\angle{-36^\circ}V$$

$$\therefore \boxed{P_{Max} = \frac{(5.44)^2}{(8)(2.8)} = 1.32W}$$

Problem 9.30

Determine the impedance Z for maximum average power transfer and the value of the maximum average power absorbed by the load in the network shown in Fig P. 9.30

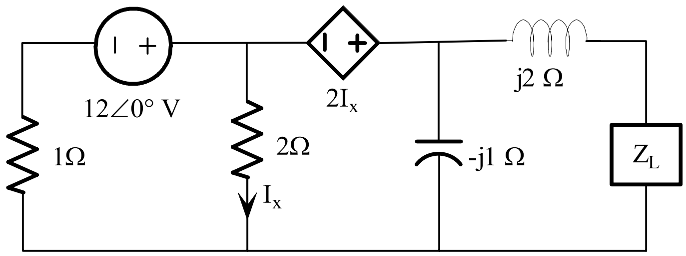
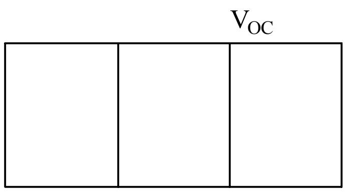


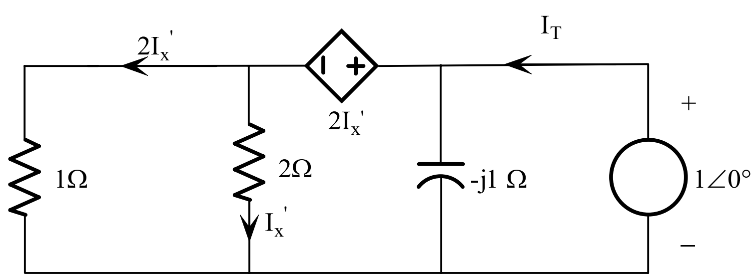
Figure P 9.30

Suggested Solution



$$\frac{V_{OC} - 2I_x - 2}{1} + \frac{V_{OC} - 2I_x}{2} + \frac{V_{OC}}{-j} = 0$$

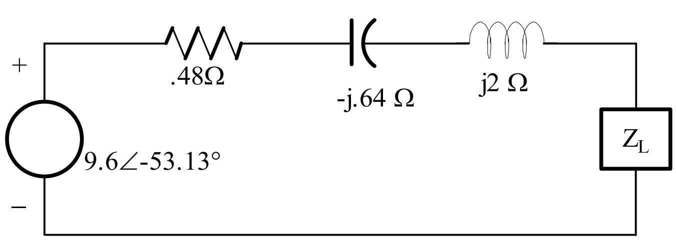
$$and \;\; I_x = \frac{V_{OC}}{4} \qquad \therefore \; V_{OC} = \frac{4.8}{3 + j4} = 9.6\angle{-53.13^\circ} V$$



$4I_x' = 1 \;\; \therefore \; I_x' = 1/4$

$$I_T = \frac{1}{-j1} + 3I_x' = \frac{3 + j4}{4}$$

$$Z_{TH} = \frac{1}{I_T} = \frac{3}{3 + j4} = .8\angle{-53.13^\circ} = .48 - j.64 \qquad \boxed{P_L = \frac{1}{2}(10)^2(.48) = 24W}$$



$Z_{TH\ Total} = .48 + j1.36$

$\therefore \; Z_L = .48 - j1.36$

$I_L = 9.6 / (.48 + .48) = 10$ A

$$\boxed{P_L = \frac{1}{2}(10)^2(.48) = 24W}$$

Problem 9.31

Determine the impedance ZL for maximum average power transfer and the value of the maximum average power absorbed by the load in the network shown in Fig P. 9.31

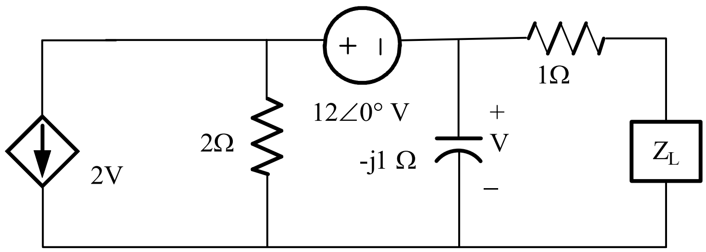
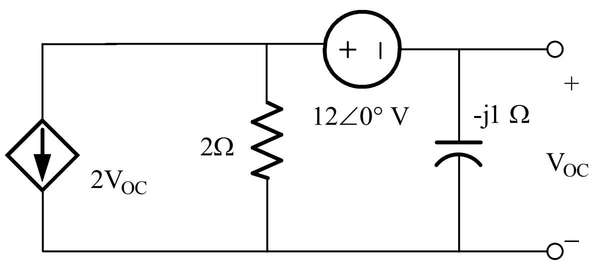


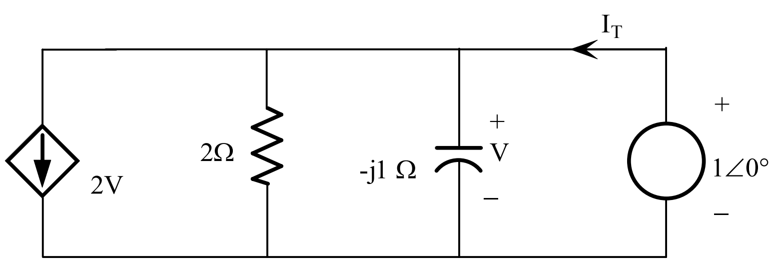
Figure P 9.31

Suggested Solution



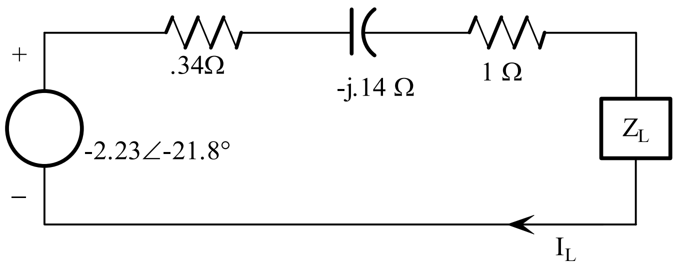
$$\frac{V_{OC}}{-j} + \frac{V_{OC}+12}{2} + 2V_{OC} = 0$$

$$\therefore V_{OC} = \frac{-12}{5+2j} = -2.23\underline{\lfloor -21.8^{\circ}}V$$



$$I_T = \frac{1}{-j} + \frac{1}{2} + 2 = j + \frac{5}{2}$$

$$Z_{TH} = \frac{1}{I_T} = \frac{1}{\frac{5}{2}+j} = .37\underline{\lfloor -21.8^{\circ}} = .34 - j.14\Omega$$



$Z_L = 1.34 + j.14\Omega$

$I_L = 2.23 / [2(1.34)] = .83$ A

$P_L = 1(.83)^2(1.34) / 2 = .46$ W

Problem 9.32

Find the impedance Z_L for the maximum average power transfer and the value of the maximum average power transferred to Z_L for the circuit shown in Fig P 9.32

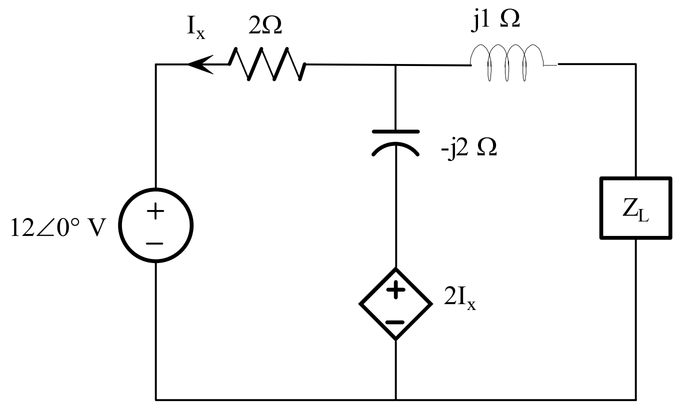
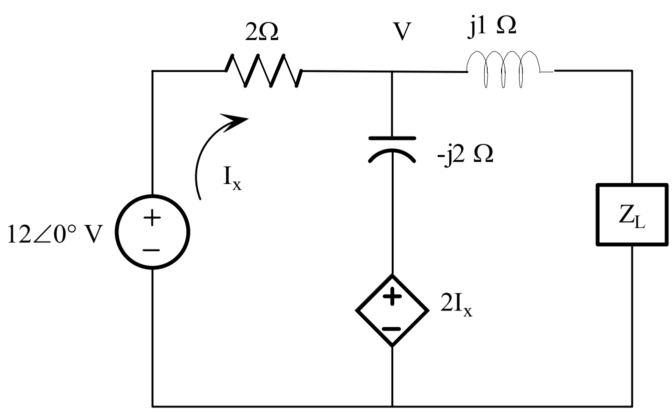


Figure P 9.32

Suggested Solution



Solving for V_{OC} :

$$\frac{12\angle 0^\circ - V}{2} = \frac{V - 2I_x}{-2j}$$

$$I_x = \frac{12 - V}{2}$$

Solving, $V = 7.6\angle -18.4^\circ = V_{OC}$

Then, we find I_{SC} :

$$\frac{12 - V}{2} + \frac{2I_x - V}{-2j} = \frac{V}{j}$$

Solving, $I_{SC} = 17.0\angle 45^\circ$

Then, $Z_{TH} = \dfrac{7.6\angle -18.4^\circ}{17.0\angle 45^\circ} = .45\angle -63.4^\circ = .2 - .4j$

So, $\boxed{Z_L = .2 + .4j}$

$P = \dfrac{1}{2}(17)^2(.2) = \boxed{28.9W}$

Problem 9.33

Determine the impedance Z_L for the maximum average power transfer and the value of the maximum average power absorbed by the load in the network shown in Fig P 9.33

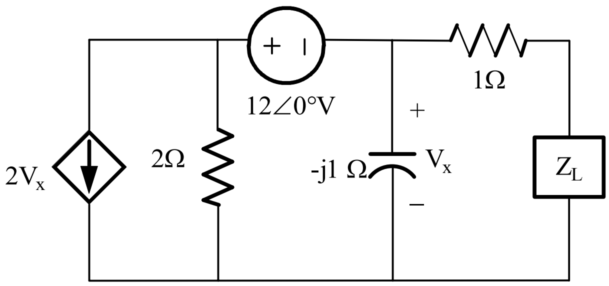
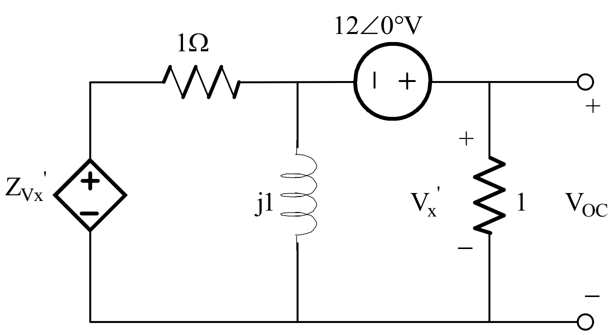


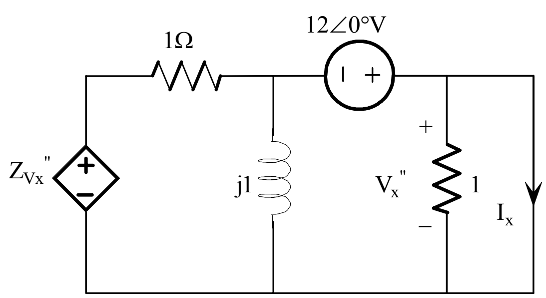
Figure P 9.33

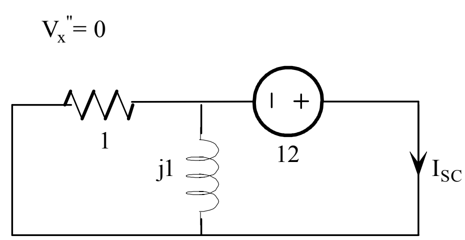
Suggested Solution



$$\frac{V_{OC}-12-2V_{OC}}{1}-\frac{V_{OC}-12}{j1}+\frac{V_{OC}}{1}=0$$

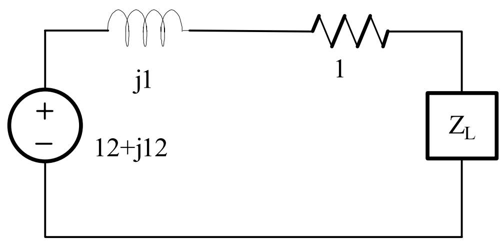
$$V_{OC}=12+j12V$$





$$I_{SC}=\frac{12}{\frac{j}{1+j}}=\frac{12(1+j)}{j}=\frac{12+j12}{j}$$

$$Z_{TH}=\frac{V_{OC}}{I_{SC}}=j1\Omega \quad \text{Then}$$



Z_L = 1- j Ω

$$I_L = \frac{12 + j12}{1 + j + 1 - j} = 8.4853\angle 45^\circ$$

And $\boxed{P_{load} = \frac{1}{2}(8.4853)^2(1) = 36W}$

Problem 9.34

Compute the rms value of the voltage given by the waveform shown in Fig P 9.34

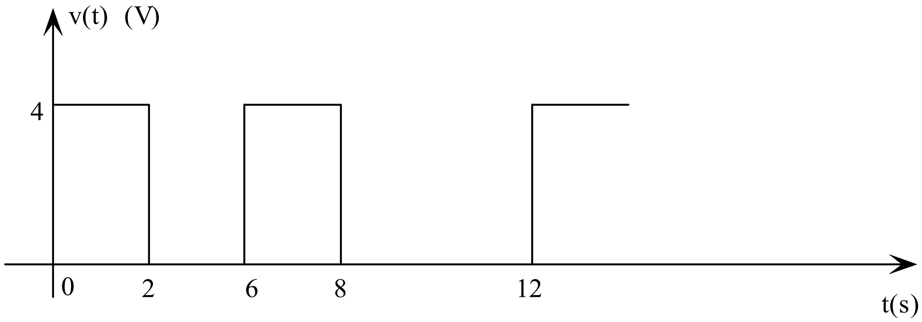


Figure P 9.34

Suggested Solution

$$V_{RMS} = \sqrt{\frac{1\int_0^T v^2(t)dt}{\text{T}}} \quad \text{where } v(t) = \left\{ \begin{smallmatrix} 4 & 0 \le t \le 2 \\ 0 & 2 \le t \le 6 \end{smallmatrix} \right\} \ \& \ T = 6$$

$$V_{RMS} = \sqrt{\frac{1[\int_0^2 4^2 dt]}{6}} = \sqrt{\frac{16(t|_0^2)}{6}} = \sqrt{\frac{32}{6}}$$

$$\boxed{V_{RMS} = 2.31 V_{RMS}}$$

$$V_{RMS} = \sqrt{\frac{1\int_0^T v^2(t)dt}{\text{T}}} \quad \text{where } v(t) = \left\{ \begin{smallmatrix} 4 & 0 \le t \le 2 \\ 0 & 2 \le t \le 6 \end{smallmatrix} \right\} \ \& \ T = 6$$

$$V_{RMS} = \sqrt{\frac{1[\int_0^2 4^2 dt]}{6}} = \sqrt{\frac{16(t|_0^2)}{6}} = \sqrt{\frac{32}{6}}$$

$$\boxed{V_{RMS} = 2.31 V_{RMS}}$$

Problem 9.35

Find the rms values of the waveform shown in Fig P 9.35

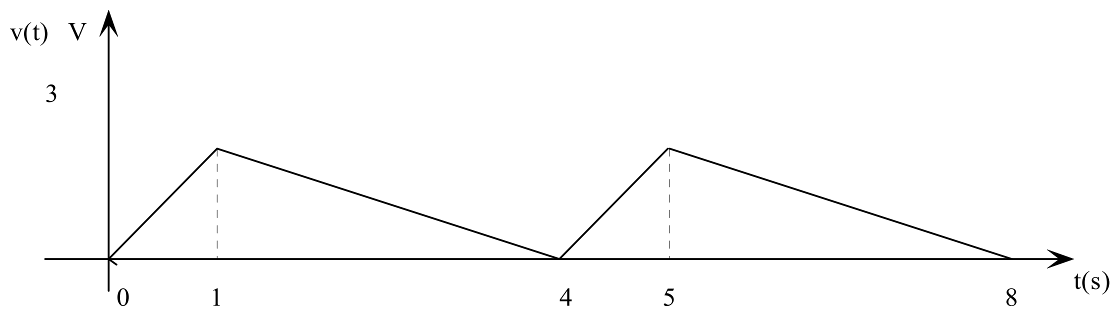


Figure P 9.35

Suggested Solution

$$V_{rms} = [\frac{1}{4}(\int_0^1 (3t)^2\,dt + \int_1^4 (4-t)^2\,dt]^{\frac{1}{2}}$$

$$= [\frac{1}{4}(3t^3\,|_0^1 + (16t - 4t^2 + \frac{t^3}{3})|_1^4)]^{\frac{1}{2}}$$

$$\boxed{= \sqrt{3}V}$$

Problem 9.36

Calculate the rms value of the waveform shown in Fig P 9.36

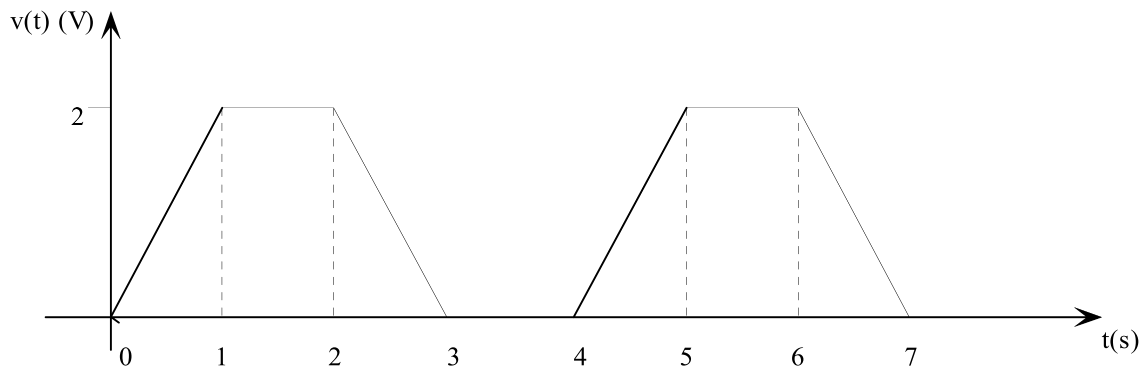


Figure P 9.36

Suggested Solution

$$I_{rms} = \sqrt{\frac{1[\int_0^1 (2t)^2\,dt + \int_1^2 2^2\,dt + \int_2^3 (6-2t)^2\,dt}{4}}$$

$$= \sqrt{\frac{1[\frac{4}{3}t^3|_0^1 + 4 + (36t - 12t^2 + \frac{4}{3}t^3)|_2^3}{4}}$$

$$= \boxed{1.29A}$$

Problem 9.37

Calculate the rms value of the waveform shown in Fig P 9.37

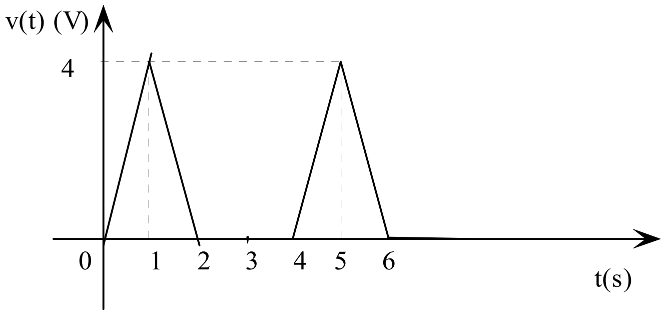


Figure P 9.37

Suggested Solution

$$I_{rms} = \sqrt{\frac{1}{4}[\int_0^1 (4t)^2\,dt + \int_1^2 (8-4t)^2\,dt]}$$

$$= \sqrt{\frac{1}{4}\left(\frac{16}{3} + \left(64t - \frac{64t^2}{2} + \frac{16t^3}{3}\right)\Big|_1^2\right)} = \boxed{1.63V}$$

Problem 9.38

The current waveform in Fig P 9.38 is flowing through a 5-Ω resistor. Find the average power absorbed by the resistor.

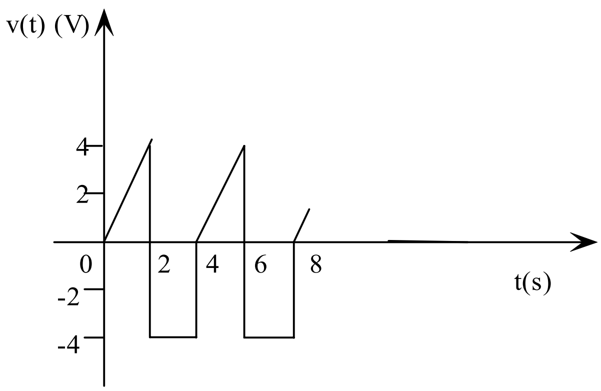


Figure P 9.38

Suggested Solution

A 5Ω resistor passes current shown in Fig P 9.38

$$I_{rms} = \sqrt{\frac{1}{4}\left[\int_0^2 (2t)^2\,dt + \int_2^4 -4^2\,dt\right]}$$

$$= \sqrt{\frac{1}{4}\left(\frac{4}{3} + 3\Big|_0^2 + 16t\Big|_2^4\right)} = 3.27A$$

Then, $\boxed{P = (3.27)^2(5) = 53.3W}$

Problem 9.39

Calculate the rms value of the waveform shown in Fig P 9.39

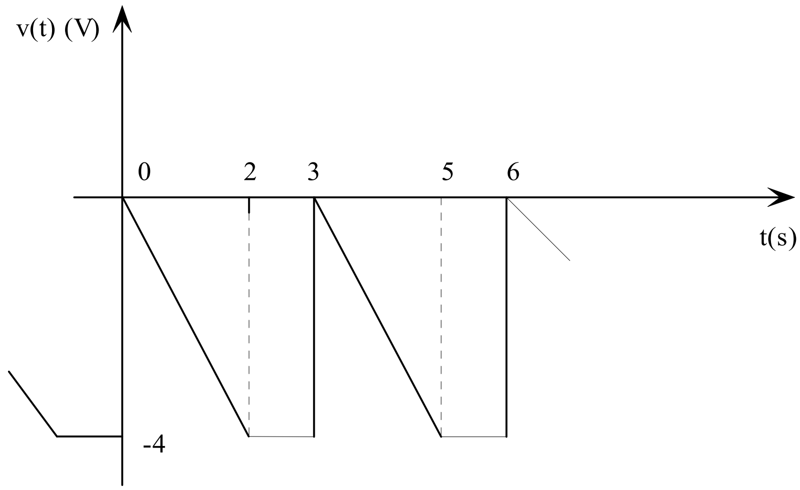


Figure P 9.39

Suggested Solution

Find rms value.

$$V_{rms} = \sqrt{\frac{1}{3}\left[\int_0^2 (-2t)^2\,dt + \int_2^3 -4^2\,dt\right]}$$

$$= \sqrt{\frac{1}{3}\left(\frac{4}{3}t^3\Big|_0^2 + 16t\,\Big|_2^3\right)} = \boxed{2.98V}$$

Problem 9.40

Compute the rms value of the waveform in Fig 9.40

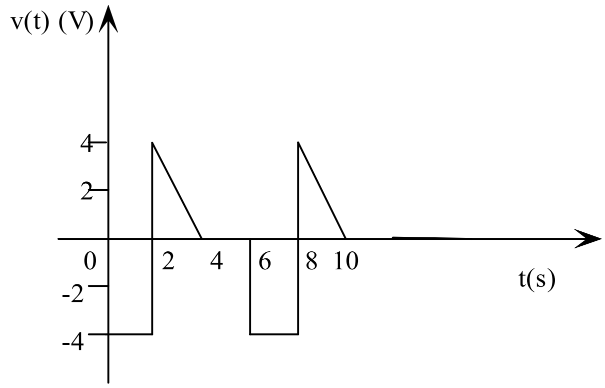


Figure P 9.40

Suggested Solution

Find rms value

$$V_{rms} = \sqrt{\frac{1}{6}\left[\int_0^2 -4^2\,dt + \int_2^4 (8-2t)^2\,dt\right]}$$

$$= \sqrt{\frac{1}{6}\left[16t\,|_0^2 + \left(64t - 16t^2 + \frac{4}{3}t^3\right)|_2^4\right]}$$

$$= \sqrt{\frac{1}{6}\frac{128}{3}} = \boxed{2.67V}$$

Problem 9.41

Calculate the rms value of the waveform shown in Fig 9.41

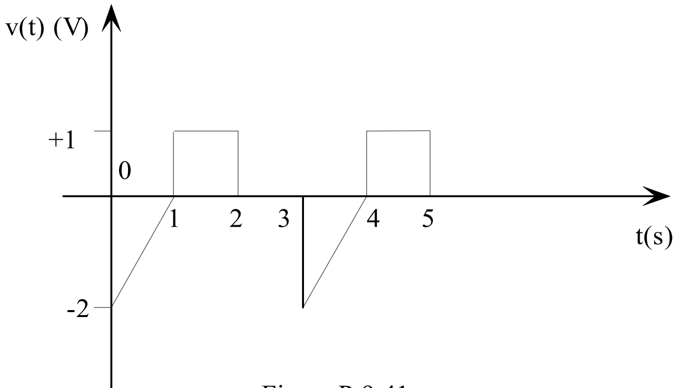


Figure P 9.41

Suggested Solution

Find rms value:

$$I_{rms} = \sqrt{\frac{1}{3}\left[\int_0^1 (2t-2)^2\,dt + \int_1^2 1^2\,dt\right]}$$

$$= \sqrt{\frac{1}{3}\left[\left(\frac{4}{3}t^3 - \frac{8t^2}{2} + 4t\right)\Big|_0^1 + t\,\Big|_1^2\right]}$$

$$= \sqrt{\frac{1}{3}\left(\frac{7}{3}\right)} = \boxed{0.88A}$$

Problem 9.42

Calculate the rms value of the waveform shown in Fig 9.42

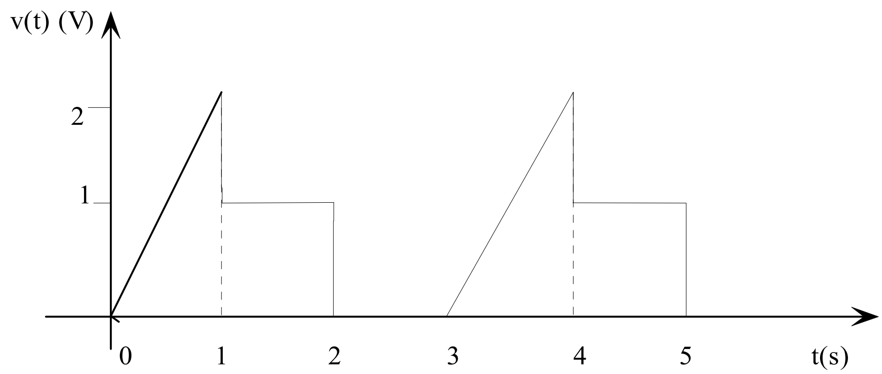


Figure P 9.42

Suggested Solution

Find rms value:

$$I_{rms} = \sqrt{\frac{1}{3}\left[\int_0^1 (2t)^2\,dt + \int_1^2 1^2\,dt\right]}$$

$$= \sqrt{\frac{1}{3\left(\frac{4}{3}t^3 \,|_0^1 + t\,|_1^2\right)}} = \boxed{0.88A}$$

Problem 9.43

An industrial load consumes 100 kW at 0.8 pf lagging. If an ammeter in the transmission line indicates that the load current is 284 A rms, find the load voltage.

Suggested Solution

A load consumes 100 kW at pf=.8 lag

$I_{load} = 284A$ rms. Find load voltage.

$$V_{rms} = \frac{100,000W}{(.8)(284A)} = \boxed{440V}$$

Problem 9.44

An industrial load that consumes 80 kW is supplied by the power company through a transmission line with 0.1 Ω resistance, with 84kW. If the voltage at the load is 440 Vrms, find the power factor at the load.

Suggested Solution

A load consumes 80 kW. 1 B source supplies 84 kW. $R_{line} = 0.1\Omega, V_{load} = 440V_{rms}$. And load p.f.

$P_S = P_{load} + I^2 R_{line} = 84kW = I^2(.1) + 80kW$

Yeilds $I = 200A_{rms}$. Then, PF = $\dfrac{80{,}000}{(440)(200)} = \boxed{.91Lag}$

Problem 9.45

The power company supplies 80 kW to an industrial load. The load draws 220 A rms from the transmission line. If the load voltage is 440 V rms and the load power factor is 0.8 lagging, find the losses in the transmission line.

Suggested Solution

$P_S = 80kW, I_L = 220A_{rms}, V_{load} = 440V_{rms}$ *with* p.f.= 0.8 lag. Find line loss.

$P_S = V_{rms} I_{rms} pf = (440)(220)(.8) = 72.44kW$

$P_{line} = P_S - P_L = 80k - 72.44k = \boxed{2.56kW}$

Problem 9.46

An industrial plant with an inductive load consumes 10 kW of power from a 220 V rms line. If the load power factor is 0.8, what is the angle by which the load voltage leads the load current?

Suggested Solution

$$\theta = \cos^{-1} 0.8 = \boxed{36.87°}$$

Problem 9.47

The power company must generate 100 kW in order to supply an industrial load with 94 kW through a transmission line with 0.09 Ω resistance. If the load power factor is 0.83 lagging, find the load voltage.

Suggested Solution

$$P_S = I^2 R_{line} + P_L$$

$$100kW = I^2(.09) + 94kW$$

$$I^2 = \frac{600}{.09}$$

$$\therefore I_{rms} = 258.2A$$

$$V_{rms} = \frac{P_L}{I_{rms}Pf} = \frac{94000}{(258.2)(.83)} = \boxed{440V}$$

Problem 9.48

An industrial load operates at 30 kW, 0.8 pf lagging. The load voltage is 220∠0° V rms. The real and reactive power losses in the transmission-line feeder are 1.8 kW and 2.4 kvar, respectively. Find the inpedance of the transmission line and the input voltage to the line.

Suggested Solution

$P_{load} = 30kW$ as pf =.8 lag. $\text{V}_\text{L} = 220\angle 0° V_{rms}$

$P_{line} = 1.8kW + 2.4k\,\text{var}$. Find $\text{R}_\text{line}, V_S$

$$I_L = \frac{P_L}{V_{rms}.pf} = \frac{30000}{220 \times .8} = 170.45\angle \cos^{-1} .8$$

$$= 170.45\angle -36.9°$$

$$Then, R_{line} = \frac{P}{I^2}$$

$$R_{line} = \frac{1800}{170.45^2} = .062\Omega$$

$$L_{line} = \frac{P}{I^2} = \frac{2400}{170.45^2} = .083j$$

$$\boxed{Z_{line} = .062 + .083j}$$

$$V_S = V_{load} + I_L Z_L = 220\angle 0° + (170.45\angle -36.9°)Z_L$$

$$\boxed{= 237\angle 1.19°}$$

Problem 9.49

A transmission line with impedance 0.08 – *j* 0.25 Ω is used to deliver power to a load. The load is inductive and the3 load voltage is 220 ∠0° V rms at 60 Hz. If the load requires

Suggested Solution

$$Z_{line} = .08 + j.25$$

$$V_{Load} = 220\angle 0° \text{ at 60 Hz}$$

$$P_{load} = 12kW \qquad P_{Line} = 560kW \quad \text{Find pf at load.}$$

$$P_{line} = 560 = I^2(.08) \text{ yeilds } I = 83.67A$$

$$P_{load} = IV\cos\theta = 12kW = (220)(83.67)\cos\theta$$

$$\theta = \cos^{-1}\frac{12000}{(220)(83.67)} = \boxed{.65 = pf\ \ lagging}$$

Problem 9.50

Find the source voltage in the network shown in Fig P 9.50

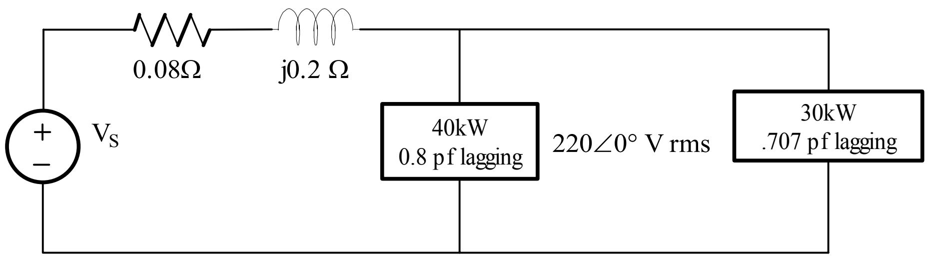


Figure P 9.50

Suggested Solution

$$I_{L_1} = \frac{40000}{(220)(.8)} = 227.27A \qquad I_{L_2} = \frac{30000}{(220)(.707)} = 177.1A$$

$$\theta_1 = \cos^{-1}(.8) = 36.87° \qquad \theta_2 = \cos^{-1}(.707) = 45°$$

$$I_L = I_{L_1} + I_{L_2} = 227.27\angle -36.87° + 177.1\angle -45°$$

$$= 403\angle -40.43° A$$

$$V_{line} = I_L(.8 + j2) = (403.37\angle -40.43°)(.22\angle 68.2°)$$

$$= 88.74\angle 27.77° V$$

$$V_S = V_{line} + 220\angle 0° = \boxed{301.37\angle 7.89°}$$

Problem 9.51

Given the network in Fig P 9.51, compute the input source voltage and the input power factor.

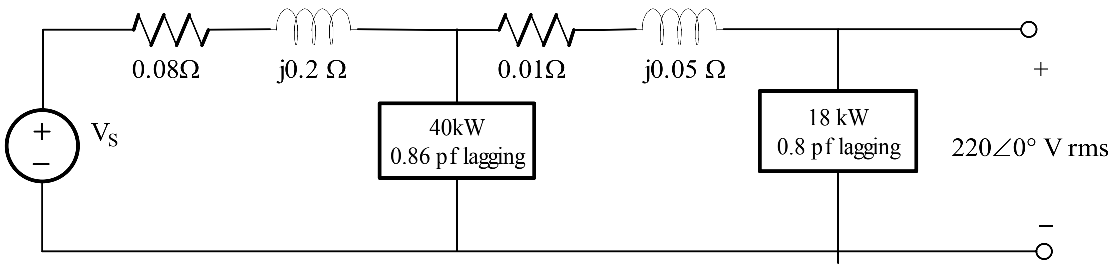


Figure P 9.51

Suggested Solution

$$\theta_L = \cos^{-1} .8 = 36.87°$$

$$I_{L_2} = \frac{80000}{(220)(.8)} = 102.27A$$

$$I_{L_2} = 102.27\angle -36.87° A$$

$$V_{L_1} = (.01 + j.05)(102.27\angle -36.87°) + 220\angle 0°$$

$$= 223.91\angle .89° V$$

$$\theta_1 = \cos^{-1} .86 = 30.68° lagging$$

$$I_{L_1} = \frac{40000}{(223.91)(.86)} = 207.72A$$

$$I_{L_1} = 207.72\angle(-30.68° + .89°) = 207.72\angle -29.79° A$$

$$I_L = I_{L_1} + I_{L_2} = 207.72\angle -29.79° + 102.27\angle -36.87° A$$

$$= 309.47\angle -32.12° A$$

$$V_S = (309.47\angle -32.12°)(.08 + j.2) + 223.91\angle .89°$$

$$= 281.02\angle 8.75° V$$

$$\theta_S = \theta_{VS} - \theta_{iS} = 8.75° - (-32.12°) = 40.87°$$

$$P_{Fsource} = \cos(40.87°) = \boxed{.756 lagging}$$

$$\theta_L = \cos^{-1} .8 = 36.87°$$

$$I_{L_2} = \frac{80000}{(220)(.8)} = 102.27 A$$

$$I_{L_2} = 102.27\angle -36.87° A$$

$$V_{L_1} = (.01 + j.05)(102.27\angle -36.87°) + 220\angle 0°$$

$$= 223.91\angle .89° V$$

$$\theta_1 = \cos^{-1} .86 = 30.68° lagging$$

$$I_{L_1} = \frac{40000}{(223.91)(.86)} = 207.72 A$$

$$I_{L_1} = 207.72\angle(-30.68° + .89°) = 207.72\angle -29.79° A$$

$$I_L = I_{L_1} + I_{L_2} = 207.72\angle -29.79° + 102.27\angle -36.87° A$$

$$= 309.47\angle -32.12° A$$

$$V_S = (309.47\angle -32.12°)(.08 + j.2) + 223.91\angle .89°$$

$$= 281.02\angle 8.75° V$$

$$\theta_S = \theta_{VS} - \theta_{iS} = 8.75° - (-32.12°) = 40.87°$$

$$P_{Fsource} = \cos(40.87°) = \boxed{.756 lagging}$$

Problem 9.52

Given the network in Fig P 9.52, determine the input voltage Vs.

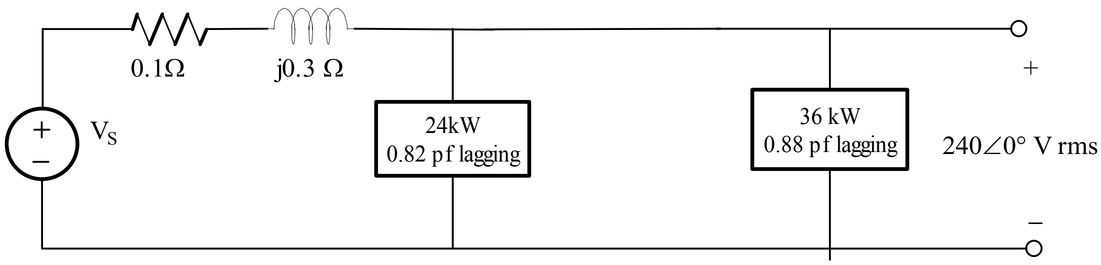
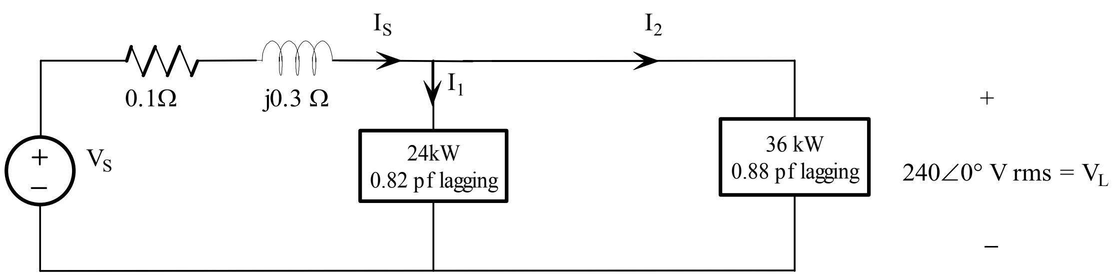


Figure P 9.52

Suggested Solution



$V_S = V_L + I_S\left(.01 + j.03\right)$

$I_S = I_1 + I_2$

Find I_1 : $I_1 = \dfrac{P_1}{|V_L|\left(pf_1\right)} \angle -\cos^{-1}(.82) = 121.95\angle -34.92° A_{rms}$

Find I_2 : $I_2 = \dfrac{P_2}{|V_L|\left(pf_2\right)} \angle -\cos^{-1}(.88) = 170.45\angle -28.36° A_{rms}$

Find I_S : $I_S = I_1 + I_2 = 250 - j150.76$

$I_S = 291.94\angle -31.09° A_{rms}$

Find V_S : $V_S = V_L + I_S\left(0.1 + j0.3\right) = V_L + I_S\left[0.316\angle 71.57°\right]$

$V_S = 240 + 92.25\angle 40.48° V_{rms}$

$\boxed{V_S = 315.90\angle 10.93° V_{rms}}$

Problem 9.53

Find the input source voltage and the power factor of the source for the network shown in Fig P 9.53

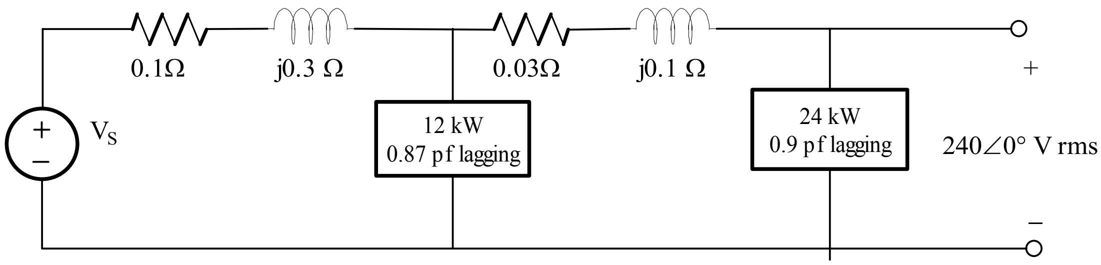
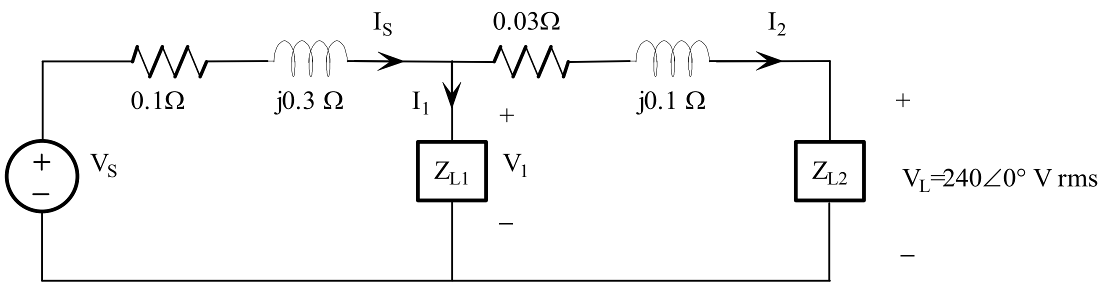


Figure P 9.53

Suggested Solution



Load 1: 12kW 0.87 pf lagging

Load 1: 24kW 0.90 pf lagging

$V_S = V_L + I_2[0.03 + j0.1] + I_S[0.1 + j0.3]$

$I_S = I_1 + I_2$

$I_2 = \dfrac{P_2}{V_L(pf_2)} \angle -\cos^{-1}(pf_2) = 111.11\angle -25.84° A_{rms}$

$V_1 = V_L + (0.03 + j0.1)I_2 = 240 + (0.104\angle 73.30°)(111.11\angle -25.84°)$

$= 240 + 11.60\angle 47.46° = 247.99\angle 1.98° V_{rms}$

$V_S = V_1 + (0.1 + j0.3)I_S \qquad I_S = I_1 + I_2$

$|I_1| = \dfrac{P_1}{V_1(pf_1)} = \dfrac{12000}{(247.99)(0.87)} = 55.62 A_{rms}$

$\theta_{V_1} - \theta_{I_1} = \cos^{-1}(0.87) = 29.54° \Rightarrow \theta_{I_1} = -27.56°$

$I_1 = 55.62\angle -27.56° A_{rms}$

$I_S = 166.71\angle -26.41° A_{rms}$

$V_S = 247.99\angle 1.98° + (0.1 + j0.3)166.71\angle -26.41°$

$$\boxed{V_S = 288.16\angle 8.48° V_{rms}}$$

$$\theta_S = \theta_{Vs} - \theta_{is} = 8.48° - (-26.41°) = 34.89°$$

$$\boxed{pf_{source} = \cos(34.89°) = 0.82 lagging}$$

Problem 9.54

Use Kirchhoff's laws to compute the source voltage of the network shown in Fig P 9.54.

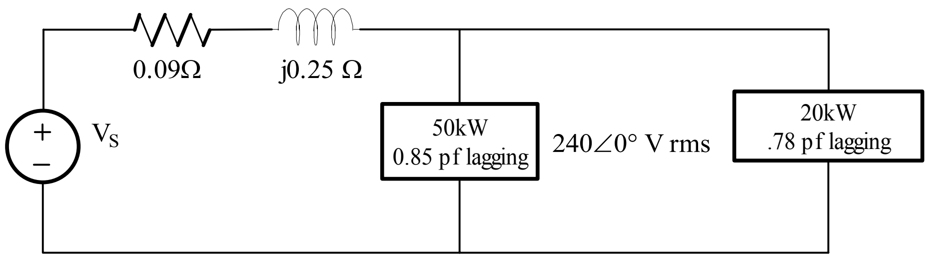
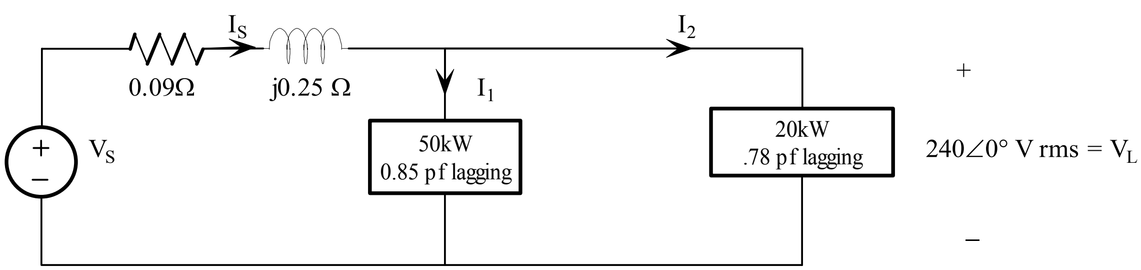


Figure P 9.54

Suggested Solution



Find I_1 :

$$|I_1| = \frac{P_1}{|V_L|(pf_1)} = \frac{50,000}{240(0.85)} = 245.10$$

$$\theta_{I_1} = -\cos^{-1}(0.85) = -31.79°$$

$$I_1 = 245.10\angle -31.79° A_{rms}$$

Find I_1 :

$$|I_2| = \frac{P_2}{|V_L|(pf_2)} = \frac{20,000}{240(0.78)} = 106.84$$

$$\theta_{I_2} = -\cos^{-1}(0.78) = -38.74°$$

$$I_2 = 106.84\angle -38.74° A_{rms}$$

Find I_S :

$$I_S = I_1 + I_2 = 351.40\angle -33.90° A_{rms}$$

Find V_S :

$$V_S = I_S(0.09 + j0.25) + 240$$

$$= (351.40\angle -33.90°)(0.266\angle 70.20°) + 240$$

$$= 93.37\angle 36.30° + 240$$

$$\boxed{V_S = 320.06\angle 9.95° V_{rms}}$$

Problem 9.55

Given the network in Fig P9.55, determine the input voltage V5.

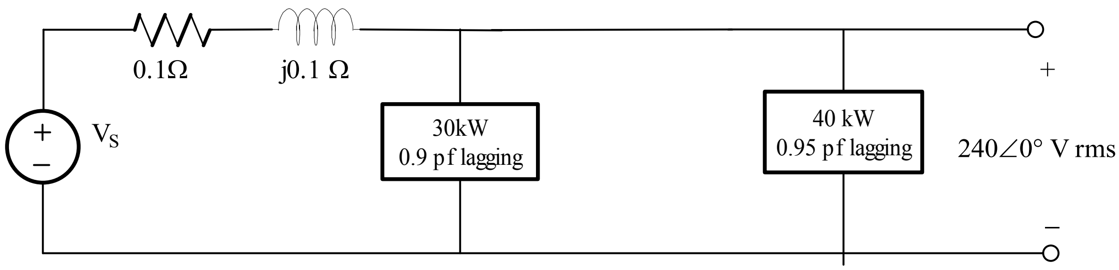
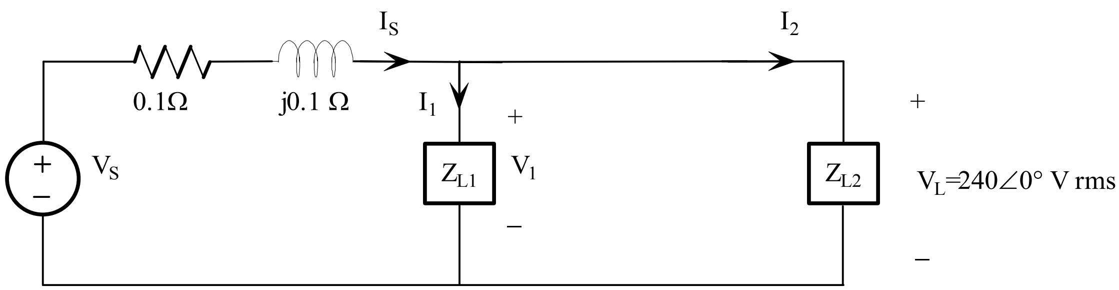


Figure P 9.55

Suggested Solution



Load 1: 30 kVA   0.9 pf lagging

Load 2: 40 kVA   0.95 pf lagging

$V_S = V_L + I_S(0.1 + j0.1)$ $I_S = I_1 + I_2$

Find I_1 : $|I_1| = \dfrac{P_1}{|V_L|(pf_1)} = \dfrac{|S_1|}{|V_L|} = \dfrac{30{,}000}{240} = 125A_{rms}$

$\theta_{I_1} = -\cos^{-1}(0.9) = -25.84°$

$I_1 = 125\angle -25.84° A_{rms}$

Find I_2 : $|I_2| = \dfrac{P_2}{|V_L|(pf_2)} \angle -\cos^{-1}(pf_2) = 175.44\angle -18.19° A_{rms}$

Find I_S : $I_S = I_1 + I_2 = 299.79\angle -21.38° A_{rms}$

$V_S = 240 + (0.1 + j0.1)I_S$

$= 240 + \left[0.1\sqrt{2}\angle 45\right]\left[299.79\angle -21.38\right]$

$= 279.24\angle 3.48° V_{rms}$

$\boxed{V_S = 279.24\angle 3.48° V_{rms}}$

Problem 9.56

What value of capacitance must be placed in parallel with the 18-kW load in Problem 9.51 in order to raise the power factor of this load to 0.9 lagging?

Suggested Solution

$$P_{old} = 18000W \qquad \theta_{old} = \cos^{-1}.8 = 36.87°$$

$$Q_{old} = P_{old}\tan\theta = 18000\tan 36.87 = 13500VAR$$

$$S_{old} = 18000 + j13500VA$$

$$P_{Fnew} = .9 \qquad \therefore \theta_{new} = 25.84°$$

$$S_{new} = 18000 + j18000\tan 25.84 = 18000 + j8718$$

$$S_{cap} = S_{new} - S_{old} = -j4782$$

$$C = \frac{4782}{(377)(220)^2} = \boxed{262\mu F}$$

Problem 9.57

An industrial load is supplied through a transmission line that has a line impedance of 0.1 + j0.2 Ω. The 60-Hz line voltage at the load is 480∠0° V rms. The load consumes 124 kW at 0.75pf lagging. What value of capacitance when placed in parallel with the load will change the power factor to 0.9 lagging?

Suggested Solution

$$P_{old} = 124kW \qquad \theta_{old} = \cos^{-1}.75 = 41.41°$$

$$Q_{old} = P_{old}\tan\theta = 124000\tan 41.41 = 109358VAR$$

$$S_{old} = 124000 + j109358VA$$

$$P_{Fnew} = .9 \qquad \therefore \theta_{new} = 25.84°$$

$$S_{new} = 124000 + j124000\tan 25.84 = 124000 + j60056$$

$$S_{cap} = S_{new} - S_{old} = -j49302$$

$$C = \frac{49302}{(377)(480)^2} = \boxed{567.6\mu F}$$

Problem 9.58

A plant consumes 60 kW at a power factor of 0.5 lagging from a 220-V rms 60-Hz line. Determine the value of the capacitor which when placed in parallel with the load will change the load power factor to 0.9 lagging.

Suggested Solution

$$I_L = \frac{60,000}{(220)(0.5)} = 545.45\angle \cos^{-1} 0.5A$$

$$Q_{old} = 60,000 \tan(-60°) = 103,923VAR$$

$$S_{old} = 60,000 + j103,923VA$$

$$P_{Fnew} = .9 \qquad \therefore \theta_{new} = 25.84°$$

$$S_{new} = 60,000 + j60,000 \tan 25.84 = 60000 + j29056$$

$$S_{cap} = S_{new} - S_{old} = 29056 - 103,923 = -74866$$

$$C = \frac{74866}{(377)(220)^2} = \boxed{4103\mu F}$$

Problem 9.59

A small plant has a bank of induction motors that consume 64 kW at a pf of 0.68 lagging. The 60-Hz line voltage across the motors is $220\angle 0°$V rms. The local power company has tolad the plant to raise the pf to 0.92 lagging. What value of capacitance is required?

Suggested Solution

$$P_{old} = 64kW\angle 47.2°$$

$$Q_{old} = P_{old}\tan 47.2° = 69,000VAR$$

$$S_{old} = 64000 + 69,000j$$

$$Q_{new} = \tan^{-1}.42 = 23.1°$$

$$S_{new} = 64000 + j64000\tan 21.3° = 64000 + 27,264j$$

$$Then, S_{cap} = S_{new} - S_{old} = -41.744j$$

$$C = \frac{S_{cap}}{\omega V_{rms}^2}\frac{41.744}{(377)(220)^2} = \boxed{2,288\mu F}$$

Problem 9.60

The 60-Hz line voltage for a 60-kW, 0.76-pf lagging industrial load is 440∠0° V rms. Find the value of the capacitance which when placed in parallel with the load will raise the power factor to 0.9 lagging.

Suggested Solution

$P_{old} = 60,000\angle 40.5°$

$Q_{old} = 60,000 \tan 40.5 = 51,300 VAR$

$S_{old} = 60,000 + j51,300 VA$

With the capacitor in place, pf=.9 θ=25.8°

$S_{new} = 60,000 + j60,000 \tan 25.8°$

$= 60,000 + j29,000 VA$

$S_{cap} = S_{new} - S_{old} = -j22,300$

$$C = \frac{22,300}{(377)(440)^2} = \boxed{305 \mu F}$$

Problem 9.61

A particular load has a pf of 0.8 lagging. The power delivered to the load is 40 kW from a 220-V rms 60-Hz line. What value of capacitance placed in parallel with the load will raise the pf to 0.9 lagging?

Suggested Solution

$$I_L = \frac{40,000}{(220)(0.8)} = 227.3\angle -36.9°A$$

$$Q_{old} = 40,000 \tan(-36.9°) = 30,000 VAR$$

$$S_{old} = 40,000 + j30,000$$

With the capacitor, $P_{Fnew} = .9, \theta_{new} = 25.8°$

$$S_{new} = 40,000 + j40,000 \tan 25.8° = 40,000 + j20,000$$

$$So, S_{cap} = S_{new} - S_{old} = -10,000j$$

$$C = \frac{S_{cap}}{\omega (V_{rms})^2} = \frac{10,000}{(377)(220)^2} = \boxed{586\mu F}$$

Problem 9.62

An industrial load consumes 44 kW at 0.82 pf lagging from a 220∠0° V rms, 60-Hz line. A bank of capacitors totaling 900 μF is available. If these capacitors are placed in parallel with the load, what is the new power factor of the total load?

Suggested Solution

$$S_{cap} = -j(220)^2(377)(900\mu F) = -j16,422$$

$$Then, S_{old} = 44,000 + j\left(44,000\tan\left(\cos^{-1}.82\right)\right)$$

$$44,000 + j30,712VA$$

$$S_{new} = S_{old} + S_{cap} = 44,000 + j14,300VA$$

$$\theta_{new} = \tan^{-1}\frac{14,300}{44,000} = 18°$$

$$pf_{new} = .95lag$$

Problem 9.63

A bank of induction motors consumes 36 kW at 0.78 pf lagging from a 60-Hz, $220\angle 0°$ V-rms line. If 500μF of capacitors is placed in parallel with the load, find the new power factor of the combined load.

Suggested Solution

$$I_L = \frac{36kW}{(220\angle 0°)(.78)} = 209.8\angle -38.7°$$

$$I_C = \frac{220\angle 0°}{\frac{1}{j\omega_C}} = 41.5j$$

$$Then, I_T = I_L + I_C = 209.8\angle -38.7° + 41.5j = 186.7\angle -28.7°$$

$$Finally, pf = \cos(28.7°) = \boxed{.88(lag)}$$

Problem 9.64

A single-phase three-wire 60-Hz circuit serves three loads, as shown in Fig P9.64. Determine $I_{aA,} I_{nN}, I_C$, and the energy use over a 24-hour period in kilowatt-hours.

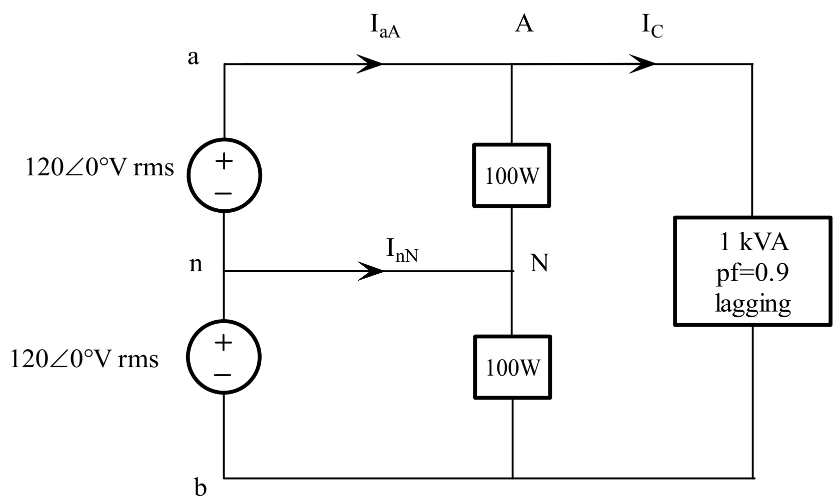


Figure P9.64

Suggested Solution

$$I_{AN} = \frac{P_{AN}}{V_{AN}} = \frac{100}{120\angle 0°} = 0.833\angle 0°\, Arms$$

$$I_{NB} = \frac{P_{NB}}{V_{NB}} = \frac{100}{120\angle 0°} = 0.833\angle 0°\, Arms$$

$$I_{nN} = I_{NB} - I_{AN} = 0\, Arms$$

$$I_C = \frac{S_C}{240}\angle -\cos^{-1}(0.9) = \frac{1000}{240}\angle -25.84° = 4.17\angle -25.84°\, Arms$$

$$I_{aA} = I_C + I_{AN} = 4.17\angle -25.84° + 0.833\angle 0°$$

$$\boxed{\begin{aligned} I_{aA} &= 4.86\angle -21.60°\, Arms \\ I_{nN} &= 0\, Arms \\ I_C &= 4.17\angle -25.84°\, Arms \end{aligned}}$$

$$\text{Power } usage = 100 + 100 + 1000(0.9) = 1.1kW$$

$$E = Energy = \int p(t)\,dt = \overline{p(t)}\Delta t$$

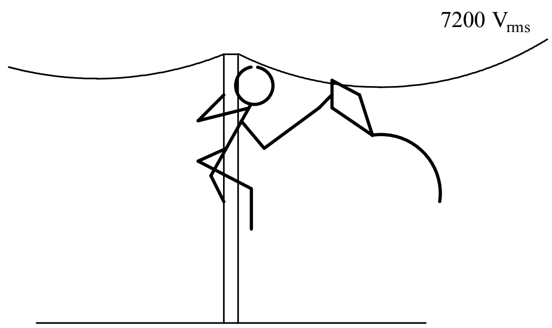
$$\Delta t = 24 hours$$
$$E = (1.1)(24) = 26.4kW \cdot hr$$

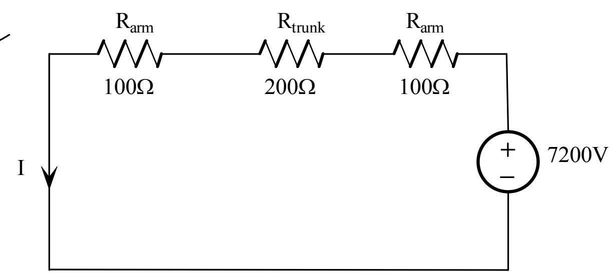
$$\boxed{E = 26.4kW \cdot hr}$$

Problem 9.65

A man and his son are flying a kite. The kite becomes entangled in a 7200-V power line close to a power pole. The man crawls up the pole to remove the kite. While trying to remove the kite, the man accidentally touches the 7200-V line. Assuming the power pole is well grounded, what is the potential current through the man's body?

Suggested Solution





Assuming shoes provided a high resistance!

$$I = \frac{7200}{400} = 18A$$

Problem 9.66

A number of 120-V household fixtures are to be used to provide lighting for a large room. The total lighting load is 8 kW. The National Electric Code requires that no circuit breaker be large than 20 A with a 25% safety margin. Determine the number of identical branch circuits needed for this requirement.

Suggested Solution

$P_L = 8kW \qquad V_L = 129V_{rms} \qquad I_L = \frac{P_L}{V_L} = 66.7\,A_{rms}$

$\#\ \text{of branches} = \frac{I_L}{20} = 3.33$

$\#\ \text{of breakers} = (1.25)(\#\ \text{of branches}) = 4.17$

$\boxed{\#\ \text{of breakers} = 5}$

Problem 9.67

A 5.1-kW household range is designed to operate on a 240-V rms sinusoidal voltage, as shown in Fig. P9.67. However, the electrician has mistakenly connected the range to 120 V rms, as shown in Fig P9.67. What is the effect of this error?

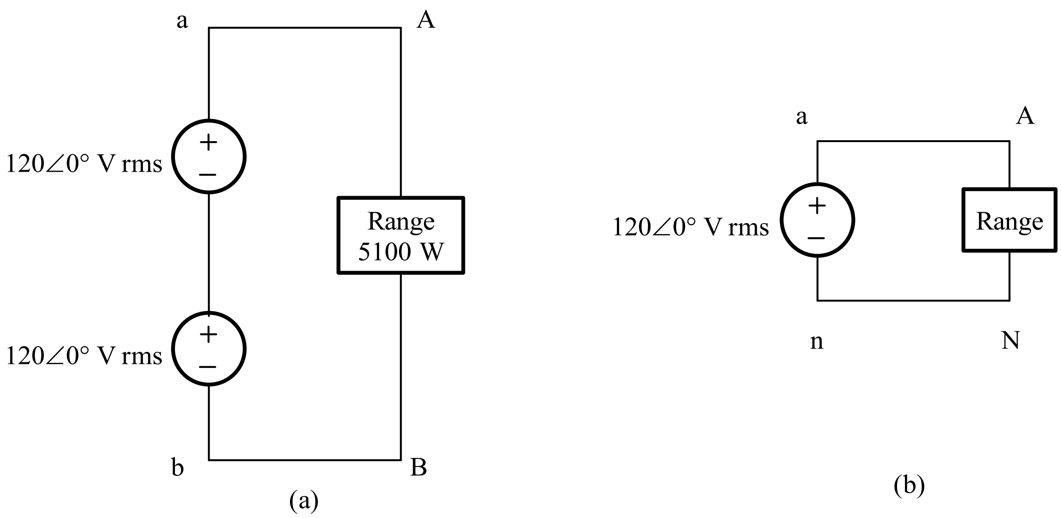


Figure P 9.67

Suggested Solution

$$R_{HE} = \frac{V^2}{P} = \frac{240^2}{5100} = 11.29\Omega$$

Now using 120V and the same R_{HE}.

$$P = \frac{120^2}{11.29} = 1275W$$

The heating element does not get as hot!

Problem 9.68

In order to test a light socket, a woman, while standing on cushions that insulate her from the ground, sticks her finger into the socket, as shown in Fig P9.68. The tip of her finger makes contact with one side of the line, and the side of her finger makes contact with the other side of the line. Assuming that any portion of a limb has a resistance of 95 Ω, is there any current in the body? Is there any current in the vicinity of the heart?

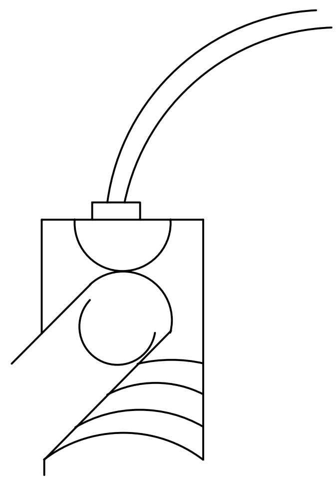
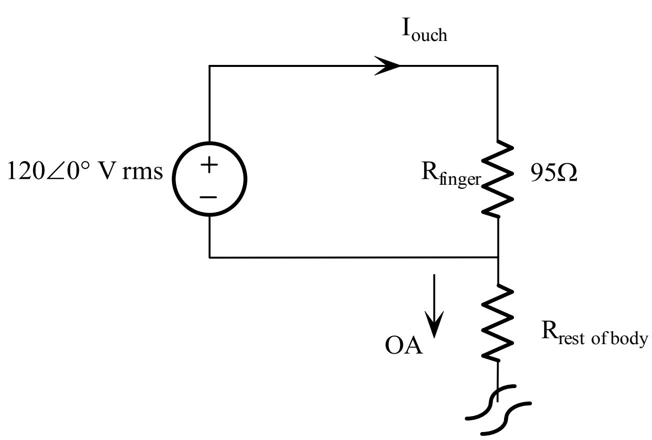


Figure P 9.68

Suggested Solution



$I_{ouch} = 120 / 95 = 1.26\ A_{rms}$

No current near the heart!

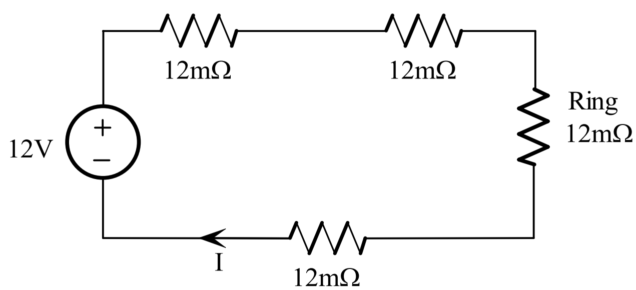
Problem 9.69

An inexperienced mechanic is installing a 12-V battery in a car. The negative terminal has been connected. He is currently tightening the bolts on the positive terminal. With a tight grip on the wrench, he turns it so that the gold ring on his finger makes contact with the frame of the car. This situation is modeled in Fig. P9.69, where we assume that the resistance of the wrench is negligible and the resistance of the contact is as follows:

$$R_1 = R_{\text{bolt to wrench}} = 0.012\Omega$$
$$R_2 = R_{\text{wrench to ring}} = 0.012\Omega$$
$$R_3 = R_{\text{ring}} = 0.012\Omega$$
$$R_4 = R_{\text{ring to frame}} = 0.012\Omega$$

What power is quickly dissipated in the gold ring, and what is the impact of this power dissipation?

Suggested Solution



$$I = \frac{12}{48 \times 10^{-3}} = 250A$$

$$P_{ring} = I^2 R_{ring} = \boxed{750W}$$

Ring will get very hot!

Finger will be burned!

Ring may spot weld to frame!

Problem 9FE-1

An industrial load consumes 120 kW at 0.707 pf lagging and is connected to a $480\angle 0°$ V rms line. Determine the value of the capacitor which, when connected in parallel with the load, will raise the power factor to 0.95 lagging.

Suggested Solution

$$\theta_{old} = \cos^{-1}.707 = 45°$$

$$Q_{old} = P_{old}\tan\theta_{old} = 120,000(1) = 120,000$$

So, $S_{old} = 120,000 + j120,000VA$

$$\theta_{new} = \cos^{-1}\left(0.95\right) = 18.19°$$

$$Q_{new} = P_{new}\tan\theta_{new} = 120,000\tan 18.19°$$

$$= 39.431VAR$$

$$Q_{cap} = Q_{new} - Q_{old} = -\omega C V_{rms}^2$$

$$= 39,431 - 120,000 = -\omega C V^2$$

$$\frac{80569}{\left(377\right)\left(480\right)^2} = \boxed{C = 927.6\mu F}$$

Problem 9FE-2

Determine the average and rms values of the following waveform.

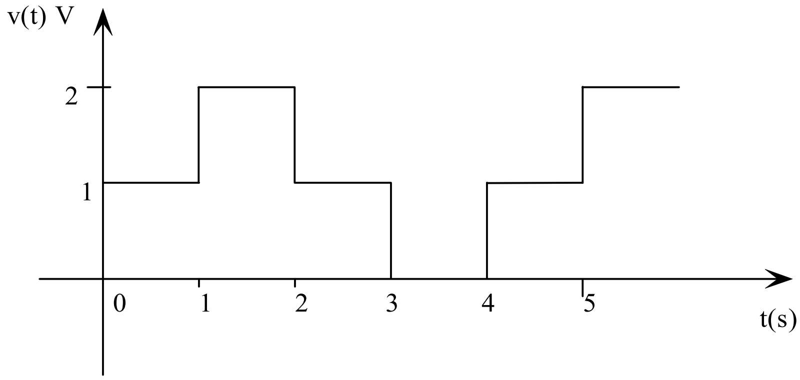


Figure 9FE-2

Suggested Solution

$$V_{ave} = \frac{1+2+1}{4} = 1V$$

$$V_{rms} = \left[\frac{1}{4}\left(\int_0^1 1dt + \int_1^2 4dt + \int_2^3 1dt\right)\right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{4}(1+4+1)\right]^{\frac{1}{2}} = \boxed{1.22V_{rms}}$$

Problem 9FE-3

Find the impedance Z_L in the network in Fig 9PFE-3 for maximum average power transfer.

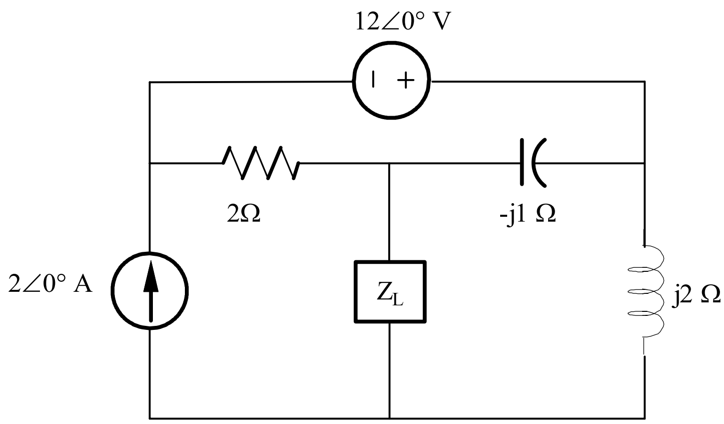
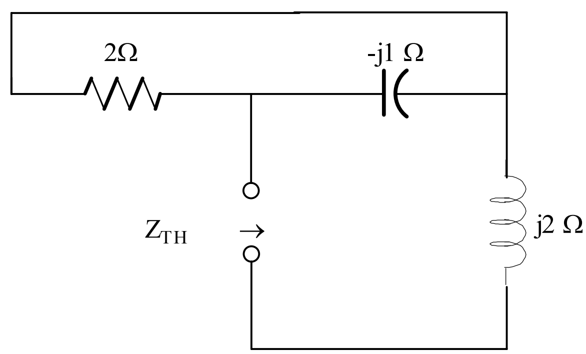


Figure 9PFE-3

Suggested Solution



$$Z_{TH} = \frac{(2)(-j1)}{2-j1} + j2$$

$$= \frac{-j2(2+j1)}{5} + j2$$

$$= \frac{-j4+2}{5} + j2$$

$$= 0.4 + j1.2\Omega$$

$$\therefore Z_L = 0.4 - j1.2\Omega$$

Problem 9FE-4

A rms-reading volmeter is connected to the output of the op-amp shown in Fig. 9PFE-4. Determine the meter reading.

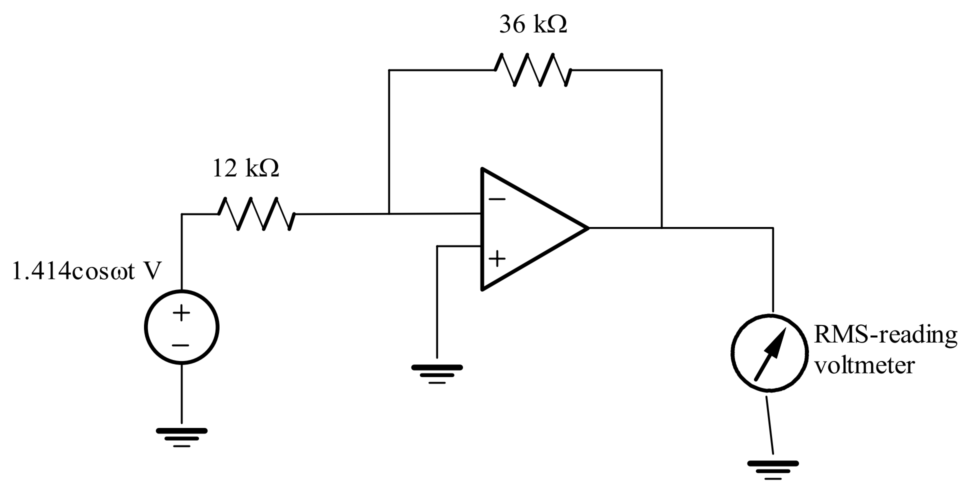


Figure 9PFE-4

Suggested Solution

$$Gain = \frac{-36k}{12k} = -3$$

$$\therefore \text{Volmeter reading} = \left| \frac{1.414}{\sqrt{2}}(-3) \right| = 3 \text{ volts rms}$$

Problem 9FE-5

Determine the average power delivered to the resistor in Fig. 9PFE-5a if the current waveform is shown in Fig. 9PFE-5b.

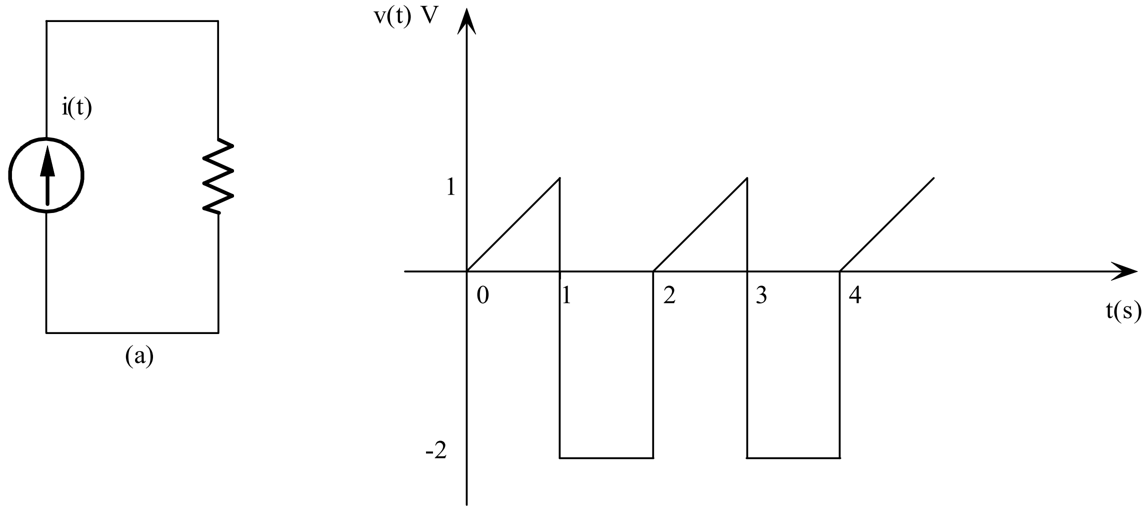


Figure 9PFE-5

Suggested Solution

$$I_{rms} = \left\{ \frac{1}{2} \left(\int_0^1 (t)^2 \, dt + \int_1^2 (-2)^2 \, dt \right) \right\}^{\frac{1}{2}}$$

$$= \left\{ \frac{1}{2} \left(\frac{t^3}{3} \Big|_0^1 + 4t \Big|_1^2 \right) \right\}^{\frac{1}{2}}$$

$$= 1.47 A$$

Then

$$\boxed{P_{ave} = I_{rms}^2 (4) = 8.67W}$$

Problem 10.1

Sketch a phasor representation of a balanced three-phase system containing both phase voltages and line voltages if $\mathbf{V}_{an} = 100\angle 45°$ V rms . Label all magnitudes and assume an *abc*-phase sequence.

Suggested Solution

Phase voltages:

$$\mathbf{V}_{an} = 100\angle 45° \text{ V rms}$$

$$\mathbf{V}_{bn} = 100\angle -75° \text{ V rms}$$

$$\mathbf{V}_{cn} = 100\angle -195° \text{ V rms}$$

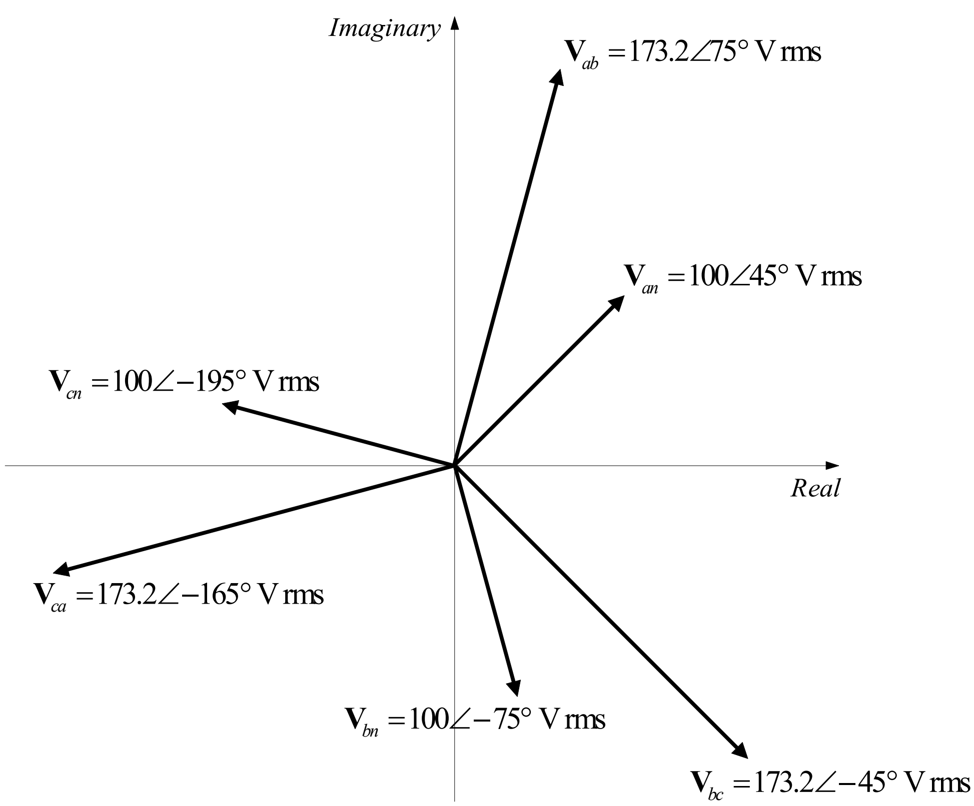
Line voltages:

Magnitude $= \sqrt{3} \times$ phase voltage magnitude
Angle = phase voltage angle $+30°$

$$\mathbf{V}_{ab} = 173.2\angle 75° \text{ V rms}$$

$$\mathbf{V}_{bc} = 173.2\angle -45° \text{ V rms}$$

$$\mathbf{V}_{ca} = 173.2\angle -165° \text{ V rms}$$



Problem 10.2

A positive-sequence three-phase balanced wye voltage source has a phase voltage of $\mathbf{V}_{an} = 100\angle 20°$ V. Determine the line voltages of the source.

Suggested Solution

$\mathbf{V}_{ab}$ leads $\mathbf{V}_{an}$ by $30°$. Therefore,

$\mathbf{V}_{ab} = 100\sqrt{3}\angle 50° = 173.2\angle 50°$ V rms ,

$\mathbf{V}_{bc} = 173.2\angle -70°$ V rms , and

$\mathbf{V}_{ca} = 173.2\angle -190°$ V rms .

Problem 10.3

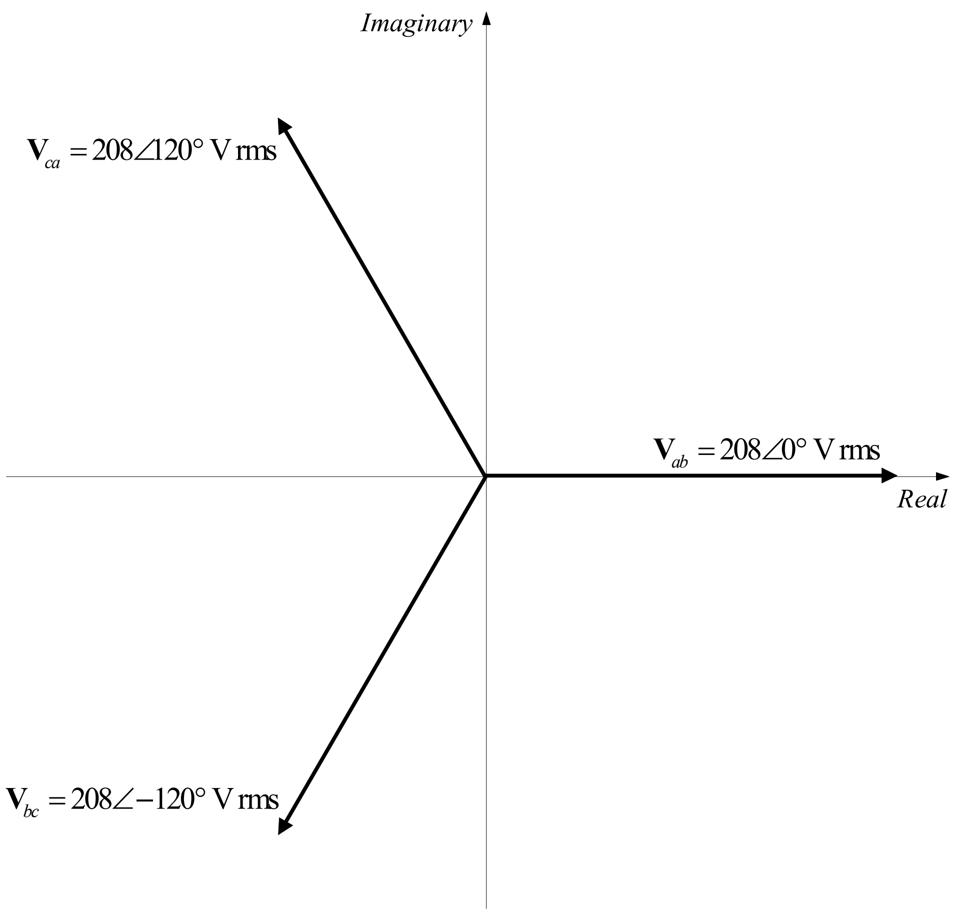
Sketch a phasor representation of an *abc*-sequence balanced three-phase Δ-connected source, including $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, and $\mathbf{V}_{ca}$ if $\mathbf{V}_{ab} = 208\angle 0°$ V rms.

Suggested Solution

$\mathbf{V}_{ab} = 208\angle 0°$ V rms

$\mathbf{V}_{bc} = 208\angle -120°$ V rms

$\mathbf{V}_{ca} = 208\angle 120°$ V rms



Problem 10.4

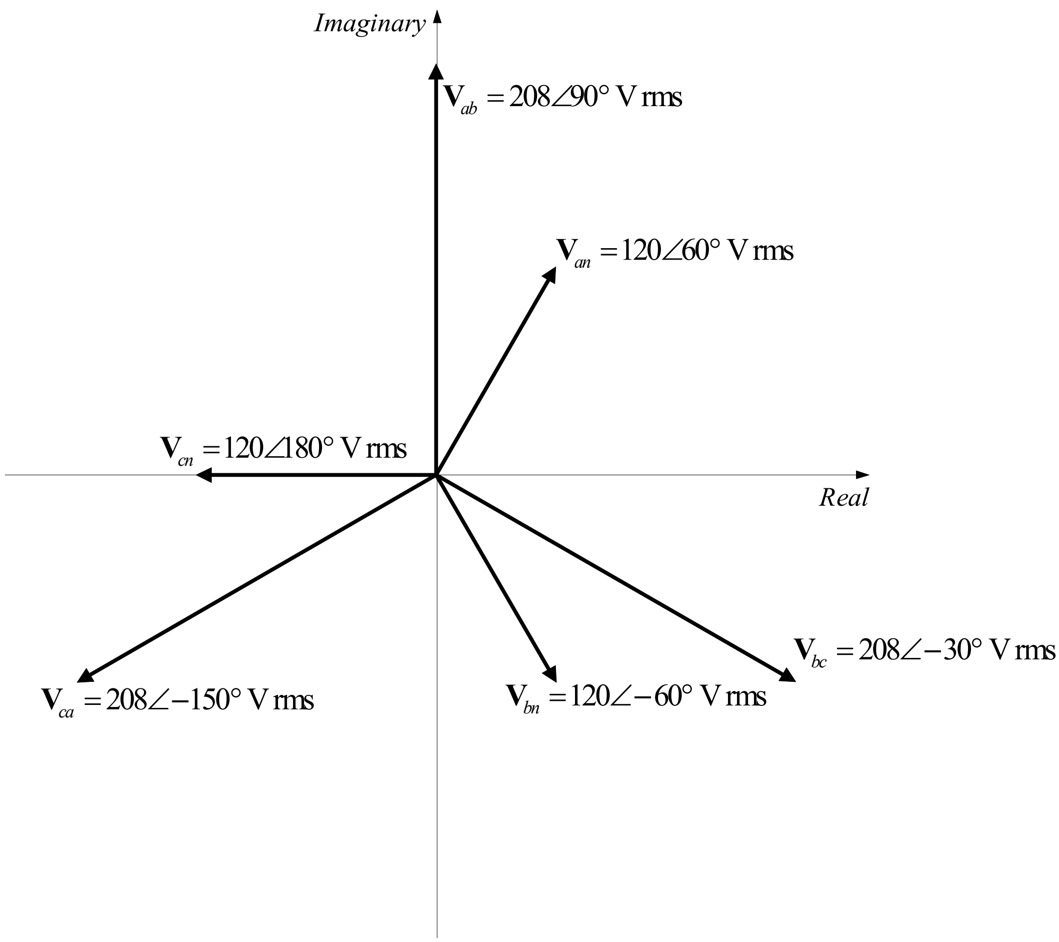
Sketch a phasor representation of a balanced three-phase system containing both phase voltages and line voltages if $\mathbf{V}_{an} = 120\angle 60°$ V rms. Label all magnitudes and assume an *abc*-phase sequence.

Suggested Solution

$\mathbf{V}_{an} = 120\angle 60°$ V rms $\qquad$ $\mathbf{V}_{ab} = 208\angle 90°$ V rms

$\mathbf{V}_{bn} = 120\angle -60°$ V rms $\qquad$ $\mathbf{V}_{bc} = 208\angle -30°$ V rms

$\mathbf{V}_{cn} = 120\angle 180°$ V rms $\qquad$ $\mathbf{V}_{ca} = 208\angle -150°$ V rms



Problem 10.5

A positive-sequence three-phase balanced wye voltage source has a phase voltage of $\mathbf{V}_{an} = 240\angle 90° \text{ V rms}$. Determine the line voltages of the source.

Suggested Solution

$$\mathbf{V}_{ab} = \sqrt{3} \cdot |\mathbf{V}_{an}| \angle\left(\theta_{\mathbf{V}_{an}} + 30°\right) = 415.7\angle 120° \text{ V rms}$$

$$\mathbf{V}_{bc} = 415.7\angle 0° \text{ V rms}$$

$$\mathbf{V}_{ca} = 415.7\angle -120° \text{ V rms}$$

Problem 10.6

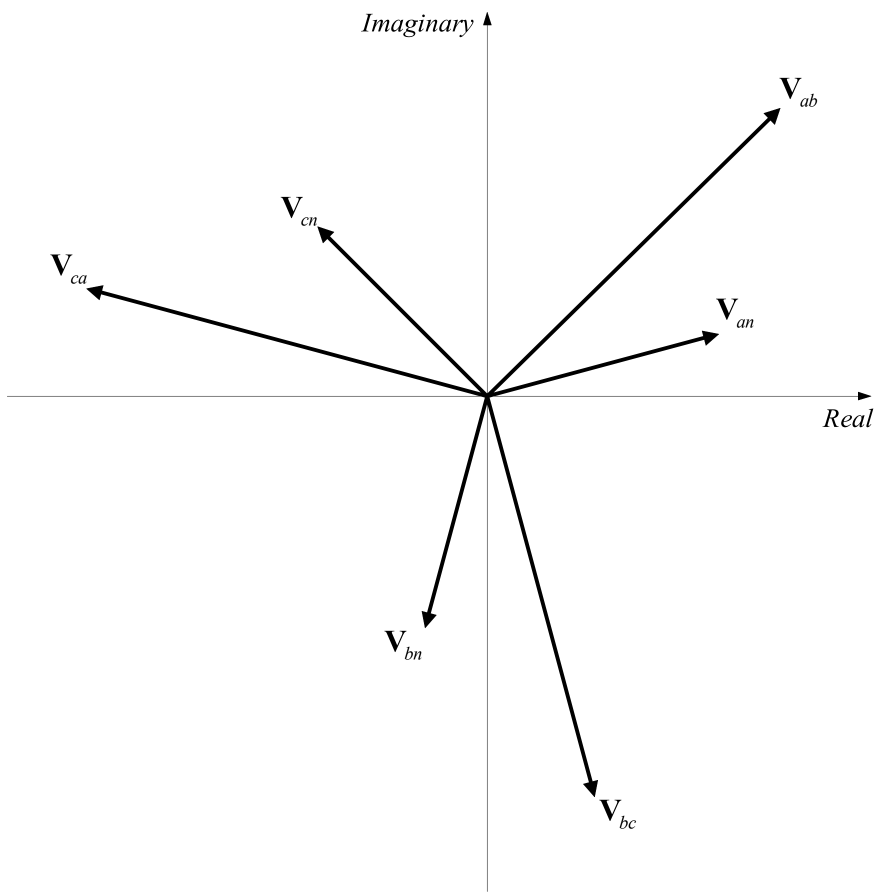
Sketch a phasor representation of a balanced three-phase system containing both phase voltages and line voltages if $\mathbf{V}_{ab} = 208\angle 45°\ \text{V rms}$. Label all phasors and assume an *abc*-phase sequence.

Suggested Solution

$$\mathbf{V}_{ab} = 208\angle 45°\ \text{V rms} \qquad \mathbf{V}_{an} = \frac{|\mathbf{V}_{ab}|}{\sqrt{3}}\angle\left(\theta_{\mathbf{V}_{ab}} - 30°\right) = 120\angle 15°\ \text{V rms}$$

$$\mathbf{V}_{bc} = 208\angle -75°\ \text{V rms} \qquad \mathbf{V}_{bn} = 120\angle -105°\ \text{V rms}$$

$$\mathbf{V}_{ca} = 208\angle 165°\ \text{V rms} \qquad \mathbf{V}_{cn} = 120\angle 135°\ \text{V rms}$$



Problem 10.7

A positive-sequence balanced three-phase wye-connected source with a phase voltage of 100 V supplies power to a balanced wye-connected load. The per phase load impedance is $40 + j10\,\Omega$. Determine the line currents in the circuit if $\angle \mathbf{V}_{an} = 0°$.

Suggested Solution

$$\mathbf{I}_{an} = \frac{\mathbf{V}_{an}}{\mathbf{Z}_{\phi}} = \frac{100 + j0}{40 + j10} = 2.43\angle -14° \text{ A rms}$$

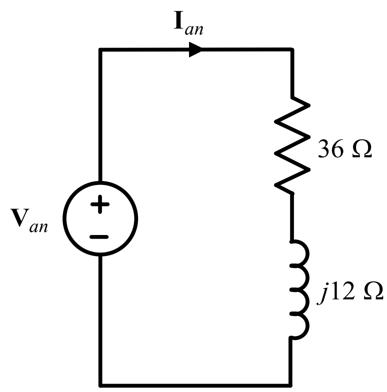
$$\mathbf{I}_{bn} = 2.43\angle -134° \text{ A rms}$$

$$\mathbf{I}_{cn} = 2.43\angle -254° \text{ A rms}$$

Problem 10.8

A positive-sequence balanced three-phase wye-connected source supplies power to a balanced wye-connected load. The magnitude of the line voltages is 150 V. If the load impedance per phase is $36+j12\,\Omega$, determine the line currents if $\angle \mathbf{V}_{an} = 0°$.

Suggested Solution



$$\mathbf{V}_{an} = \frac{150}{\sqrt{3}}\angle 0° = 86.6\angle 0° \text{ V rms} \quad \Rightarrow \quad \mathbf{I}_{an} = \frac{86.6+j0}{36+j12} = 2.28\angle -18.43° \text{ A rms}$$

$$\mathbf{I}_{bn} = 2.28\angle -138.43° \text{ A rms}$$

$$\mathbf{I}_{cn} = 2.28\angle -258.43° \text{ A rms}$$

Problem 10.9

An *abc*-sequence balanced three-phase wye-connected source supplies power to a balanced wye-connected load. The line impedance per phase is $1+j0\ \Omega$, and the load impedance per phase is $20+j20\ \Omega$. If the source line voltage $\mathbf{V}_{ab}$ is $100\angle 0°$ V rms, find the line currents.

Suggested Solution

$$\mathbf{V}_{an} = \frac{100}{\sqrt{3}}\angle\left(0°-30°\right) = 57.74\angle -30° \text{ V rms}$$

$$\mathbf{I}_{aA} = \frac{57.74\angle -30°}{\left(20+j20\right)+\left(1+j0\right)} = \frac{57.74\angle -30°}{21+j20} = 1.99\angle -73.6° \text{ A rms}$$

$$\mathbf{I}_{bB} = 1.99\angle -193.6° \text{ A rms}$$

$$\mathbf{I}_{cC} = 1.99\angle -313.6° \text{ A rms}$$

Problem 10.10

In a three-phase balanced wye-wye system, the source is an *abc*-sequence set of voltages with $\mathbf{V}_{an} = 120\angle 60^\circ$ V rms. The per phase impedance of the load is $12 + j16\,\Omega$. If the line impedance per phase is $0.8 + j1.4\,\Omega$, find the line currents and the load voltages.

Suggested Solution

Line currents:

$$\mathbf{I}_{AN} = \frac{\mathbf{V}_{an}}{\mathbf{Z}_\phi + \mathbf{Z}_L} = \frac{120\angle 60^\circ}{(12 + j16) + (0.8 + j1.4)} = \frac{120\angle 60^\circ}{21.6\angle 53.7^\circ} = 5.56\angle 6.3^\circ \text{ A rms}$$

$$\mathbf{I}_{BN} = 5.56\angle -113.7^\circ \text{ A rms}$$

$$\mathbf{I}_{CN} = 5.56\angle 126.3^\circ \text{ A rms}$$

Load voltages:

$$\mathbf{V}_{AN} = \frac{\mathbf{Z}_\phi}{\mathbf{Z}_\phi + \mathbf{Z}_L} \cdot \mathbf{V}_{an} = \frac{12 + j16}{12.8 + j17.4} \cdot (120\angle 60^\circ) = 111.1\angle 59.4^\circ \text{ V rms}$$

$$\mathbf{V}_{BN} = 111.1\angle -60.6^\circ \text{ V rms}$$

$$\mathbf{V}_{CN} = 111.1\angle 179.4^\circ \text{ V rms}$$

Problem 10.11

An *abc*-sequence set of voltages feeds a balanced three-phase wye-wye system. The line and load impedances are $0.6 + j1\,\Omega$ and $18 + j14\,\Omega$, respectively. If the load voltage on the *a* phase is $\mathbf{V}_{AN} = 114.47\angle 18.99°$ V rms, determine the voltages at the line input.

Suggested Solution

$$\mathbf{I}_a = \frac{\mathbf{V}_{AN}}{18 + j14} = 5.02\angle -18.88° \text{ A rms}$$

$$\mathbf{V}_{an} = \mathbf{I}_a\left(\mathbf{Z}_{line} + \mathbf{Z}_{load}\right) = \mathbf{I}_a\mathbf{Z}_{line} + \mathbf{V}_{AN} = \left(5.02\angle -18.88°\right)\left(0.6 + j1\right) + 114.47\angle 18.99° = 120\angle 20° \text{ V rms}$$

$$\mathbf{V}_{bn} = 120\angle -100° \text{ V rms}$$

$$\mathbf{V}_{cn} = 120\angle -220° \text{ V rms}$$

Problem 10.12

In a balanced three-phase wye-wye system, the source is an *abc*-sequence set of voltages. The load voltage on the *a* phase is $\mathbf{V}_{AN} = 108.58\angle 79.81^\circ$ V rms, $\mathbf{Z}_{\text{line}} = 1 + j1.4\,\Omega$, and $\mathbf{Z}_{\text{load}} = 10 + j13\,\Omega$. Determine the input sequence of voltages.

Suggested Solution

$$\mathbf{I}_a = \frac{\mathbf{V}_{AN}}{\mathbf{Z}_{load}} = 6.62\angle 27.38^\circ \text{ A rms}$$

$$\mathbf{V}_{an} = \mathbf{I}_a \mathbf{Z}_{line} + \mathbf{V}_{AN} = \mathbf{I}_a \left(\mathbf{Z}_{line} + \mathbf{Z}_{load}\right) = 120\angle 80^\circ \text{ V rms}$$

$$\mathbf{V}_{bn} = 120\angle -40^\circ \text{ V rms}$$

$$\mathbf{V}_{cn} = 120\angle -160^\circ \text{ V rms}$$

Problem 10.13

A balanced *abc*-sequence of voltages feeds a balanced three-phase wye-wye system. The line and load impedances are $0.6 + j0.9\,\Omega$ and $8 + j12\,\Omega$, respectively. The load voltage on the *a* phase is $\mathbf{V}_{AN} = 116.63\angle 10°\ \text{V rms}$. Find the line voltage $\mathbf{V}_{ab}$.

Suggested Solution

$$\mathbf{V}_{an} = \mathbf{V}_{AN} \cdot \frac{\mathbf{Z}_{line} + \mathbf{Z}_{load}}{\mathbf{Z}_{load}} = \left(116.63\angle 10°\right)\frac{8.6 + j12.9}{8 + j12} = 125.5\angle 10°\ \text{V rms}$$

$$\mathbf{V}_{ab} = \sqrt{3} \cdot V_{an}\angle\left(\theta_{\mathrm{V}_{an}} + 30°\right) = 217.4\angle 40°\ \text{V rms}$$

Problem 10.14

In a balanced three-phase wye-wye system, the source is an *abc*-sequence set of voltages. $\mathbf{Z}_{line} = 1 + j1.8\ \Omega$, $\mathbf{Z}_{load} = 14 + j12\ \Omega$, and the load voltage on the *a* phase is $\mathbf{V}_{AN} = 398.1\angle 17.99°\ \text{V rms}$. Find the line voltage $\mathbf{V}_{ab}$.

Suggested Solution

$$\mathbf{I}_a = \frac{\mathbf{V}_{AN}}{\mathbf{Z}_{load}} = \frac{398.1\angle 17.99°}{14 + j12} = 21.59\angle -22.61°\ \text{A rms}$$

$$\mathbf{V}_{an} = \mathbf{I}_a\left(\mathbf{Z}_{line} + \mathbf{Z}_{load}\right) = \left(21.59\angle -22.61°\right)\left(15 + j13.8\right) = 440\angle 20°\ \text{V rms}$$

$$\Rightarrow \qquad \mathbf{V}_{ab} = 440\sqrt{3}\angle 50° = 762.1\angle 50°\ \text{V rms}$$

Problem 10.15

An *abc*-phase sequence balanced three-phase source feeds a balanced load. The system is connected wye-wye and $\angle \mathbf{V}_{an} = 0°$. The line impedance is $0.5 + j0.2\,\Omega$, the load impedance is $16 + j10\,\Omega$, and the total power absorbed by the load is 1836.54 W. Determine the magnitude of the source voltage $\mathbf{V}_{an}$.

Suggested Solution

$$P_{\phi,load} = \frac{1836.54}{3} = 612.18 \text{ W} \qquad \mathbf{Z}_L = R_L + jX_L = 16 + j10\ \Omega$$

$$I_{aA}^2 = \frac{612.18}{R_L} = \frac{612.18}{16} = 38.26\ \text{W}/\Omega \quad \Rightarrow \quad I_{aA} = 6.19 \text{ A rms}$$

The total impedance is $16.5 + j10.2 = 19.4\angle 31.72°\ \Omega$

Therefore, since $\angle \mathbf{V}_{an} = 0°$, $\mathbf{I}_{aA} = 6.19\angle -31.72°$ A rms

and

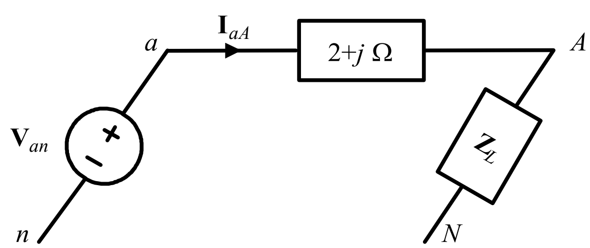
$V_{an} = 6.19 \times 19.4 = 120$ V rms

Problem 10.16

In a balanced three-phase wye-wye system, the total power loss in the lines is 272.57 W. $\mathbf{V}_{an} = 105.28\angle 31.65°$ V rms and the power factor of the load is 0.77 lagging. If the line impedance is $2 + j1\,\Omega$, determine the load impedance.

Suggested Solution

If the total line loss is 272.57 W, then the loss per phase is $\frac{272.57}{3} = 90.86$ W



$$P_{\phi,loss} = I_{aA}^2 \times \mathrm{Re}\{\mathbf{Z}_{line}\} = 2 \cdot I_{aA}^2 = 90.86 \text{ W} \quad \Rightarrow \quad I_{aA} = 6.74 \text{ A rms}$$

Then,

$$Z_L = \frac{105.28}{6.74} = 15.62\ \Omega$$

$$\theta_{\mathbf{Z}_L} = \cos^{-1}(0.77) = 39.8° \qquad \text{(lagging pf)}$$

So,

$$\mathbf{Z}_L = 15.62\angle 39.8° = 12 + j10\ \Omega$$

Problem 10.17

In a balanced three-phase wye-wye system the load impedance is $8 + j4\,\Omega$. The source has phase sequence *abc* and $\mathbf{V}_{an} = 120\angle 0°$ V rms. If the load voltage is $\mathbf{V}_{AN} = 111.62\angle -1.33°$ V rms , determine the line impedance.

Suggested Solution

$$\mathbf{I}_{aA} = \frac{111.62\angle -1.33°}{8 + j4} = 12.48\angle -27.9° \text{ A rms}$$

$$\mathbf{V}_{line} = 120\angle 0° - 111.62\angle -1.33° = 8.8\angle 17.12° \text{ V rms}$$

$$\mathbf{Z}_{line} = \frac{8.8\angle 17.12°}{12.48\angle -27.9°} = 0.71\angle 45.02° = 0.5 + j0.5\ \Omega$$

Problem 10.18

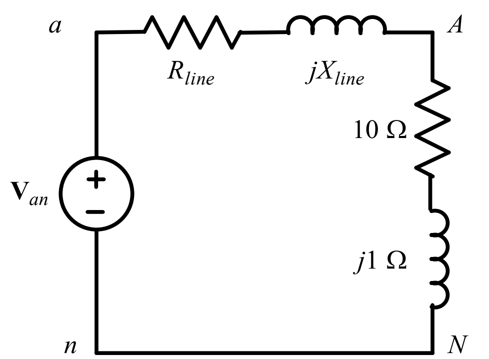
In a balanced three-phase wye-wye system the load impedance is $10 + j1\,\Omega$. The source has phase sequence *abc* and the line voltage $\mathbf{V}_{ab} = 220\angle 30°$ V rms. If the load voltage $\mathbf{V}_{AN} = 120\angle 0°$ V rms, determine the line impedance.

Suggested Solution

$\mathbf{Z}_{Y} = 10 + j1\,\Omega$

$$\mathbf{V}_{an} = \frac{|\mathbf{V}_{ab}|}{\sqrt{3}}\angle\left(\theta_{\mathbf{V}_{ab}} - 30°\right) = \frac{220}{\sqrt{3}}\angle\left(30° - 30°\right) = 127.02\angle 0° \text{ V rms}$$

Per phase Y circuit:



$$\mathbf{V}_{AN} = \left[\frac{\mathbf{Z}_{Y}}{\mathbf{Z}_{Y} + \mathbf{Z}_{line}}\right]\mathbf{V}_{an} \quad \Rightarrow \quad \mathbf{Z}_{line} = \mathbf{Z}_{Y}\left[\frac{\mathbf{V}_{an}}{\mathbf{V}_{AN}} - 1\right] = \left(10 + j1\right)\left[\frac{127.02\angle 0°}{120\angle 0°} - 1\right] = 0.59\angle 5.71°\ \Omega$$

Problem 10.19

In a balanced three-phase wye-wye system, the load impedance is $20 + j12\ \Omega$. The source has an *abc*-phase sequence and $\mathbf{V}_{an} = 120\angle 0°$ V rms. If the load voltage is $\mathbf{V}_{AN} = 111.49\angle -0.2°$ V rms, determine the magnitude of the line current if the load is suddenly short circuited.

Suggested Solution

$$\mathbf{I}_{aA} = \frac{111.49\angle -0.2°}{20 + j12} = 4.78\angle -31.16° \text{ A rms}$$

$$\mathbf{V}_{line} = 120\angle 0° - 111.49\angle -0.2° = 8.52\angle 2.62° \text{ V rms}$$

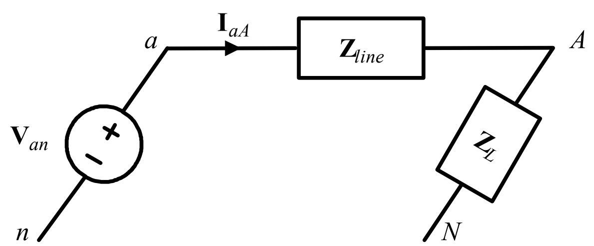
$$\mathbf{Z}_{line} = \frac{\mathbf{V}_{line}}{\mathbf{I}_{aA}} = \frac{8.52\angle 2.62°}{4.78\angle -31.16°} = 1.78\angle 33.78°\ \Omega$$

$$\therefore \qquad \left|\mathbf{I}_{aA_{SC}}\right| = \frac{120}{1.78} = 67.42 \text{ A rms}$$

Problem 10.20

In a balanced three-phase wye-wye system, the source is an *abc*-sequence set of voltages and $\mathbf{V}_{an} = 120\angle 50°$ V rms. The load voltage on the *a* phase is $110.65\angle 29.03°$ V rms and the load impedance is $16 + j20\ \Omega$. Find the line impedance.

Suggested Solution



$\mathbf{V}_{an} = 120\angle 50°$ V rms $\qquad\qquad \mathbf{V}_{AN} = 110.65\angle 29.03°$ V rms

$\mathbf{Z}_L = 16 + j20 = 25.6\angle 51.34°\ \Omega$

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{AN}}{\mathbf{Z}_L} = \frac{110.65\angle 29.03°}{25.6\angle 51.34°} = 4.32\angle -22.31° \text{ A rms}$$

Then,

$$\mathbf{Z}_{line} = \frac{\mathbf{V}_{line}}{\mathbf{I}_{aA}} = \frac{120\angle 50° - 110.65\angle 29.03°}{4.32\angle -22.31°} = 9.94\angle 139.4°\ \Omega$$

Problem 10.21

In a balanced three-phase wye-wye system, the source is an *abc*-sequence set of voltages and $\mathbf{V}_{an} = 120\angle 40°$ V rms. If the *a*-phase line current and line impedance are known to be $7.10\angle -10.28°$ A rms and $0.8 + j1\ \Omega$, respectively, find the load impedance.

Suggested Solution

$$\mathbf{Z}_T = \frac{120\angle 40°}{7.10\angle -10.28°} = 16.9\angle 50.28° = 10.8 + j13\ \Omega$$

$$\mathbf{Z}_L = \mathbf{Z}_T - \mathbf{Z}_{line} = (10.8 + j13) - (0.8 + j1) = 10 + j12\ \Omega$$

Problem 10.22

An *abc*-sequence set of voltages feeds a balanced three-phase wye-wye system. If $\mathbf{V}_{an} = 440\angle 30°$ V rms, $\mathbf{V}_{AN} = 413.28\angle 29.78°$ V rms, and $\mathbf{Z}_{\text{line}} = 1.2 + j1.5\ \Omega$, find the load impedance.

Suggested Solution

$$\mathbf{I}_a = \frac{\mathbf{V}_{an} - \mathbf{V}_{AN}}{\mathbf{Z}_{line}} = \frac{440\angle 30° - 413.28\angle 29.78°}{1.2 + j1.5} = 13.94\angle -17.93° \text{ A rms}$$

$$\mathbf{Z}_{load} = \frac{\mathbf{V}_{AN}}{\mathbf{I}_a} = \frac{413.28\angle 29.78°}{13.94\angle -17.93°} = 19.95 + j21.93\ \Omega$$

Problem 10.23

In a three-phase balanced positive-sequence system a delta-connected source supplies power to a wye-connected load. If the line impedance is $0.2 + j0.4\ \Omega$, the load impedance $6 + j4\ \Omega$, and the source phase voltage $\mathbf{V}_{ab} = 208\angle 40°$ V rms, find the magnitude of the line voltage at the load.

Suggested Solution

$\mathbf{V}_{ab} = 208\angle 40°$ V rms $\quad\Rightarrow\quad$ $\mathbf{V}_{an} = 120\angle 10°$ V rms

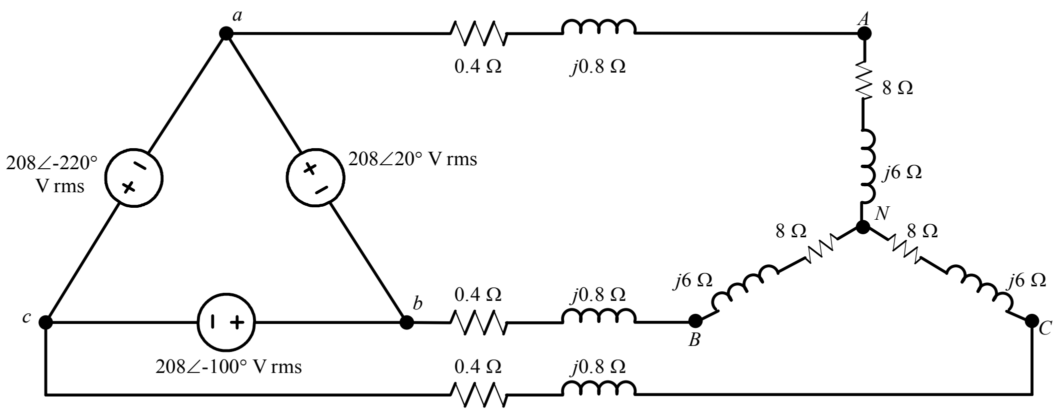
Then,

$$\mathbf{V}_{AN} = \frac{6 + j4}{6.2 + j4.4}\left(120\angle 10°\right) = 113.8\angle 8.33° \text{ V rms}$$

$$\left|\mathbf{V}_{AB}\right| = \sqrt{3}\left|\mathbf{V}_{AN}\right| = \sqrt{3}\left(113.8\right) = 197.28 \text{ V rms}$$

Problem 10.24

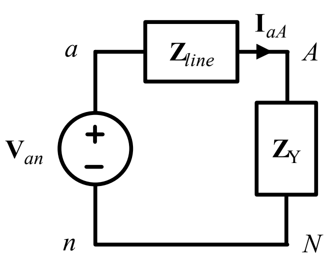
Given the network shown, compute the line currents and the magnitude of the phase voltage at the load.



Suggested Solution

$\mathbf{V}_{ab} = 208\angle 20°$ V rms $\qquad \mathbf{Z}_{line} = 0.4 + j0.8\ \Omega \qquad \mathbf{Z}_{Y} = 8 + j6\ \Omega$

Per phase Y circuit:



$$\mathbf{V}_{an} = \frac{|\mathbf{V}_{ab}|}{\sqrt{3}}\angle\left(\theta_{\mathbf{V}_{ab}} - 30°\right) = 120\angle -10° \text{ V rms}$$

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{\mathbf{Z}_{line} + \mathbf{Z}_{Y}} = \frac{120\angle -10°}{8.4 + j6.8} = 11.10\angle -49.00° \text{ A rms}$$

$$\mathbf{I}_{bB} = 11.10\angle -169° \text{ A rms}$$

$$\mathbf{I}_{cC} = 11.10\angle 71° \text{ A rms}$$

$$|\mathbf{V}_{AN}| = |\mathbf{I}_{aA}||\mathbf{Z}_{Y}| = 11.10 \times |(8 + j6)| = 11.10 \times 10 = 111 \text{ V rms}$$

$$|\mathbf{V}_{BN}| = |\mathbf{V}_{CN}| = |\mathbf{V}_{AN}| = 111 \text{ V rms}$$

Problem 10.25

In a balanced three-phase delta-wye system the source had an *abc*-phase sequence. The line and load impedances are $0.6 + j0.3\ \Omega$ and $12 + j7\ \Omega$, respectively. If the line current $\mathbf{I}_{aA} = 9.6\angle -20^\circ$ A rms, determine the phase voltages of the source.

Suggested Solution

$$\mathbf{V}_{an} = \left(9.6\angle -20^\circ\right)\left(12.6 + j7.3\right) = 139.78\angle 10.09^\circ \text{ V rms}$$

$$\mathbf{V}_{ab} = 139.78\sqrt{3}\angle\left(10.09^\circ + 30^\circ\right) = 242.11\angle 40.09^\circ \text{ V rms}$$

$$\mathbf{V}_{bc} = 242.11\angle\left(40.09^\circ - 120^\circ\right) = 242.11\angle -79.91^\circ \text{ V rms}$$

$$\mathbf{V}_{ca} = 242.11\angle\left(40.09^\circ + 120^\circ\right) = 242.11\angle 160.09^\circ \text{ V rms}$$

Problem 10.26

An *abc*-phase-sequence three-phase balanced wye-connected 60-Hz source supplies a balanced delta-connected load. The phase impedance in the load consists of a $20-\Omega$ resistor in series with a 50-mH inductor, and the phase voltage at the source is $\mathbf{V}_{an} = 120\angle 20°$ V rms. If the line impedance is zero, find the line currents in the system.

Suggested Solution

$$\mathbf{Z}_{\Delta} = 20 + j377(0.05)\ \Omega$$

$$\mathbf{Z}_{line} = 0\ \Omega$$

$$\mathbf{V}_{an} = 120\angle 20°\text{ V rms}$$

$$\Delta \rightarrow \text{Y}: \quad \mathbf{Z}_{\text{Y}} = \frac{\mathbf{V}_{AN}}{\mathbf{I}_{aA}} = \frac{\mathbf{V}_{AB}/\sqrt{3}}{\mathbf{I}_{AB}\sqrt{3}} = \frac{\mathbf{Z}_{\Delta}}{3} = 6.67 + j6.21\ \Omega$$

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{\mathbf{Z}_{\text{Y}}} \quad \Rightarrow \quad \mathbf{I}_{aA} = 13.10\angle -23.28°\text{ A rms}$$

$$\mathbf{I}_{bB} = 13.10\angle -143.28°\text{ A rms}$$

$$\mathbf{I}_{cC} = 13.10\angle 96.72°\text{ A rms}$$

Problem 10.27

An *abc*-phase-sequence three-phase balanced wye-connected source supplies a balanced delta-connected load. The impedance per phase in the delta load is $12 + j9\ \Omega$. The line voltage at the source is $\mathbf{V}_{ab} = 120\sqrt{3}\angle 40°$ V rms. If the line impedance is zero, find the line currents in the balanced wye-delta system.

Suggested Solution

$$\mathbf{Z}_{line} = 0 \quad \Rightarrow \quad \mathbf{V}_{ab} = \mathbf{V}_{AB}$$

$$\mathbf{I}_{AB} = \frac{120\sqrt{3}\angle 40°}{12 + j9} = 13.86\angle 3.13° \text{ A rms}$$

$$\therefore \quad \mathbf{I}_{aA} = 13.86\sqrt{3}\angle\left(3.13° - 30°\right) = 24.01\angle -26.87° \text{ A rms},$$

$$\mathbf{I}_{bB} = 24.01\angle -146.87° \text{ A rms}, \text{ and}$$

$$\mathbf{I}_{cC} = 24.01\angle 93.13° \text{ A rms}$$

Problem 10.28

An *abc*-phase-sequence three-phase balanced wye-connected source supplies power to a balanced delta-connected load. The impedance per phase in the load is $14 + j12\ \Omega$. If the source voltage for the *a* phase is $\mathbf{V}_{an} = 120\angle 80°$ V rms, and the line impedance is zero, find the phase currents in the wye-connected source.

Suggested Solution

$$\mathbf{V}_{an} = 120\angle 80° \text{ V rms} \Rightarrow \mathbf{V}_{ab} = 120\sqrt{3}\angle 110° \text{ V rms}$$

$$\mathbf{Z}_{line} = 0 \quad \Rightarrow \quad \mathbf{V}_{ab} = \mathbf{V}_{AB}$$

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{load}} = \frac{120\sqrt{3}\angle 110°}{14 + j12} = 11.27\angle 69.4° \text{ A rms},$$

$$\mathbf{I}_{bB} = 19.52\angle -80.6° \text{ A rms} \text{ and}$$

$$\mathbf{I}_{cC} = 19.52\angle 159.4° \text{ A rms}$$

Problem 10.29

An *abc*-phase-sequence three-phase balanced wye-connected source supplies a balanced delta-connected load. The impedance per phase of the delta load is $10 + j8\ \Omega$. If the line impedance is zero and the line current in the *a* phase is known to be $\mathbf{I}_{aA} = 28.10\angle -28.66°$ A rms, find the load voltage $\mathbf{V}_{AB}$.

Suggested Solution

If $\mathbf{I}_{aA} = 28.1\angle -28.66°\ \text{A rms}$, then

$$\mathbf{I}_{AB} = \frac{28.1}{\sqrt{3}}\angle\left(-28.66° + 30°\right) = 16.22\angle 1.34°\ \text{A rms}$$

$$\mathbf{V}_{AB} = \mathbf{I}_{AB} \cdot \mathbf{Z}_{\Delta} = \left(16.22\angle 1.34°\right)\left(10 + j8\right) = 207.8\angle 40°\ \text{V rms}$$

Problem 10.30

An *abc*-phase-sequence three-phase balanced wye-connected source supplies power to a balanced delta-connected load. The impedance per phase of the delta load is $14 + j11\ \Omega$. If the line impedance is zero and the line current in the *a* phase is $\mathbf{I}_{aA} = 20.22\angle 31.84°$ A rms, find the voltages of the balanced source.

Suggested Solution

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = \frac{14}{3} + j\frac{11}{3} = 5.93\angle 38.2°\ \Omega$$

$$\begin{aligned}\mathbf{V}_{an} &= \mathbf{I}_{aA} \cdot \mathbf{Z}_Y \\ &= (20.22\angle 31.84°)(5.93\angle 38.2°) \\ &= 120\angle 70°\ \text{V rms}\end{aligned}$$

$$\mathbf{V}_{bn} = 120\angle -50°\ \text{V rms}$$

$$\mathbf{V}_{cn} = 120\angle 190°\ \text{V rms}$$

Problem 10.31

In a balanced three-phase wye-delta system, the source has an *abc*-phase sequence and $\mathbf{V}_{an} = 120\angle 0°$ V rms. If the line current is $\mathbf{I}_{aA} = 4.8\angle 20°$ A rms, find the load impedance per phase in the delta.

Suggested Solution

If $\mathbf{V}_{an} = 120\angle 0°$, then $\mathbf{V}_{AB} = 120\sqrt{3}\angle 30°$ V rms

and if $\mathbf{I}_{aA} = 4.8\angle 20°$, then $\mathbf{I}_{AB} = \frac{4.8}{\sqrt{3}}\angle 50°$ A rms

$$\therefore\ \mathbf{Z}_{load} = \frac{\mathbf{V}_{AB}}{\mathbf{I}_{AB}} = 70.48 - j25.65\ \Omega$$

Problem 10.32

In a balanced three-phase wye-delta system, the source has an *abc*-phase sequence and $\mathbf{V}_{an} = 120\angle 40°$ V rms. The line and load impedance are $0.5 + j0.4\ \Omega$ and $24 + j18\ \Omega$, respectively. Find the delta currents in the load.

Suggested Solution

Using $\Delta \to \text{Y}$ conversion, $\mathbf{Z}_{\text{Y}} = \frac{1}{3}\mathbf{Z}_{\Delta} = 8 + j6\ \Omega$

$$\mathbf{I}_{aA} = \frac{120\angle 40°}{8.5 + j6.4} = 11.28\angle 3.02°\ \text{A rms}$$

Then,

$$\mathbf{I}_{AB} = \frac{11.28}{\sqrt{3}}\angle\left(3.02° + 30°\right) = 6.51\angle 33.02\ \text{A rms}\ ,$$

$\mathbf{I}_{BC} = 6.51\angle -86.98°$ A rms , and

$\mathbf{I}_{CA} = 6.51\angle 153.02°$ A rms

Problem 10.33

In a three-phase balanced delta-delta system, the source has an *abc*-phase sequence. The line and load impedances are $0.3 + j0.2\ \Omega$ and $9 + j6\ \Omega$, respectively. If the load current in the delta is $\mathbf{I}_{AB} = 15\angle 40°$ A rms, find the phase voltages of the source.

Suggested Solution

Converting to wye sources, $\mathbf{Z}_{\text{Y}} = \dfrac{\mathbf{Z}_{\Delta}}{3}$

$$\mathbf{I}_{aA} = 15\sqrt{3}\angle\left(40° - 30°\right) = 26\angle 10°\ \text{A rms}$$

$$\begin{aligned}\mathbf{V}_{an} &= \left(\mathbf{I}_{an}\right)\left(\mathbf{Z}_{line} + \mathbf{Z}_{\text{Y}}\right)\\ &= \left(26\angle 10°\right)\left(3.3 + j2.2\right)\\ &= 103\angle 43.7°\ \text{V rms}\end{aligned}$$

Then,

$$\begin{aligned}\mathbf{V}_{ab} &= 103\sqrt{3}\angle 73.7°\\ &= 178.5\angle 73.7°\ \text{V rms,}\end{aligned}$$

$$\mathbf{V}_{bc} = 178.5\angle -46.3°\ \text{V rms, and}$$

$$\mathbf{V}_{ca} = 178.5\angle 193.7°\ \text{V rms}$$

Problem 10.34

In a balanced three-phase delta-delta system, the source has an *abc*-phase sequence. The phase angle for the source voltage is $\angle \mathbf{V}_{ab} = 40°$ and $\mathbf{I}_{ab} = 4\angle 15°$ A rms. If the total power absorbed by the load is 1400 W, find the load impedance.

Suggested Solution

$$P_{L_{total}} = 1400 \text{ W} \quad \Rightarrow \quad P_\phi = \frac{1400}{3} = 466.67 \text{ W}$$

Since $\angle \mathbf{V}_{ab} = 40°$ and $\mathbf{I}_{ab} = 4\angle 15°$ A rms $\quad \Rightarrow \quad$ No line impedance

$$P_\phi = |\mathbf{V}_{AB}||\mathbf{I}_{AB}|\cos(40° - 15°)$$

$$|\mathbf{V}_{AB}| = \frac{466.67}{4\cos 25°} = 128.73 \text{ V rms}$$

$$\mathbf{Z}_L = \frac{\mathbf{V}_{AB}}{\mathbf{I}_{AB}} = \frac{128.73\angle 40°}{4\angle 15°} = 32.18\angle 25° \ \Omega$$

Problem 10.35

A three-phase load impedance consists of a balanced wye in parallel with a balanced delta. What is the equivalent wye load and what is the equivalent delta load if the phase impedance of the wye and delta are $6 + j3\ \Omega$ and $15 + j12\ \Omega$, respectively?

Suggested Solution

Equivalent Y:

$$\mathbf{Z}_{\mathrm{Y}}^{'} = \frac{\mathbf{Z}_{\Delta}^{'}}{3}$$

$$\mathbf{Z}_{total} = \frac{(6+j3)(5+j4)}{(6+j3)+(5+j4)} = \frac{18+j39}{11+j7} = 2.77 + j1.78\ \Omega$$

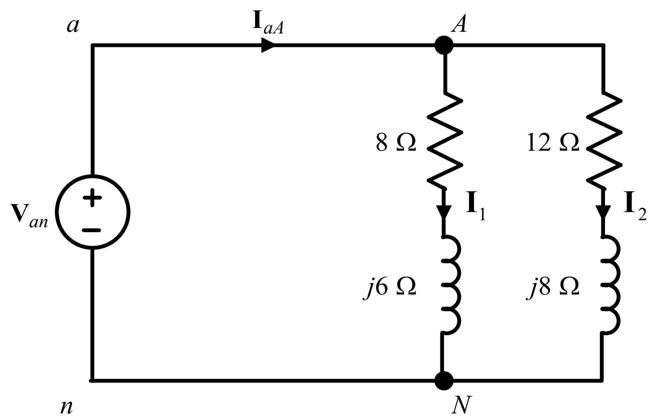
Equivalent Δ:

$$\mathbf{Z}_{\Delta} = 3\mathbf{Z}_{\mathrm{Y}} = 3(2.77 + j1.78) = 8.31 + j5.34\ \Omega$$

Problem 10.36

In a balanced three-phase system, the *abc*-phase-sequence source is wye connected and $\mathbf{V}_{an} = 120\angle 20°$ V rms. The load consists of two balanced wyes with phase impedances of $8 + j6\ \Omega$ and $12 + j8\ \Omega$. If the line impedance is zero, find the line currents and the phase currents in each load.

Suggested Solution



$$\mathbf{I}_1 = \frac{120\angle 20°}{8 + j6} = 12\angle -16.87° \text{ A rms}$$

$$\mathbf{I}_2 = \frac{120\angle 20°}{12 + j8} = 8.32\angle -13.69° \text{ A rms}$$

$$\mathbf{I}_{aA} = \mathbf{I}_1 + \mathbf{I}_2 = 20.3\angle -15.57° \text{ A rms}$$

The other currents are shifted by $-120°$ and $-240°$

Problem 10.37

In a balanced three-phase system, the source is a balanced wye with an *abc*-phase sequence and $\mathbf{V}_{ab} = 208\angle 60°$ V rms. The load consists of a balanced wye with a phase impedance of $8 + j5\ \Omega$ in parallel with a balanced delta with a phase impedance of $21 + j12\ \Omega$. If the line impedance is $1.2 + j1\ \Omega$, find the phase currents in the balanced wye load.

Suggested Solution

Converting $\mathbf{Z}_\Delta$ to $\mathbf{Z}'_Y$: $\quad \mathbf{Z}'_Y = 7 + j4\ \Omega$

Then, $\mathbf{Z}_L = (8 + j5)\|(7 + j4) = 4.35\angle 30.8°\ \Omega$

$$\mathbf{V}_{ab} = 208\angle 60° \text{ V rms} \quad \Rightarrow \quad \mathbf{V}_{an} = \frac{208}{\sqrt{3}}\angle(60° - 30°) = 120\angle 30° \text{ V rms}$$

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{\mathbf{Z}_L + \mathbf{Z}_{line}} = \frac{120\angle 30°}{5.9\angle 33.2°} = 20.3\angle -3.2° \text{ A rms}$$

$$\mathbf{V}_{AN} = \mathbf{I}_{aA}\mathbf{Z}_L = (20.33\angle -3.2°)(4.35\angle 30.8°) = 88.4\angle 27.6° \text{ V rms}$$

$$\mathbf{I}_{AN} = \frac{\mathbf{V}_{AN}}{\mathbf{Z}_Y} = \frac{88.4\angle 27.6°}{8 + j5} = 9.37\angle -4.4° \text{ A rms}$$

$$\mathbf{I}_{BN} = 9.37\angle 115.6° \text{ A rms}$$

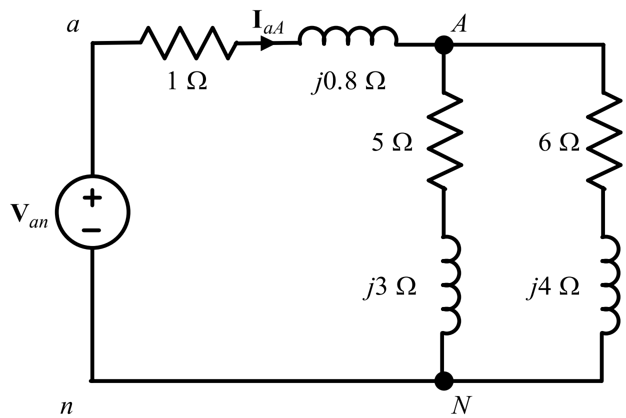
$$\mathbf{I}_{CN} = 9.37\angle -124.4° \text{ A rms}$$

Problem 10.38

In a balanced three-phase system, the source is a balanced wye with an *abc*-phase sequence and $\mathbf{V}_{ab} = 208\angle 50°$ V rms. The load is a balanced wye in parallel with a balanced delta. The phase impedance of the wye is $5 + j3\ \Omega$ and the phase impedance of the delta is $18 + j12\ \Omega$. If the line impedance is $1 + j0.8\ \Omega$, find the line currents and the phase currents in the loads.

Suggested Solution

$$\mathbf{Z}'_{\mathrm{Y}} = \frac{\mathbf{Z}_{\Delta}}{3} = 6 + j4\ \Omega$$



$$\mathbf{V}_{an} = \frac{208}{\sqrt{3}}\angle(50° - 30°) = 120\angle 20°\ \text{V rms}$$

$$\mathbf{Z}_L = \frac{(5 + j3)(6 + j4)}{(5 + j3) + (6 + j4)} = 3.22\angle 32.18°\ \Omega$$

$$\mathbf{I}_{aA} = \frac{120\angle 20°}{\mathbf{Z}_{line} + \mathbf{Z}_L} = \frac{120\angle 20°}{4.5\angle 33.94°} = 26.67\angle -13.94°\ \text{A rms}$$

$$\mathbf{V}_{AN} = \mathbf{I}_{aA}\mathbf{Z}_{load} = 85.88\angle 18.24°\ \text{V rms}$$

$$\mathrm{Y}_{load}: \quad \mathbf{I}_{AN} = \frac{\mathbf{V}_{AN}}{5 + j3} = 14.73\angle -12.72°\ \text{A rms}$$

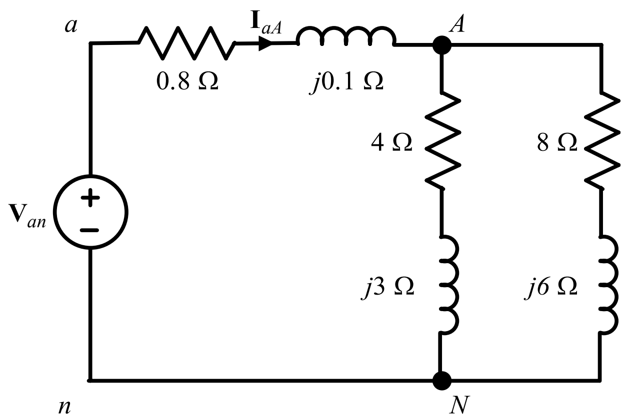
$$\Delta_{load}: \quad \mathbf{V}_{AB} = 85.88\sqrt{3}\angle 48.24°\ \text{V rms} \quad \Rightarrow \quad \mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{18 + j12} = 6.88\angle 14.55°\ \text{A rms}$$

The other currents are shifted by $-120°$ and $-240°$

Problem 10.39

In a balanced three-phase system the source, which has an *abc*-phase sequence, is connected in delta and $\mathbf{V}_{ab} = 208\angle 55°$ V rms. There are two loads connected in parallel. Load 1 is connected in wye and the phase impedance is $4 + j3\ \Omega$. Load 2 is connected in wye and the phase impedance is $8 + j6\ \Omega$. Compute the delta currents in the source if the line impedance connecting the source to the loads is $0.2 + j0.1\ \Omega$.

Suggested Solution



$$\mathbf{V}_{an} = \frac{208}{\sqrt{3}}\angle(55° - 30°) = 120\angle 25° \text{ V rms}$$

$$\mathbf{Z}_L = \frac{(4 + j3)(8 + j6)}{(4 + j3) + (8 + j6)} = 3.33\angle 36.87° = 2.66 + j2.0\ \Omega$$

$$\mathbf{I}_{aA} = \frac{120\angle 25°}{\mathbf{Z}_{line} + \mathbf{Z}_L} = \frac{120\angle 25°}{2.86 + j2.1} = 33.8\angle -11.29° \text{ A rms}$$

$$\mathbf{I}_{ba} = \frac{33.8}{\sqrt{3}}\angle(-11.29° + 30°) = 19.51\angle 18.71° \text{ A rms}$$

$$\mathbf{I}_{cb} = 19.51\angle(18.71° - 120°) = 19.51\angle -101.29° \text{ A rms}$$

$$\mathbf{I}_{ac} = 19.51\angle(18.71° - 240°) = 19.51\angle -221.29° \text{ A rms}$$

Problem 10.40

In a balanced three-phase system, the source has an *abc*-phase sequence and is connected in delta. There are two parallel wye-connected loads. The phase impedance of load 1 and load 2 is $4+j4\,\Omega$ and $10+j4\,\Omega$, respectively. The line impedance connecting the source to the loads is $0.3+j0.2\,\Omega$. If the current in the *a* phase of load 1 is $\mathbf{I}_{AN_1}=10\angle 20°$ A rms, find the delta currents in the source.

Suggested Solution

By current division:

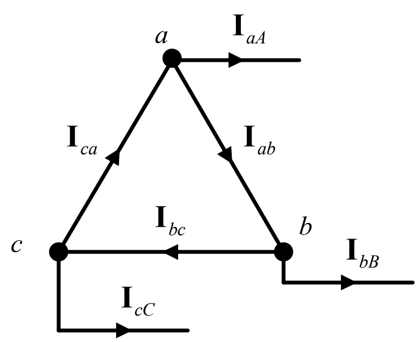
$$\mathbf{I}_{AN_1}=\mathbf{I}_{aA}\cdot\frac{10+j4}{(4+j4)+(10+j4)} \quad\Rightarrow\quad \mathbf{I}_{aA}=15\angle 27.9°\text{ A rms}$$

Then, since a source:

$$\mathbf{I}_{ab}=-\frac{|\mathbf{I}_{aA}|}{\sqrt{3}}\angle(\theta+30°)=\frac{15}{\sqrt{3}}\angle -122.1°=8.64\angle -122.1°\text{ A rms}$$

$$\mathbf{I}_{bc}=8.64\angle -242.1°\text{ A rms}$$

$$\mathbf{I}_{ca}=8.64\angle -2.1°\text{ A rms}$$

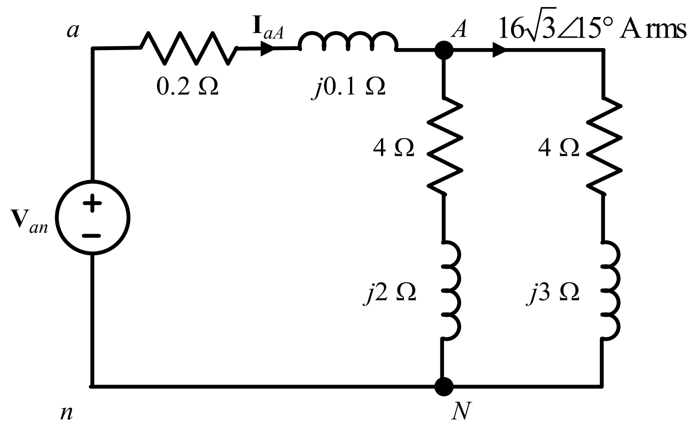


Problem 10.41

In a balanced three-phase system, the source has an *abc*-phase sequence and is connected in delta. There are two loads connected in parallel. The line connecting the source to the loads has an impedance of $0.2 + j0.1\ \Omega$. Load 1 is connected in wye and the phase impedance is $4 + j2\ \Omega$. Load 2 is connected in delta, and the phase impedance is $12 + j9\ \Omega$. The current $\mathbf{I}_{AB}$ in the delta load is $16\angle 45°$ A rms. Find the phase voltages of the source.

Suggested Solution

The Y equivalent circuit is:



Using current division:

$$\frac{\mathbf{I}_{aA}\left(4 + j2\right)}{8 + j5} = 16\sqrt{3}\angle 15° \quad \Rightarrow \quad \mathbf{I}_{aA} = 58.43\angle 20.44° \text{ A rms}$$

$$\mathbf{V}_{an} = \mathbf{I}_{aA}\left(0.2 + j0.1\right) + \left(16\sqrt{3}\angle 15°\right)\left(4 + j3\right) = 151.36\angle 51.46° \text{ V rms}$$

$\therefore \quad \mathbf{V}_{ab} = 262.16\angle 81.46°$ V rms ,

$\mathbf{V}_{bc} = 262.16\angle -38.54°$ V rms , and

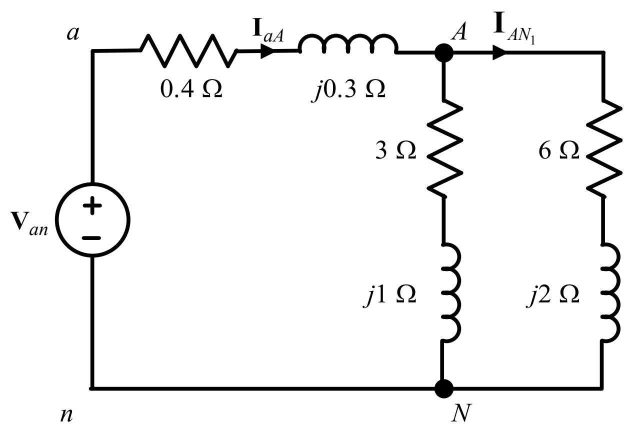
$\mathbf{V}_{ca} = 262.16\angle -158.54°$ V rms

Problem 10.42

In a balanced three-phase system, the source has an *abc*-phase sequence and is connected in delta. There are two loads connected in parallel. Load 1 is connected in wye and has a phase impedance of $6 + j2\ \Omega$. Load 2 is connected in delta and has a phase impedance of $9 + j3\ \Omega$. The line impedance is $0.4 + j0.3\ \Omega$. Determine the phase voltages of the source if the current in the *a* phase of load 1 is $\mathbf{I}_{AN_1} = 12\angle 30°$ A rms.

Suggested Solution

The Y equivalent circuit is:



Using current division:

$$\frac{\mathbf{I}_{aA}(3 + j1)}{9 + j3} = 12\angle 30° \text{ A rms} \quad \Rightarrow \quad \mathbf{I}_{aA} = 36\angle 30° \text{ A rms}$$

$$\mathbf{V}_{an} = (\mathbf{I}_{aA})(0.4 + j0.3) + (12\angle 30°)(6 + j2) = 93.14\angle 51.93° \text{ V rms}$$

$$\mathbf{V}_{ab} = 93.14\sqrt{3}\angle(51.93° + 30°) = 161.32\angle 81.93° \text{ V rms}$$

$$\mathbf{V}_{bc} = 161.32\angle(81.93° - 120°) = 161.32\angle -38.07° \text{ V rms}$$

$$\mathbf{V}_{ca} = 161.32\angle(81.93° - 240°) = 161.32\angle -158.07° \text{ V rms}$$

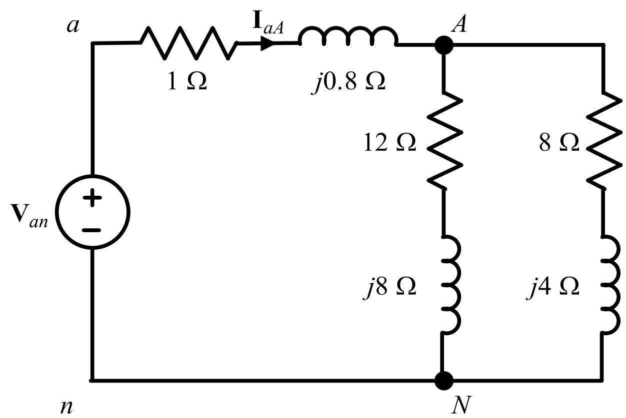
Problem 10.43

A balanced three-phase delta-connected source supplies power to a load consisting of a balanced delta in parallel with a balanced wye. The phase impedance of the delta is $24 + j12\ \Omega$, and the phase impedance of the wye is $12 + j8\ \Omega$. The *abc*-phase-sequence source voltages are $\mathbf{V}_{ab} = 440\angle 60°\ \text{V rms}$, $\mathbf{V}_{bc} = 440\angle -60°\ \text{V rms}$ and $\mathbf{V}_{ca} = 440\angle -180°\ \text{V rms}$, and the line impedance per phase is $1 + j0.8\ \Omega$. Find the line currents and the power absorbed by the wye-connected load.

Suggested Solution

$$\mathbf{V}_{ab} = 440\angle 60°\ \text{V rms} \quad \Rightarrow \quad \mathbf{V}_{an} = \frac{440}{\sqrt{3}}\angle\left(60° - 30°\right) = 254\angle 30°\ \text{V rms}$$

Equivalent single-phase (*a*-phase) diagram:



$$\mathbf{Z}_L = \frac{(12 + j8)(8 + j4)}{20 + j12} = 5.54\angle 29.3°\ \Omega$$

$$\mathbf{Z}_T = \mathbf{Z}_{line} + \mathbf{Z}_L = 6.8\angle 30.9°\ \Omega$$

$$\mathbf{I}_{aA} = \frac{254\angle 30°}{6.8\angle 30.9°} = 37.35\angle -1°\ \text{A rms} \quad \Rightarrow \quad \mathbf{I}_{bB} = 37.35\angle -121°\ \text{A rms} \ \text{and}\ \mathbf{I}_{cC} = 37.35\angle 119°\ \text{A rms}$$

$$\mathbf{V}_{AN} = \mathbf{I}_{aA}\mathbf{Z}_L = 206.93\angle 28.3°\ \text{V rms}$$

$$\mathbf{I}_{AN-\text{Y}} = \frac{\mathbf{V}_{AN}}{\mathbf{Z}_\text{Y}} = \frac{206.93\angle 28.3°}{12 + j8} = 14.37\angle -5.4°\ \text{A rms}$$

$$P_{\text{Y},load} = 3(14.37)^2(12) = 7.434\ \text{kW}$$

Problem 10.44

An *abc*-sequence wye-connected source having a phase-*a* voltage of $120\angle 0°$ V rms is attached to a wye-connected load having an impedance of $80\angle 70°\ \Omega$. If the line impedance is $4\angle 20°\ \Omega$, determine the total complex power produced by the voltage source and the real and reactive power dissipated by the load.

Suggested Solution

$$\mathbf{I}_{aA} = \frac{120\angle 0°}{80\angle 70° + 4\angle 20°} = \frac{120\angle 0°}{31.12 + j76.55} = 1.45\angle -67.88° \text{ A rms}$$

$$\mathbf{V}_L = \mathbf{I}_{aA}\mathbf{Z}_{\text{Y}} = (1.45\angle -67.88°)(80\angle 70°) = 116.18\angle 2.12° \text{ V rms}$$

$$\mathbf{S}_{\phi,S} = \mathbf{VI}^* = (120\angle 0°)(1.45\angle 67.88°) = 174\angle 67.88° \text{ VA}$$

$$\mathbf{S}_{T,S} = 3\mathbf{S}_{\phi,S} = 522\angle 67.88° \text{ VA}$$

$$\mathbf{S}_{\phi,L} = (116.18\angle 2.12°)(1.45\angle 62.88°) = 168.46\angle 70° = 57.62 + j158.30 \text{ VA}$$

$$\mathbf{S}_L = 3\mathbf{S}_{\phi,L} = 505.38\angle 70° = 172.86 + j474.9 \text{ VA}$$

$$\Rightarrow \quad P_L = 172.86 \text{ W} \quad \text{and} \quad Q_L = 474.9 \text{ VAR}$$

Problem 10.45

The magnitude of the complex power (apparent power) supplied by a three-phase balanced wye-wye system is 3600 VA. The line voltage is 208 V rms. If the line impedance is negligible and the power factor angle of the load is $25°$, determine the load impedance.

Suggested Solution

$$|\mathbf{S}| = \sqrt{3} V_L I_L \quad \Rightarrow \quad I_L = \frac{3600}{208\sqrt{3}} = 9.99 \text{ A rms}$$

$$\mathbf{Z}_{\text{Y}} = \frac{208/\sqrt{3}}{9.99}\angle 25° = \frac{120}{9.99}\angle 25° = 12.01\angle 25° = 10.88 + j5.08\ \Omega$$

Problem 10.46

A balanced three-phase wye-wye system has two parallel loads. Load 1 is rated at 3000 VA, 0.7 pf lagging, and load 2 is rated at 2000 VA, 0.75 pf leading. If the line voltage is 208 V rms, find the magnitude of the line current.

Suggested Solution

$$\mathbf{S}_1 = 3000\angle \cos^{-1}(0.7) = 3000\angle 45.57° = 2100 + j2142.43 \text{ VA}$$

$$\mathbf{S}_2 = 2000\angle -\cos^{-1}(0.75) = 2000\angle -41.41° = 1500 - j1322.88 \text{ VA}$$

$$\mathbf{S}_T = \mathbf{S}_1 + \mathbf{S}_2 = 3600 + j819.55 = 3692.11\angle 12.82° \text{ VA}$$

$$|\mathbf{I}_L| = \frac{3692.11}{208\sqrt{3}} = 10.25 \text{ A rms}$$

Problem 10.47

Two industrial plants represent balanced three-phase loads. The plants receive their power from a balanced three-phase source with a line voltage of 4.6 kV rms. Plant 1 is rated at 300 kVA, 0.8 pf lagging and plant 2 is rated at 350 kVA, 0.84 pf lagging. Determine the power line current.

Suggested Solution

$$\mathbf{S}_1 = 300\angle\cos^{-1}(0.8) = 300\angle 36.87° = 240 + j180 \text{ kVA}$$

$$\mathbf{S}_2 = 350\angle\cos^{-1}(0.84) = 350\angle 32.86° = 294 + j189.9 \text{ kVA}$$

$$\mathbf{S}_T = \mathbf{S}_1 + \mathbf{S}_2 = 534 + j369.9 = 649.6\angle 34.71° \text{ kVA}$$

$$|\mathbf{I}_L| = \frac{649.6}{4.6\sqrt{3}} = 81.53 \text{ A rms}$$

Problem 10.48

A cluster of loads is served by a balanced three-phase source with a line voltage of 4160 V rms. Load 1 is 240 kVA at 0.8 pf lagging and load 2 is 160 kVA at 0.92 pf lagging. A third load is unknown except that it has a power factor of unity. If the line current is measured and found to be 62 A rms, find the complex power of the unknown load.

Suggested Solution

$$\left|\mathbf{S}_T\right| = \sqrt{3}\,V_L I_L = \sqrt{3}\left(4160\right)\left(62\right) = 446730.54 \text{ VA}$$

$$\mathbf{S}_1 = 240\angle\cos^{-1}\left(0.8\right) = 192000 + j144000 \text{ VA}$$

$$\mathbf{S}_2 = 160\angle\cos^{-1}\left(0.92\right) = 147200 + j62707 \text{ VA}$$

$$\left|\left(192000 + 147200 + P_3\right) + j\left(144000 + 62707 + 0\right)\right| = \left|\mathbf{S}_T\right|$$

$$\Rightarrow \quad \sqrt{\left(339200 + P_3\right)^2 + \left(206707\right)^2} = 446730.54$$

$$\therefore \quad P_3 = 56831 \text{ W}$$

$$\mathbf{S}_3 = P_3 + j0 = 56831\angle 0° \text{ VA}$$

Problem 10.49

A balanced three-phase source serves two loads:

Load 1: 36 kVA at 0.8 pf lagging

Load 2: 18 kVA at 0.6 pf lagging

The line voltage at the load is 208 V rms at 60 Hz. Find the line current and the combined power factor at the load.

Suggested Solution

$V_{ab} = 208$ V rms

$$\mathbf{S}_1 = 36\angle\cos^{-1}(0.8) = 36\angle 36.87° = 28.8 + j21.6 \text{ kVA}$$

$$\mathbf{S}_2 = 18\angle\cos^{-1}(0.6) = 18\angle 53.13° = 10.8 + j14.4 \text{ kVA}$$

$$\mathbf{S}_L = \mathbf{S}_1 + \mathbf{S}_2 = 39.6 + j36.0 = 53.52\angle 42.27° \text{ kVA}$$

$$|\mathbf{S}_L| = \sqrt{3}V_{ab}I_{aA} \quad \Rightarrow \quad |\mathbf{I}_{aA}| = \frac{53.52\times 10^3}{208\sqrt{3}} = 148.56 \text{ A rms}$$

$$pf_L = \cos\theta = \cos(42.27°) = 0.74 \text{ lagging}$$

Problem 10.50

A balanced three-phase source serves the following loads:

Load 1: 48 kVA at 0.9 pf lagging

Load 2: 24 kVA at 0.75 pf lagging

The line voltage at the load is 208 V rms at 60 Hz. Determine the line current and the combined power factor at the load.

Suggested Solution

$V_{ab} = 208 \text{ V rms}$

$$\mathbf{S}_1 = 48\angle\cos^{-1}(0.9) = 48\angle 25.84° = 43.2 + j20.92 \text{ kVA}$$

$$\mathbf{S}_2 = 24\angle\cos^{-1}(0.75) = 24\angle 41.41° = 18 + j15.87 \text{ kVA}$$

$$\mathbf{S}_L = \mathbf{S}_1 + \mathbf{S}_2 = 61.2 + j36.79 = 71.41\angle 31.02° \text{ kVA}$$

$$|\mathbf{S}_L| = \sqrt{3}V_{ab}I_{aA} \quad \Rightarrow \quad |\mathbf{I}_{aA}| = \frac{71.41\times 10^3}{208\sqrt{3}} = 198.21 \text{ A rms}$$

$$pf_L = \cos\theta = \cos(31.02°) = 0.86 \text{ lagging}$$

Problem 10.51

A small shopping center contains three stores that represent three balanced three-phase loads. The power lines to the shopping center represent a three-phase source with a line voltage of 13.8 kV rms. The three loads are:

Load 1: 500 kVA at 0.8 pf lagging

Load 2: 400 kVA at 0.85 pf lagging

Load 3: 300 kVA at 0.90 pf lagging

Find the magnitude of the power line current.

Suggested Solution

$V_{ab} = 13.8$ kV rms

$$\mathbf{S}_1 = 500\angle\cos^{-1}\left(0.8\right) = 500\angle 36.87° = 400 + j300 \text{ kVA}$$

$$\mathbf{S}_2 = 400\angle\cos^{-1}\left(0.85\right) = 400\angle 31.79° = 340 + j210.72 \text{ kVA}$$

$$\mathbf{S}_3 = 300\angle\cos^{-1}\left(0.9\right) = 300\angle 25.84° = 270 + j130.76 \text{ kVA}$$

$$\mathbf{S}_T = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 = 1010 + j641.48 = 1196.5\angle 32.42° \text{ kVA}$$

$$\left|\mathbf{S}_T\right| = \sqrt{3}\,V_{ab} I_{aA} \quad \Rightarrow \quad \left|\mathbf{I}_{aA}\right| = \frac{1196.5}{13.8\sqrt{3}} = 50.1 \text{ A rms}$$

Problem 10.52

The following loads are served by a balanced three-phase source:

Load 1: 18 kVA at 0.8 pf lagging

Load 2: 8 kVA at 0.8 pf leading

Load 3: 12 kVA at 0.75 pf lagging

The load voltage is 208 V rms at 60 Hz. If the line impedance is negligible, find the power factor of the source.

Suggested Solution

$$\mathbf{S}_1 = 18\angle\cos^{-1}(0.8) = 18\angle 36.87° = 14.40 + j10.80 \text{ kVA}$$

$$\mathbf{S}_2 = 8\angle -\cos^{-1}(0.8) = 8\angle -36.87° = 6.40 - j4.80 \text{ kVA}$$

$$\mathbf{S}_3 = 12\angle\cos^{-1}(0.75) = 12\angle 41.41° = 9.00 + j7.94 \text{ kVA}$$

$$\mathbf{S}_T = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 = 29.80 + j13.94 = 32.90\angle 25.07° \text{ kVA}$$

$$pf = \cos\theta = \cos(25.07°) = 0.91 \text{ lagging}$$

Problem 10.53

A balanced three-phase source supplies power to three loads. The loads are:

Load 1: 30 kVA at 0.8 pf lagging

Load 2: 24 kW at 0.6 pf leading

Load 3: unknown

If the line voltage and total complex power at the load are 208 V rms and $60\angle 0°$ kVA, respectively, find the unknown load.

Suggested Solution

$V_{AB} = 208$ V rms pf at source = 1.0

$$\mathbf{S}_T = \sqrt{3}V_{AB}I_{aA}\angle\cos^{-1}(1.0) = 60.0\angle 0° \text{ kVA} \quad \Rightarrow \quad I_{aA} = \frac{60\times 10^3}{208\sqrt{3}} = 166.8 \text{ A rms}$$

$$\mathbf{S}_1 = 30\angle\cos^{-1}(0.8) = 30\angle 36.87° = 24 + j18 \text{ kVA}$$

$$\mathbf{S}_2 = \frac{24}{0.6}\angle -\cos^{-1}(0.6) = 40\angle -53.13° = 24 - j32 \text{ kVA}$$

$$P_T = P_1 + P_2 + P_3 \quad \Rightarrow \quad P_3 = 60.0 - 24 - 24 = 12.0 \text{ kW}$$

$$Q_T = Q_1 + Q_2 + Q_3 \quad \Rightarrow \quad Q_3 = 0 - 18 + 32 = 14 \text{ kVAR}$$

$$\mathbf{S}_3 = 12.0 + j14 \text{ kVA} = 18.44 \text{ kVA at 0.65 pf lagging}$$

Problem 10.54

A balanced three-phase source serves the following loads:

Load 1: 18 kVA at 0.8 pf lagging

Load 2: 10 kVA at 0.7 pf leading

Load 3: 12 kW at unity pf

Load 4: 16 kVA at 0.6 pf lagging

The magnitude of the line voltage at the load is 208 V rms at 60 Hz, and the line impedance is $0.02 + j0.04\ \Omega$. Find the magnitude of the line voltage and power factor at the source.

Suggested Solution

$|\mathbf{V}_{AB}| = 208$ V rms $\qquad \mathbf{Z}_{line} = 0.02 + j0.04\ \Omega$

$$\mathbf{S}_1 = 18\angle\cos^{-1}(0.8) = 18\angle 36.87° = 14.4 + j10.8 \text{ kVA}$$

$$\mathbf{S}_2 = 10\angle -\cos^{-1}(0.7) = 10\angle -45.57° = 7.0 - j7.14 \text{ kVA}$$

$$\mathbf{S}_3 = \frac{12}{1.0}\angle 0° = 12\angle 0° = 12 + j0 \text{ kVA}$$

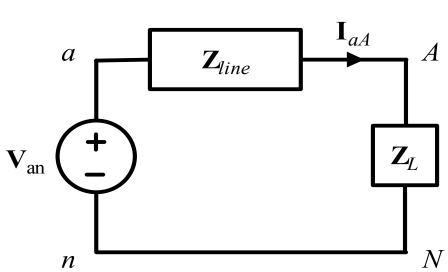
$$\mathbf{S}_4 = 16\angle\cos^{-1}(0.6) = 16\angle 53.13° = 9.6 + j12.8 \text{ kVA}$$

$$\mathbf{S}_L = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 + \mathbf{S}_4 = 43 + j16.46 = 46.04\angle 20.95° \text{ kVA}$$

$$\mathbf{S}_L = 3\frac{V_{AN}^2}{\mathbf{Z}_L^*} \quad \Rightarrow \quad \mathbf{Z}_L = \left(3\frac{\mathbf{V}_{AN}^2}{\mathbf{S}_L}\right)^* \qquad V_{AN} = \frac{V_{AB}}{\sqrt{3}} = \frac{208}{\sqrt{3}} = 120 \text{ V rms}; \quad \text{Let } \angle\mathbf{V}_{AN} = 0°$$

Then, $$\mathbf{Z}_L = \left[3\frac{(V_{AN}\angle 0°)^2}{\mathbf{S}_L}\right]^* = \left(3\frac{V_{AN}^2}{\mathbf{S}_L^*}\right) = \left(3\frac{120^2}{46.04\angle -20.95°}\right) = 0.94\angle 20.95° = 0.88 + j0.34\ \Omega$$

Per-phase Y circuit:



$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{AN}}{\mathbf{Z}_L} = \frac{120\angle 0°}{0.94\angle 20.95°} = 127.66\angle -20.95° \text{ A rms}$$

$$\mathbf{V}_{an} = \mathbf{I}_{aA}\left[\mathbf{Z}_{line} + \mathbf{Z}_L\right] = 124.72\angle 1.94° \text{ V rms}$$

$$|\mathbf{V}_{ab}| = \sqrt{3}|\mathbf{V}_{an}| = 216.02 \text{ V rms}$$

$$pf = \cos\left(\theta_{\mathbf{V}_{an}} - \theta_{\mathbf{I}_{aA}}\right) = \cos(22.89°) = 0.92 \text{ lagging}$$

Problem 10.55

A balanced three-phase source supplies power to three loads. The loads are:

Load 1: 18 kW at 0.8 pf lagging

Load 2: 10 kVA at 0.6 pf leading

Load 3: unknown

If the line voltage at the load is 208 V rms, the magnitude of the total complex power is 41.93 kVA and the combined power factor at the load is 0.86 lagging, find the unknown load.

Suggested Solution

$V_{AB} = 208$ V rms $\qquad pf_L = 0.86$ lagging

$$\left|\mathbf{S}_L\right| = \sqrt{3}\,V_{AB} I_{aA} = 41.93 \text{ kVA} \quad \Rightarrow \quad I_{aA} = \frac{41.93\times 10^3}{208\sqrt{3}} = 116.39 \text{ A rms}$$

$$\theta_{\mathbf{S}_L} = \cos^{-1}(0.86) = 30.68°$$

$$\mathbf{S}_L = 41.93\angle 30.68° = 36.06 + j21.40 \text{ kVA} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3$$

$$\mathbf{S}_1 = \frac{18}{0.8}\angle\cos^{-1}(0.8) = 22.5\angle 36.87° = 18 + j13.5 \text{ kVA}$$

$$\mathbf{S}_2 = 10\angle -\cos^{-1}(0.6) = 10\angle -53.13° = 6 - j8 \text{ kVA}$$

$$\begin{aligned}\mathbf{S}_3 &= \mathbf{S}_L - \mathbf{S}_1 - \mathbf{S}_2 \\ &= (36.06 + j21.40) - (18 + j13.5) - (6 - j8) \\ &= 12.06 + j15.9 \text{ kVA} \\ &= 19.96 \text{ kVA at } 0.60 \text{ pf lagging}\end{aligned}$$

Problem 10.56

A balanced three-phase source supplies power to three loads. The loads are:

Load 1: 20 kVA at 0.6 pf lagging

Load 2: 12 kW at 0.75 pf lagging

Load 3: unknown

If the line voltage at the load is 208 V rms, the magnitude of the total complex power is 35.52 kVA and the combined power factor at the load is 0.88 lagging, find the unknown load.

Suggested Solution

$V_{AB} = 208$ V rms $\qquad\qquad pf_L = 0.88$ lagging

$$\left|\mathbf{S}_L\right| = \sqrt{3}\,V_{AB} I_{aA} = 35.52 \text{ kVA} \quad \Rightarrow \quad I_{aA} = \frac{35.52\times 10^3}{208\sqrt{3}} = 98.6 \text{ A rms}$$

$$\theta_{\mathbf{S}_L} = \cos^{-1}\left(0.88\right) = 28.36°$$

$$\mathbf{S}_L = 35.52\angle 28.36° = 31.26 + j16.87 \text{ kVA} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3$$

$$\mathbf{S}_1 = 20\angle\cos^{-1}\left(0.6\right) = 20\angle 53.13° = 12 + j16 \text{ kVA}$$

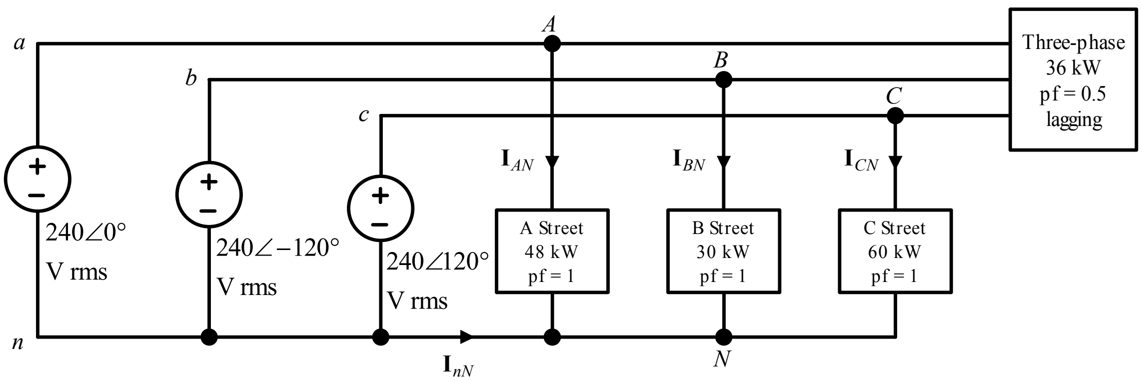
$$\mathbf{S}_2 = \frac{12}{0.75}\angle\cos^{-1}\left(0.75\right) = 16\angle 41.41° = 12 + j10.58 \text{ kVA}$$

$$\begin{aligned}\mathbf{S}_3 &= \mathbf{S}_L - \mathbf{S}_1 - \mathbf{S}_2 \\ &= \left(31.26 + j16.87\right) - \left(12 + j16\right) - \left(12 + j10.58\right) \\ &= 7.26 - j9.71 \text{ kVA} \\ &= 12.13 \text{ kVA at 0.60 pf leading}\end{aligned}$$

Problem 10.57

A standard practice for utility companies is to divide its customers into single-phase users and three-phase users. The utility must provide three-phase users, typically industries, with all three phases. However, single-phase users, residential and light commercial, are connected to only one phase. To reduce cable costs, all single-phase users in a neighborhood are connected together. This means that even if the three-phase users present perfectly balanced loads to the power grid, the single-phase loads will never be in balance, resulting in current flow in the neutral connection.

Consider the 60-Hz, *abc*-sequence network shown. With a line voltage of $416\angle 30°$ V rms, phase *a* supplies the single-phase users on A Street, phase *b* supplies B Street and phase *c* supplies C Street. Furthermore, the three-phase industrial load, which is connected in delta, is balanced. Find the neutral current.



Suggested Solution

$P_A = 48,000 = V_{an} I_{AN}$

$$\mathbf{I}_{AN} = \frac{P_A}{\mathbf{V}_{an}} = 200\angle 0° \text{ A rms}$$

$$\mathbf{I}_{BN} = \frac{P_B}{\mathbf{V}_{BN}} = 125\angle -120° \text{ A rms}$$

$$\mathbf{I}_{CN} = \frac{P_C}{\mathbf{V}_{CN}} = 250\angle -240° \text{ A rms}$$

$$\mathbf{I}_{Nn} = \mathbf{I}_{AN} + \mathbf{I}_{BN} + \mathbf{I}_{CN}$$
$$= 108.97\angle 83.41° \text{ A rms}$$

$$\therefore \quad \mathbf{I}_{nN} = 108.97\angle -96.59° \text{ A rms}$$

Problem 10.58

A three-phase *abc*-sequence wye-connected source with $\mathbf{V}_{an} = 220\angle 0°$ V rms supplies power to a wye-connected load that consumes 150 kW of power at a pf of 0.8 lagging. Three capacitors are found that each have an impedance of $-j2.0\ \Omega$, and they are connected in parallel with the previous load in a wye configuration. Determine the power factor of the combined load as seen by the source.

Suggested Solution

$\mathbf{V}_{an} = 220\angle 0°$ V rms $\qquad P_{1\phi} = 50$ kW $\qquad pf = 0.8$ lagging $\qquad \mathbf{Z}_C = -j2\ \Omega$

Original situation per phase:

$$\begin{aligned}\mathbf{S}_{old} &= \frac{50}{0.8}\angle\cos^{-1}(0.8)\\ &= 62.5\angle 36.87°\\ &= 50 + j37.5 \text{ kVA}\end{aligned}$$

Corrected situation:

$$P_{new} = P_{old} = 50 \text{ kW}$$

$$\begin{aligned}Q_{new} &= Q_{old} + Q_C\\ &= \left(37.5\times 10^3\right) + Q_C\end{aligned}$$

$$Q_C = -\frac{\left|\mathbf{V}_{an}\right|^2}{\left|\mathbf{Z}_C\right|} = -24.2 \text{ kVAR}$$

$$Q_{new} = 13.3 \text{ kVAR}$$

$$\mathbf{S}_{new} = 51.74\angle 14.90° \text{ kVA}$$

$$pf_{new} = \cos\theta_{new} = \cos(14.90°) = 0.97 \text{ lagging}$$

Problem 10.59

A three-phase *abc*-sequence wye-connected source with $\mathbf{V}_{an} = 220\angle 0°$ V rms supplies power to a wye-connected load that consumes 150 kW of power at a pf of 0.8 lagging. Three capacitors are found that each have an impedance of $-j2.0\ \Omega$, and they are connected in parallel with the previous load in a delta configuration. Determine the power factor of the combined load as seen by the source.

Suggested Solution

$\mathbf{V}_{an} = 220\angle 0°$ V rms $\qquad P_{1\phi} = 50$ kW $\qquad pf = 0.8$ lagging $\qquad \mathbf{Z}_{C\Delta} = -j2\ \Omega$

$$\mathbf{Z}_{CY} = \frac{\mathbf{Z}_{C\Delta}}{3} = -j\frac{2}{3}\ \Omega$$

Original situation per phase:

$$\begin{aligned}\mathbf{S}_{old} &= \frac{50}{0.8}\angle\cos^{-1}(0.8)\\ &= 62.5\angle 36.87°\\ &= 50 + j37.5\text{ kVA}\end{aligned}$$

Corrected situation:

$$P_{new} = P_{old} = 50\text{ kW}$$

$$\begin{aligned}Q_{new} &= Q_{old} + Q_C\\ &= \left(37.5\times 10^3\right) + Q_C\end{aligned}$$

$$Q_C = -\frac{\left|\mathbf{V}_{an}\right|^2}{\left|\mathbf{Z}_{CY}\right|} = -72.6\text{ kVAR}$$

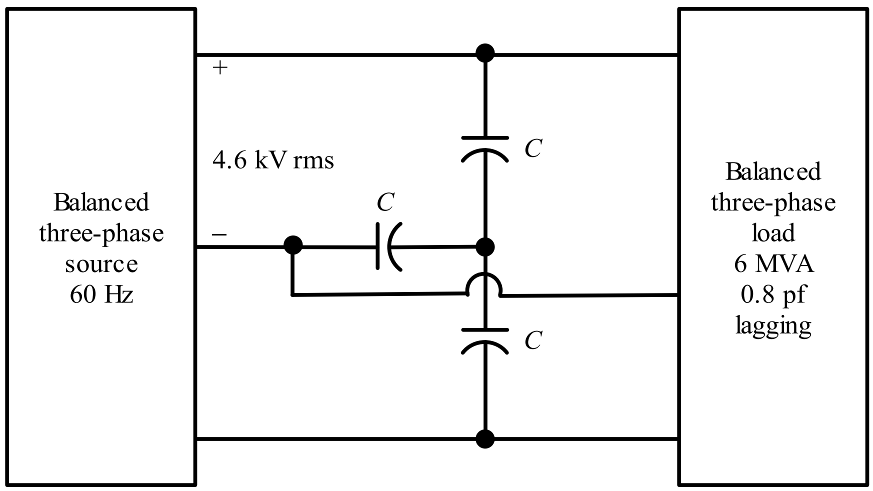
$$Q_{new} = -35.1\text{ kVAR}$$

$$\mathbf{S}_{new} = 61.09\angle -35.07°\text{ kVA}$$

$$pf_{new} = \cos\theta_{new} = \cos(-35.07°) = 0.82\ \text{ leading}$$

Problem 10.60

Find C in the network shown such that the total load has a power factor of 0.9 lagging.



Suggested Solution

$V_{ab} = 4.6$ kV rms $\quad |\mathbf{S}_{L3\phi}| = 6$ MVA $\quad pf_{old} = 0.8$ lagging

$pf_{new} = 0.9$ lagging $\quad f = 60$ Hz

$$\mathbf{S}_{L1\phi} = \frac{\mathbf{S}_{L3\phi}}{3} = 2\angle\cos^{-1}0.8 = 1.60 + j1.20 \text{ MVA}$$

Old situation per phase:

$$\mathbf{S}_{old} = 1.60 + j1.20 \text{ MVA}$$

New situation:

$$\begin{aligned}\mathbf{S}_{new} &= \frac{1.60}{0.9}\angle\cos^{-1}(0.9)\\ &= 1.60 + j0.77 \text{ kVA}\\ &= P_{old} + j(Q_{old} + Q_C)\end{aligned}$$

$$Q_C = 0.77 - 1.20 = -0.43 \text{ MVAR}$$

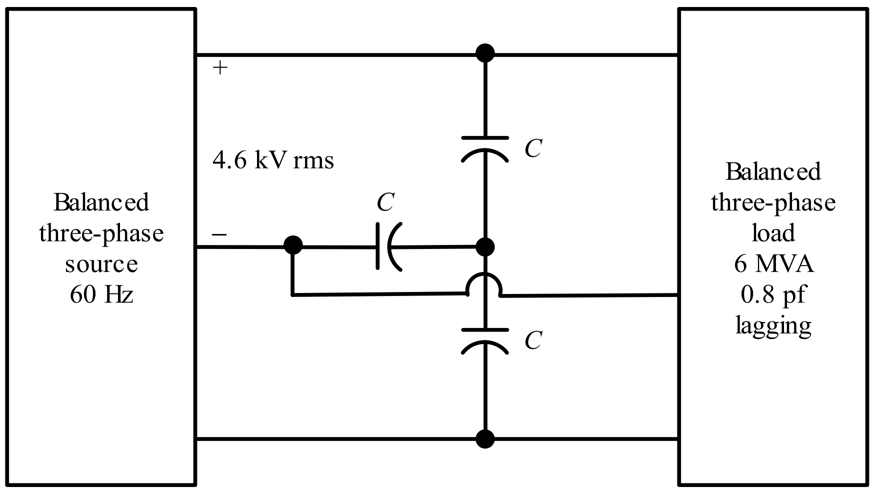
But,

$$Q_C = -\omega C_Y V_{an}^2 = -\frac{\omega C_Y V_{ab}^2}{3}$$

$\therefore \quad C_Y = 161.7\ \mu\text{F}$

Problem 10.61

Find C in the network shown such that the total load has a power factor of 0.9 leading.



Suggested Solution

$pf_{new} = 0.9$ leading $\qquad f = 60$ Hz

Original complex power per phase:

$$\mathbf{S}_{old} = 2\angle 36.87° = 1.6 + j1.2 \text{ MVA}$$

Corrected complex power per phase:

$$\mathbf{S}_{new} = \frac{1.6}{0.9}\angle -\cos^{-1}(0.9)$$
$$= 1.6 - j0.77 \text{ MVA}$$

$$Q_{old} + Q_C = Q_{new} \quad \Rightarrow \quad Q_C = -0.77 - 1.20 = -1.97 \text{ MVAR}$$

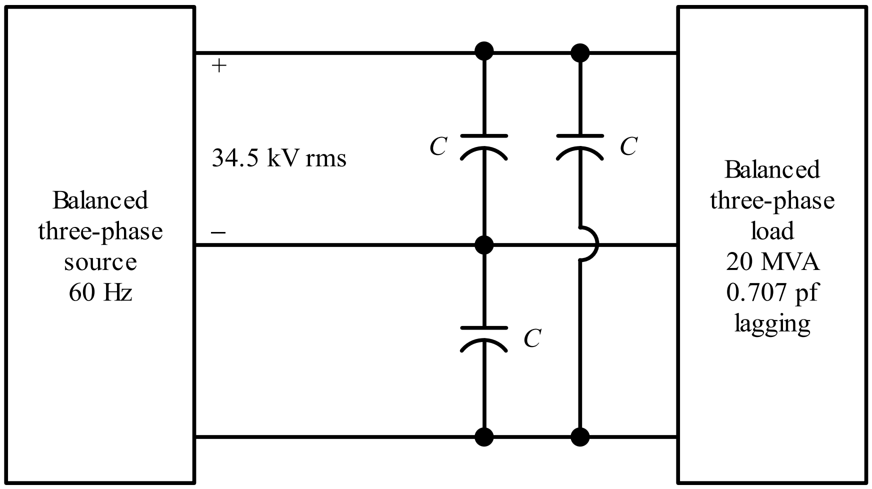
But,

$$Q_C = -\omega C_Y V_{an}^2 = -\frac{\omega C_Y V_{ab}^2}{3}$$

$$\therefore \quad C_Y = 740.9\ \mu\text{F}$$

Problem 10.62

Find C in the network shown such that the total load has a power factor of 0.92 leading.



Suggested Solution

$V_{ab} = 34.5 \text{ kV rms}$ $\quad f = 60 \text{ Hz}$ $\quad S_{3\phi} = 20 \text{ MVA}$

$pf_{old} = 0.707 \text{ lagging}$ $\quad pf_{new} = 0.92 \text{ leading}$

Old complex power per phase:

$$\mathbf{S}_{old} = \frac{20}{3}\angle \cos^{-1}(0.707) = 4.71 + j4.71 \text{ MVA}$$

New complex power per phase:

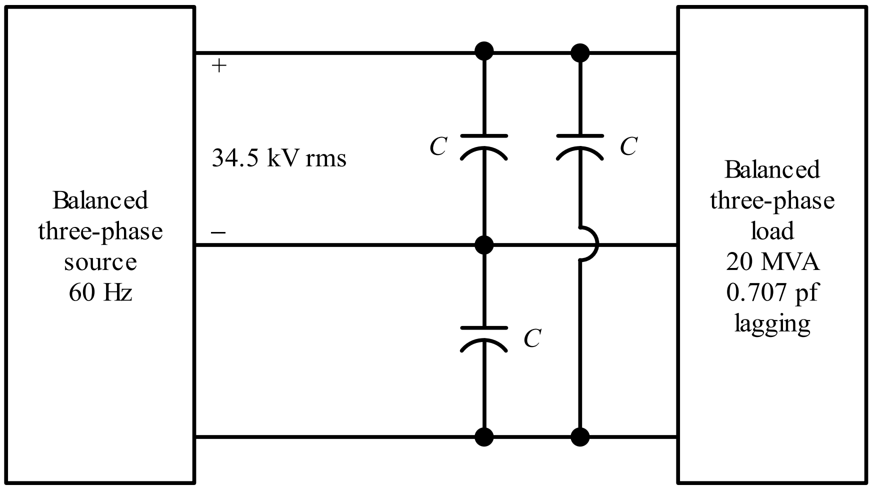
$$\mathbf{S}_{new} = \frac{4.71}{0.92}\angle - \cos^{-1}(0.92) = 4.71 - j2.01 \text{ MVA}$$

$Q_C = -\omega C_\Delta V_{ab}^2 = Q_{new} - Q_{old} = -6.72 \text{ MVAR}$

$C_\Delta = 15.0\ \mu\text{F}$

Problem 10.63

Find C in the network shown such that the total load has a power factor of 0.92 lagging.



Suggested Solution

$V_{ab} = 34.5$ kV rms $\qquad f = 60$ Hz $\qquad S_{3\phi} = 20$ MVA

$pf_{old} = 0.707$ lagging $\qquad pf_{new} = 0.92$ lagging

Old complex power per phase:

$$\mathbf{S}_{old} = \frac{20}{3}\angle\cos^{-1}(0.707) = 4.71 + j4.71 \text{ MVA}$$

New complex power per phase:

$$\mathbf{S}_{new} = \frac{4.71}{0.92}\angle\cos^{-1}(0.92) = 4.71 + j2.01 \text{ MVA}$$

$Q_C = -\omega C_\Delta V_{ab}^2 = Q_{new} - Q_{old} = -2.70$ MVAR

$C_\Delta = 6.0\ \mu$F

Problem 10FE-1

A wye-connected load consists of a series *RL* impedance. Measurements indicate that the rms voltage across each element is 84.85 V. If the rms line current is 6 A, find the total complex power for the three-phase load configuration.

Suggested Solution

$\mathbf{V}_L = 84.85 + j84.85 = 120\angle 45° \text{ V}$

$$\begin{aligned}\mathbf{S}_T &= 3\left[(6)^2\left(\frac{84.85}{6}\right) + j(6)^2\left(\frac{84.85}{6}\right)\right] \\ &= 1527.3 + j1527.3 \\ &= 2160\angle 45° \text{ VA}\end{aligned}$$

or

$$\mathbf{S}_T = 3(120\angle 45°)(6) = 2160\angle 45° \text{ VA}$$

Problem 10FE-2

A balanced three-phase delta-connected load consists of an impedance of $12 + j12\ \Omega$. If the line voltage at the load is measured to be 230 V rms, find the magnitude of the line current and the total real power absorbed by the three-phase configuration.

Suggested Solution

$$\left|\mathbf{I}_\phi\right| = \left|\frac{230}{12 + j12}\right| = 13.55 \text{ A rms}$$

$$\left|\mathbf{I}_L\right| = \sqrt{3}\left|\mathbf{I}_\phi\right| = 23.47 \text{ A rms}$$

$$P_{total} = \sqrt{3}\left|\mathbf{V}_L\right|\left|\mathbf{I}_L\right|\cos 45° = 6.61 \text{ kW}$$

or

$$P_{total} = 3\left|\mathbf{I}_\phi\right|^2 R_\phi = 3(13.55)^2(12) = 6.61 \text{ kW}$$

Problem 10FE-3

Two balanced three-phase loads are connected in parallel. One load with a phase impedance of $24+j18\ \Omega$ is connected in delta, and the other load has a phase impedance of $6+j4\ \Omega$ and is connected in wye. If the line-to-line voltage is 208 V rms, determine the line current.

Suggested Solution

Assume $208\angle 0°$ V rms

For Δ:

$$\mathbf{I}_\phi = \frac{208\angle 0°}{24+j18} = 6.933\angle -36.87° \text{ A rms}$$

$$\mathbf{I}_L = \left(\sqrt{3}\right)\left(\angle -30°\right)\left(6.933\angle -36.87°\right) = 12\angle -66.87° \text{ A rms}$$

For Y:

$$\mathbf{I}_\mathrm{L} = \frac{\frac{208}{\sqrt{3}}\angle -30°}{6+j4} = 16.64\angle -63.69° \text{ A rms}$$

Total line current:

$$\begin{aligned}\mathbf{I}_L &= 12\angle -66.87° + 16.64\angle -63.69° \\ &= 12.09 - j25.96 \\ &= 28.64\angle -65° \text{ A rms}\end{aligned}$$

or

Convert $\Delta \rightarrow \mathrm{Y} \quad \Rightarrow \quad 8+j6\ \Omega$

$$\therefore \qquad \mathbf{Z}_{total} = \frac{(6+j4)(8+j6)}{(6+j4)+(8+j6)} = \frac{24+j68}{14+j10} = 4.19\angle 35.06°\ \Omega$$

Then

$$\mathbf{I}_L = \frac{\frac{208}{\sqrt{3}}\angle -30°}{4.19\angle 35.06°} = 28.66\angle -65.06° \text{ A rms}$$

Problem 10FE-4

The total complex power at the load of a three-phase balanced system is $24\angle 30°$ kVA. Find the real power per phase.

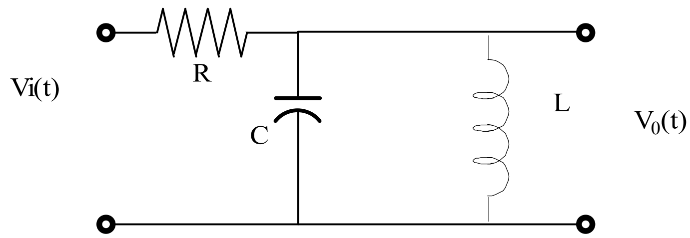
Suggested Solution

$$\begin{aligned}\mathbf{S} &= 24,000\angle 30° \text{ VA} \\ &= 20,785 + j12,000 \text{ VA}\end{aligned}$$

$$P_\phi = \frac{20,785}{3} = 6.928 \text{ kW}$$

Problem 11.1

Determine the driving point impedance at the input terminals of the network shown in fig P11.1 as a function of s.



Suggested Solution

$$Z(s) = R + \left(\frac{1}{SC} \,||\, SL\right) = R + \frac{\frac{L}{C}}{\frac{1}{SC} + SL} = R + \frac{SL}{1 + S^2 LC} = \frac{S^2 LCR + SL + R}{S^2 LC + 1}$$

Problem 11.2

Determine the voltage transfer function Vo(s)/Vc(s) as a function of s for the network shown in fig p11.1.

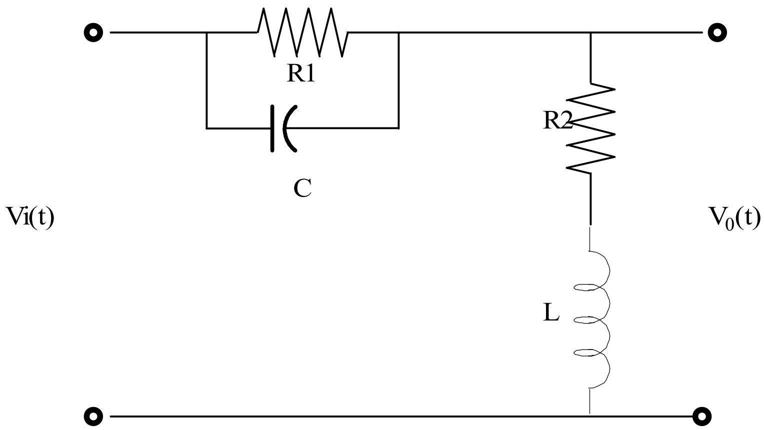
Suggested Solution

$$\frac{V_O}{V_i}(S)=\frac{(\frac{1}{SC}\,||\,SL)}{R+(\frac{1}{SC}\,||\,SL)}=\frac{\frac{\frac{L}{C}}{\frac{1}{SC}+SL}}{R+\frac{\frac{L}{C}}{\frac{1}{SC}+SL}}=\frac{\frac{L}{C}}{\frac{R}{SC}+RSL+\frac{L}{C}}=\frac{SL}{S^2LCR+SL+R}$$

$$\frac{SL}{S^2LCR+SL+R}$$

Problem 11.3

Determine the voltage transfer function Vo(s)/Vc(s) as a function of s for the network shown in fig p11.3.



Suggested Solution

$$\frac{V_O}{V_i}(S) = \frac{(R_2 + SL)}{(R_2 + SL) + \dfrac{\frac{R_1}{SC}}{R_1 + \frac{1}{SC}}} = \frac{(R_2 + SL)}{(R_2 + SL) + \dfrac{R_1}{SCR_1 + 1}}$$

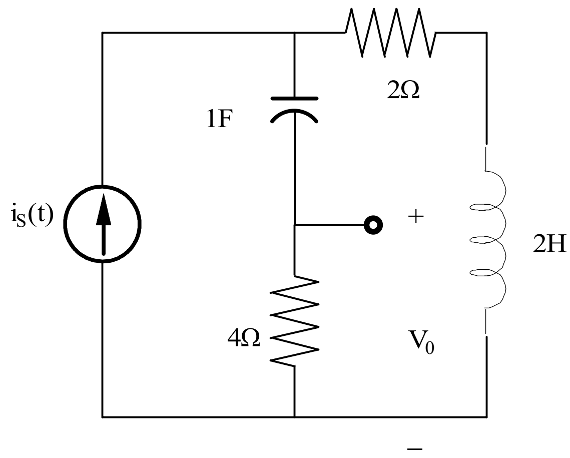
$$= \frac{(R_2 + SL)(SCR_1 + 1)}{R_2 + SCR_1R_2 + S^2LCR_1 + SL + R_1} = \frac{(R_2 + SL)(SCR_1 + 1)}{S^2LCR_1 + S(L + CR_1R_2) + R_1 + R_2}$$

$$= \frac{(S + \frac{R_2}{L})(S + \frac{1}{CR_1})}{S^2 + \left[\frac{1}{R_1C} + \frac{R_2}{L}\right]S + \frac{R_1 + R_2}{LCR_1}}$$

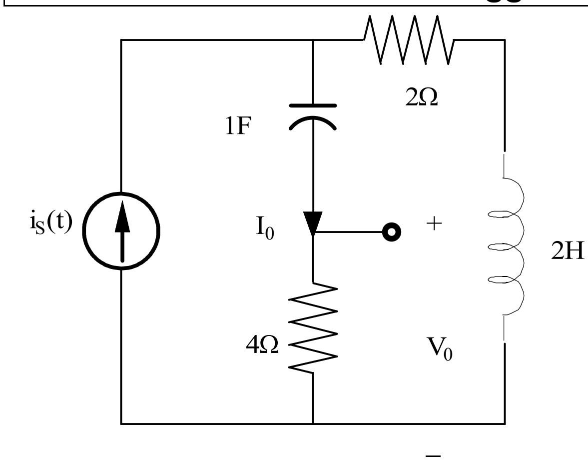
$$\frac{(S + \frac{R_2}{L})(S + \frac{1}{CR_1})}{S^2 + \left[\frac{1}{R_1C} + \frac{R_2}{L}\right]S + \frac{R_1 + R_2}{LCR_1}}$$

Problem 11.4

Find the transfer impedance Vo(s)/Is(s) for the network shown in fig 11.4.



Suggested Solution



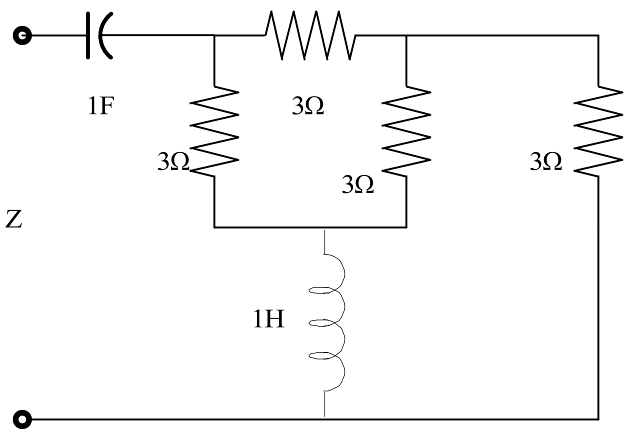
$$I_O = I_S\left[\frac{2S+2}{2S+2+4+\frac{1}{S}}\right]$$

$$\frac{I_O}{I_S} = \frac{2S^2+2S}{2S^2+6S+1}, V_O = 4I_O, SO \quad \frac{V_O}{I_S} = \frac{8S(S+1)}{2S^2+6S+1}$$

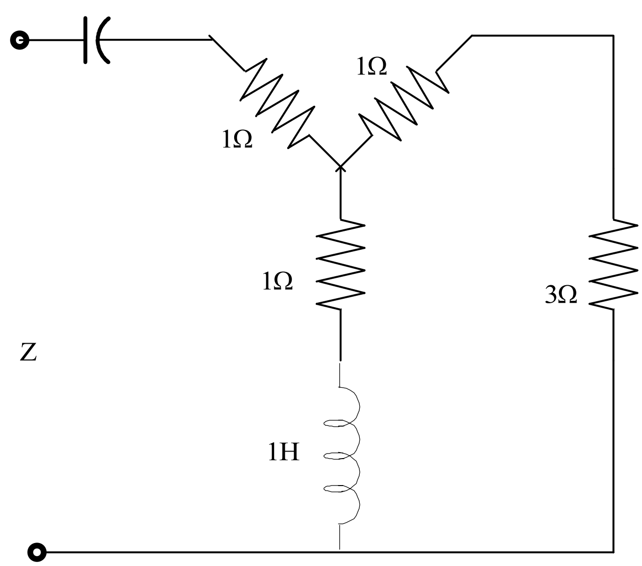
$$\frac{8S(S+1)}{2S^2+6S+1}$$

Problem 11.5

Find the driving point impedance at the input terminal of the circuit in fig 11.5.



Suggested Solution



$$Z_i(s) = \frac{4(1+S)}{5+S} + 1 + \frac{1}{S} = \frac{5(S+1)^2}{S(S+5)}$$

Problem 11.6

Draw the bode plot for the network function.

Suggested Solution

$$H(jw) = \frac{jw5+1}{jw20+1}$$

Problem 11.7

Draw the bode plot for the network function.

$$H(jw) = \frac{jw2+1}{jw10+1}$$

Suggested Solution

$$H(jw) = \frac{jw2+1}{jw10+1}$$

Problem 11.8

Draw the bode plot for the network function.

$$H(jw) = \frac{10(10jw+1)}{(100jw+1)(jw+1)}$$

Suggested Solution

$$H(jw) = \frac{10(10jw+1)}{(100jw+1)(jw+1)}$$

Zeros: 0.1 r/s

Poles: 0.01 & 1 r/s

H(jo) = 10 = 20dB

Problem 11.9

Draw the bode plot for the network function.

$$H(jw) = \frac{jw}{(jw+1)(0.1jw+1)}$$

Suggested Solution

$$H(jw) = \frac{jw}{(jw+1)(0.1jw+1)}$$

Zero :dc

Poles: 1& 10 r/s

Problem 11.10

Draw the bode plot for the network function.

$$H(jw) = \frac{10jw+1}{(jw)(0.1jw+1)}$$

Suggested Solution

$$H(jw) = \frac{10jw+1}{(jw)(0.1jw+1)}$$

zeros : 0.1 r/s

poles : dc & 10 r/s

Problem 11.11

Sketch the magnitude characteristic of the bode plot for the transfer function.

$$H(jw) = \frac{10}{(jw)(0.1jw+1)}$$

Suggested Solution

Problem 11.12

Sketch the magnitude characteristic of the bode plot for the transfer function.

$$H(jw)=\frac{20(0.1jw+1)}{(jw)(0.1jw+1)(0.01jw+1)}$$

Suggested Solution

Problem 11.13

Sketch the magnitude characteristic of the bode plot for the transfer function.

$$H(jw) = \frac{100(jw)}{(jw+1)(jw+10)(jw+50)}$$

Suggested Solution

$$H(jw) = \frac{\frac{1}{S}(jw)}{(jw+1)(\frac{jw}{10}+1)(\frac{jw}{50}+1)}$$

Problem 11.14

Draw the bode plot for the network function.

$$H(jw) = \frac{16}{(jw)^2(jw2+1)}$$

Suggested Solution

$$H(jw) = \frac{16}{(jw)^2(jw2+1)}$$

zeros: none

poles: 2@dc, 10 r/s

also: 16=24.1dB

Problem 11.15

Sketch the magnitude characteristic of the bode plot for the transfer function.

$$H(jw) = \frac{640(jw+1)(0.01jw+1)}{(jw)^2(jw+10)}$$

Suggested Solution

$$H(jw) = \frac{640(jw+1)(0.01jw+1)}{(jw)^2(jw+10)} = \frac{64(jw+1)(0.01jw+1)}{(jw)^2(0.1jw+1)}$$

poles: 2@dc, 10 r/s

zeroe at : 1 &100 r/s

also: 64=36.1dB

Problem 11.16

Sketch the magnitude characteristic of the bode plot for the transfer function.

$$H(jw) = \frac{10^5(5jw+1)^2}{(jw)^2(jw+10)(jw+100)^2}$$

Suggested Solution

$$H(jw) = \frac{10^5(5jw+1)^2}{(jw)^2(jw+10)(jw+100)^2}$$

$$H(jw) = \frac{(5jw+1)^2}{(jw)^2(0.1jw+1)(0.01jw+1)^2}$$

poles: 2@dc, 10 r/s & 2@ 100r/s

zeroe at : 2@ 0.2r/s

also: 64=36.1dB

Problem 11.17

Sketch the magnitude characteristic of the bode plot for the transfer function.

$$G(j\omega) = \frac{10j\omega}{(j\omega+1)(j\omega+10)^2}$$

Suggested Solution

$$G(j\omega) = \frac{10j\omega}{(j\omega+1)(j\omega+10)^2} = \frac{0.1j\omega}{(j\omega+1)(0.1j\omega+1)^2}$$

poles at : 1 and 2@ 10 r/s

zeros at : dc

also : 0.1 = - 20 dB

Problem 11.18

Sketch the magnitude characteristic of the bode plot for the transfer function.

$$H(j\omega) = \frac{100(j\omega)^2}{(j\omega+1)(j\omega+10)^2(j\omega+50)}$$

Suggested Solution

$$H(j\omega) = \frac{100(j\omega)^2}{(j\omega+1)(j\omega+10)^2(j\omega+50)}$$

$$H(j\omega) = \frac{(1/50)(j\omega)^2}{(j\omega+1)(0.1j\omega+1)^2(0.02j\omega+1)}$$

poles :1, 50 and 2@ 10 r/s

zeros: 2 @ dc

also: 1/50 = - 34 dB

Problem 11.19

Sketch the magnitude characteristic of the bode plot for the transfer function.

$$G(j\omega) = \frac{-\omega^2 10^4}{(j\omega+1)^2 (j\omega+10)(j\omega+100)^2}$$

Suggested Solution

$$G(j\omega) = \frac{-\omega^2 10^4}{(j\omega+1)^2 (j\omega+10)(j\omega+100)^2} = \frac{-(1/10)\omega^2}{(j\omega+1)^2 (0.1j\omega+1)(0.01j\omega+1)^2}$$

poles : 2@ 1 r/s, 1@ 10 r/s, 2@ 100 r/s

zeros: 2 @ dc

also: 0.1 = - 20 dB

Problem 11.20

Sketch the magnitude characteristic of the bode plot for the transfer function.

$$G(j\omega) = \frac{64(j\omega+1)^2}{-j\omega^3(0.1j\omega+1)}$$

Suggested Solution

$$G(j\omega) = \frac{64(j\omega+1)^2}{-j\omega^3(0.1j\omega+1)}$$

poles : 3@dc, 1@ 10 r/s

zeros: 2 @ 1 r/s

also: 64 = 36.1 dB

Problem 11.21

Sketch the magnitude characteristic of the bode plot for the transfer function.

$$G(j\omega) = \frac{-\omega^2}{(j\omega+1)^3}$$

Suggested Solution

$$G(j\omega) = \frac{-\omega^2}{(j\omega+1)^3}$$

poles: 3 @ 1 r/s

zeros: 2@ dc

Problem 11.22

Draw the Bode plot for the network function.

$$H(j\omega) = \frac{72(j\omega + 2)}{j\omega\left[(j\omega)^2 + 2.4j\omega + 144\right]}$$

Suggested Solution

$$H(j\omega) = \frac{72(j\omega + 2)}{j\omega\left[(j\omega)^2 + 2.4j\omega + 144\right]}$$

$$H(j\omega) = \frac{(0.5j\omega + 1)}{j\omega\left[(j\omega/12)^2 + (j\omega/60) + 1\right]}$$

zeros : 1@ 2 r/s

simple pole : 1 @ dc

complex poles T = (1/12)s \$ 2τξ=1/60

so Wo = 12 r/s ξ = 0.1

Problem 11.23

Sketch the magnitude characteristics of the Bode plot for the transfer function.

$$G(j\omega) = \frac{10^4 (j\omega + 1)\left[-\omega^2 + 6j\omega + 225\right]}{j\omega(j\omega + 450)(j\omega + 50)^2}$$

Suggested Solution

$$G(j\omega) = \frac{10^4 (j\omega + 1)\left[-\omega^2 + 6j\omega + 225\right]}{j\omega(j\omega + 450)(j\omega + 50)^2}$$

$$G(j\omega) = \frac{2(j\omega + 1)\left[(j\omega/15)^2 + (2/75)j\omega + 1\right]}{(j\omega)(j\omega/450 + 1)(0.02j\omega + 1)^2}$$

poles : dc 2@ 50 and 450 r/s

simple pole : 1 r/s

complex poles T = (1/15)s \$ 2τξ=2/75

so Wo = 15 r/s ξ = 0.2

also: 2=6dB

Problem 11.24

Sketch the magnitude characteristic of the Bode plot for the transfer function.

$$H(j\omega) = \frac{81(jw + 0.1)}{j\omega\left[-\omega^2 + 3.6j\omega + 81\right]}$$

Suggested Solution

$$H(j\omega) = \frac{0.1\left(\dfrac{j\omega}{0.1} + 1\right)}{j\omega\left[\dfrac{-(\omega)^2}{81} + \dfrac{3.6}{81}j\omega + 1\right]}$$

Problem 11.25

Sketch the magnitude characteristic of the bode plot for the transfer function.

$$H(j\omega) = \frac{6.4(j\omega)}{(j\omega + 1)(1 - \omega^2 + 8j\omega + 64)}$$

Suggested Solution

$$H(j\omega) = \frac{0.1(j\omega)}{(j\omega + 1)(-\frac{\omega^2}{64} + \frac{1}{8}j\omega + 1)}$$

POLES : 0.5r/s & 2@10r/s
ZEROS: 0.1 r/s

Problem 11.26

Determine H(jw) if the amplitude characteristic for H(jw) is shown in fig 26.

Suggested Solution

$$\left|H(jo)\right| = -40dB = 0.01$$

$$H(j\omega) = \frac{10j\omega + 1}{100(2j\omega + 1)(0.1j\omega + 1)^2}$$

Poles: 10r/s & 2@ 80r/s
Zeroes:1&120r/s

Problem 11.27

Find H(jw) if its magnitude characteristic is shown in fig 27.

Suggested Solution

$$\left|H(jo)\right| = 40dB = 100$$

$$H(j\omega) = \frac{100(j\omega+1)(\frac{j\omega}{120}+1)}{(0.1j\omega+1)(\frac{j\omega}{80}+1)^2}$$

poles: 10 r/s $ 2 @ 80 r/s

zeroes : 0.1 r/s

H(jO) = 40 dB = 100

Problem 11.28

Find H(jw) if its magnitude characteristic is shown in fig 28.

Suggested Solution

There is a zero at w =10r/s, A pole at w=0 and A double pole at w=20r/s.
Since the initial slope of –20dB /dec cuts the odB line at w=0, the gain is 10.
.:

$$H(j\omega) = \frac{10(\frac{j\omega}{10}+1)}{j\omega(\frac{j\omega}{20}+1)^2}$$

Problem 11.29

Find H(jw) if its amplitude characteristic is shown in fig 29.

Suggested Solution

The initial slope of –20dB/dec will cut the odB line at w=40r/s. Therefore the gain is 40. The zeros are at w=50r/s and w=1000r/s. The pole are ar w=0 and there is a double pole at w=400.:

$$H(j\omega) = \frac{40(\frac{j\omega}{50}+1)(\frac{j\omega}{1000}+1)}{j\omega(\frac{j\omega}{100}+1)^2}$$

Problem 11.30

Find H(jw) if its magnitude characteristics is shown in fig 11.30.

Suggested Solution

Poles: 0.8,100 $ 2@700 r/s
Zeros: dc , 12 r/s

$$|H(j0.05)| = -20dB = 0.1$$

$$H(jw) = \frac{K(jw)\left(\frac{jw}{12}+1\right)}{(1.25jw+1)\left(\frac{jw}{100}+1\right)\left(\frac{jw}{100}+1\right)^2}$$

$$|H(j0.05)| = \frac{K(0.05)(1)}{(1)} = 0.1 \Rightarrow K = 2$$

$$H(jw) = \frac{2(jw)\left(\frac{jw}{12}+1\right)}{(1.25jw+1)\left(\frac{jw}{100}+1\right)\left(\frac{jw}{100}+1\right)^2}$$

Problem 11.31

Find H(jw) if its amplitude characteristics is shown in fig 11.31.

Suggested Solution

The initial slope indicates that the gain is 1,there are zeroes at w=0 and w=30. The poles are at w=1, w=100 and a double pole w=8/5

$$H(jw) = \frac{1(jw)\left(\frac{jw}{30}+1\right)}{(jw+1)\left(\frac{jw}{100}+1\right)\left(\frac{jw}{8}+1\right)^2}$$

Problem 11.32

Given the magnitude characteristics in fig 11.32, find G(jw).

Suggested Solution

Poles : 2@ dc, 1@200r/s

Zeros: 1.5 $ 10 r/s

$$\left|H(j0.02)\right| = 80dB = 10^4$$

$$H(jw) = \frac{K(0.1jw+1)\left(\frac{2jw}{3}+1\right)}{(jw)^2\left(\frac{jw}{80}+1\right)\left(\frac{jw}{200}+1\right)}$$

$$\left|H(j0.02)\right| = 10^4 \Rightarrow k = 400$$

$$H(jw) = \frac{400(0.1jw+1)\left(\frac{2jw}{3}+1\right)}{(jw)^2\left(\frac{jw}{80}+1\right)\left(\frac{jw}{200}+1\right)}$$

Problem 11.33

Find G(jw) if the amplitude characteristic fir this function is shown in fig 11.33.

Suggested Solution

Zeros: 2@100 r/s
Simple poles: dc $ 900 r/s
Complex poles : τ=1/20s $ ζ=0.1
So Wo=20 r/s
$ 2ξζ=0.01
Also,

$$H(j0.8) = 20dB = 10$$

$$H(jw) = \frac{k(0.1jw+1)^2}{(jw)\left(\frac{jw}{900}+1\right)\left(\left(\frac{jw}{20}\right)^2 + \frac{jw}{100}+1\right)}$$

$$H(j0.8) = 10 \Rightarrow K = 8$$

$$H(jw) = \frac{0.8(0.1jw+1)^2}{(jw)\left(\frac{jw}{900}+1\right)\left(\left(\frac{jw}{20}\right)^2 + \frac{jw}{100}+1\right)}$$

Problem 11.34

A series of RLC circuit resonates at 2000 rad/s. If C= 20μF and it is known that the impedance at resonance is 2.4 ohm, compute the value of L, the Q of the circuit and the bandwidth.

Suggested Solution

$$W_O = 2000\frac{r}{s} = \frac{1}{\sqrt{LC}} \Rightarrow L = 12.5mH$$

$$THEN \quad \text{Q=}\frac{W_O L}{R} = 10.42$$

$$BW = \frac{W_O}{Q} = \frac{2000}{10.42} = 192\frac{r}{s}$$

Problem 11.35

A series of resonant circuit has a Q of 120 and a resonant frequency of 60,000 rad/s. Determine the half-power frequencies and the bandwidth of the circuit.

Suggested Solution

$$BW = \frac{W_O}{Q} = \frac{60K}{120} = 500\frac{r}{s}$$

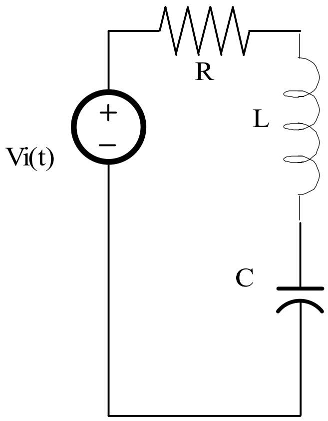
$$W_{LO,HI} = W_O\left(\sqrt{\left(\frac{1}{2Q}\right)^2 + 1} \pm \frac{1}{2Q}\right)$$

$$W_{HI} = 60.25Kr/s \quad \text{ALSO BW}=W_{HI} = W_{LO}$$

$$W_{LO} = 59.75Kr/s$$

Problem 11.36

Given the series RLC circuit in fig 11.36 if R=10ohm , find the values of L and C such that the network will have a resonant frequency of 100 kHz and a bandwidth of 1kHz.



Suggested Solution

$$W_O = \frac{1}{\sqrt{LC}} = 200\Pi \frac{Kr}{s}$$

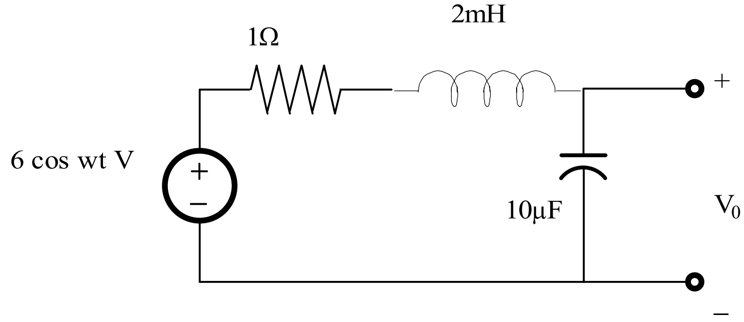
$$Q=\frac{W_O L}{R} = \frac{(100K)(2\Pi)L}{10} = 100 \Rightarrow L = 1.59mH$$

$$W_O = \frac{1}{\sqrt{LC}} \Rightarrow C = 1.59nF$$

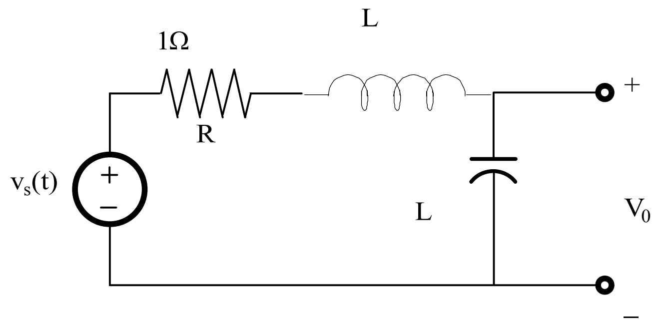
$$C = 1.59nF$$

Problem 11.37

Given the network in fig 11.37, find Wo ,Q, Wmax and Vo(max).



Suggested Solution



$$W_O = \frac{1}{\sqrt{LC}} = 7071\frac{r}{s} \qquad \text{Q=}\frac{W_O L}{R} = 14.14$$

$$W_{\max} = W_O\sqrt{(1 - \frac{1}{2Q^2})} = 7062\frac{r}{s}$$

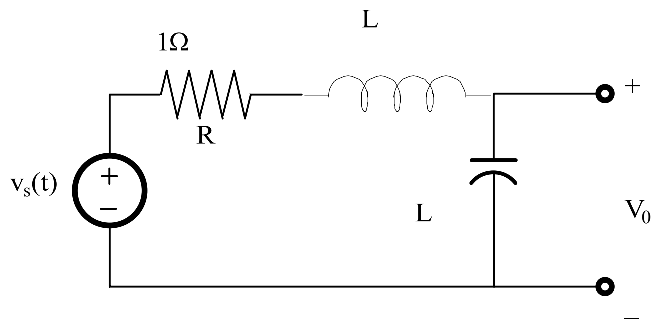
$$|V_O|_{\max} = \frac{Q|V_S|}{\sqrt{1 - \frac{1}{4Q^2}}} = 84.89V$$

$$W_O = 7071\frac{r}{s} \quad \text{Q=14.14} \quad W_{\max} = 7062\frac{r}{s} \quad |V_O|_{\max} = 84.89V$$

Problem 11.38

Repeat the problem 11.37 if the value of R is changed to 0.1ohm.

Suggested Solution



$$W_O = \frac{1}{\sqrt{LC}}$$

$$Q = \frac{W_O L}{R}$$

$$W_{max} = W_O \sqrt{(1 - \frac{1}{2Q^2})}$$

$$W_O = 7071 \frac{r}{s} \quad Q=141 \quad W_{max} = 7071 \frac{r}{s} \quad |V_O|_{max} = 846V$$

Problem 11.39

A series RLC circuit is driven by a signal generator. The resonant frequency of the network is known to be 1600 rad/s and at that frequency the impedance seen by the signal generator is 5ohm. If C=20μF, find L,Q and the bandwidth.

Suggested Solution

$$W_O = \frac{1}{\sqrt{LC}} \Rightarrow L = 19.5mH$$

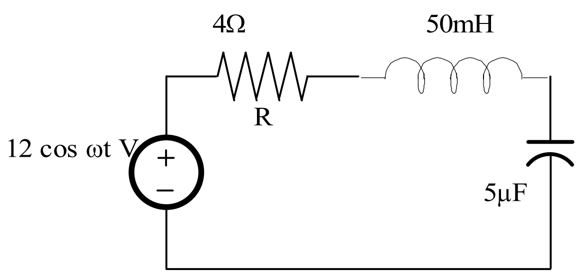
$$BW = R/L = 256r/s$$

$$Q = \frac{W_O}{BW} = 6.25$$

$$L = 19.5mH$$
$$BW = 256r/s$$
$$Q = 6.25$$

Problem 11.40

A variable frequency voltage source drives the network in fig 11.40. Determine the resonant frequency ,Q,BW and the average power dissipated by the network at resonance.



Suggested Solution

$$W_O = \frac{1}{\sqrt{LC}} \Rightarrow 2\frac{kr}{s}$$

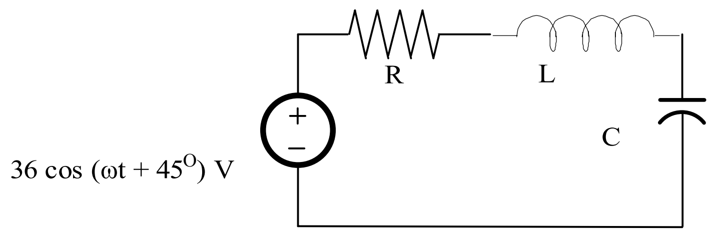
$$Q = \frac{W_O L}{R} = 25$$

$$BW = \frac{W_O}{Q} = 80r/s$$

$$P = 18W$$

Problem 11.41

In the network in fig 11.41 the inductor value is 30mH, and the circuit is driven by a variable frequency source. If the magnitude of the current at resonance is 12A,Wo=1000rad/s and L=10mH, find C, Q and the bandwidth of the circuit.



Suggested Solution

$$I_O = \frac{36}{R}$$

$$I_O = R = 3\Omega$$

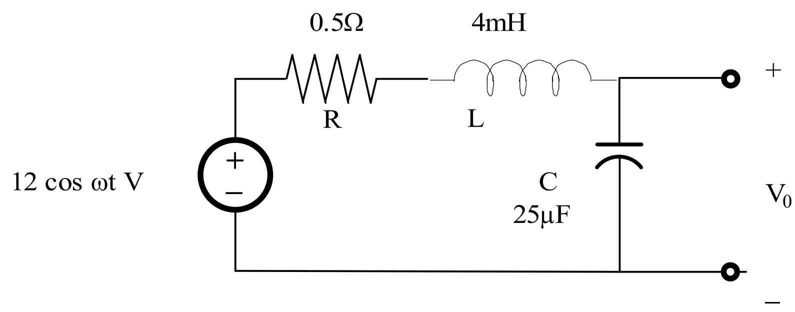
$$W_O = \frac{1}{\sqrt{LC}} \Rightarrow 1\frac{kr}{s} \Rightarrow C = 10nF$$

$$Q = \frac{W_O L}{R} = 3.33$$

$$BW = \frac{W_O}{Q} = 300r/s$$

Problem 11.42

Given the network in fig 11.42, find Vo(max).



Suggested Solution

$$W_O = \frac{1}{\sqrt{LC}} \Rightarrow 3162\frac{kr}{s}$$

$$BW = \frac{W_O}{Q} = 25.98$$

$$|V_O|_{\max} = \frac{Q|V_S|}{\sqrt{1-\frac{1}{4Q^2}}} = 305.1V$$

Problem 11.43

A parallel RLC resonant circuit with a resonant frequency of 20,000rad/s has admittance at resonance of 1ms. If the capacitance of the network is 5μF, find the values of R and L.

Suggested Solution

$$R = \frac{1}{0.001} = 1K\Omega$$

$$L = \frac{1}{W_O{}^2 C} = 500\mu H$$

Problem 11.44

A parallel RLC resonant circuit with a resistance of 200ohm. If it is known that the bandwidth is 80rad/s, find the values of the parameters L and C.

Suggested Solution

$$BW = \frac{1}{RC} \Rightarrow 80 = \frac{1}{200C} \Rightarrow C = 62.5\mu F$$

$$W_{HE} = B_W + W_{LO} = 80r/s$$

$$W_O = \frac{1}{\sqrt{LC}} \Rightarrow L = 22.73mH$$

$$L = 22.73mH$$

Problem 11.45

A parallel RLC circuit, which is driven by a variable frequency 2-a current source, has the following values: R=1Kohm,L=100mH, and C=10μF. Find the bandwidth of the network, the half-power frequencies , and the voltage across the network at the half-power frequencies.

Suggested Solution

$$BW = \frac{1}{RC} \Rightarrow 100r/s \qquad W_O = \frac{1}{\sqrt{LC}} \Rightarrow 1kr/s$$

$W_O{}^2 = W_{HI}W_{LO}$ and $W_{HI} - W_{LO} = BW$

$$W_{LO} = \frac{W_O{}^2}{W_{HI}}$$

$$W_{HI}{}^2 - BWW_{HI} - W_O{}^2 = 0$$

$$W_{HI} = \frac{BW + \sqrt{BW^2 + 4}W_O{}^2}{2} \Rightarrow 1051.25r/s$$

$$W_{LO} = 951.25r/s$$

AT RESONANCE $|V_O| = 2KV$ AND ½ POWER FREQUENCIES

$$|V_O| = \frac{2KV}{\sqrt{2}} 1.41KV$$

Problem 11.46

A parallel RLC circuit, which is driven by a variable-frequency 10-A source, has the following parameters: R=500ohm, L=0.5 mH, and C=20µF. Find the resonant frequency, the BW, and the average power dissipated at the half-power frequencies.

Suggested Solution

$W_O = \frac{1}{\sqrt{LC}} \Rightarrow 10\frac{kr}{s}$ $\qquad BW = \frac{1}{RC} \Rightarrow 100r/s$ $\qquad$ Q$=\frac{W_O}{BW} = 100$

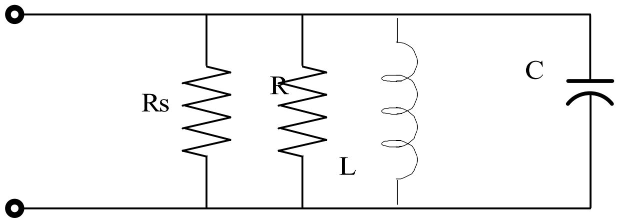
$W = W_O$

$P = 25KW$

$P_{LC} = P_{HI} = 12.5KW$

Problem 11.47

Consider the network in fig 11.47. if R=2Kohm, L=20mH, C=50µF, and Rs=infinity, determine the resonant frequency Wo, the Q of the network, and the bandwidth of the network. What impact does an Rs of 10Kohm have on the quantities determined.



Suggested Solution

$W_O = \frac{1}{\sqrt{LC}} = 1000\frac{r}{s}$ $BW = \frac{1}{RC} \Rightarrow 10r/s$ $Q = \frac{W_O}{BW} = 100$

$R_S = 10K\Omega$

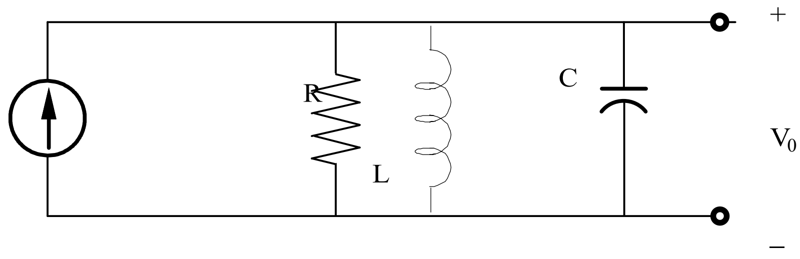
$R_S \parallel R = 1.67K\Omega$

$BW = 12r/s$

$Q = 83.3$

Problem 11.48

The source in the network in fig 11.48 is i(t) = cos1000t + cos1500t A.R=200ohm and C=500μF. If Wo =1000rad/s, find L, Q, and the BW. Compute the o/p voltage Vo(t) and discuss the magnitude of the output voltage at the two input frequencies.



Suggested Solution

$$L = \frac{1}{W_O{}^2 C} = 2mH \qquad BW = \frac{1}{RC} = 10r/s \qquad Q = \frac{W_C}{BW} = 100$$

$$Z_{EQ} = R \,||\, \frac{L/C}{j(WL - \frac{1}{WC})}$$

$$|Z_{EQ}| = \frac{\frac{RL}{C}}{\sqrt{\left(\frac{L}{C}\right)^2 + \left(WCR - \frac{R}{WC}\right)^2}}$$

$$Z_{EQ} = 2.4\angle{-89.3^0}\Omega$$

$$|Z_{EQ}| = 2.4$$

$$\therefore |V_{O2}| = 2.4$$

Problem 11.49

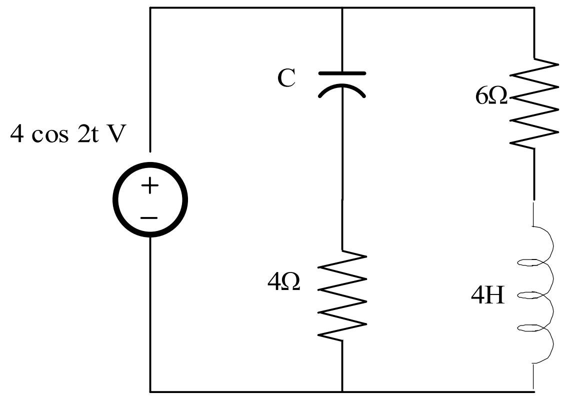
Determine the parameters of a parallel resonant circuit which has the following properties: Wo = 2Mrad/sec, BW = 20 krad/sec, and an impedance of 2000ohm.

Suggested Solution

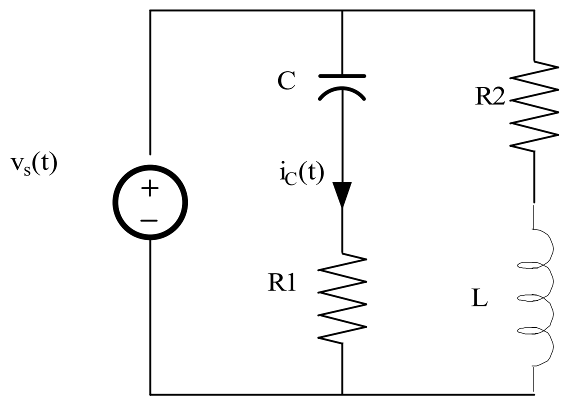
$W_O = \dfrac{1}{\sqrt{LC}} \Rightarrow C = 25nF$ $\qquad BW = \dfrac{1}{RC} \Rightarrow L = 10\mu H$

Problem 11.50

Determine the value of C in the network shown in fig 11.50 in order for the circuit to be in resonance.



Suggested Solution



AT RESONANCE Vs(t) AND Is(t) ARE IN PHASE.
SO, THE IMPEDANCE SEEN BY THE SOURCE, Zs, IS PURELY RESISTIVE.

$$Z_S = \left(R_1 + \frac{1}{jwc}\right) || \left(R_2 + \frac{1}{jwL}\right) = R_{EQ} + jo$$

$$Z_S = \left(4 + \frac{1}{j2c}\right) || \left(6 + j8\right)$$

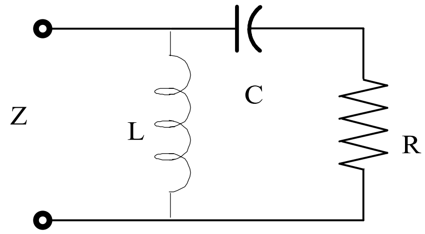
$$Z_S = \frac{24 + \frac{4}{C} + j(32 - \frac{3}{C})}{10 + j(8 - \frac{1}{2C})} = \frac{N(jw)}{D(jw)} = R_{eq}$$

IF Zs IS RESISTIVE, THEN THE PHASE ANGLES OF N(jw) AND D(jw) MUST BE EQUAL.

$$\frac{32 - 3/C}{24 + 4/C} \Rightarrow 64C^2 - 25C + 1 = 0 \Rightarrow C = 45.2mF, 345.4mF$$

Problem 11.51

Determine the equation for the nonzero resonant frequency of the impedance shown in fig 11.51.



Suggested Solution

$$Z = jwL \,||\left(R + \frac{1}{jwc} \right) = \frac{jwLR + L/C}{R + j(WL - I/LC)} = \frac{N\underline{|\theta_N}}{D\underline{|\theta_D}}$$

FOR RESONANCE, Z=Req +jo , OR, $\theta_N = \theta_D$

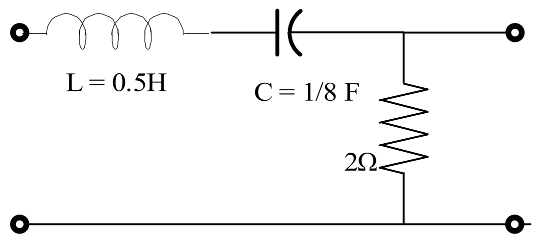
IF $\theta_N = \theta_D$,

$$\frac{W_O LR}{L/C} = \frac{W_O - 1/W_O C}{R} \Rightarrow \frac{1}{W_O CR}$$

$$W_O{}^2 = \frac{1}{RC(\frac{L}{R} - RC)} \Rightarrow W_O = \sqrt{\frac{1}{LC - (RC)^2}} r/s$$

Problem 11.52

Determine the new parameters of the network shown in fig 11.52 if
Znew = 10000 Zold.



Suggested Solution

$L = 0.5H$

$C = 1/8F$

$R = 2\Omega$

$Z_{NEW} = 10000 Z_{OLD}$

$KM = 10000$

Problem 11.53

Determine the new parameters of the network shown in fig 11.52 if
Wnew = 10000 Wold.

Suggested Solution

$L = 0.5H$

$C = 1/8F$

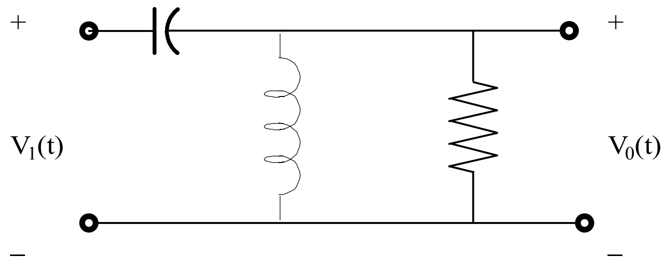
$R = 2\Omega$

$W_{NEW} = 10000 W_{OLD}$

$KF = 10000$

Problem 11.54

Given the network in fig 11.54 sketch the magnitude characteristic of the transfer function.



Suggested Solution

$$Z_{EQ} = jwL \,||\, (R) = \frac{jwL}{1 + jwL/R}$$

$$G_V(jw) = \frac{V_O}{V_1} = \frac{Z_{EQ}}{Z_{EQ} + jwL/C} = \frac{jwL}{jwL + \frac{1}{jWC}[jwL/R]}$$

$$G_V(jw) = \frac{(jw)^2}{(jw)^2 + 10jw + 100}$$

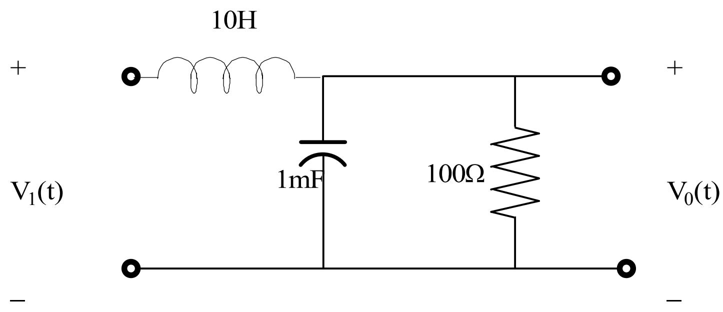
FIND FORM, $$G_V(jw) = \frac{0.01(jw)^2}{(\frac{jw}{10})^2 + \frac{jw}{10} + 1}$$

NETWORK IS A HIGHPASS FILTER.

Problem 11.55

Given the network in fig 11.55, sketch the magnitude characteristic of the transfer function.

$$G_V(jw) = \frac{V_O}{V_1}(jw)$$



Suggested Solution

LET $Z_{EQ} = (R) \parallel jwL = \dfrac{R}{1 + jwRC}$

$$\frac{V_O}{V_i} = \frac{Z_{EQ}}{Z_{EQ} + jwL} = \frac{R}{R + jwL(1 + jwRC)}$$

$$\frac{V_O}{V_i} = \frac{1}{(jw)^2 LC + \frac{jwL}{R} + 1}$$

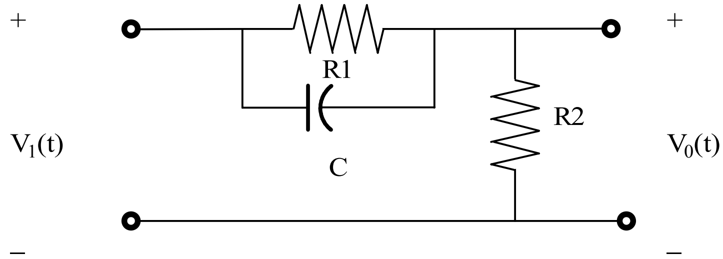
$$\frac{V_O}{V_i} = \frac{100}{(jw)^2 + jw(10) + 1}$$

$$\frac{V_O}{V_i} = \frac{1}{(\frac{jw}{10})^2 + \frac{jw}{10} + 1}$$

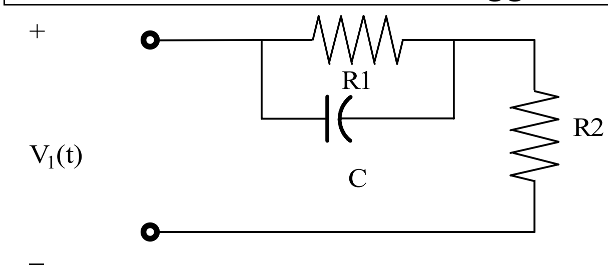
NETWORK IS A LOW-PASS FILTER.

Problem 11.56

Determine what type of filter the network shown in fig 11.56 represents by determining the voltage transfer function.



Suggested Solution



$$Z_{EQ} = R_1 \,||\, \frac{1}{jwc} = \frac{R_1}{1 + jwR_1c}$$

$$\frac{V_O}{V_i} = \frac{R_2}{R_2 + Z_{EQ}} = \frac{(1 + jwR_1c)R_2}{R_1 + R_2 + jwR_1R_2c}$$

$$\frac{V_O}{V_i} = \left(\frac{R_2}{R_1 + R_2}\right)\left(\frac{(1 + jwR_1c)}{(1 + jwRc)}\right) WHERE$$

$$R = R_1 \,||\, R_2 < R_1$$

$$W_Z = \frac{1}{RC}$$

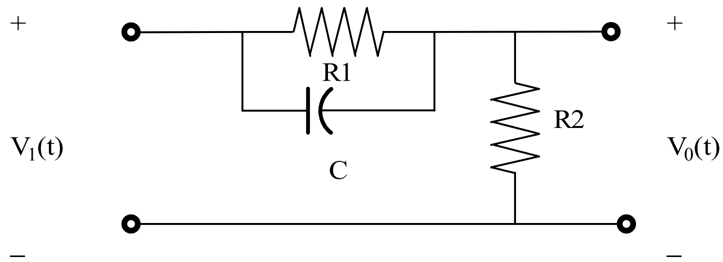
$$W_P = \frac{1}{RC}$$

$$W_P > W_Z$$

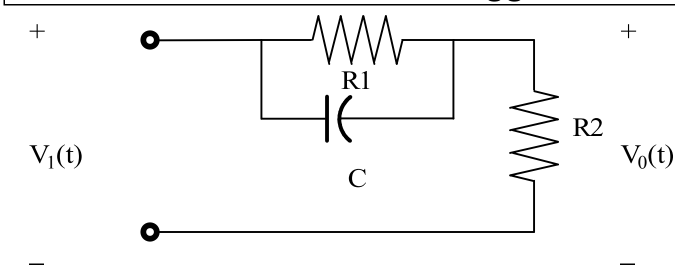
NETWORK IS A HIGHER FILTER.

Problem 11.57

Determine what type of filter the network shown in fig 11.57 represents by determining the voltage transfer function.



Suggested Solution



$$Z_{EQ} = R_1 \parallel \frac{1}{jwL} = \frac{jwLR_1}{R_1 + jwL}$$

$$G_V = \frac{V_O}{V_i} = \frac{R_2}{R_2 + Z_{EQ}} = \frac{(R_1 + jwL)R_2}{R_1R_2 + jwLR_1 + jwLR_1}$$

$$G_V = \frac{1 + \frac{jwL}{R_1}}{1 + \frac{jwL}{R}}$$

$$R = R_1 \parallel R_2 < R_1$$

$$W_Z = \frac{R_1}{L}$$

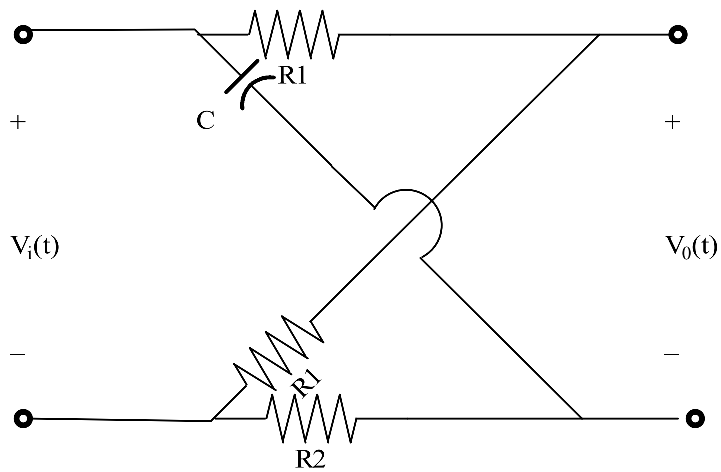
$$W_P = \frac{R}{L}$$

$$W_P < W_Z$$

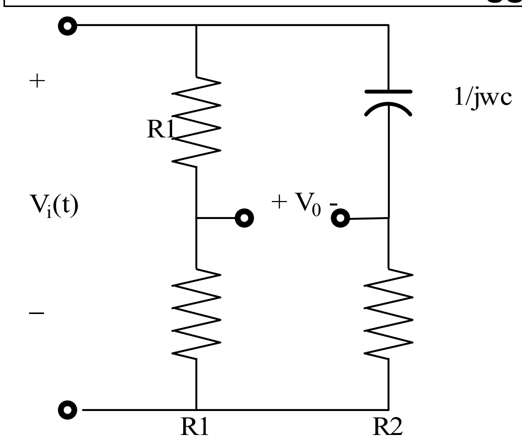
NETWORK IS A LOW-PASS FILTER.

Problem 11.58

Given the lattice network shown in fig 11.58 determine what type of filter this network represents by determining the voltage transfer function.



Suggested Solution



$$V_O = V_i\left[\frac{R_1}{R_1+R_1} - \frac{R_1}{R_2+\dfrac{1}{jwc}}\right] = V_i\left[\frac{1}{2} - \frac{jwcR_2}{1+jwcR_2}\right] = \frac{1}{2}V_i\left[\frac{1-jwcR_2}{1+jwcR_2}\right]$$

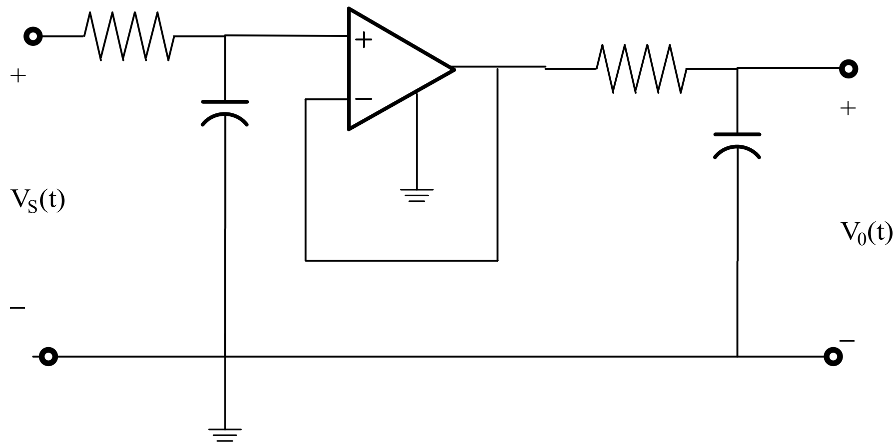
THEREFORE

$$\left|\frac{V_O}{V_i}\right| = \frac{1}{2}\frac{\sqrt{1+(wcR_2)^2}}{\sqrt{1+(wcR_2)^2}} = 0.5$$

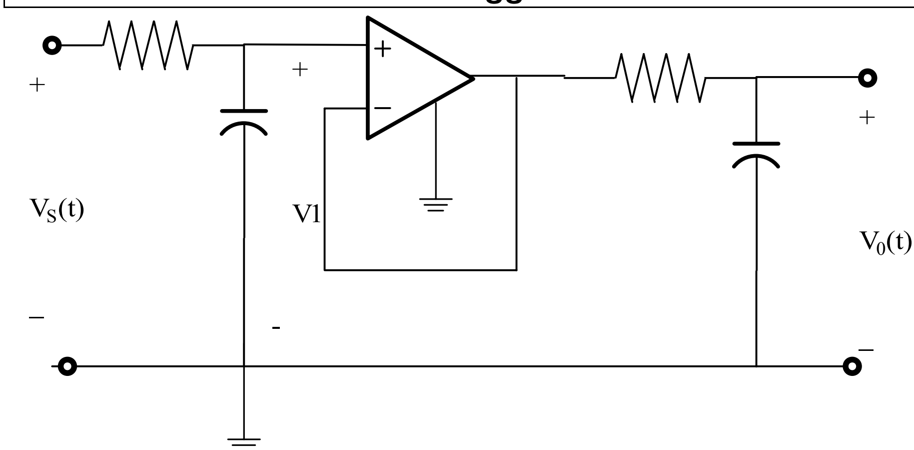
AN ALL PASS FILTER.

Problem 11.59

Given the network shown in fig 11.59 and employing the voltage follower analyzed in chapter 3 determine the voltage transfer function and its magnitude characteristic. What type of filter does the network represent?



Suggested Solution



$$R = 1\Omega$$

$$C = 1F$$

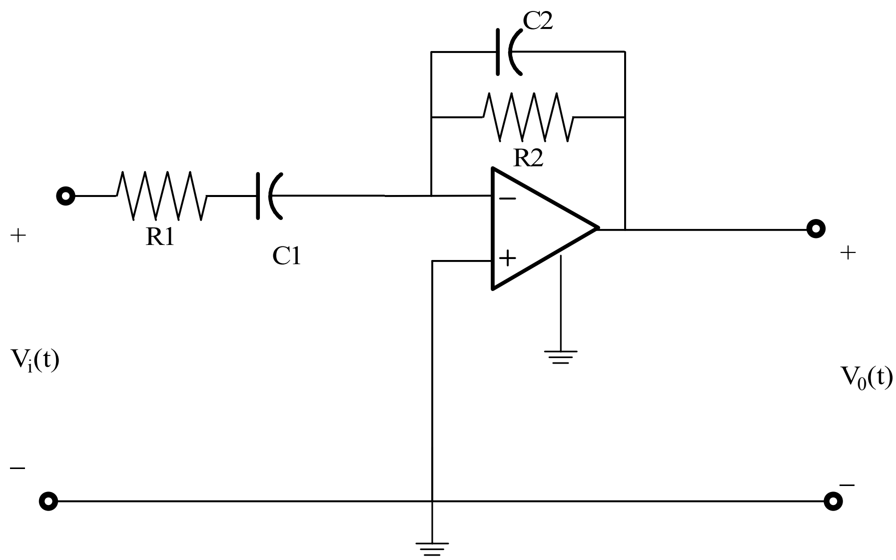
$$\frac{V_1}{V_i} = \frac{1/jw}{(1/jw)+1} = \frac{1}{1+jw}$$

$$\frac{V_O}{V_1} = \left(\frac{1}{1+jw}\right)^2$$

SECOND ORDER LOW-PASS

Problem 11.60

Determine the voltage transfer function and its magnitude characteristic for the network shown in fig 11.60 and identify the filter properties.



Suggested Solution

$$\frac{V_O}{V_i}(jw) = \frac{-Z_2(jw)}{Z_1(jw)}$$

$$Z_2(jw) = \frac{R_2 / jwc_2}{R_1 + \frac{1}{jwc_2}}$$

$$Z_1(jw) = R_1 + \frac{1}{jwc_1}$$

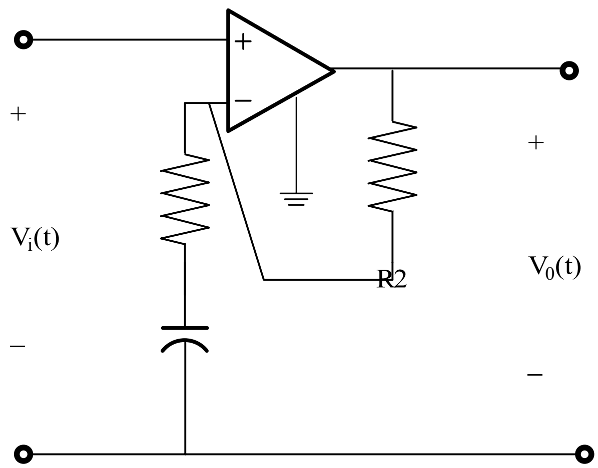
$$\therefore \frac{V_O}{V_i}(jw) = \frac{jwc_1 R_2}{(jwc_1 R_1 + 1)(jwc_2 R_2 + 1)}$$

THIS IS A 2ND ORDER BAND PADD FILTER.

Problem 11.61

Given the network in fig 11.61 find the transfer function and determine what type of filter the network represents.

$$\frac{V_O}{V_i}(jw)$$



Suggested Solution

$$\frac{V_O}{V_i}(jw) = 1 + \frac{Z_2}{Z_1}$$

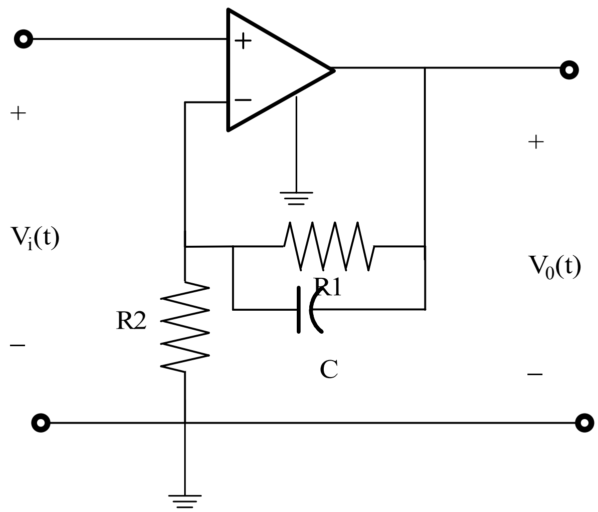
$$Z_2 = R_2 \qquad\qquad Z_1 = R_1 + \frac{1}{jwc}$$

$$\therefore \frac{V_O}{V_i}(jw) = 1 + \frac{jwcR_2}{(jwc_1R_1 + 1)} = \frac{(jwc(R_1 + R_2) + 1)}{(jwc_1R_1 + 1)}$$

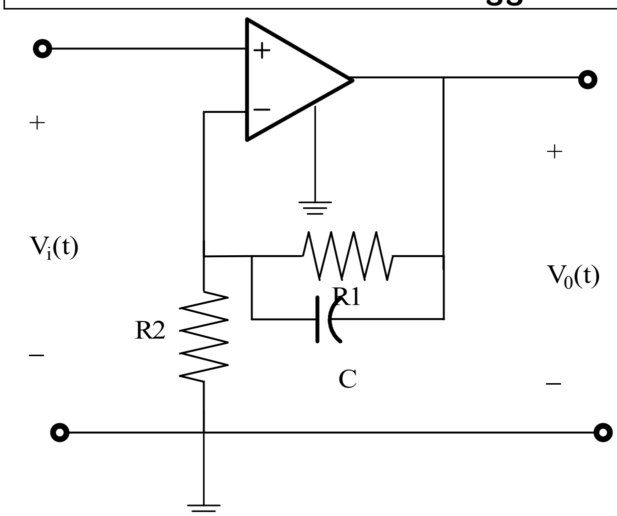
this is a high pass filter

Problem 11.62

Repeat problem 11.54 for the network shown in fig 11.62.



Suggested Solution



$$Z_{eq} = R_1 \,||\, \frac{1}{jwc} = \frac{R_1}{1 + jwcR_1}$$

USING STANDARD NON-INVERTING GAIN EQUATION,

$$G_V = \frac{V_O}{V_i} = 1 + \frac{Z_{eq}}{R_2} = \frac{R_1 + R_2 + jwcR_1R_2}{R_2(1 + jwcR_1)}$$

$$G_V = (1+\frac{R_1}{R_2})\left[\frac{(1+jwcR)}{(1+jwcR_1)}\right]$$

$$R = R_1 \,||\, R_2$$

$$W_Z = \frac{1}{CR}$$

$$W_P = \frac{1}{CR_1}$$

SINCE R<R1, $W_Z > W_P$

NETWORK IS A LOW-PASS FILTER.

Problem 11.63

In all OTA problem, the specifications are g_m - I_{abc} sensitivity = 20, maximum g_m = 1mS with range of 4 decades.
For the circuit in figure 11.50, find g_m and I_{abc} values required for a simulated resistance of 10Kohm.

Suggested Solution

$$i_o = g_m(-v_{in}) \;\&\; i_o = -i_{in}$$

$$v_{in} = -\frac{i_o}{g_m} = \frac{i_{in}}{g_m}$$

$$R_{eq} = \frac{v_{in}}{i_{in}} = \frac{1}{g_m}$$

$$\therefore g_m = 100\mu s \text{ and } \mathrm{I}_{\mathrm{ARC}} = 5\mu A$$

Problem 11.64

In all OTA problem, the specifications are g_m - I_{abc} sensitivity = 20, maximum g_m = 1mS with range of 4 decades.
For the circuit in figure 11.53, find g_m and I_{abc} values required for a simulated resistance of 10Kohm. Repeat for 750Kohm.

Suggested Solution

$$i_{o1} = -i_1 = -g_{m1} v_1^-$$

$$R_{eq} = \frac{1}{g_{m1}}$$

$$i_{o2} = -i_2 = -g_{m2} v_2^-$$

$$R_{eq} = \frac{1}{g_{m2}}$$

For 10kΩ

$$g_{m1} = g_{m2} = \frac{1}{10k} = 100\mu s \text{ and } \mathrm{I}_{\mathrm{ARC}} = 5\mu A$$

For 750kΩ

$$g_{m1} = g_{m2} = 1.33\mu s \text{ and } \mathrm{I}_{\mathrm{ARC}} = 66\mu A$$

Problem 11.65

Use the summing circuit in figure 11.51 to design a circuit that realizes the following function.
Vo = 7V1 + 3V2

Suggested Solution

$$i_{o1} + i_{o2} = i_o = i_{o3} = g_3 v_o$$

$$g_1 v_1 + g_2 v_2 = g_3 v_o$$

$$\therefore v_o = \frac{g_1}{g_3} v_1 + \frac{g_2}{g_3} v_2$$

$$\frac{g_1}{g_3} = 7, \frac{g_2}{g_3} = 3$$

$\therefore$ arbitrarily select

$$g_3 = 100 \mu s$$

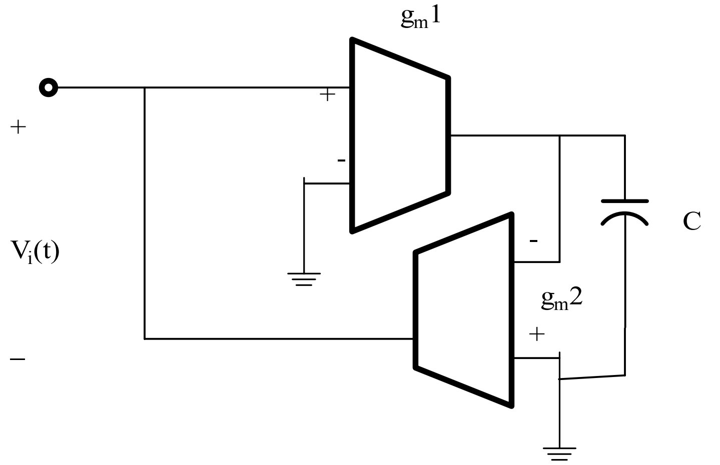
$$g_2 = 700 \mu s$$

$$g_1 = 300 \mu s$$

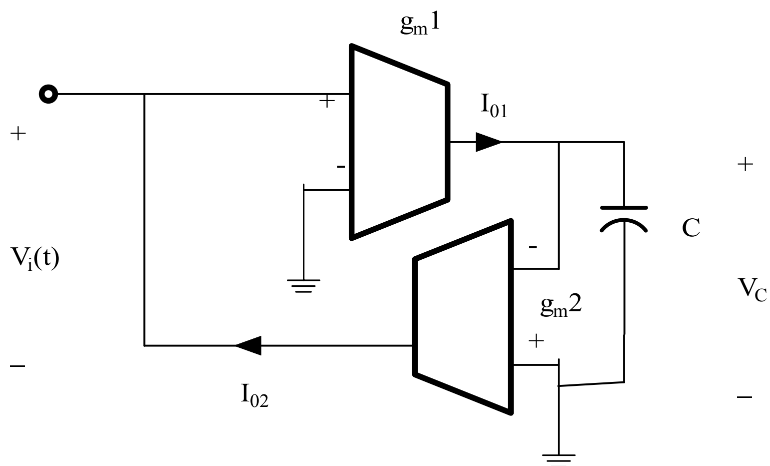
$$and \;\; \mathrm{I_{ARC1}} = 5 \mu A, \mathrm{I_{ARC2}} = 35 \mu A \text{ and } \mathrm{I_{ARC3}} = 15 \mu A$$

Problem 11.66

Prove that the circuit in figure p11.66 is a simulated inductor. Find the inductance in terms of C, g_{m1} and g_{m2}.



Suggested Solution



$$i_{o1} = g_{m1}v_{in} = c\frac{dv_i}{dt}$$

$$i_{o2} = -g_{m2}v_i = i_{in}$$

$$\therefore g_{m1}v_{in} = c\frac{dv_i}{dt} = -\frac{c}{g_{m2}}\frac{di_{o2}}{dt} = \frac{c}{g_{m2}}\frac{di_{in}}{dt}$$

$$v_{in} = \frac{c}{g_{m1}g_{m2}}\frac{di_{in}}{dt} \quad \text{or} \quad i_{eq} = \frac{c}{g_{m1}g_{m2}}$$

Problem 11.67

In the Tow-Thomas biquad in figure 11.57, C1=20pF, C2 =10 pF, g_m = 10μS , g_{m2} = 80μS, g_{m3} =10μS . find the low pass filter transfer function for the Vi1 – V02 i/p- o/p pair. Plot the corresponding Bode plot.

Suggested Solution

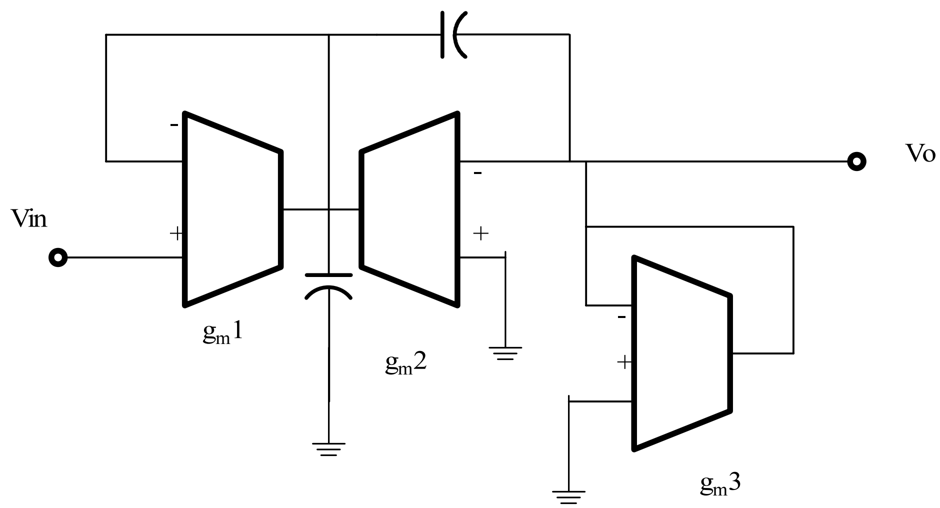
$$w_o = \sqrt{\frac{g_1 g_2}{c_1 c_2}} = 2Mr/s$$

$$\frac{w_o}{Q} = \frac{g_3}{c_2} = 1Mr/s$$

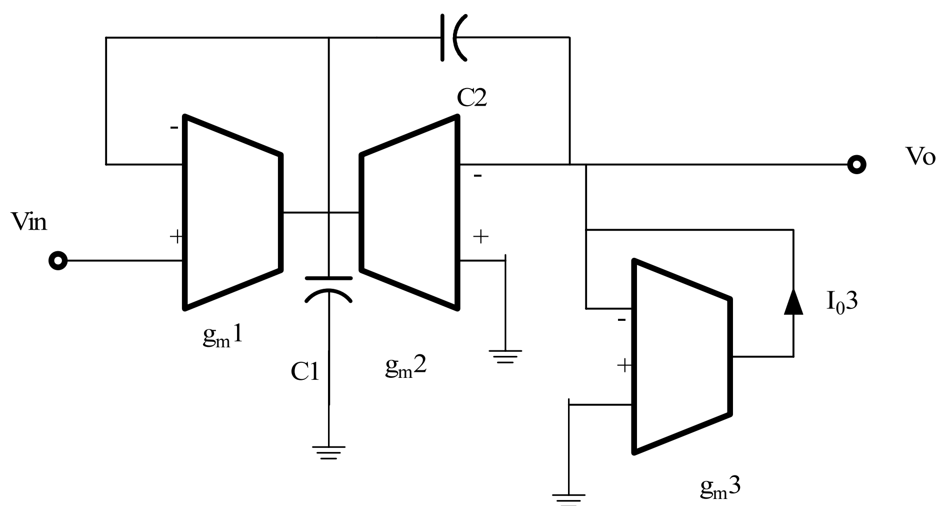
$$\frac{v_{o2}}{v_{i1}} = \frac{w_o^{\;2}}{s^2 + \frac{w_o}{Q}s + w_o^{\;2}} = \frac{4*10^{12}}{s^2 + 10^6 s + 4*10^{12}}$$

Problem 11.68

Find the transfer function of the OTA filter in figure 11.68. Express Wo and Q in terms of the capacitances and transconductances. What kind of filter is it.



Suggested Solution



$$I_{o1} = g_1(V_{in} - V_x)$$
$$I_{o2} = -g_2 V_o$$
$$I_{o3} = -g_3 V_o$$
$$I_{o1} + I_{o2} + (V_o - V_x)\, jwc_2 = jwc_1 V_x$$
$$I_{o3} = (V_o - V_x)\, jwc_2$$

making substitutions.

$$g_1(V_{in} - V_x) - g_2 V_o + jwc_2 V_o - jwc_2 V_x = jwc_1 V_x$$
$$-g_3 V_o = jwc_2 V_o - jwc_2 V_x$$

from the last eq

$$jwc_2 V_x = g_3 V_o + jwc_2 V_o$$
$$V_x = (1 + \frac{g_3}{jwc_2}) V_o$$

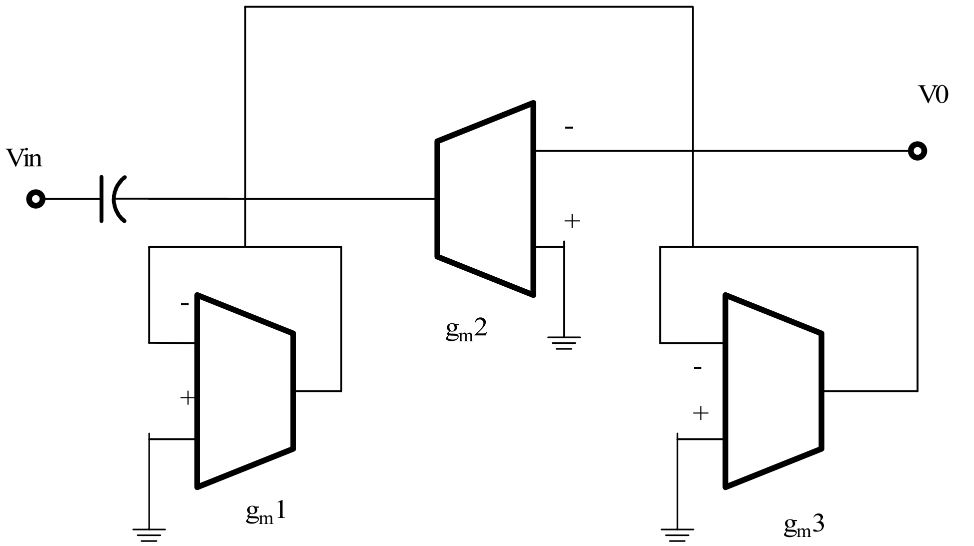
then

$$g_1 V_{in} - g_1(1 + \frac{g_3}{jwc_2}) V_o - g_2 V_o + jwc_2 V_o - jwc_2(1 + \frac{g_3}{jwc_2}) V_o = jwc_1(1 + \frac{g_3}{jwc_2}) V_o$$
$$g_1 V_{in} = \left[\frac{jwc_2(g_1 + g_2 + g_3) + g_1 g_3 - w^2 c_1 c_2 + jwg_1 g_3}{jwc_1} \right]$$
$$jwc_1 g_1 V_{in} = \left[-w^2 c_1 c_2 + jwc_2(g_1 + g_2 + g_3) + g_3 c_1 \right] V_o$$
$$\frac{V_o}{V_{in}} = \frac{jwc_1 g_1}{-w^2 + jw[\frac{(g_1 + g_2 + g_3) + g_3 c_1 / c_2}{c_1}] + \frac{g_1 g_3}{c_1 c_2}}$$
$$w_o = \sqrt{\frac{g_1 g_3}{c_1 c_2}}$$
$$Q = \frac{\sqrt{g_1 g_2 c_1 c_2}}{c_2(g_1 + g_2 + g_3) + g_3 c_1}$$

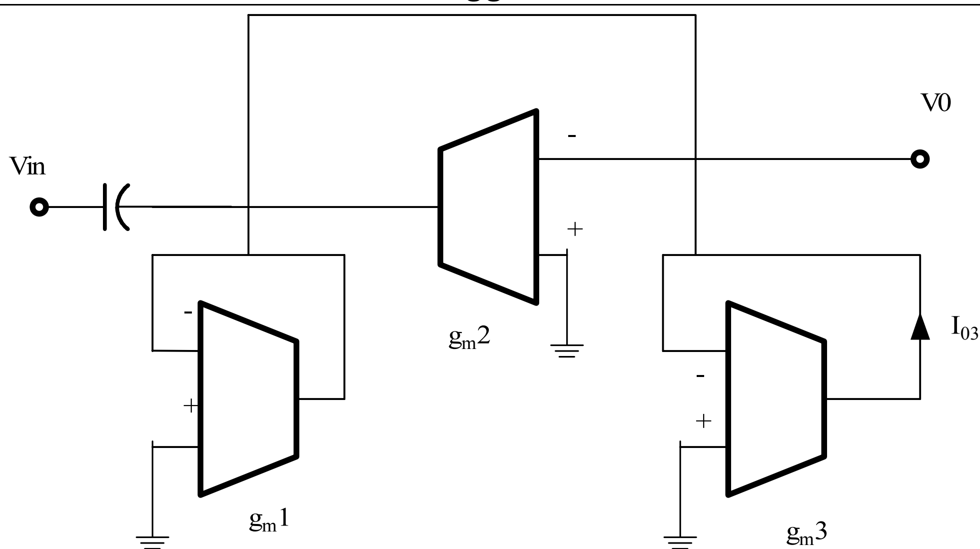
This is a band pass filter.

Problem 11.69

Find the transfer function of the OTA filter in fig 11.69. Express Wo and Q in terms of the capacitances and transconductances. What kind of filters is it?



Suggested Solution



$$I_{o1} = -g_1 V_x$$

$$I_{o2} = -g_2 V_o$$

$$(V_{in} - V_x)\, jwc_1 + I_{o1} + I_{o2} + (V_o - V_x)\, jwc_2 = 0$$

$$jwc_1 V_{in} - jwc_1 V_x - g_1 V_x - g_2 V_o + jwc_2 V_o - jwc_2 V_x = 0$$

$$jwc_1 V_{in} - (jwc_1 + g_1 + jwc_2) V_x + (jwc_2 - g_2) V_o = 0$$

in addition

$$(V_o - V_x) jwc_2 = -g_3 V_o$$
$$or$$
$$jwc_2 V_x = (jwc_2 + g_3) V_o$$
$$V_x = \left(\frac{jwc_2 + g_3}{jwc_2}\right) V_o = \left(1 + \frac{g_3}{jwc_2}\right) V_o$$

then,

$$jwc_1 V_{in} = \left[(jwc_1 + g_1 + jwc_2)\left(1 + \frac{g_3}{jwc_2}\right) - jwc_2 + g_2 \right] V_o$$
$$jwc_1 V_{in} = \left[\frac{-w^2 c_1 c_2 + jwc_2((g_1 + g_2 + g_3) + g_3 c_1 / c_2) + g_1 g_3}{jwc_2} \right] V_o$$

$$\frac{V_o}{V_{in}} = \frac{-w^2 c_1 c_2}{-w^2 c_1 c_2 + jwc_2(g_1 + g_2 + g_3 + g_3(c_1 / c_2)) + g_1 g_3}$$
$$\frac{V_o}{V_{in}} = \frac{-w^2}{-w^2 + jw(\frac{(g_1 + g_2 + g_3 + g_3(c_1 / c_2))}{c_1}) + \frac{g_1 g_3}{c_1 c_2}}$$

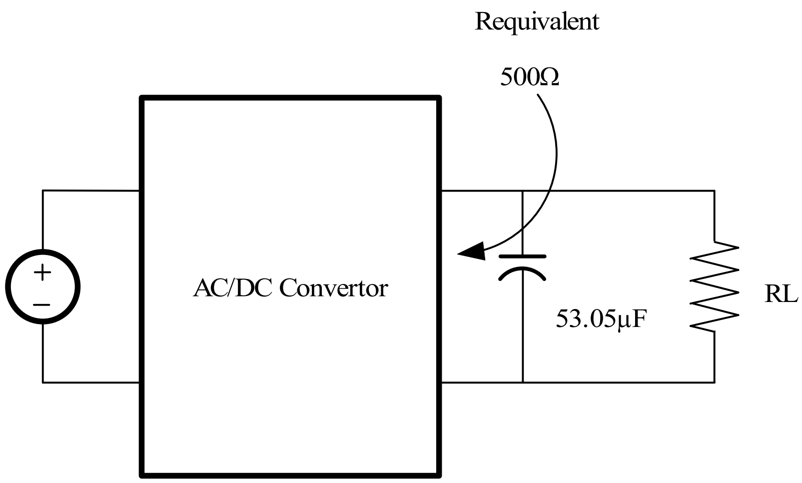
Therefore,

$$w_o = \sqrt{\frac{g_1 g_3}{c_1 c_2}}$$
$$Q = \frac{\sqrt{g_1 g_2 c_1 c_2}}{c_2(g_1 + g_2 + g_3) + g_3 c_1}$$

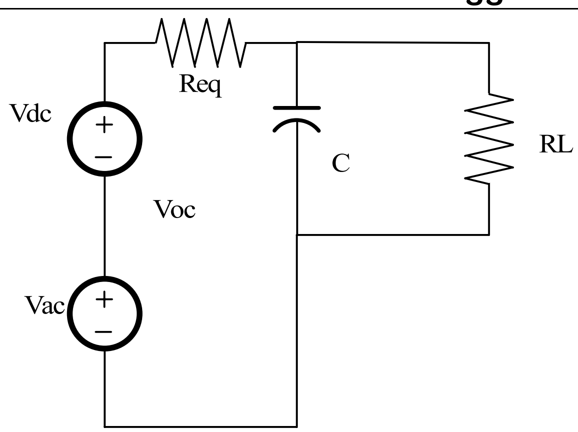
A high pass filter.

Problem 11.70

Refer to the ac/dc converter low-pass filter application of example 11.25. if we put the converter to use powering a calculator, the load current can be modeled by a resistor as shown in fig 11.25. The load resistor will affect both the magnitude of the dc component of Vof and the pole frequency. Plot both the pole frequency and the ratio of the 60-Hz component of the o/p voltage to the dc component of Vof versus Rl for 100ohm less than equal to Rl, which is less than equal to 100 kohm. Comment on the advisable limitations on Rl if (a)the dc component of Vof is to remain within 20 % of its 9-v ideal value: (b) the 60 Hz component of Vof remains less than 15% of the dc component.



Suggested Solution



THE CIRCUIT BELOW MODELS THE SITUATION DESCRIBED IN THE TEXT.

$$H = \frac{V_O}{V_S} = \frac{R_L}{R_L + R_{eq} + jwcR_L} = \frac{R_L}{R_L + R_{eq}}\left[\frac{1}{1 + jwcR}\right]$$

$$WHERE \quad \text{R=}R_L \parallel R_{eq}$$

$$a).H(jo) = \frac{R_L}{R_L + R_{eq}} \geq 0.8$$

$$R_L \geq 2K\Omega$$

$$b).\left|\frac{H(j377)}{H(jo)}\right| < 0.15$$

$$OR, R > 330\Omega$$

$$Since, R = R_{eq} \parallel R_L$$

$$R_L > 970.6\Omega$$

Requirement a) is more stringent.

Problem 11.71

Referring to Ex 11.28 design a notch filter for the tape deck for use in Europe, where power utilities generate at 50 Hz.

Suggested Solution

$R_{tape} = 50\Omega$

$R_{amp} = 1K\Omega$

$W_Z = \frac{1}{\sqrt{LC}} = 2\prod(50)$

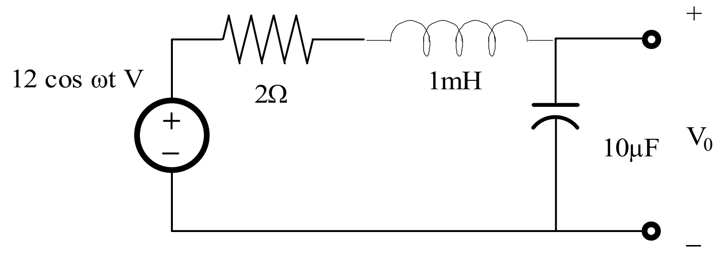
CHOOSE YEILDS,

C=100μF

L=101mH

Problem 11FE-1

Determine the resonant frequency of the circuit in fig 11PFE-1 and find the voltage Vo at resonance.



Suggested Solution

$$w_o = \frac{1}{\sqrt{LC}} = 10kr/s$$

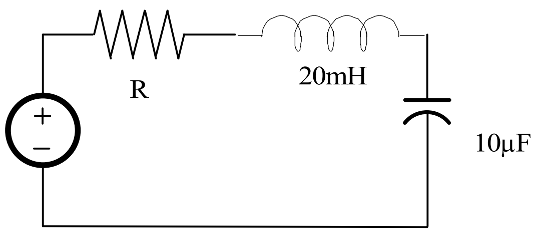
$$I = \frac{12\angle 0^0}{2} = 6\angle 0^0$$

$$v_c = \frac{I}{jwc} = \frac{6\angle -90^0}{10^{-4}10^{-5}} = 60\angle -90^0 v$$

$$also \quad |v_c| = Qv_s = \frac{w_o L}{R}(12) = 60v$$

Problem 11FE-2

Given the series circuit in fig 11PFE-2, determine the resonant frequency and find the value of R so that the BW of the network about the resonant frequency is 200 r/s.



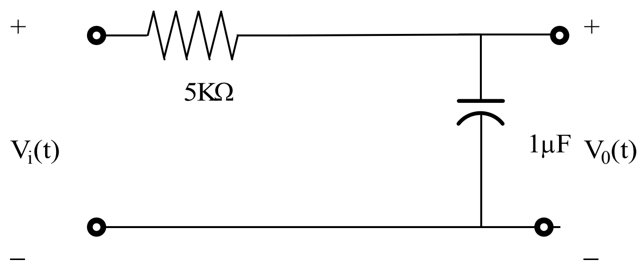
Suggested Solution

$$w_o = \frac{1}{\sqrt{LC}} = 1kr/s$$

$$Q = \frac{w_o L}{R} \text{ and BW=} \frac{w_o}{Q} \therefore BW = \frac{R}{L} \text{ and } 200 = \frac{\text{R}}{0.00020} \therefore R = 4\Omega$$

Problem 11FE-3

Given the low-pass filter circuit in fig 11PFE-3, determine the frequency in Hz at which the output is down from the dc, or very low frequency, output.



Suggested Solution

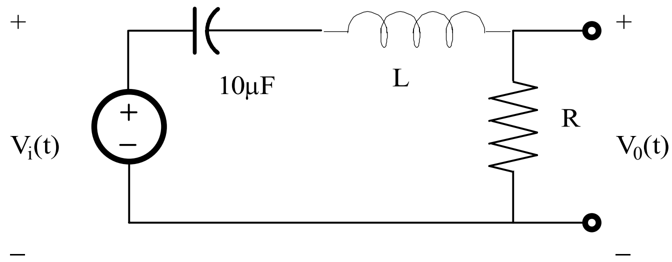
$$\frac{v_o}{v_{in}} = \frac{1/jwc}{R + 1/jwc} = \frac{1}{1 + jwRc} \therefore 3dB point$$

$$w = \frac{1}{RC} = 200 r/s$$

$$f_{3dB} = 31.83 Hz$$

Problem 11FE-4

Given the band-pass shown in fig 11PFE-4, find the components L and R necessary to provide a resonant frequency of 1000 r/s and a BW of 100 r/s.



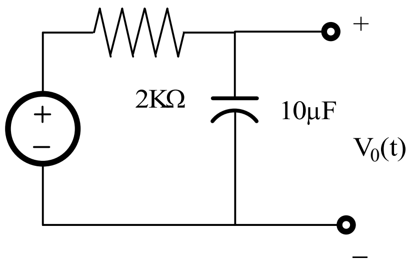
Suggested Solution

$$w = \frac{1}{\sqrt{LC}} \Rightarrow L = 100mH$$

$$BW = \frac{R}{L} \Rightarrow R = 10\Omega$$

Problem 11FE-5

Given the low-pass shown in fig 11PFE-5, find the half-power frequency and the gain of this circuit, if the source frequency is 8Hz.



Suggested Solution

$$w = \frac{1}{RC} = 50r/s$$

$$f = \frac{50}{2\prod} = 7.96Hz$$

The dc gain is unity and the gain at the Half Power Frequency 7.98 is approximate to 8 Hz is by definition 0.707 of the dc gain.

Problem 12.1

Find the Laplace Transform of the function

$$f(t) = te^{-\alpha t}\delta(t-1)$$

Suggested Solution

$$F(s) = \int_0^\infty te^{-at}\delta(t-1)e^{-st}dt$$

$$let$$
$$g(t) = te^{-at}$$
$$F(s) = \int_0^\infty g(t)\delta(t-1)e^{-st}dt = g(1)e^{-s(1)} = e^{-(s+a)}$$

Problem 12.2

Find the Laplace transform of the function

$$f(t) = te^{-a(t-1)}\delta(t-1)$$

Suggested Solution

$$f(t) = te^{-a(t-1)}\delta(t-1)$$

let

$$g(t) = te^{-(a-1)t}$$

$$F(s) = \int_0^\infty g(t)\delta(t-1)e^{-st}dt = g(1)e^{-s(1)} = e^{-s}$$

$$F(s) = e^{-s}$$

Problem 12.3

If

$$f(t) = e^{-at}\cos(\omega t)$$

show

$$F(s) = \frac{s+a}{(s+a)^2 + (\omega)^2}$$

Suggested Solution

$$F(s) = \int_0^\infty e^{-at}e^{-st}\cos(\omega t)dt = \int_0^\infty e^{-(a+s)}[\frac{e^{-j\omega t} + e^{-j\omega t}}{2}]dt$$

$$F(s) = \frac{1}{2}[\int_0^\infty e^{-(s+a-j\omega)t}dt + \int_0^\infty e^{-(s+a+j\omega)t}dt]$$

$$= \frac{1}{2}[\frac{-1}{s+a-j\omega}e^{-(s+a-j\omega)t}\Big|_0^\infty - \frac{1}{s+a+j\omega}e^{-(s+a+j\omega)t}\Big|_0^\infty]$$

$$= \frac{1}{2}[\frac{-1}{s+a-j\omega} + \frac{1}{s+a+j\omega}]$$

$$= \frac{1}{2}[\frac{s+a+j\omega + s+a-j\omega}{(s+a-j\omega)(s+a+j\omega)}]$$

$$F(s) = \frac{s+a}{(s+a)^2 + (\omega)^2}$$

$$F(s) = \frac{s+a}{(s+a)^2 + (\omega)^2}$$

Problem 12.4

Find F(s) if

$$f(t) = e^{-at}\sin(\omega t)u(t-1)$$

Suggested Solution

$$f(t) = e^{-at}\sin(\omega t)u(t-1)$$

$$F(s) = e^{-s}L[e^{-a(t+1)}\sin(\omega)(t+1)]$$

$$= e^{-(s+a)}L[e^{-at}\sin\omega(t+1)]$$

$$= e^{-(s+a)}L[e^{-at}(\sin\omega t\cos\omega + \cos\omega t\sin\omega)]$$

$$= e^{-(s+a)}[\frac{\omega\cos\omega}{(s+a)^2+(\omega)^2} + \frac{(s+a)\sin\omega}{(s+a)^2+(\omega)^2}]$$

$$F(s) = e^{-(s+a)}\left[\frac{\omega\cos\omega}{(s+a)^2+(\omega)^2} + \frac{(s+a)\sin\omega}{(s+a)^2+(\omega)^2}\right]$$

Problem 12.5

If

$$f(t) = t\cos(\omega t)u(t-1)$$

find $F(s)$

Suggested Solution

$$f(t) = t\cos(\omega t)u(t-1),$$

$$F(s) = e^{-s}L[(t+1)\cos\omega(t+1)]$$
$$= e^{-s}L[(t+1)(\cos\omega t\cos\omega - \sin\omega t\sin\omega)]$$
$$= e^{-s}L[t\cos\omega t\cos\omega + \cos\omega t\cos\omega - t\sin\omega t\sin\omega - \sin\omega t\sin\omega]$$

$$L[\cos\omega t\cos\omega] = \frac{s\cos\omega}{s^2+\omega^2}$$
$$L[\sin\omega t\sin\omega] = \frac{s\sin\omega}{s^2+\omega^2}$$
$$L[t\cos\omega t\cos\omega] = \frac{-d}{ds}\left(\frac{s\cos\omega}{s^2+\omega^2}\right) = -\left[\frac{(s^2+\omega^2)\cos\omega - s\cos\omega(2s)}{(s^2+\omega^2)^2}\right]$$
$$L[t\sin\omega t\sin\omega] = \frac{-d}{ds}\left(\frac{\omega\sin\omega}{s^2+\omega^2}\right) = -\left[\frac{\omega\sin\omega(2s)}{(s^2+\omega^2)^2}\right]$$

$$F(s) = e^{-s}\left[\frac{2s^2\cos\omega - (s^2+\omega^2)\cos\omega}{(s^2+\omega^2)^2} + \frac{2s\omega\sin\omega}{(s^2+\omega^2)^2} + \frac{s\cos\omega}{s^2+\omega^2} + \frac{\omega\sin\omega}{s^2+\omega^2}\right]$$

$$F(s) = e^{-s}\left[\frac{2s^2\cos\omega - (s^2+\omega^2)\cos\omega}{(s^2+\omega^2)^2} + \frac{2s\omega\sin\omega}{(s^2+\omega^2)^2} + \frac{s\cos\omega}{s^2+\omega^2} + \frac{\omega\sin\omega}{s^2+\omega^2}\right]$$

Problem 12.6

Find F(s) if

$$f(t) = te^{-at}u(t-4)$$

Suggested Solution

$$f(t) = te^{-at}u(t-4)$$

$$F(s) = e^{-4s}L[(t+4)e^{-a(t+4)}]$$
$$= e^{-4(s+a)}L[(t+4)e^{-at}]$$
$$= e^{-4(s+a)}L[te^{-at} + 4e^{-at}]$$
$$= e^{-4(s+a)}[\frac{1}{(s+a)^2} + \frac{4}{(s+a)}]$$

$$F(s) = e^{-4(s+a)}[\frac{1}{(s+a)^2} + \frac{4}{(s+a)}]$$

Problem 12.7

Use the shifting Theorem to determine $\mathrm{L}\left\{f\left(t\right)\right\}$ where

$$f(t)=[e^{-(t-2)}-e^{-2(t-2)}]u(t-2)$$

Suggested Solution

$$f(t)=[e^{-(t-2)}-e^{-2(t-2)}]u(t-2)$$

The shifting Theorem states

$$L[f(t-t_0)u(t-t_0)]=e^{-t_0 s}F(s)$$

$$L[f(t-t_0)u(t-t_0)]=e^{-t_0 s}F(s)$$

let

$$g(t)=(e^{-t}-e^{-2t})u(t)$$

so

$$G(s)=\frac{1}{s+1}-\frac{1}{s+2}$$

$$F(s)=e^{-2s}G(s)$$

$$F(s)=e^{-2s}[\frac{1}{s+1}-\frac{1}{s+2}]$$

$$F(s)=\frac{e^{-2s}}{(s+1)(s+2)}$$

Problem 12.8

Use the shifting Theorem to determine L{f(t)} where

$$f(t) = [e^{-(t-2)} - e^{-(t-1)}]u(t-1)$$

Suggested Solution

$$f(t) = [e^{-(t-2)} - e^{-(t-1)}]u(t-1)$$

let

$$g(t) = (t + e^{-t})u(t-1)$$

so

$$G(s) = \frac{1}{s+1} + \frac{1}{s^2}$$

$$F(s) = e^{-s}G(s)$$

$$F(s) = e^{-2s}[\frac{1}{s+1} + \frac{1}{s^2}]$$

$$F(s) = e^{-2s}[\frac{1}{s+1} + \frac{1}{s^2}]$$

Problem 12.9

Use Property Number 5 to find L{f(t)} if

$$f(t) = e^{-at}u(t-1)$$

Suggested Solution

$$f(t) = e^{-at}u(t-1)$$

$$let$$
$$g(t) = e^{-at}$$
$$Thm\_5:$$
$$L[g(t)u(t-t_0)] = e^{-t_0 s}L[g(t+t_0)]$$
$$so$$
$$L[e^{-at}u(t-1)] = e - sL[e^{-a(t+1)}] = e^{-(s+a)}L[e^{-at}]$$
$$\sin ce$$
$$L[e^{-at}] = \frac{1}{s+a}$$
$$F(s) = \frac{e^{-(s+a)}}{s+1}$$

$$F(s) = \frac{e^{-(s+a)}}{s+1}$$

Problem 12.10

Use Property Number 6 to find L{f(t)} if

$$f(t) = te^{-at}u(t-1)$$

Suggested Solution

$$f(t) = te^{-at}u(t-1)$$

$$let$$
$$g(t) = tu(t-1)$$
$$Thm\_6:$$
$$L[e^{-at}g(t)] = G(s+a)$$
$$L[g(t)] = L[tu(t-1)] = e^{-s}L[t-1] = e^{-s}(\frac{1}{s^2}+\frac{1}{s}) = G(s)$$
$$F(s) = G(s+a) = e^{-(s+a)}(\frac{1}{(s+a)^2}+\frac{1}{(s+a)})$$

$$F(s) = G(s+a) = e^{-(s+a)}(\frac{1}{(s+a)^2}+\frac{1}{(s+a)})$$

Problem 12.11

Given the following functions F(s), find f(t),

$$F(s) = \frac{4}{(s+1)+(s+2)}$$

$$F(s) = \frac{10s}{(s+1)+(s+4)}$$

Suggested Solution

A.

$$F(s) = \frac{4}{(s+1)+(s+2)}$$

for

$s = -1$

$$\frac{4}{s+2} = 4 = k_1$$

for

$s = -2$

$$\frac{4}{s+1} = -4 = k_2$$

$$f(t) = (4e^{-t} - 4e^{-2t})u(t)$$

so

$$F(s) = \frac{4}{s+2} - \frac{4}{s+1}$$

$$f(t) = (4e^{-t} - 4e^{-2t})u(t)$$

B.

$$F(s) = \frac{10s}{(s+1)+(s+4)}$$

for

$s=-1$

$$\frac{10s}{s+4}=\frac{-10}{3}=k_1$$

for

$s=-4$

$$\frac{10s}{s+1}=\frac{40}{3}=k_2$$

so

$$F(s)=\frac{-10/3}{s+1}-\frac{40/3}{s+4}$$

$$f(t)=(\frac{-10}{3}e^{-t}+\frac{40}{3}e^{-4t})u(t)$$

$$f(t)=(\frac{-10}{3}e^{-t}+\frac{40}{3}e^{-4t})u(t)$$

Problem 12.12

Given the following functions F(s), find f(t)

$$F(s) = \frac{s+10}{(s+4)(s+6)}$$

$$F(s) = \frac{24}{(s+2)(s+8)}$$

Suggested Solution

A.

$$F(s) = \frac{s+10}{(s+4)(s+6)}$$

for

$s = -4$

$$\frac{s+10}{s+6} = 3 = k_1$$

for

$s = -6$

$$\frac{s+10}{s+4} = -2 = k_2$$

$$f(t) = (3e^{-4t} - 2e^{-6t})u(t)$$

so

$$F(s) = \frac{3}{s+4} - \frac{2}{s+6}$$

$$f(t) = (3e^{-4t} - 2e^{-6t})u(t)$$

B.

$$F(s) = \frac{24}{(s+2)(s+8)}$$

for

$s = -2$

$$\frac{24}{s+8} = 6 = k_1$$

for

$s = -8$

$$\frac{24}{s+2} = -4 = k_2$$

$$f(t) = (4e^{-2t} - 4e^{-8t})u(t)$$

so

$$F(s) = \frac{4}{s+2} - \frac{4}{s+8}$$

$$f(t) = (4e^{-2t} - 4e^{-8t})u(t)$$

Problem 12.13

Given the following functions F(s), find f(t) if

$$F(s) = \frac{s+1}{s(s+2)(s+3)}$$

$$F(s) = \frac{s^2+s+1}{s(s+2)(s+1)}$$

Suggested Solution

A.

$$F(s) = \frac{s+1}{s(s+2)(s+3)}$$

$$for\ \ s=0,\ \frac{s+1}{(s+2)(s+3)} = 1/6 = k_1$$

$$for\ \ s=-2,\ \frac{s+1}{s(s+3)} = \frac{1}{2} = k_2$$

$$for\ \ s=-3,\ \frac{s+1}{s(s+2)} = \frac{-2}{3} = k_3$$

$$so\ \ F(s) = \frac{1/6}{s} + \frac{1/2}{s+2} - \frac{2/3}{s+3}$$

$$and\quad f(t) = \left(\frac{1}{6} + \frac{1}{2}e^{-2t} - \frac{2}{3}e^{-3t}\right)u(t)$$

$$f(t) = \left(\frac{1}{6} + \frac{1}{2}e^{-2t} - \frac{2}{3}e^{-3t}\right)u(t)$$

B.

$$F(s) = \frac{s^2+s+1}{s(s+2)(s+1)}$$

$$for\ \ s=0,\ \frac{s^2+s+1}{(s+2)(s+3)} = 1/2 = k_1$$

$$for\ \ s=-1,\ \frac{s^2+s+1}{s(s+2)} = -1 = k_2$$

$$for\ \ s=-2,\ \frac{s^2+s+1}{s(s+2)} = \frac{3}{2} = k_3$$

$$so\ \ F(s) = \frac{1/2}{s} + \frac{-1}{s+1} + \frac{3/2}{s+2}$$

$$and\quad f(t) = (\frac{1}{2} + e^{-t} + \frac{3}{2}e^{-2t})u(t)$$

$$f(t) = (\frac{1}{2} + e^{-t} + \frac{3}{2}e^{-2t})u(t)$$

Problem 12.14

Given the following functions F(s), find f(t).

$$F(s) = \frac{s^2 + 5s + 4}{(s+2)(s+4)(s+6)}$$

$$F(s) = \frac{(s+3)(s+6)}{s(s^2 + 8s + 12)}$$

Suggested Solution

A.

$$F(s) = \frac{s^2 + 5s + 4}{(s+2)(s+4)(s+6)}$$

$$F(s) = \frac{(s+1)(s+4)}{(s+2)(s+4)(s+6)} = \frac{(s+1)}{(s+2)(s+6)} = \frac{A}{s+2} + \frac{B}{s+6}$$

$$A = F(s)\,(s+2)\Big|_{s=-2} = \frac{-1}{4}$$

$$B = F(s)\,(s+6)\Big|_{s=-6} = \frac{5}{4}$$

$$F(s) = \frac{5/4}{s+6} - \frac{1/4}{s+2}$$

$$f(t) = (\frac{5}{4} + e^{-6t} - \frac{1}{4}e^{-2t})u(t)$$

$$f(t) = (\frac{5}{4} + e^{-6t} - \frac{1}{4}e^{-2t})u(t)$$

B.

$$F(s) = \frac{(s+3)(s+6)}{s(s^2 + 8s + 12)}$$

$$F(s) = \frac{(s+3)(s+6)}{s(s+2)(s+6)} = \frac{(s+3)}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}$$

$$A = F(s)\,s\Big|_{s=0} = \frac{3}{2}$$

$$B = F(s)\,(s+2)\Big|_{s=-2} = \frac{-1}{2}$$

$$F(s) = \frac{3/2}{s} - \frac{1/2}{s+2} + \frac{3/2}{s+2}$$

$$f(t) = (\frac{3}{2} - \frac{1}{2}e^{-2t})u(t)$$

$$f(t) = (\frac{3}{2} - \frac{1}{2}e^{-2t})u(t)$$

Problem 12.15

Given the following functions F(s), find f(t).

$$F(s) = \frac{s^2 + 7s + 12}{(s+2)(s+4)(s+6)}$$

$$F(s) = \frac{(s+3)(s+6)}{s(s^2 + 10s + 24)}$$

Suggested Solution

A.

$$F(s) = \frac{s^2 + 7s + 12}{(s+2)(s+4)(s+6)}$$

$$F(s) = \frac{(s+3)(s+4)}{(s+2)(s+4)(s+6)} = \frac{(s+3)}{(s+6)(s+2)} = \frac{A}{s+6} + \frac{B}{s+2}$$

$$F(s) = \frac{1/4}{s+2} + \frac{3/4}{s+6}$$

$$f(t) = (\frac{1}{4}e^{-2t} + \frac{3}{4}e^{-6t})u(t)$$

$$f(t) = (\frac{1}{4}e^{-2t} + \frac{3}{4}e^{-6t})u(t)$$

B.

$$F(s) = \frac{(s+3)(s+6)}{s(s^2 + 10s + 24)}$$

$$F(s) = \frac{(s+3)(s+6)}{(s)(s+4)(s+6)} = \frac{(s+3)}{(s+6)(s+2)} = \frac{A}{s} + \frac{B}{s+4}$$

$$F(s) = \frac{3/4}{s} + \frac{1/4}{s+4}$$

$$f(t) = (\frac{3}{4} + \frac{1}{4}e^{-4t})u(t)$$

$$f(t) = (\frac{3}{4} + \frac{1}{4}e^{-4t})u(t)$$

Problem 12.16

Given the following functions F(s) find f(t)

$$F(s) = \frac{s2 + 7s + 12}{(s+2)(s+4)(s+6)}$$

$$F(s) = \frac{10(s+2)}{(s^2 + 4s + 5)}$$

Suggested Solution

A.

$$F(s) = \frac{s2 + 7s + 12}{(s+2)(s+4)(s+6)}$$

Matlab code

EDU» syms s t

EDU» ilaplace((s^2+7*s+12)/(s+2)/(s+4)/(s+6))

$$f(t) = (\frac{3}{4}e^{-6t} + \frac{1}{4}e^{-2t})$$

ans =

3/4*exp(-6*t)+1/4*exp(-2*t)

or

$$f(t) = (\frac{3}{4}e^{-6t} + \frac{1}{4}e^{-2t})$$

B.

$$F(s) = \frac{10(s+2)}{(s^2 + 4s + 5)}$$

Matlab code

ilaplace((s+3)*(s+6)/(s*(s^2+10*s+24)))

ans =

1/4*exp(-4*t)+3/4

or

$$f(t) = (\frac{3}{4} + \frac{1}{4}e^{-4t})$$

$$f(t) = (\frac{3}{4} + \frac{1}{4}e^{-4t})$$

Problem 12.17

Given the following functions F(s) find f(t)

$$F(s)=\frac{10}{(s^2+2s+2)}$$

$$F(s)=\frac{10(s+2)}{(s^2+4s+5)}$$

Suggested Solution

A.

$$F(s)=\frac{10}{(s^2+2s+2)}$$

$$F(s)=\frac{10}{s^2+2s+2}=\frac{k_1}{s+1-\alpha}+\frac{k_1}{s+1+\alpha}$$

$$f(\alpha)=\frac{-5\alpha}{s+1-\alpha}+\frac{5\alpha}{s+1+\alpha}$$

$$f(\alpha)=10e^{-t}\cos(t-90^o)u(t)$$

$$\boxed{f(\alpha)=10e^{-t}\cos(t-90^o)u(t)}$$

B.

$$F(s)=\frac{10(s+2)}{(s^2+4s+5)}$$

$$F(s)=\frac{10(s+2)}{s^2+4s+5}=\frac{k_1}{s+2-\alpha}+\frac{k_1}{s+2+\alpha}$$

so

$$f(\alpha)=\frac{5}{s+2-\alpha}+\frac{5}{s+2+\alpha}$$

$$f(\alpha)=10e^{-2t}\cos(t)u(t)$$

$$\boxed{f(\alpha)=10e^{-2t}\cos(t)u(t)}$$

Problem 12.18

Given the following functions F(s) find inverse Laplace functions.

$$F(s) = \frac{10}{(s^2 + 2s + 2)}$$

$$F(s) = \frac{10(s+2)}{(s^2 + 4s + 5)}$$

Suggested Solution

A

$$F(s) = \frac{10}{(s^2 + 2s + 2)}$$

$$F(s) = \frac{10(s+2)}{s^2 + 4s + 5} = \frac{k_1}{s+2-\alpha} + \frac{k_1}{s+2+\alpha}$$

$$for$$
$$s = -1 + \alpha$$
$$\frac{10(s+1)}{s+1+\alpha} = 5 = k_1$$
$$so$$

$$F(s) = \frac{5}{s+1-\alpha} + \frac{5}{s+1+\alpha}$$
$$f(t) = 10e^{-6}\cos tu(t)$$

$$f(t) = 10e^{-6}\cos tu(t)$$

B

$$F(s) = \frac{10(s+2)}{(s^2 + 4s + 5)}$$

$$F(s) = \frac{s+1}{s(s^2 + 4s + 5)} = \frac{k_1}{s} + \frac{k_2}{s+2-\alpha} + \frac{k_3}{s+2+\alpha}$$

$$for$$
$$s = 0$$
$$\frac{s+1}{s^2+4s+5} = 1/5 = k_1$$
$$for$$
$$s = -2+\alpha$$
$$\frac{s+1}{s(s+2+\alpha)} = 0.31\angle -108.43^o = k_2$$

$$F(s) = \frac{1/5}{s} + \frac{0.31\angle -108.43^o}{s+2+\alpha} + \frac{0.31\angle 108.43^o}{s+2+\alpha}$$
$$f(t) = (\frac{1}{5} + 0.62e^{-2t}\cos(t-108.43^o))u(t)$$

$$f(t) = (\frac{1}{5} + 0.62e^{-2t}\cos(t-108.43^o))u(t)$$

Problem 12.19

Given the following functions F(s), find f(t)

$$F(s) = \frac{s(s+6)}{(s+3)(s^2+6s+18)}$$

$$F(s) = \frac{(s+4)(s+8)}{s(s^2+8s+32)}$$

Suggested Solution

A.

$$F(s) = \frac{s(s+6)}{(s+3)(s^2+6s+18)}$$

$$F(s) = \frac{s(s+6)}{(s+3)(s^2+6s+18)} = \frac{s(s+6)}{(s+3)(s+3+3\alpha)(s+3-3\alpha)}$$

$$= \frac{k_1}{s+3} + \frac{k_2}{s+3+3\alpha} + \frac{k_3}{s+3-3\alpha}$$

at

$s = -3$

$$\frac{s(s+6)}{(s+3)(s^2+6s+18)} = -1 = k_1$$

at

$s = -3+3\alpha$

$$\frac{s(s+6)}{(s+3)(s^2+6s+18)} = 1 = k_2$$

so

$$F(s) = \frac{-1}{s+3} + \frac{1}{s+3+3\alpha} + \frac{1}{s+3-3\alpha}$$

$$f(t) = (e^{-3t} + 2e^{-3t}\cos 3t)u(t))$$

$$f(t) = (e^{-3t} + 2e^{-3t}\cos 3t)u(t))$$

B.

$$F(s) = \frac{(s+4)(s+8)}{s(s^2+8s+32)}$$

$$F(s)=\frac{(s+4)(s+8)}{(s)(s^2+8s+32)}=\frac{s(s+6)}{(s+3)(s+3+3\alpha)(s+3-3\alpha)}$$

$$=\frac{k_1}{s}+\frac{k_2}{s+4+4\alpha}+\frac{k_2}{s+4-4\alpha}$$

at

$s=0$

$$\frac{(s+4)(s+8)}{(s)(s^2+8s+32)}=1=k_1$$

at

$s=-4+4\alpha$

$$\frac{(s+4)(s+8)}{(s)(s^2+8s+32)}=\frac{-1}{2}\alpha=k_2$$

so

$$F(s)=\frac{1}{s}+\frac{1/2}{s+4+4\alpha}+\frac{\alpha/2}{s+4-4\alpha}$$

$$f(t)=(1+e^{-4t}\cos(4t-90^o)u(t))$$

$$f(t)=(1+e^{-4t}\cos(4t-90^o)u(t))$$

Problem 12.20

Given the following functions F(s) find f(t)

$$F(s) = \frac{6s+12}{(s^2+4s+5)(s^2+4s+8)}$$

$$F(s) = \frac{s(s+2)}{s^2+2s+2}$$

Suggested Solution

$$F(s) = \frac{6s+12}{((s+2)^2+1^2)((s+2)^2+2^2)}$$

$$= \frac{k_1\angle\theta_1}{s+2-j1} + \frac{k_1\angle\theta_1}{s+2+j1} + \frac{k_2\angle\theta_2}{s+2-j2} + \frac{k_2\angle\theta_2}{s+2+j2}$$

$$k_1\angle\theta_1 = F(s)(s+2-j1)\,|_{s=-2+j1} = 1\angle 0^o$$

$$k_1\angle\theta_1 = F(s)(s+2-j2)\,|_{s=-2-j2} = -1\angle 0^o$$

$$f(t) = [2e^{-2t}\cos(t) - 2e^{-2t}\cos(2t)]u(t)$$

$$f(t) = [2e^{-2t}\cos(t) - 2e^{-2t}\cos(2t)]u(t)$$

$$F(s) = \frac{s2+2s+2-2}{(s^2+2s+2)} = 1 - \frac{2}{(s^2+2s+2)}$$

$$F(s) = 1 - \frac{2}{(s+1)^2+(1)^2}$$

$$f(t) = [\delta(t) - 2e^{-t}\sin(t)]u(t)$$

$$f(t) = [\delta(t) - 2e^{-t}\sin(t)]u(t)$$

Problem 12.21

Use Matlab to solve Problem 12.19

Suggested Solution

$$F(s)=\frac{s(s+6)}{(s+3)(s^2+6s+18)}$$

Matlab Code

```
>> syms s t
>> ilaplace(s*(s+6)/((s+3)*(s^2+6*s+18)))

ans =

-exp(-3*t)+2*exp(-3*t)*cos(3*t)

>>
```

$$f(t)=2e^{-3t}\cos(3t)-e^{-3t}$$

$$f(t)=2e^{-3t}\cos(3t)-e^{-3t}$$

$$F(s)=\frac{(s+3)(s+8)}{s(s^2+8s+32)}$$

Matlab Code

```
>> syms s t
>> ilaplace((s+4)*(s+8)/(s*(s^2+8*s+32)))

ans =

1+exp(-4*t)*sin(4*t)

>>
```

$$f(t)=1+e^{-4t}\sin(4t)$$

$$f(t)=1+e^{-4t}\sin(4t)$$

Problem 12.22

Given following functions F(s), find f(t)

$$F(s) = \frac{s+1}{s^2(s+2)}$$

$$F(s) = \frac{s+3}{(s+1)^2(s+4)}$$

Suggested Solution

A.

$$F(s) = \frac{s+1}{s^2(s+4)} = \frac{k_1}{s^2} + \frac{k_2}{s} + \frac{k_3}{s+2}$$

at

$s = 0$

$$\frac{s+1}{s+2} = k_1 = \frac{1}{2}$$

$$\frac{d}{ds}[\frac{s+1}{(s+2)}] = k_2 = \frac{1}{4}$$

$$f(t) = (\frac{1}{4} + \frac{1}{2}t - \frac{1}{4}e^{-2t})u(t)$$

at

$s = -2$

$$\frac{s+1}{(s)^2} = k_3 = \frac{-1}{4}$$

so

$$F(s) = \frac{1/2}{s^2} + \frac{1/4}{s} - \frac{1/4}{s+4}$$

$$f(t) = (\frac{1}{4} + \frac{1}{2}t - \frac{1}{4}e^{-2t})u(t)$$

B.

$$F(s) = \frac{(s+4)(s+8)}{s(s^2+8s+32)}$$

$$f(t) = 1 + e^{-4t}\sin(4t)$$

$$F(s) = \frac{s+3}{(s+1)^2(s+4)} = \frac{k_1}{(s+1)^2} + \frac{k_2}{s+1} + \frac{k_3}{s+4}$$

at

$s = -1$

$$\frac{s+3}{s+4} = k_1 = \frac{2}{3}$$

$$\frac{d}{ds}[\frac{s+3}{(s+4)}] = k_2 = \frac{1}{9}$$

at

$s = -4$

$$\frac{s+3}{(s+1)^2} = k_3 = \frac{1}{9}$$

so

$$F(s) = \frac{2/3}{(s+1)^2} + \frac{1/9}{s+1} - \frac{1/9}{s+4}$$

$$f(t) = (\frac{1}{9}e^{-t} + \frac{2}{3}te^{-t} - \frac{1}{9}e^{-4t})u(t)$$

$$f(t) = (\frac{1}{9}e^{-t} + \frac{2}{3}te^{-t} - \frac{1}{9}e^{-4t})u(t)$$

Problem 12.23

Given the following functions F(s), find f(t)

$$F(s) = \frac{s+8}{s^2(s+6)}$$

$$F(s) = \frac{1}{s^2(s+1)^2}$$

Suggested Solution

A.

$$F(s) = \frac{s+8}{s^2(s+6)} = \frac{k_1}{s^2} + \frac{k_2}{s} + \frac{k_3}{s+6}$$

at

$s = 0$

$$\frac{s+8}{s+6} = k_1 = \frac{4}{3}$$

$$\frac{d}{ds}[\frac{s+8}{(s+6)}] = k_2 = \frac{-1}{18}$$

at

$s = -6$

$$\frac{s+8}{(s)^2} = k_3 = \frac{1}{18}$$

so

$$F(s) = \frac{4/3}{s^2} - \frac{1/18}{s} + \frac{1/18}{s+6}$$

$$f(t) = (\frac{4}{3}t - \frac{1}{18} + \frac{1}{18}e^{-6t})u(t)$$

$$f(t) = (\frac{4}{3}t - \frac{1}{18} + \frac{1}{18}e^{-6t})u(t)$$

B.

$$F(s) = \frac{s+8}{s^2(s+1)^2} = \frac{k_1}{s^2} + \frac{k_2}{s} + \frac{k_3}{(s+1)^2} + \frac{k_4}{s+1}$$

at

$s = 0$

$$\frac{1}{(s+1)^2} = k_1 = 1$$

$$\frac{d}{ds}\left[\frac{1}{(s+1)^2}\right] = k_2 = -2$$

at

$s = -1$

$$\frac{1}{(s)^2} = k_3 = \frac{1}{18}$$

$$\frac{d}{ds}\left[\frac{1}{(s)^2}\right] = k_2 = 1$$

so

$$F(s) = \frac{1}{s^2} - \frac{2}{s} - \frac{1}{(s+1)^2} + \frac{1}{s+1}$$

$$f(t) = (t - 2 + te^{-t} + 2e^{-t})u(t)$$

$$f(t) = (t - 2 + te^{-t} + 2e^{-t})u(t)$$

Problem 12.24

Given the following functions F(s), find f(t)

$$F(s) = \frac{s+4}{(s+2)^2}$$

$$F(s) = \frac{s+6}{s(s+1)^2}$$

Suggested Solution

$$F(s) = \frac{s+4}{(s+2)^2} = \frac{k_1}{(s+2)^2} + \frac{k_2}{s+2}$$

at

$s = -2$

$s + 2 = 2 = k_1$

$\frac{d}{ds}[s+2] = k_2 = 1$

$$f(t) = (2te^{-2t} + e^{-2t})u(t)$$

so

$$F(s) = \frac{2}{(s+2)^2} + \frac{1}{s+2}$$

$f(t) = (2te^{-2t} + e^{-2t})u(t)$

B.

$$F(s) = \frac{s+6}{s(s+1)^2} = \frac{k_1}{s} + \frac{k_2}{(s+1)^2} + \frac{k_3}{s+1}$$

at

$s = 0$

$\frac{s+6}{(s+1)^2} = k_1 = 6$

at

$s = -1$

$\frac{s+6}{s} = k_2 = -5$

$$f(t) = (6 - 5te^{-t} - te^{-t})u(t)$$

$\frac{d}{ds}[\frac{s+6}{s}] = k_3 = -6$

so

$$F(s) = \frac{6}{s} - \frac{5}{(s+1)^2} - \frac{6}{s+1}$$

$f(t) = (6 - 5te^{-t} - te^{-t})u(t)$

Problem 12.25

Given the following functions F(s) find f(t)

$$F(s) = \frac{s^2}{(s+1)^2(s+2)}$$

$$F(s) = \frac{s^2 + 9s - 20}{s(s+4)^3(s+5)}$$

Suggested Solution

A.

$$F(s) = \frac{s^2}{(s+1)^2(s+2)} = \frac{k_1}{(s+1)^2} + \frac{k_2}{s+1} + \frac{k_3}{s+2}$$

at

$s = -1$

$$\frac{s^2}{(s+2)} = k_1 = 1$$

at

$s = -2$

$$\frac{s^2}{(s+1)^2} = k_3 = 4$$

$$\frac{d}{ds}\left[\frac{s^2}{(s+1)^2}\right] = k_2 = -3$$

so

$$F(s) = \frac{1}{(s+1)^2} - \frac{3}{s+1} + \frac{4}{s+2}$$

$$f(t) = (te^{-t} - 3e^{-t} + 4te^{-2t})u(t)$$

$$f(t) = (te^{-t} - 3e^{-t} + 4te^{-2t})u(t)$$

B.

$$F(s)=\frac{s^2+9s-20}{s(s+4)^3(s+5)}=\frac{1}{s(s+4)^2}=\frac{k_1}{s}+\frac{k_2}{(s+4)^2}+\frac{k_3}{s+4}$$

at

$s=0$

$$\frac{1}{(s+4)^2}=k_1=1/10$$

at

$s=-4$

$$\frac{1}{s}=k_2=-1/4$$

$$\frac{d}{ds}[\frac{1}{s}]=k_3=-1/16$$

so

$$F(s)=\frac{1/16}{s}-\frac{1/4}{(s+4)^2}-\frac{1/16}{s+4}$$

$$f(t)=(\frac{1}{16}-\frac{t}{4}e^{-4t}-\frac{1}{16}e^{-4t})u(t)$$

$$f(t)=(\frac{1}{16}-\frac{t}{4}e^{-4t}-\frac{1}{16}e^{-4t})u(t)$$

Problem 12.26

Find f(t) if F(s) is given by expression

$$F(s) = \frac{s(s+1)}{(s+2)^3(s+3)}$$

Suggested Solution

$$F(s) = \frac{s(s+1)}{(s+2)^3(s+3)} = \frac{A}{s+3} + \frac{B}{(s+3)^3} + \frac{C}{(s+3)^2} + \frac{D}{s+2}$$

$$A = F(s)(s+3)|_{s=-3} = -6$$
$$B = F(s)(s+2)^2|_{s=-2} = 2$$

$$F(0) = 0 = \frac{-6}{3} + \frac{2}{8} + \frac{C}{4} + \frac{D}{2} \Rightarrow C + 2D = 7$$

$$f(t) = (t2e^{-2t} - 5te^{-2t} + 6e^{-2t} - 6e^{-3t})u(t)$$

$$F(-1) = 0 = \frac{-6}{2} + 2 + C + D \Rightarrow C + D = 1$$

$\therefore$

$$D = 6, C = -5$$

$$F(s) = \frac{-6}{s+3} + \frac{2}{(s+3)^3} - \frac{5}{(s+3)^2} + \frac{6}{s+2}$$

$$f(t) = (t2e^{-2t} - 5te^{-2t} + 6e^{-2t} - 6e^{-3t})u(t)$$

Problem 12.27

Find f(t) if F(s) is given by

$$F(s) = \frac{12(s+2)}{s^2(s+1)(s^2+4s+8)}$$

Suggested Solution

$$F(s) = \frac{k_1}{s^2} + \frac{k_2}{s} + \frac{k_3}{s+1} + \frac{k4}{s+2-2\alpha} + \frac{k4}{s+2+2\alpha}$$

at

$s = 0$

$$\frac{12(s+2)}{(s+1)(s^2+4s+8)} = 3 = k_1$$

$$\frac{d}{ds}\left[\frac{12(s+2)}{(s+1)(s^2+4s+8)}\right] = k_2 = -3$$

at

$s = -1$

$$\frac{12(s+2)}{(s^2)(s^2+4s+8)} = k_3 = \frac{12}{5}$$

at

$s = -2 + 2\alpha$

$$\frac{12(s+2)}{s^2(s+1)} = k_4 = \frac{1}{3}\angle -26.56^o$$

so

$$F(s) = \frac{3}{s^2} - \frac{3}{s} + \frac{12/5}{s+1} + \frac{\frac{1}{3}\angle -26.56^o}{s+2-2\alpha} + \frac{\frac{1}{3}\angle 26.56^o}{s+2+2\alpha}$$

$$f(t) = [-3 + 3t + \frac{12}{5}e^{-t} + \frac{2}{3}e^{-2t}\cos(2t - 26.56^o)]u(t)$$

$$f(t) = [-3 + 3t + \frac{12}{5}e^{-t} + \frac{2}{3}e^{-2t}\cos(2t - 26.56^o)]u(t)$$

Problem 12.28

Use Matlab to solve Problem 12.25

Suggested Solution

$$F(s) = \frac{s^2}{(s+1)^2(s+2)}$$

Matlab Code

```
>> syms s t
>>
>> ilaplace(s^2/((s+1)^2*(s+2)))

ans =

(-3+t)*exp(-t)+4*exp(-2*t)

>>
```

$f(t) = te^{-t} - 3e^{-t} + 4e^{-2t}$

$$f(t) = te^{-t} - 3e^{-t} + 4e^{-2t}$$

$$F(s) = \frac{s2 + 9s + 20}{s(s+4)^3(s+5)}$$

Matlab Code

```
>> syms s t
>> ilaplace((s^2+9*s+20)/(s*(s+4)^3*(s+5)))

ans =

-1/4*t*exp(-4*t)-1/16*exp(-4*t)+1/16

>>
```

$f(t) = 0.0625\{1 - e^{-4t} - 4te^{-4t}\}$

$$f(t) = 0.0625\{1 - e^{-4t} - 4te^{-4t}\}$$

Problem 12.29

Find the inverse Laplace transform of the following functions

$$F(s)=\frac{e^{-s}}{s+1}$$

$$F(s)=\frac{1-e^{-2s}}{s}$$

$$F(s)=\frac{1-e^{-s}}{s+2}$$

Suggested Solution

a.

$$F(s)=\frac{e^{-s}}{s+1}$$

let

$$G(s)=\frac{1}{s+1}\Rightarrow g(t)=e^{-t}u(t)$$

$$f(t)=g(t-1)\Rightarrow f(t)=e^{-(t-1)}u(t-1)$$

b.

$$F(s)=\frac{1-e^{-2s}}{s}$$

let

$$G(s)=\frac{1}{s}\Rightarrow g(t)=u(t)$$

$$f(t)=g(t)-g(t-2)\Rightarrow f(t)=u(t)-u(t-2)$$

c.

$$F(s)=\frac{1-e^{-s}}{s+2}$$

let

$$G(s)=\frac{1}{s+2}\Rightarrow g(t)=e^{-2t}u(t)$$

$$f(t)=g(t)-g(t-1)\Rightarrow f(t)=e^{-2t}u(t)-e^{-2(t-1)}u(t-1)$$

$$f(t)=e^{-(t-1)}u(t-1)$$

$$f(t)=u(t)-u(t-2)$$

$$f(t)=e^{-2t}u(t)-e^{-2(t-1)}u(t-1)$$

Problem 12.30

Find the inverse Laplace transform of the following function

$$F(s) = \frac{(s+1)e^{-s}}{s(s+2)}$$

$$F(s) = \frac{10e^{-2s}}{(s+1)(s+3)}$$

Suggested Solution

$$F(s) = \frac{(s+1)e^{-s}}{s(s+2)} = e^{-s}[\frac{k_1}{s} + \frac{k_2}{s+2}]$$

at

$s = 0$

$$\frac{s+1}{s+2} = k_1 = 1/2$$

at

$s = -2$

$$\frac{s+1}{s} = 1/2 = k_2$$

$$F(s) = e^{-s}[\frac{1/2}{s} + \frac{1/2}{s+2}]$$

$$f(t) = \frac{1}{2}u(t-1) + \frac{1}{2}e^{-2(t-1)}u(t-1)$$

$$F(s) = \frac{10e^{-2s}}{(s+1)(s+3)} = e^{-2s}[\frac{k_1}{s+1} + \frac{k_2}{s+3}]$$

at

$s = -1$

$$\frac{10}{s+3} = k_1 = 5$$

at

$s = -3$

$$\frac{10}{s+1} = k_2 = -5$$

$$F(s) = e^{-2s}[\frac{5}{s+1} - \frac{5}{s+3}]$$

$$f(t) = [5e^{-(t-2)} - 5e^{-3(t-2)}]u(t-2)$$

$$f(t) = \frac{1}{2}u(t-1) + \frac{1}{2}e^{-2(t-1)}u(t-1)$$

$$f(t) = [5e^{-(t-2)} - 5e^{-3(t-2)}]u(t-2)$$

Problem 12.31

Find the inverse Laplace transform f(t) if F(s) is

$$F(s) = \frac{se^{-s}}{(s+1)(s+2)}$$

Suggested Solution

$$F(s) = \frac{se^{-s}}{(s+1)(s+2)}$$

let

$$G(s) = \frac{s}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$A = G(s)(s+1)|_{s=-1} = -1$$

$$B = G(s)(s+2)|_{s=-2} = 2$$

$$G(s) = \frac{2}{s+2} - \frac{1}{s+1} \Rightarrow g(t) = [2e^{-2t} - e^{-t}]u(t)$$

$$f(t) = g(t-1)$$

$$f(t) = [2e^{-2(t-1)} - e^{-(t-1)}]u(t-1)$$

$$f(t) = [2e^{-2(t-1)} - e^{-(t-1)}]u(t-1)$$

Problem 12.32

Find f(t) if F(s) is given by the following functions

$F(s) = e^{-2s}[\frac{s^2+2s+3}{s(s+1)(s+2)}]$ $F(s) = \frac{(s+2)e^{-4s}}{s^2(s+1)}$

Suggested Solution

A.

$$F(s) = e^{-2s}[\frac{s^2+2s+3}{s(s+1)(s+2)}] = e^{-2s}[\frac{k_1}{s} + \frac{k_2}{(s+1)} + \frac{k_3}{(s+2)}]$$

at

$s = 0$

$$\frac{s^2+2s+3}{(s+1)(s+2)} = k_1 = \frac{3}{2}$$

at

$s = -1$

$$\frac{s^2+2s+3}{s(s+2)} = k_2 = -2$$

at

$s = -2$

$$\frac{s^2+2s+3}{s(s+1)} = k_3 = \frac{3}{2}$$

so

$$F(s) = e^{-2s}[\frac{3/2}{s} + \frac{-2}{(s+1)} + \frac{3/2}{(s+2)}]$$

$$f(t) = [\frac{3}{2} - 2e^{-(t-2)} + \frac{3}{2}e^{-2(t-2)}]u(t-2)$$

B.

$$F(s) = \frac{(s+2)e^{-4s}}{s^2(s+1)} = e^{-4s}[\frac{k_1}{s^2} + \frac{k_2}{s} + \frac{k_3}{(s+1)}]$$

at

$s = 0$

$$\frac{s+2}{s+1} = k_1 = 2$$

$$\frac{d}{ds}[\frac{s+2}{s+1}] = -1 = k_2$$

at

$s = -1$

$$\frac{s+2}{s+1} = k_3 = 1$$

so

$$F(s) = e^{-4s}[\frac{2}{s^2} - \frac{1}{s} + \frac{1}{(s+1)}]$$

$$f(t) = [2(t-4) - 1 + e^{-(t-4)}]u(t-4)$$

$$f(t) = [\frac{3}{2} - 2e^{-(t-2)} + \frac{3}{2}e^{-2(t-2)}]u(t-2)$$

$$f(t) = [2(t-4) - 1 + e^{-(t-4)}]u(t-4)$$

Problem 12.33

Find f(t) if F(s) is given by following functions

$$F(s) = e^{-s}[\frac{2(s+1)}{(s+3)(s+2)}] \qquad F(s) = \frac{10(s+2)e^{-2s}}{(s+4)(s+1)}$$

Suggested Solution

A.

$$F(s) = e^{-s}[\frac{2(s+1)}{(s+3)(s+2)}] = e^{-s}[\frac{k_1}{s+2} + \frac{k_2}{s+3}]$$

at

$s = -2$

$$\frac{2(s+1)}{(s+3)} = k_1 = -2$$

at

$s = -3$

$$\frac{2(s+1)}{(s+2)} = k_2 = 4$$

so

$$F(s) = e^{-2s}[\frac{-2}{s+2} + \frac{4}{s+3}]$$

$$f(t) = [-2e^{-(t-1)} + 4e^{-3(t-1)}]u(t-1)$$

B

$$F(s) = \frac{10(s+2)e^{-2s}}{(s+4)(s+1)} = e^{-2s}[\frac{k_1}{s+1} + \frac{k_2}{s+4}]$$

at

$s = -1$

$$\frac{10(s+2)}{(s+4)} = k_1 = \frac{10}{3}$$

at

$s = -4$

$$\frac{10(s+2)}{(s+1)} = k_2 = 4$$

so

$$F(s) = e^{-2s}[\frac{10/3}{s+1} + \frac{4}{s+4}]$$

$$f(t) = [\frac{10}{3}e^{-(t-2)} + \frac{20}{3}e^{-4(t-2)}]u(t-2)$$

$$f(t) = [-2e^{-(t-1)} + 4e^{-3(t-1)}]u(t-1)$$

$$f(t) = [\frac{10}{3}e^{-(t-2)} + \frac{20}{3}e^{-4(t-2)}]u(t-2)$$

Problem 12.34

Solve the following differential equations using Laplace Transform

$$\frac{dx(t)}{dt} + 3x(t) = e^{-2t}$$

$$\frac{dx(t)}{dt} + 3x(t) = 2u(t)$$

Suggested Solution

A.

$$\frac{dx(t)}{dt} + 3x(t) = e^{-2t}, x(0) = 1$$

$$\frac{dx(t)}{dt} \Leftrightarrow sX(s) - x(0) = \frac{1}{s+2} \Rightarrow X(s)(s+3) = \frac{1}{s+2} + 1 = \frac{s+3}{s+2}$$

$$X(s) = \frac{1}{s+2} \Rightarrow x(t) = e^{-2t}u(t)$$

B.

$$\frac{dx(t)}{dt} + 3x(t) = 2u(t), x(0) = 2$$

$$\frac{dx(t)}{dt} \Leftrightarrow sX(s) - 2 + 3X(s) \Rightarrow X(s)(s+3) = \frac{2}{s} + 2 = \frac{2(s+1)}{s}$$

$$x(t) = \frac{2}{3}[1 + 2e^{-3t}]u(t)$$

$$x(t) = e^{-2t}u(t)$$

$$x(t) = \frac{2}{3}[1 + 2e^{-3t}]u(t)$$

Problem 12.35

Solve the following differential equations using Laplace transform

A. $\frac{d^2y(t)}{dt^2}+\frac{2dy(t)}{dt}+y(t)=e^{-2t}, y(0)=y'(0)=0$

B. $\frac{d^2y(t)}{dt^2}+\frac{4dy(t)}{dt}+4y(t)=u(t), y(0)=0; y'(0)=1$

Suggested Solution

A.

$$\frac{d^2y(t)}{dt^2}+\frac{2dy(t)}{dt}+y(t)=e^{-2t}, y(0)=y'(0)=0$$

$$s^2Y(s)+2sY(s)+Y(s)=Y(s)[s^2+2s+1]=\frac{1}{s+2}$$

$$Y(s)=\frac{1}{(s+1)^2(s+2)}=\frac{A}{(s+1)^2}+\frac{B}{(s+1)}+\frac{C}{(s+2)}; C=Y(s)(s+2)|_{s=-2}=1$$

$$A=Y(s)(s+1)^2|_{s=-1}=1$$

$$Y(0)=\frac{1}{2}=1+B+\frac{1}{2}\Rightarrow B=-1$$

$$Y(s)=\frac{1}{(s+1)^2}-\frac{1}{(s+1)}+\frac{1}{(s+2)}$$

$$y(t)=(te^{-t}-e^{-t}+e^{-2t})u(t)$$

B.

$$\frac{d^2y(t)}{dt^2}+\frac{4dy(t)}{dt}+4y(t)=u(t), y(0)=0; y'(0)=1$$

$$s^2Y(s)-sy'(0)-y(0)+4sY(s)-4y(0)+4Y(s)=\frac{1}{s}$$

$$Y(s)[s^2+4s+4]=\frac{1}{s}+s=\frac{s^2+1}{s}$$

$$Y(s)=\frac{s^2+1}{s(s+2)^2}=\frac{A}{s}+\frac{B}{(s+2)^2}+\frac{C}{s+2}$$

$$A=Y(s)s|_{s=0}=\frac{1}{4}$$

$$B=Y(s)(s+2)^2|_{s=-2}=-2.5$$

$$Y(-1)=-2=-\frac{1}{4}-\frac{5}{2}+C\Rightarrow C=\frac{3}{4}$$

$$Y(s)=\frac{1}{4}[\frac{1}{s}-\frac{10}{(s+2)^2}+\frac{3}{s+2}]$$

$$y(t)=\frac{1}{4}[1-10te^{-2t}+3e^{-2t}]u(t)$$

$$y(t)=(te^{-t}-e^{-t}+e^{-2t})u(t)$$

$$y(t) = \frac{1}{4}[1 - 10te^{-2t} + 3e^{-2t}]u(t)$$

Problem 12.36

Use Laplace transform to find y(t) if

$$\frac{dy(t)}{dt} + 5y(t) + 4\int_0^t y(x)dx = u(t), y(0) = 0, t > 0$$

Suggested Solution

In Laplace terms

$$sY(s) + 5Y(s) + \frac{4}{5}Y(s) = \frac{1}{s}$$

$$Y(s)[s^2 + 5s + 4] = 1$$

$$so$$

$$Y(s) = \frac{1}{s^2 + 5s + 4} = \frac{1}{(s+1)(s+4)} = \frac{k_1}{s+1} + \frac{k_2}{s+4} \Rightarrow k_1 = 1/3; k_2 = -1/3$$

$$so$$

$$Y(s) = \frac{1/3}{s+1} - \frac{1/3}{s+4}$$

$$y(t) = [\frac{1}{3}e^{-t} - \frac{1}{3}e^{-4t}]u(t)$$

$$y(t) = [\frac{1}{3}e^{-t} - \frac{1}{3}e^{-4t}]u(t)$$

Problem 12.37

Solve the integrodifferential equations using Laplace transforms.

$$\frac{dy(t)}{dt} + 2y(t) + \int_0^t y(\lambda)d\lambda = 1 - e^{-2t}, y(0) = 0, t > 0$$

Suggested Solution

In Laplace terms

$$sY(s) + 2Y(s) + \frac{1}{s}Y(s) = \frac{1}{s} - \frac{1}{s+2}$$

$$Y(s)[s + 2 + \frac{1}{s}] = \frac{1}{s} - \frac{1}{s+2}$$

so

$$Y(s) = \frac{k_1}{(s+1)^2} + \frac{k_2}{s+1} + \frac{k_3}{s+2} \Rightarrow$$

at

$$s = -1$$

$$\frac{2}{s+2} = 2 = k_1$$

$$\frac{d}{ds}[\frac{2}{s+2}] = -2 = k_2$$

at

$$s = -2$$

$$\frac{2}{(s+1)^2} = 2 = k_3$$

so

$$F(s) = \frac{2}{(s+1)^2} - \frac{2}{s+1} + \frac{2}{s+2}$$

$$f(t) = [2te^{-t} - 2e^{-t} + 2e^{-2t}]u(t)$$

$$f(t) = [2te^{-t} - 2e^{-t} + 2e^{-2t}]u(t)$$

Problem 12.38

Determine the y(t) in the following equation if all initial conditions are zero.

$$\frac{d^3y(t)}{dt^3} + 4\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} = 10e^{-2t}$$

Suggested Solution

$$\frac{d^3y(t)}{dt^3} + 4\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} = 10e^{-2t}$$

All intial conditions are zero.

$$s^3Y(s) + 4s^2Y(s) + 3sY(s) = \frac{10}{s+2}$$

$$Y(s) = \frac{10}{s(s+2)(s+1)(s+3)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+1} + \frac{D}{s+3}$$

$$A = Y(s)s\,|_{s=0} = 5/3$$

$$B = Y(s)(s+2)\,|_{s=-2} = 5$$

$$C = Y(s)(s+1)\,|_{s=-1} = -5$$

$$D = Y(s)(s+3)\,|_{s=-3} = -5/3$$

$$Y(s) = \frac{5/3}{s} + \frac{5}{s+2} - \frac{5}{s+1} - \frac{5/3}{s+3}$$

$$y(t) = (\frac{5}{3} + 5e^{-2t} - 5e^{-t} - \frac{5}{3}e^{-3t})u(t)$$

$$y(t) = (\frac{5}{3} + 5e^{-2t} - 5e^{-t} - \frac{5}{3}e^{-3t})u(t)$$

Problem 12.39

Find f(t) using convolution if F(s) is

$$F(s) = \frac{1}{(s+1)(s+2)}$$

Suggested Solution

$$F(s) = \frac{1}{(s+1)(s+2)}$$

let

$$F_1(s) = \frac{1}{s+1}$$

$$F_2(s) = \frac{1}{s+2}$$

$$f_1(t) = e^{-t}u(t)$$

$$f_2(t) = e^{-2t}u(t)$$

$$f(t) = \int_0^t e^{-(t-\lambda)}e^{-2\lambda}d\lambda = e^{-t}\int_0^t e^{-\lambda}d\lambda = e^{-t}[e^{-\lambda}]_t^0$$

$$f(t) = e^{-t}[1 - e^{-t}]$$

$$f(t) = (e^{-t} - e^{-2t})u(t)$$

$$f(t) = (e^{-t} - e^{-2t})u(t)$$

Problem 12.40

Use convolution to find f(t) if

$$F(s)=\frac{1}{(s+1)(s+2)^2}$$

Suggested Solution

$$F(s)=\frac{1}{(s+1)(s+2)^2}$$

let

$$F_1(s)=\frac{1}{(s+2)^2}$$

$$F_2(s)=\frac{1}{s+1}$$

$$f_1(t)=te^{-2t}u(t)$$

$$f_2(t)=e^{-t}u(t)$$

$$f(t)=\int_0^t e^{-(t-\lambda)}\lambda e^{-2\lambda}d\lambda=e^{-t}\int_0^t \lambda e^{-\lambda}d\lambda=e^{-t}[-e^{-\lambda}-\lambda e^{-\lambda}]_t^0$$

$$f(t)=e^{-t}[-e^{-t}-te^{-t}+1]$$

$$f(t)=(e^{-t}-te^{-2t}-e^{-2t})u(t)$$

$$f(t)=(e^{-t}-te^{-2t}-e^{-2t})u(t)$$

Problem 12.41

Determine the intial and final value of F(s) given by the expression

$$F(s)=\frac{2(s+2)}{s(s+1)}$$

$$F(s)=\frac{2(s^2+2s+6)}{s(s+1)}$$

$$F(s)=\frac{2s^2}{s(s+1)(s^2+2s+2)}$$

Suggested Solution

A.

$$F(s)=\frac{2(s+2)}{s(s+1)}$$

for

$$t=0^+, \lim_{s\to\infty} sf(s)=\lim_{s\to\infty}[\frac{2(s+2)}{s(s+1)}]=\boxed{2}$$

$$t\to\infty, \lim_{s\to 0} sf(s)=\lim_{s\to 0}[\frac{2(s+2)}{s(s+1)}]=\boxed{4}$$

B.

$$F(s)=\frac{2(s^2+2s+6)}{s(s+1)}$$

for

$$t=0^+, \lim_{s\to\infty} sf(s)=\lim_{s\to\infty}[\frac{2(s^2+2s+6)}{s(s+1)}]=\boxed{2}$$

$$t\to\infty, \lim_{s\to 0} sf(s)=\lim_{s\to 0}[\frac{2(s^2+2s+6)}{s(s+1)}]=\boxed{0}$$

C.

$$F(s)=\frac{2s^2}{s(s+1)(s^2+2s+2)}$$

for

$$t=0^+, \lim_{s\to\infty} sf(s)=\lim_{s\to\infty}[\frac{2s^2}{s(s+1)(s^2+2s+2)}]=\boxed{2}$$

$$t\to\infty, \lim_{s\to 0} sf(s)=\lim_{s\to 0}[\frac{2s^2}{s(s+1)(s^2+2s+2)}]=\boxed{0}$$

Problem 12.42

Find the initial and final value of the time function f(t) if F(s) is given as

$$F(s)=\frac{10(s+2)}{(s+1)(s+3)}$$

$$F(s)=\frac{(s^2+2s+4)}{(s+1)(s^3+4s^2+8s+10)}$$

$$F(s)=\frac{2s}{(s^2+2s+2)}$$

Suggested Solution

A.

$$F(s)=\frac{10(s+2)}{(s+1)(s+3)}$$

for

$$t=0^+, \lim_{s\to\infty} sf(s)=\lim_{s\to\infty}[\frac{10(s+2)}{(s+1)(s+3)}]=\boxed{10}$$

$$t\to\infty, \lim_{s\to 0} sf(s)=\lim_{s\to 0}[\frac{10(s+2)}{(s+1)(s+3)}]=\boxed{0}$$

B.

$$F(s)=\frac{(s^2+2s+4)}{(s+1)(s^3+4s^2+8s+10)}$$

for

$$t=0^+, \lim_{s\to\infty} sf(s)=\lim_{s\to\infty}[\frac{(s^2+2s+4)}{(s+1)(s^3+4s^2+8s+10)}]=\boxed{0}$$

$$t\to\infty, \lim_{s\to 0} sf(s)=\lim_{s\to 0}[\frac{(s^2+2s+4)}{(s+1)(s^3+4s^2+8s+10)}]=\boxed{0}$$

C.

$$F(s)=\frac{2s}{(s^2+2s+2)}$$

for

$$t=0^+, \lim_{s\to\infty} sf(s)=\lim_{s\to\infty}[\frac{2s^2}{s(s+1)(s^2+2s+2)}]=\boxed{2}$$

$$t\to\infty, \lim_{s\to 0} sf(s)=\lim_{s\to 0}[\frac{2s^2}{s(s+1)(s^2+2s+2)}]=\boxed{0}$$

Problem 12.43

Find the final values of the time function f(t) if F(s) is given as

$$F(s) = \frac{10(s+1)}{(s+2)(s+3)}$$

$$F(s) = \frac{10}{s^2 + 4s + 4}$$

Suggested Solution

A.

$$F(s) = \frac{10(s+1)}{(s+2)(s+3)}$$

for

$$t = 0^+, \lim_{s\to\infty} sf(s) = \lim_{s\to\infty}[\frac{10(s+1)}{(s+2)(s+3)}] = \boxed{10}$$

$$t \to \infty, \lim_{s\to 0} sf(s) = \lim_{s\to 0}[\frac{10(s+1)}{(s+2)(s+3)}] = \boxed{0}$$

B.

$$F(s) = \frac{10}{s^2 + 4s + 4}$$

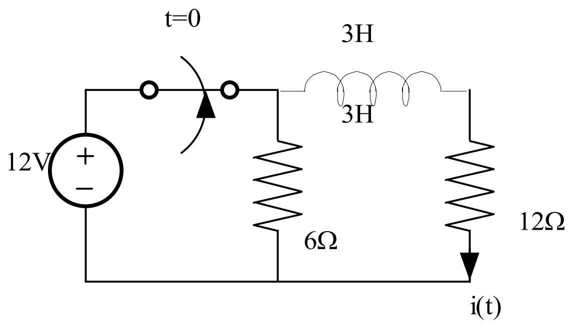
for

$$t = 0^+, \lim_{s\to\infty} sf(s) = \lim_{s\to\infty}[\frac{10}{s^2 + 4s + 4}] = \boxed{0}$$

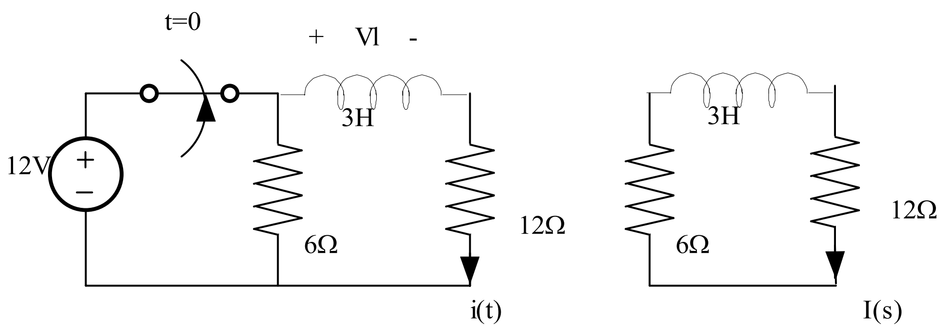
$$t \to \infty, \lim_{s\to 0} sf(s) = \lim_{s\to 0}[\frac{10}{s^2 + 4s + 4}] = \boxed{0}$$

Problem 12.44

In the network in fig, the switch opens at t =0. Use laplace transform to find I(t) for t>0.



Suggested Solution



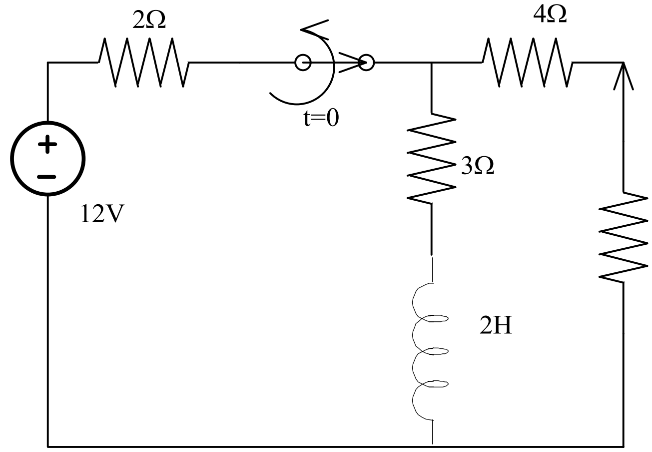
$$\begin{aligned} &for \\ &t<0 \\ &V_L = 0V \\ &and \\ &i(0^-) = i(0^+) = 1A \end{aligned}$$

$$\begin{aligned} &18I(s) + 3sI(s) - 3i(0) = 0 \\ &I(s)[3s+18] = 3 \\ &I(s) = \frac{1}{s+6} \\ &i(t) = e^{-6t}u(t) \end{aligned}$$

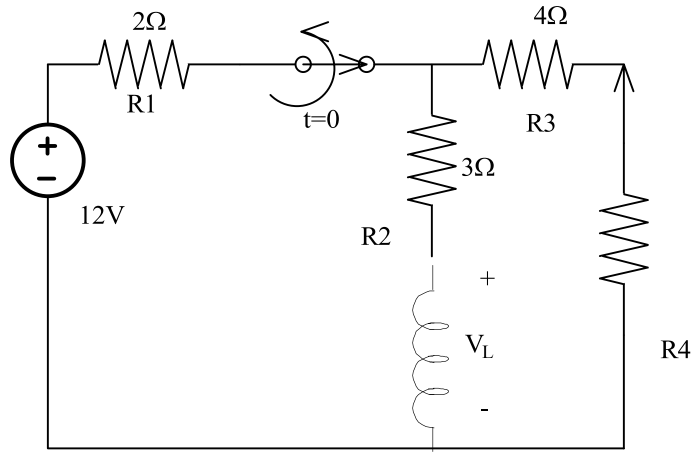
$$i(t) = e^{-6t}u(t)$$

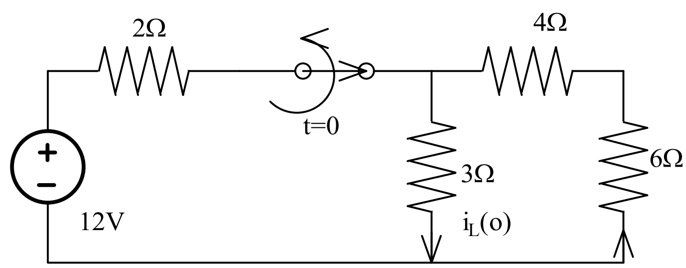
Problem 12.45

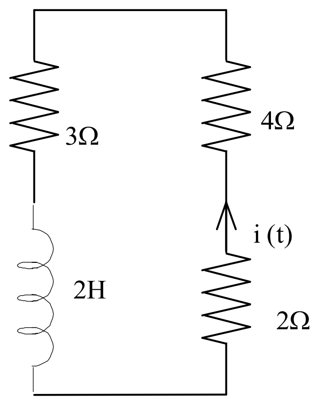
The switch in the circuit opens at t=0. Find I(t) for t>0 using Laplace transforms.



Suggested Solution







$$3i(t)+4i(t)+2i(t)+2\frac{di(t)}{dt}=0$$

$$9I(s)+2I(s)-2i(0^{+})=0$$

$$I(s)=[s+\frac{9}{2}]=4$$

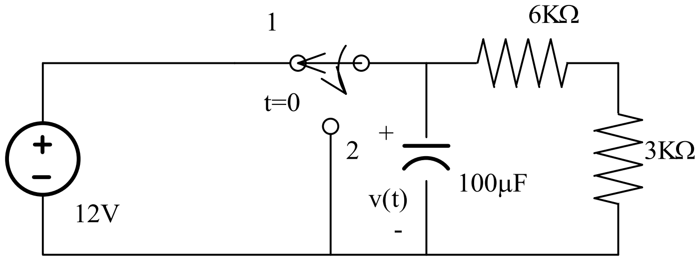
$$I(s)=\frac{4}{s+\frac{9}{2}}$$

$$i(t)=4e^{-\frac{9}{2}t}u(t)A$$

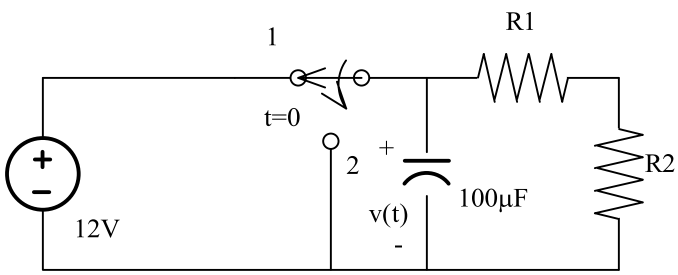
$$i(t)=4e^{-\frac{9}{2}t}u(t)A$$

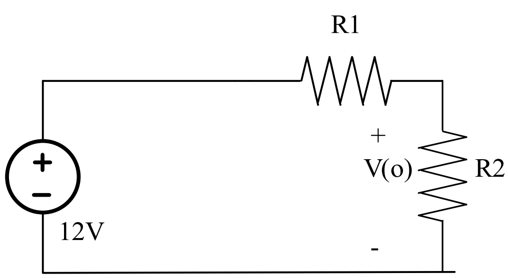
Problem 12.46

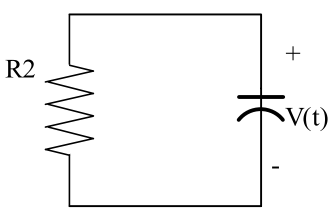
In the circuit in fig, the switch moves from position 1 to 2 at t =0. Use Laplace transforms to find v(t) for t>0.



Suggested Solution







$$\frac{V(t)}{R}+\frac{Cdv(t)}{dt}=0$$

$$\frac{V(s)}{R}+CV(s)-Cv(0)=0$$

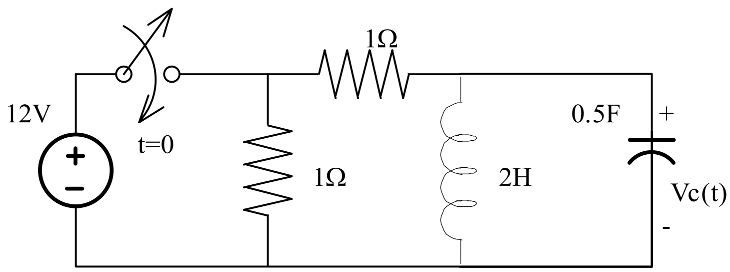
$$V(s)[s+\frac{1}{RC}]=v(0)\Rightarrow V(s)=\frac{4}{s+5}$$

$$v(t)=4e^{-5t}u(t)V$$

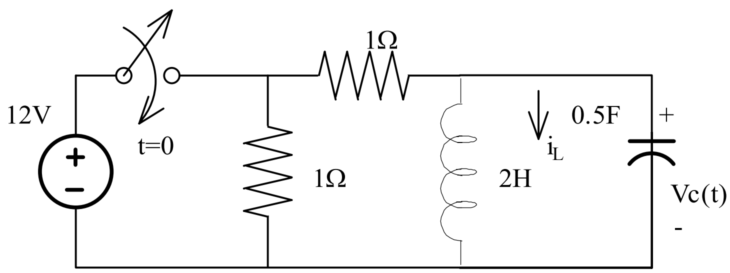
$$v(t)=4e^{-5t}u(t)V$$

Problem 12.47

In the network the switch closes at t=0. Use Laplace transforms to find Vc(t) for t>0.



Suggested Solution



$$12 - Vc(t) = \frac{1}{L}\int_0^t Vc(x)dx + C\frac{dVc(t)}{dt}$$

$$\frac{12}{s} - Vc(s) = \frac{1}{2s}Vc(s) + \frac{s}{2}Vc(s)$$

$$\frac{12}{s} = Vc(s)[1 + \frac{1}{2s} + \frac{s}{2}]$$

$$\frac{12}{s} = Vc(s)[\frac{2s + 1 + s^2}{2s}]$$

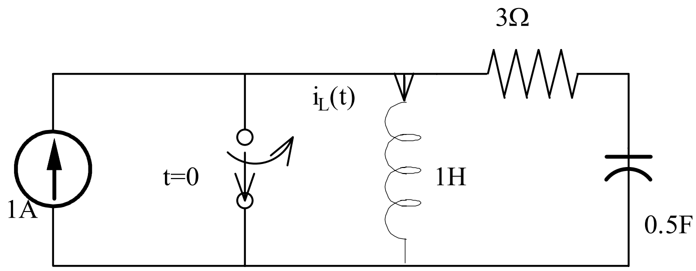
$$Vc(s) = \frac{24}{2s + 1 + s^2} = \frac{24}{(s+1)^2}$$

$$\therefore Vc(t) = 24te^{-t}u(t)V$$

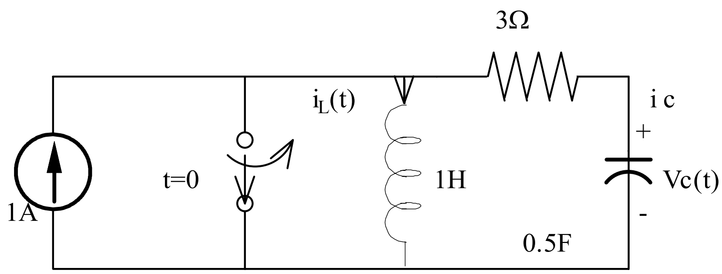
$$\therefore Vc(t) = 24te^{-t}u(t)V$$

Problem 12.48

In the network the switch opens at t=0. Use Laplace transforms to find iL(t) for t>0.



Suggested Solution



$For\_t > 0$

$$L\frac{di_L}{dt} = 3i_C + \frac{1}{C}\int_0^t i_C(t)dt$$

and

$$i_L + i_C = 1 \Rightarrow i_C = 1 - i_L$$

now,

$$sI_L(s) = 3 - 3I_L(s) + 2(\frac{1 - I_L(s)}{s})$$

$$I_L(s)[s + \frac{2}{s}] = 3 + \frac{2}{s}$$

$$I_L(s) = \frac{3s+2}{s^2+3s+2} = \frac{A}{s+2} + \frac{B}{s+1} \Rightarrow A = 4, B = -1$$

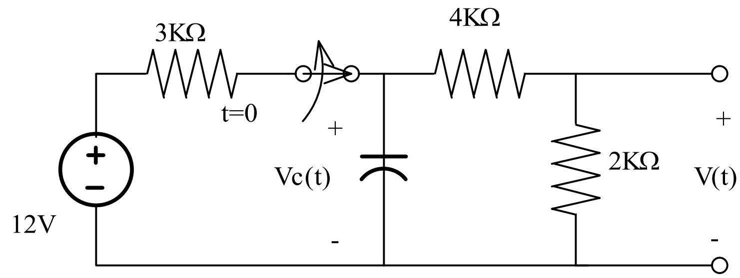
$$I_L(s) = \frac{4}{s+2} - \frac{1}{s+1}$$

$$i_L(t) = (4e^{-2t} - e^{-t})u(t)$$

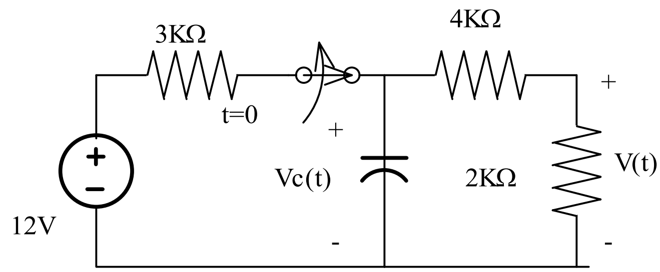
$$i_L(t) = (4e^{-2t} - e^{-t})u(t)$$

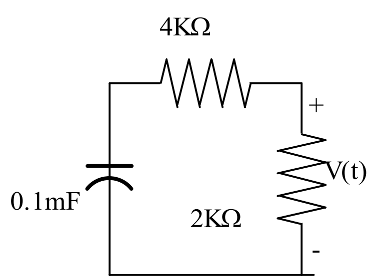
Problem 12.49

In the network the switch opens at t=0. Use Laplace transforms to find V0(t) for t>0.



Suggested Solution





$$Vc(0^-) = 12(\frac{2+4}{2+4+3}) = 8v = Vc(0^+)$$

$$v_0(0^+) = v_c(0^+) = \frac{8}{3}v$$

$$6ki(t) + \frac{1}{0.1m}\int_0^t i(t)dt - v_c(0) = 0$$

$$0.6I(s) + \frac{I(s)}{s} - \frac{0.8m}{s} = 0$$

$$I(s) = \frac{0.8}{0.6s+1}mA = \frac{4/3}{s+\frac{1}{0.6}}$$

$$i(t) = \frac{4}{3}e^{-t/0.6}u(t)mA$$

$$V_0(t) = 2ki(t) = \frac{8}{3}e^{-t/0.6}V$$

$$i(t) = \frac{4}{3} e^{-t/0.6} u(t) mA$$

Problem 12FE-1

The output function of a network is expressed using Laplace transforms in the following form.

$$V_0(s) = \frac{12}{s(s+1)(s+2)}$$

Find the output as a function of time $v_0(t)$.

Suggested Solution

$$V_0(s) = \frac{12}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$$

$$\frac{12}{(s+1)(s+2)}\Big|_{s=0} = A = 6$$

$$\frac{12}{s(s+2)}\Big|_{s=-1} = B = -12$$

$$\frac{12}{s(s+1)}\Big|_{s=-2} = C = 6$$

$$V_0(t) = (6 - 12e^{-t} + 6e^{-2t})u(t)V$$

$$V_0(t) = (6 - 12e^{-t} + 6e^{-2t})u(t)V$$

Problem 12FE-2

The Laplace transform function representing the output voltage of a network is expressed as

$$V_0(s) = \frac{120}{s(s+10)(s+20)}$$

Determine the time domain function and the value of the $v_0(t)$ at t=100mSec.

Suggested Solution

$$V_0(s) = \frac{120}{s(s+10)(s+20)} = \frac{A}{s} + \frac{B}{s+10} + \frac{C}{s+20}$$

$$\frac{120}{(s+10)(s+20)}\Big|_{s=0} = A = 0.6$$

$$\frac{120}{s(s+20)}\Big|_{s=-10} = B = -1.2$$

$$\frac{120}{s(s+10)}\Big|_{s=-20} = C = 0.6$$

$$V_0(t) = (0.6 - 1.2e^{-10t} + 0.6e^{-20t})u(t)V$$

$$V_0(t)\big|_{t=0.1\sec} = 0.24V$$

$$V_0(t)\big|_{t=0.1\sec} = 0.24V$$

Problem 12FE-3

The Laplace transform function for the output voltage of a network is expressed in the following form

$$V_0(s) = \frac{12(s+2)}{s(s+1)(s+3)(s+4)}$$

Determine the final value i.e. $v_o(t)$ as $t \to \infty$, of this voltage.

Suggested Solution

$$V_0(s) = \frac{12(s+2)}{s(s+1)(s+3)(s+4)}$$

and

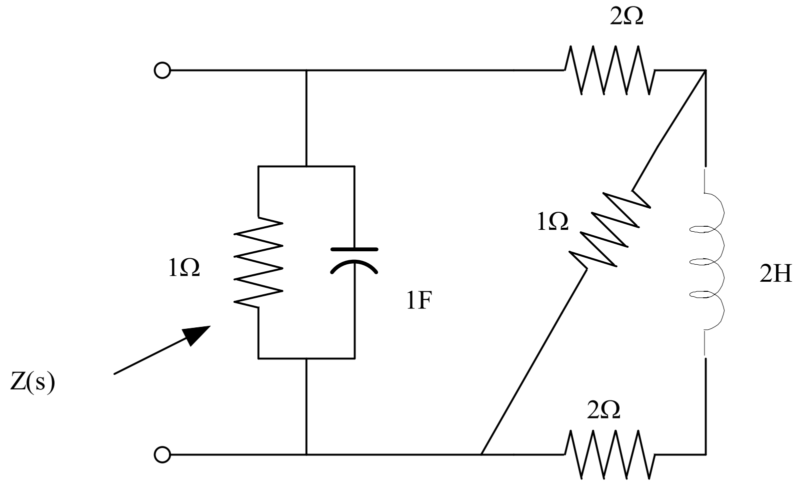
$$V_0(\infty) = sV_0(s)$$

$$V_0(\infty) = \frac{12(s+2)}{s(s+1)(s+3)(s+4)}\Big|_{s=0} = 2v$$

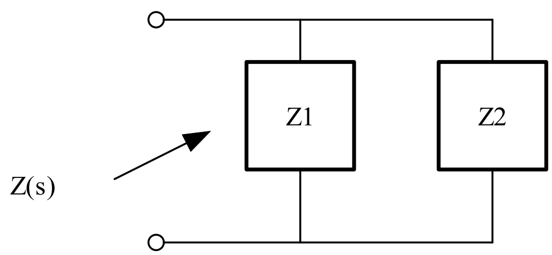
$$V_0 = 2v$$

Problem 13.1

Find the input impedance Z(s) of the network in fig 3.1.



Suggested Solution



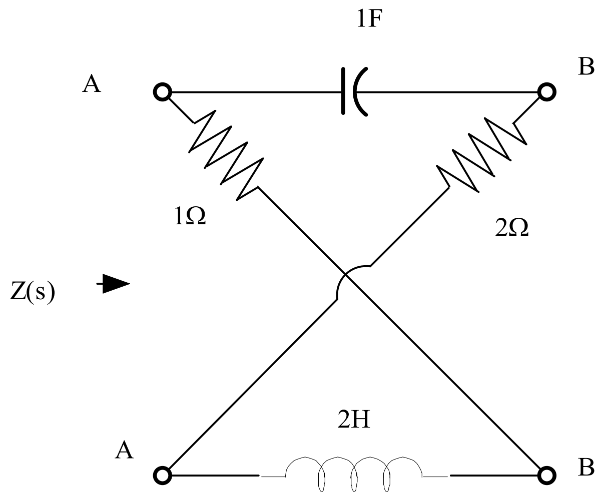
$$Z_1(S) = \frac{1(\frac{1}{S})}{1+(\frac{1}{S})} = \frac{1}{S+1}$$

$$Z_2(S) = \frac{6S+8}{2S+3}$$

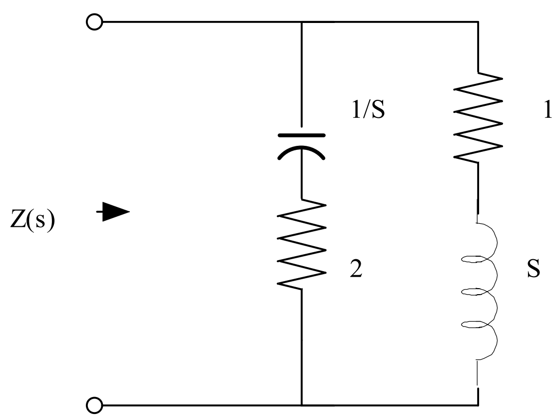
$$Z(S) = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{6S+8}{6S^2+16S+11}$$

Problem 13.2

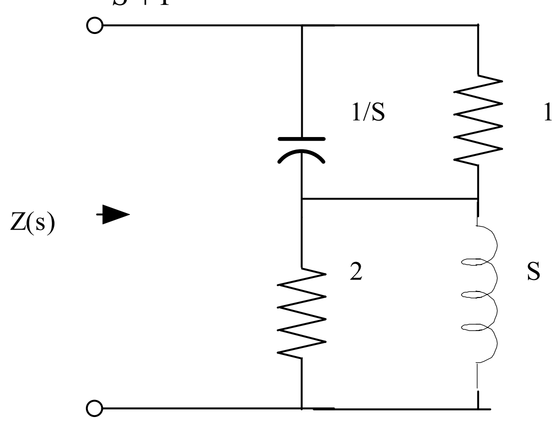
Find the input impedance Z(s) of the network in fig 3.2
(a). when the terminals B-B' are open circuited and (b). when the terminals B-B' are closed circuited.



Suggested Solution



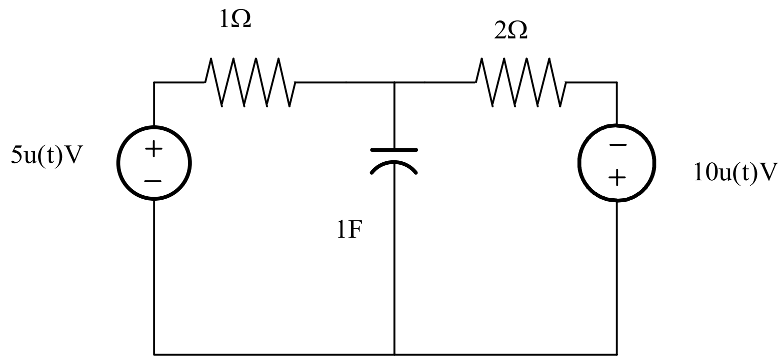
$$Z(S) = \frac{2S+1}{S+1}$$



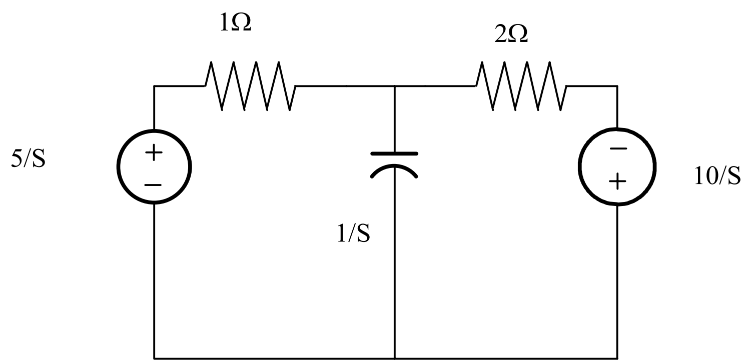
$$Z(S) = \frac{2S+1}{S+1}$$

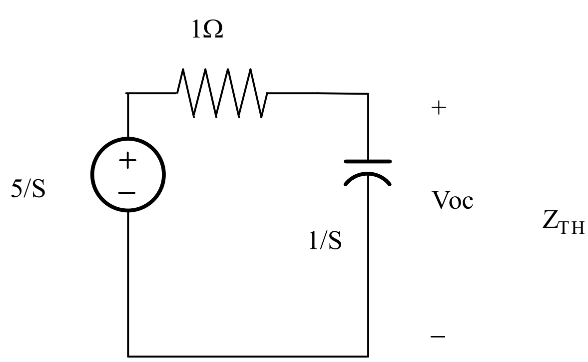
Problem 13.3

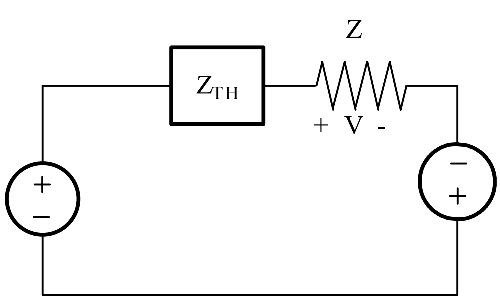
Use lap lace transforms to find v(t) for t>0 in the network shown in fig. Assume zero initial conditions.



Suggested Solution







$$V_{OC} = \frac{5}{S}\left[\frac{1/S}{1+1/S}\right] = \frac{5}{S}\left[\frac{1}{1+S}\right]$$

$$Z_{TH} = 1 \parallel (1/S) = \frac{1}{1+S}$$

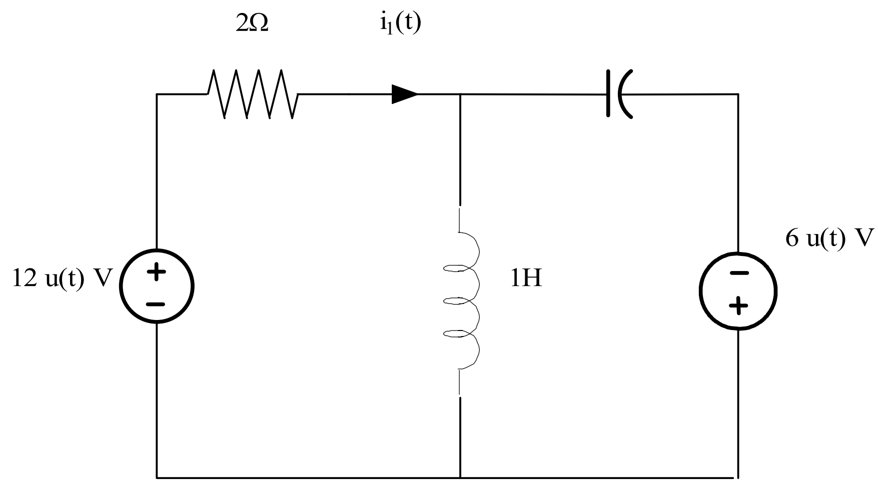
$$V_S = \left(V_{OC} + \frac{10}{S}\right)\left(\frac{2}{2+Z_{TH}}\right) = \frac{10}{S}\left[\frac{2S+3}{1+S}\right]\left[\frac{S+1}{3+2S}\right]$$

$$V_S = 10$$

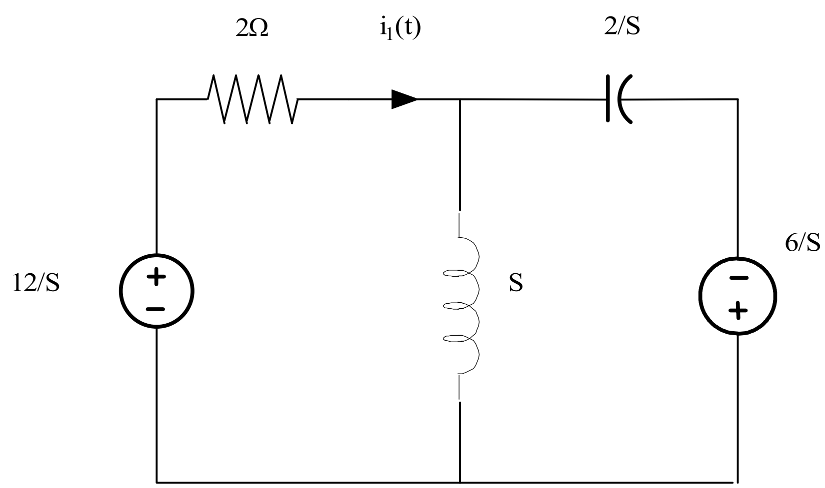
$$V(t) = 10u(t)V$$

Problem 13.4

Use lap lace transforms and node analysis to find i1(t) for t>0 in the network shown in fig. Assume zero initial conditions.



Suggested Solution

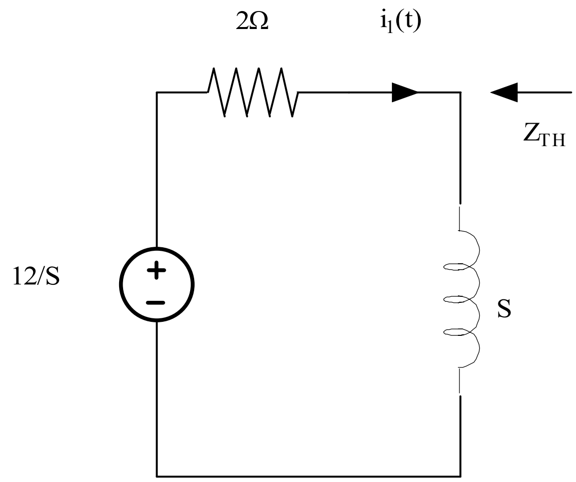


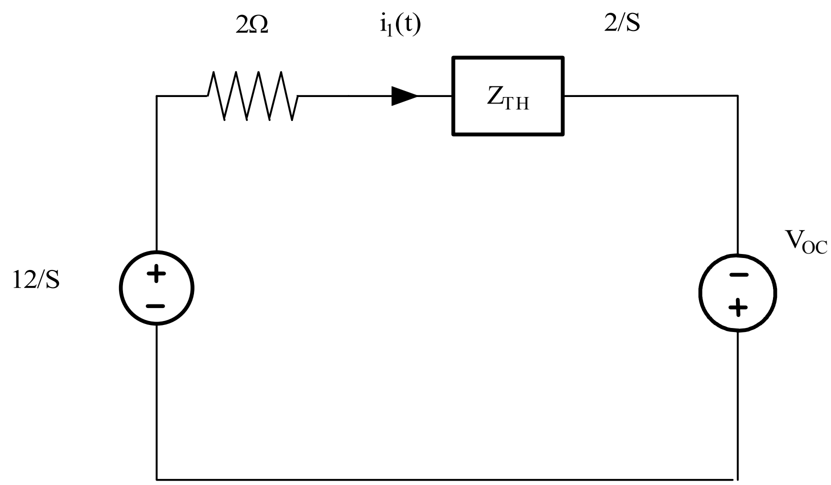
$$Z_{TH} = S \,||\, (2/S) = \frac{2S}{2+S^2} \qquad V_{OC} = \frac{6}{S}\left[\frac{S}{1+2/S}\right] = \left[\frac{6}{2+S^2}\right]$$

$$I_1 = \frac{\frac{12}{S}+V_{OC}}{Z+Z_{TH}} = \frac{\frac{12}{S}+\left[\frac{6}{2+S^2}\right]}{2+\frac{2S}{2+S^2}} = \frac{9S^2+12}{S(S^2+S+2)}$$

$$I_1 = \frac{A}{S} + \frac{K\lfloor\theta}{S-\frac{1}{2}-j\frac{\sqrt{7}}{2}} + \frac{K\lfloor -\theta}{S-\frac{1}{2}+j\frac{\sqrt{7}}{2}}$$

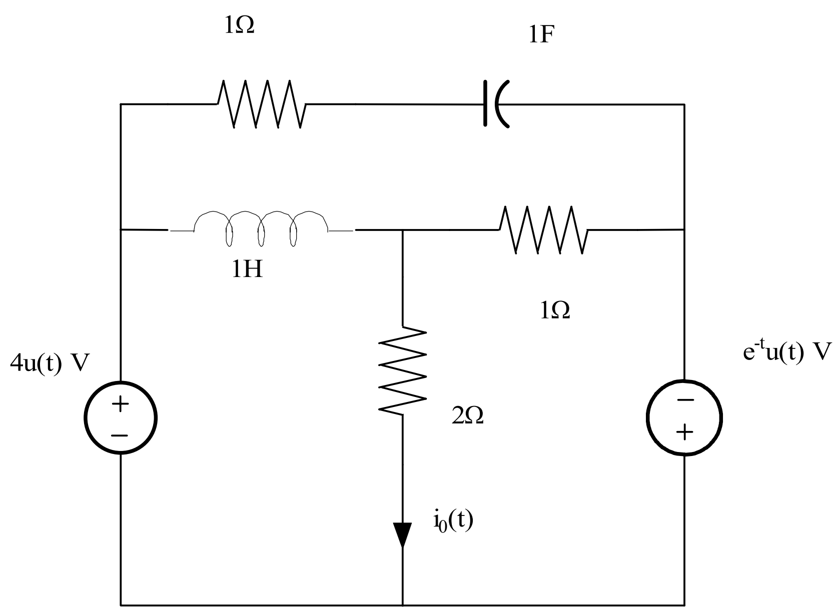
$$i_1(t) = \left[6 + 16.12e^{-t/2} COS(\sqrt{7}t/2 + 26.37^0) \right] u(t) V$$



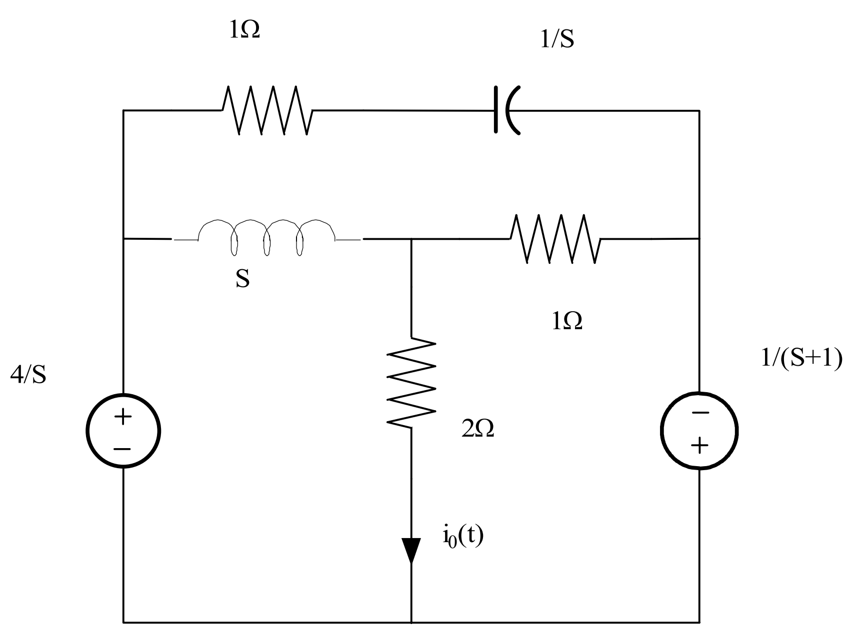


Problem 13.5

For the network shown in fig 13.5 find io(t),t>0.



Suggested Solution



$$V\left(\frac{1}{S}+\frac{1}{2}+1\right)=\frac{4}{S^2}-\frac{1}{S+1}$$

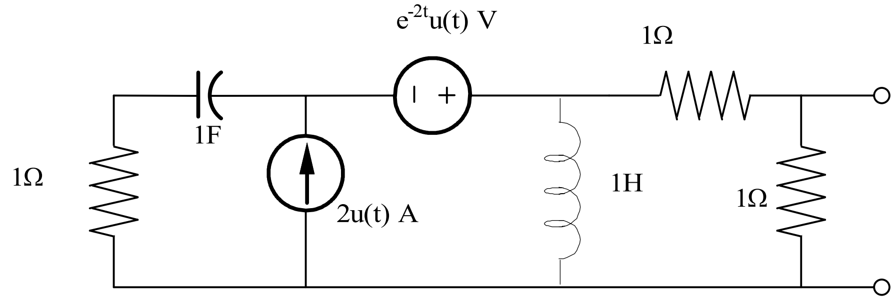
$$V=\frac{2(-S^2+4S+4)}{S(S+1)(3S+2)}$$

$$I_O=0.5V=\frac{(-S^2+4S+4)}{S(S+1)(3S+2)}$$

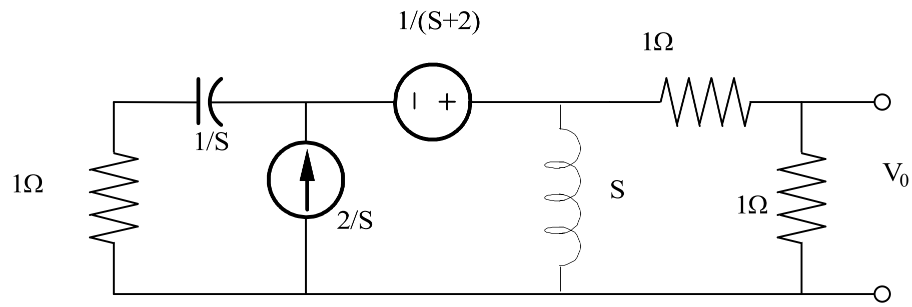
$$i_O(t)=(2-16.12e^{-t}-4/3e^{-2t/3})u(t)V$$

Problem 13.6

For the network shown in fig 13.5 find Vo(t),t>0.



Suggested Solution



KCL at the Supernode is:

$$\frac{V_1}{1+1/S}+\frac{V_2}{S}+\frac{V_2}{2}=\frac{2}{3}$$

$$V_1+\frac{1}{S+2}=V_2$$

SOLVING FOR V2 YIELDS:

$$V_2=\frac{2(3S^2+6S+4)}{(S+2)(3S^2+6S+2)}\Rightarrow V_O=\frac{S^2+2S+4/3}{(S+2)(S+0.5-j.646)(S+0.5+j.646)}$$

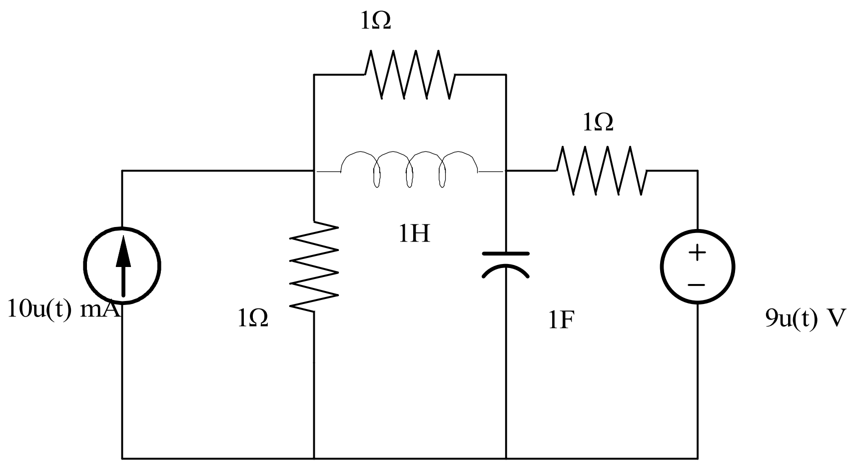
$$=\frac{K_1}{S+2}+\frac{K_2}{(S+0.5-j.646)}+\frac{K_2}{(S+0.5+j.646)}$$

$$K_1=0.5, K_2=0.316\angle{-37.76^0}$$

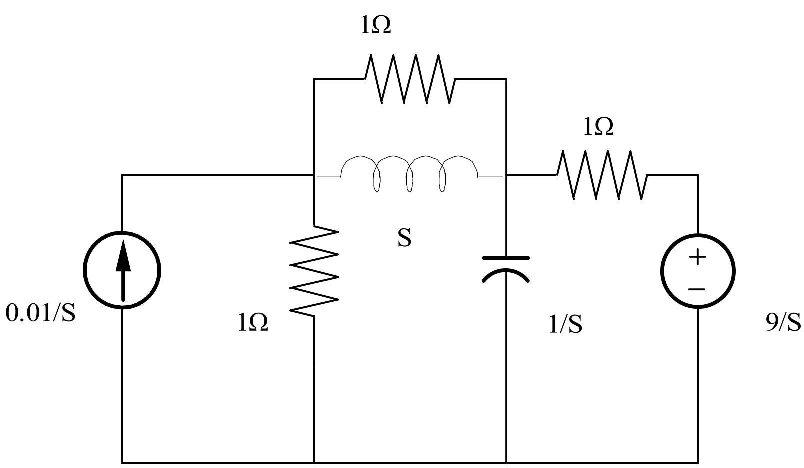
$$\therefore V_o(t)=\left(0.5e^{-2t}+0.632e^{-2t}COS(0.646t-37.76^0)\right)u(t)V$$

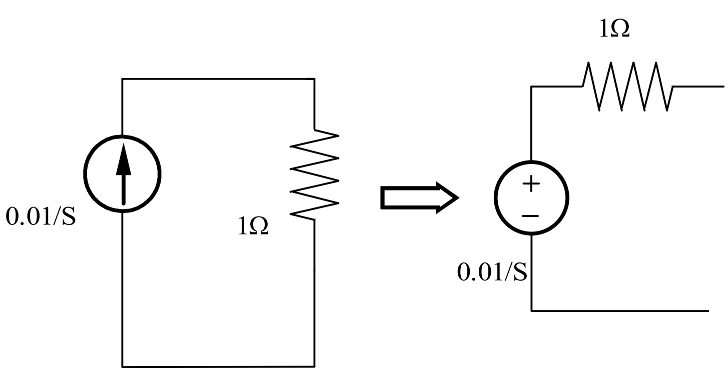
Problem 13.7

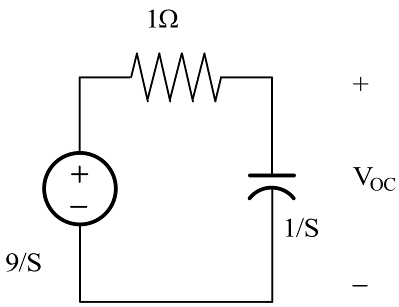
For the network shown in fig 13.5 find Vo(t),t>0.

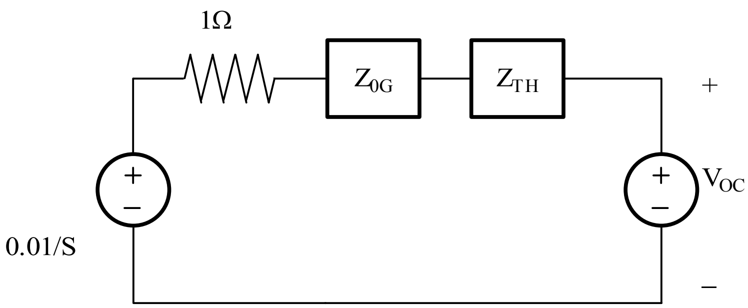


Suggested Solution









$$V_{OC} = \frac{9}{S}\left(\frac{1/S}{1+1/S}\right) = \frac{9}{S(S+1)}$$

$$Z_{TH} = 1 \| 1/S = \frac{1}{S+1}$$

$$V = \left(\frac{10^{-2}}{S} - V_{OC}\right)\left(\frac{Z_{EQ}}{Z_{EQ}+Z_{TH}+1}\right) = \frac{0.005(S-899)}{(S+1)^2}$$

$$V = \frac{A}{(S+1)^2} + \frac{B}{S+1}$$

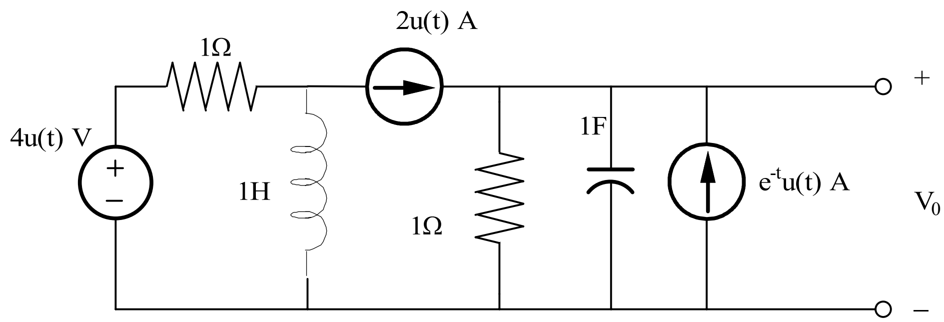
$$A = -4.5$$

$$V(899) = 0 = \frac{A}{(900)^2} + \frac{B}{900} \Rightarrow B = 5*10^{-3}$$

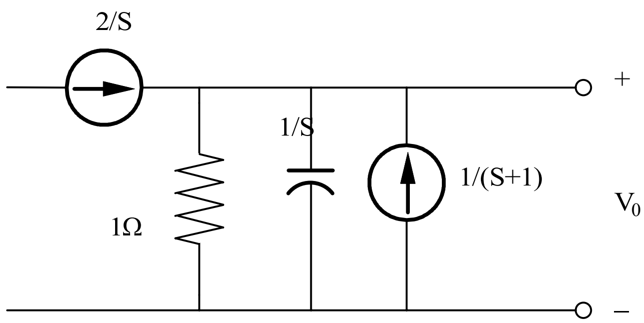
$$V(t) = (5e^{-t} - 4500te^{-t})u(t)mV$$

Problem 13.8

Find Vo(t),t>0 in the network shown in fig using node equations.



Suggested Solution



$$\frac{2}{S}+\frac{1}{S+1}=\frac{V_O}{1}+SV_O=V_O(S+1)$$

$$V_O=\frac{2+3S}{S(S+1)^2}=\frac{A}{S}+\frac{B}{(S+1)^2}+\frac{C}{S+1}$$

$$A=2$$

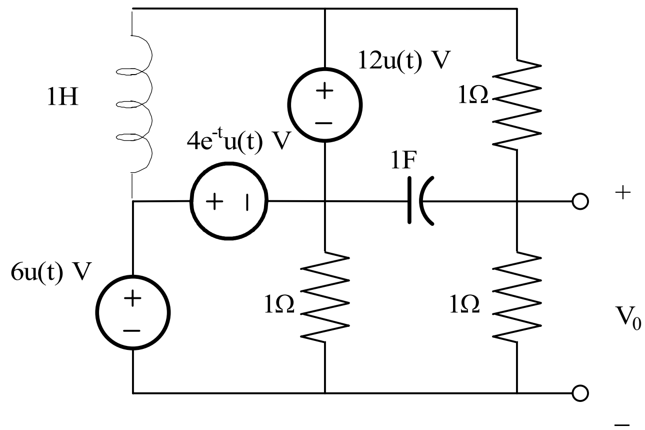
$$B=1$$

LET S=-2, V_O(-2)=2=-0.5A+B-C $\Rightarrow$ C=-2

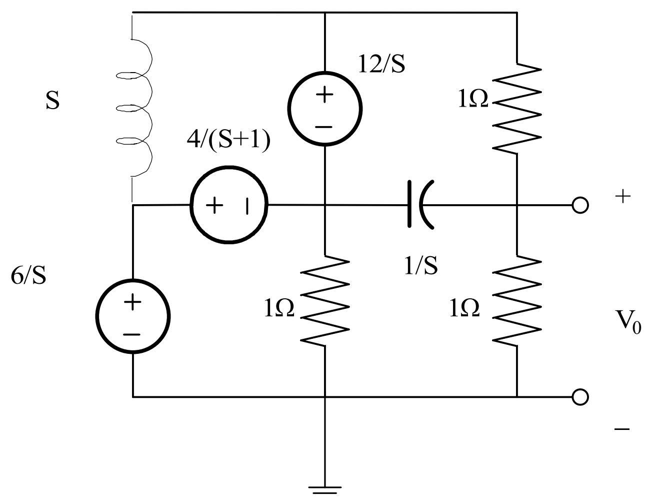
$$V_O(t)=\left[2+2e^{-t}+te^{-t}\right]u(t)V$$

Problem 13.9

Find Vo(t),t>0 in the network shown in fig using node equations.



Suggested Solution



$$KCL: V_3 - V_O + (V_2 - V_O)S = V_O \Rightarrow V_O = \frac{V_3 + SV_2}{S+2}$$

$$V_2 = \frac{6}{S} - \frac{4}{S+1} = \frac{2S+6}{S(S+1)}$$

$$V_3 - V_2 + \frac{12}{S} = \frac{14S+18}{S(S+1)}$$

$$V_O = \frac{2(S+9)}{S(S+2)}$$

$$V_O = \frac{A}{S} + \frac{B}{(S+2)}$$

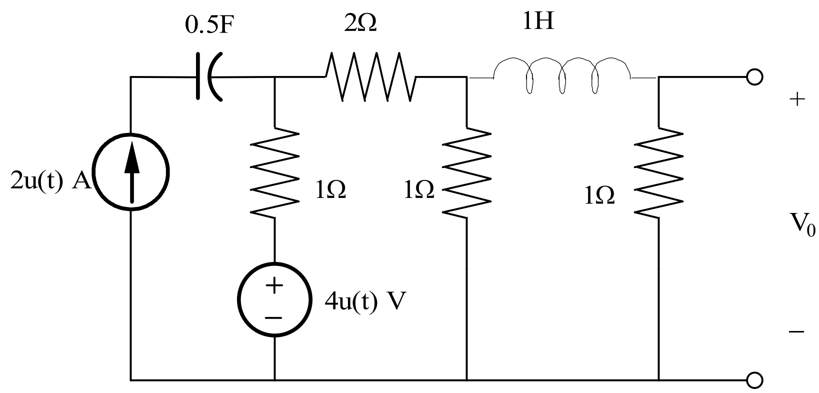
$$A = 9$$

$$B = -7$$

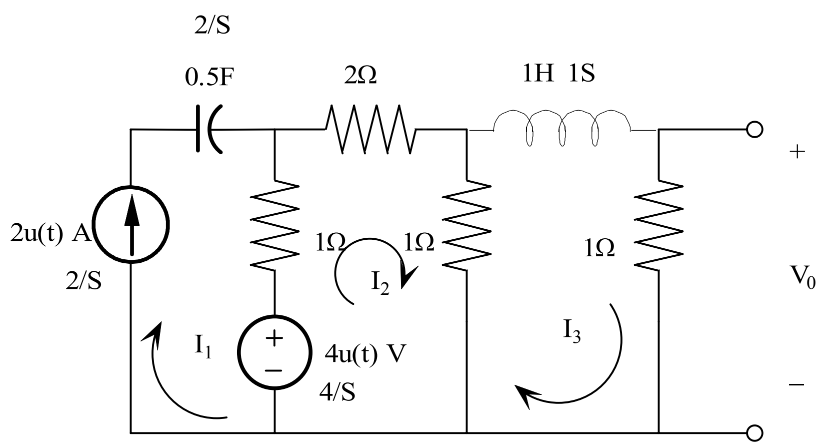
$$V_O(t) = \left[9 - 7e^{-2t}\right]u(t)V$$

Problem 13.10

For the network shown in fig find Vo(t),t>0 using mesh equations.



Suggested Solution



BY MESH ANALYSIS,

$-I_1+4I_2-I_3=4/S$

$-I_2+I_3(S+2)=0$

GIVEN

$$I_1=2/S, I_3 \text{ SOLVES TO } \frac{1.5}{S(S+7/4)}=V_O=I_3$$

$$V_O(S)=\frac{1.5}{S(S+7/4)}=\frac{K_1}{S}+\frac{K_2}{(S+7/4)}$$

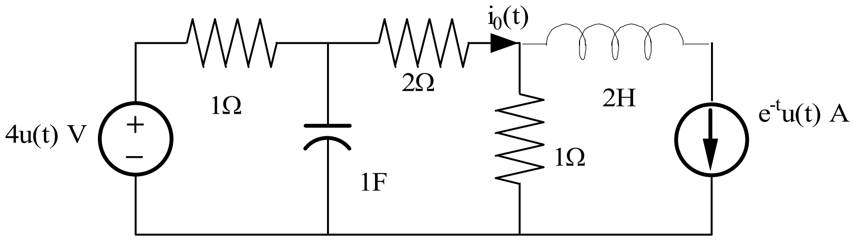
$$K_1 = 6/7, K_2 = -6/7$$

$$V_O(S) = \frac{6}{7}\left(\frac{1}{S} - \frac{1}{(\text{S+7/4})}\right)$$

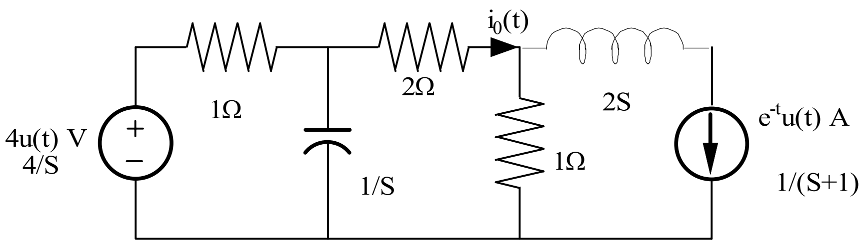
$$V_O(t) = \left[\frac{6}{7}(1 - e^{-7/4t})\right]u(t)V$$

Problem 13.11

For the network shown in fig find Vo(t),t>0 using mesh equations.



Suggested Solution



$$\frac{4}{S} - V_1 = V_1 S + I_O$$

$$I_O = V_2 + \frac{1}{(S+1)} = \frac{V_1 - V_2}{2}$$

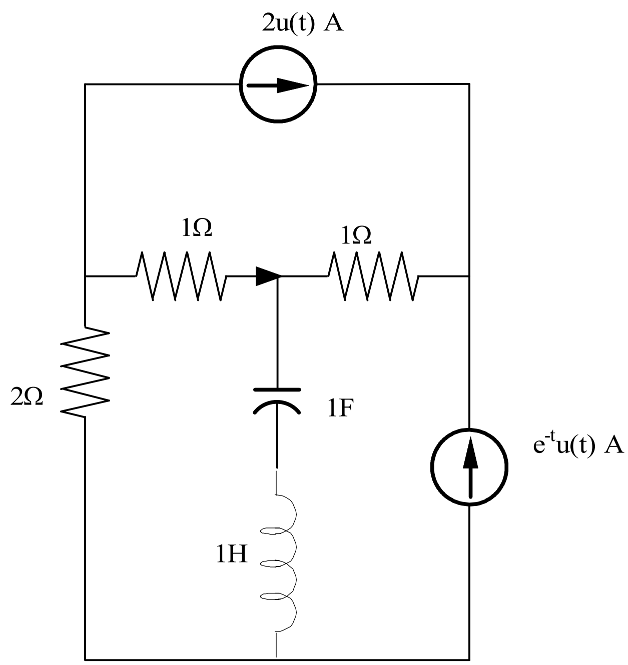
YIELDS :

$$I_O(S) = \left(\frac{1}{S} - \frac{2}{(3S+4)} \right)$$

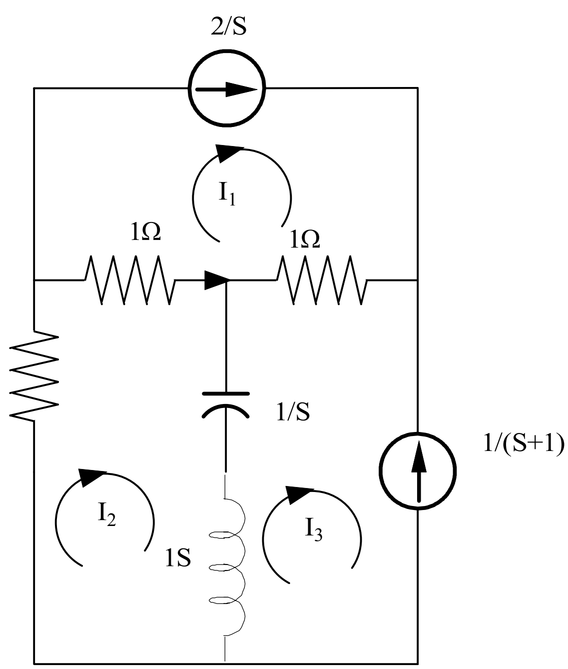
$$I_O(t) = \left[(1 - \frac{2}{3} e^{-4/3t}) \right] u(t) A$$

Problem 13.12

Use loop equations to find Vo(t),t>0 in the network shown in fig.



Suggested Solution



$$I_1 = \frac{2}{S}, I_3 = -\frac{1}{(\text{S+1})}$$

$$2I_2 + (I_2 - I_1) + (S + \frac{1}{S})(I_2 - I_3) = 0$$

$$I_2(2 + 1 + S + \frac{1}{S}) = \frac{2}{S} + \frac{S}{(\text{S+1})} + \frac{1}{\text{S(S+1)}}$$

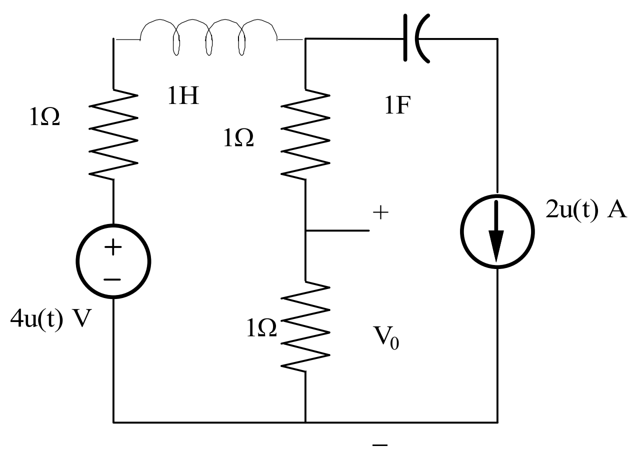
$$I_2(S) = \frac{S^2 + 2S + 3}{(\text{S+1})(S^2 + 3S + 1)}$$

$$I_O = I_2 - I_1 = \frac{-(S^3 + 6S^2 + 8S + 2)}{\text{S(S+1)(S+2.62)(S+0.381)}} = -\frac{2}{S} - \frac{2}{(\text{S+1})} + \frac{1.28}{S + 2.62} + \frac{1.72}{S + 0.38}$$

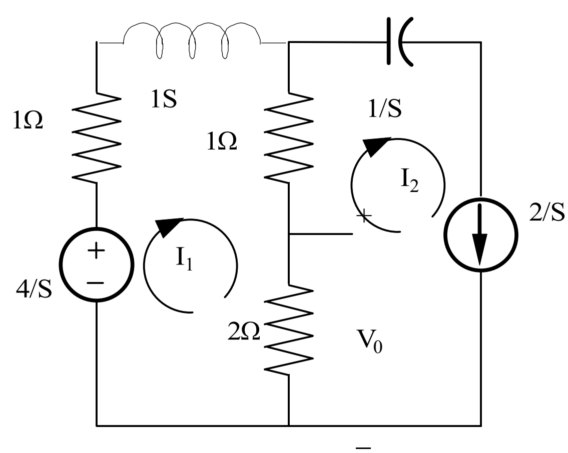
$$l_O(t) = (-2 - 2e^{-t} + 1.28e^{-2.62t} + 1.72e^{-0.38t})u(t)A$$

Problem 13.13

Use loop equations to find Vo(t),t>0 in the network shown in fig.



Suggested Solution



Mesh eq's:

$$\frac{4}{S} = (S+4)I_1 - 3I_2$$

$$I_2 = \frac{2}{S}$$

$$I_1 = \frac{10}{S(S+4)}$$

$$V_O = (I_1 - I_2)2$$

$$V_O = \left(\frac{10}{S(S+4)} - \frac{2}{S} \right) 2 = \frac{-2S+2}{S(S+4)} 2 = \frac{-4S+4}{S(S+4)}$$

$$V_O = \left(\frac{B}{(S+4)} + \frac{A}{S} \right)$$

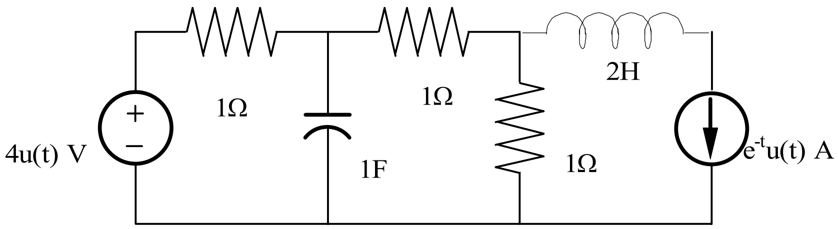
$$A = 1$$

$$B = -5$$

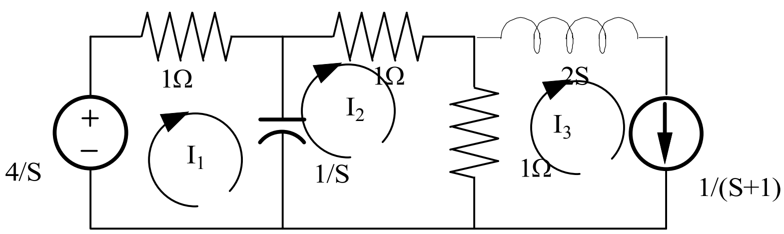
$$\underline{V_O(t) = (1 + 5e^{-4t})u(t)V}$$

Problem 13.14

Use mesh equations to find io(t),t>0 in the network shown in fig.



Suggested Solution



$$I_O = I_2$$

$$I_3 = \frac{1}{S+1}$$

$$\frac{4}{S} = (1/S+1)I_1 - I_2/S$$

$$0 = -(1/S)I_1 + I_2(3+1/S) - I_3$$

SOLVING 1 FOR I1 & PUT INTO 2.

$$I_1 = \frac{I_2+4}{(S+1)} \Rightarrow I_2\left[3S+1-\frac{1}{S+1}\right] = \frac{S}{(S+1)} + \frac{4}{S+1} = \frac{S+4}{S+1}$$

SO

$$I_O = \frac{\frac{S+4}{S+1}}{\frac{3S^2+4S}{S+1}} = \frac{(4+S)/3}{S(S+4/3)} = \frac{B}{(S+4/3)} + \frac{A}{S}$$

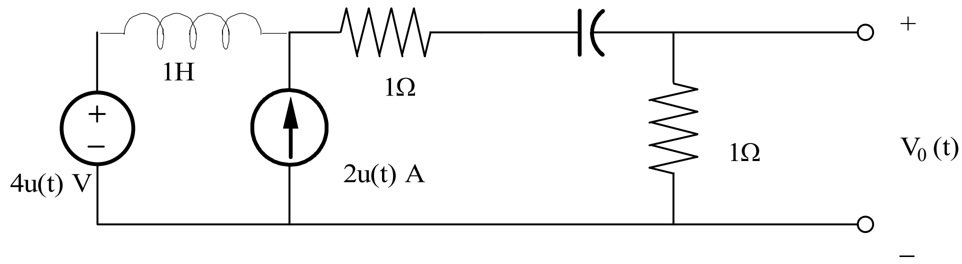
$$A = 1$$

$$B = -\frac{2}{3}$$

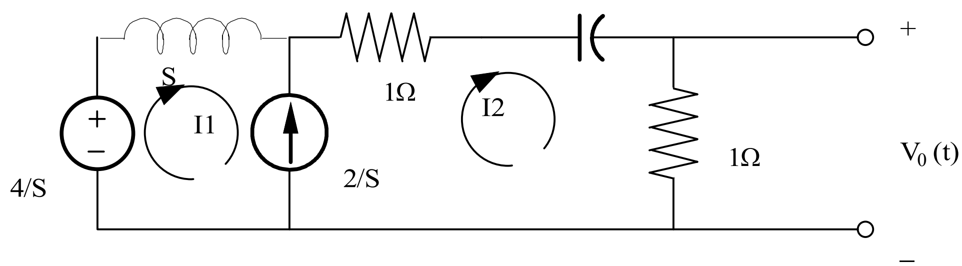
$$i_O(t) = (1 - \frac{2}{3}e^{-4t/3})u(t)V$$

Problem 13.15

Use loop equations to find Vo(t),t>0 in the network shown in fig.



Suggested Solution



MESH EQUATIONS:

$$V_O = I_2$$

$$\frac{4}{S} = SI_1 + I_2(2 + \frac{2}{S}) \;\&\; I_2 - I_1 = \frac{2}{S}$$

SO

$$I_1 = I_2 - \frac{2}{S}$$

AND

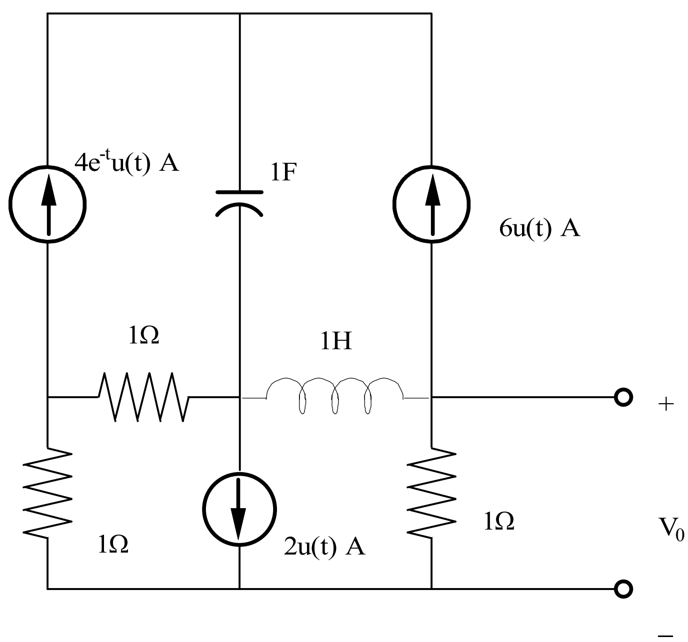
$$\frac{4}{S} = -2 + I_2(2 + \frac{2}{S} + S) \Rightarrow I_2 = \frac{2S+4}{S^2+2S+2} = \frac{K\angle\theta}{S+1-j1} + \frac{K\angle-\theta}{S+1+j1}$$

$$K\angle\theta = \frac{2(-1+j1)}{j2} = \sqrt{2}\angle-45^0$$

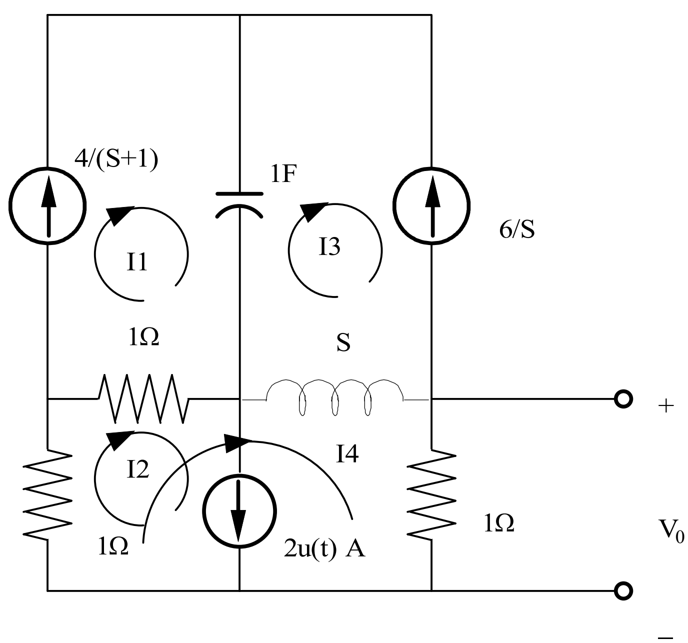
$$V_O(t) = 2\sqrt{2}e^{-t}COS(t-45^0)u(t)V$$

Problem 13.16

Use loop analysis to find Vo(t),t>0 in the network shown in fig.



Suggested Solution



$$I_1 = \frac{4}{S+1}$$

$$I_2 = \frac{2}{S}$$

$$I_3 = \frac{-6}{S}$$

KVL FOR LOOP 4 IS

$$1(I_2 - I_1) + 1(I_2 - I_1 + I_4) + S(-I_3 + I_4) + I_4 = 0$$

$$I_4 = \frac{-(6S^2 + 6S + 4)}{S(S+1)(S+3)}$$

$$1(I_2 - I_1) + 1(I_2 - I_1 + I_4) + S(-I_3 + I_4) + I_4 = 0$$

$$I_4 = \frac{-(6S^2 + 6S + 4)}{S(S+1)(S+3)} = \frac{-4/3}{S} + \frac{2}{S+1} - \frac{20/3}{S+3}$$

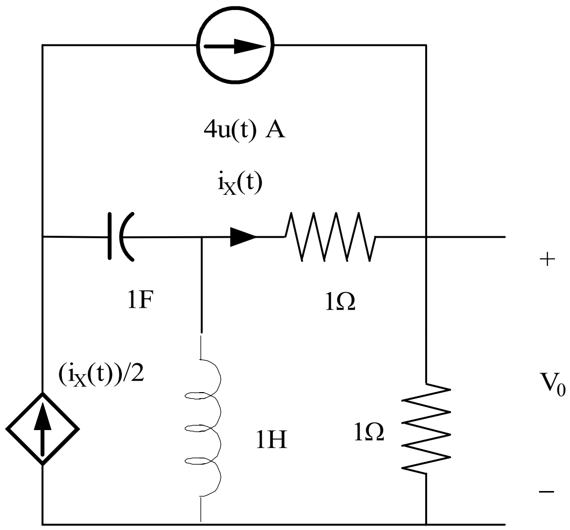
SINCE

$$V_O(t) = (1)I_4$$

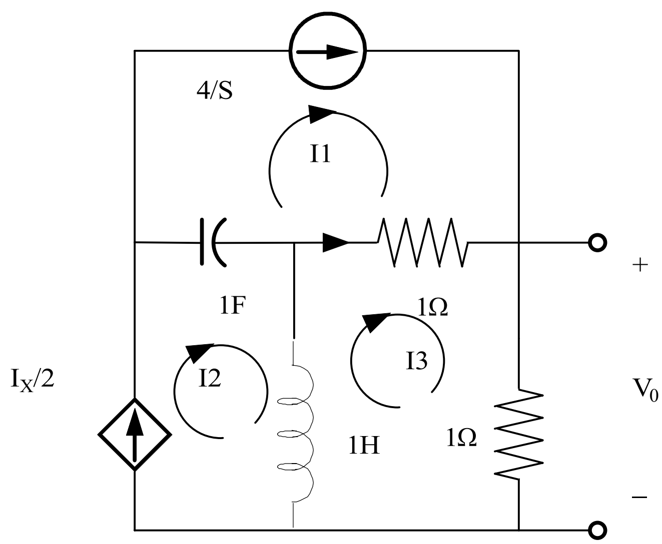
$$V_O(t) = (-\frac{4}{3} + 2e^{-t} - \frac{20}{3}e^{-3t})u(t)V$$

Problem 13.17

Use mesh analysis to find Vo(t),t>0 in the network shown in fig.



Suggested Solution



$$I_1 = \frac{4}{S}$$

$$I_2 = \frac{I_X}{2}$$

$$I_X = I_3 - \frac{4}{S}$$

THEN KVL FOR THE MESH – 3:

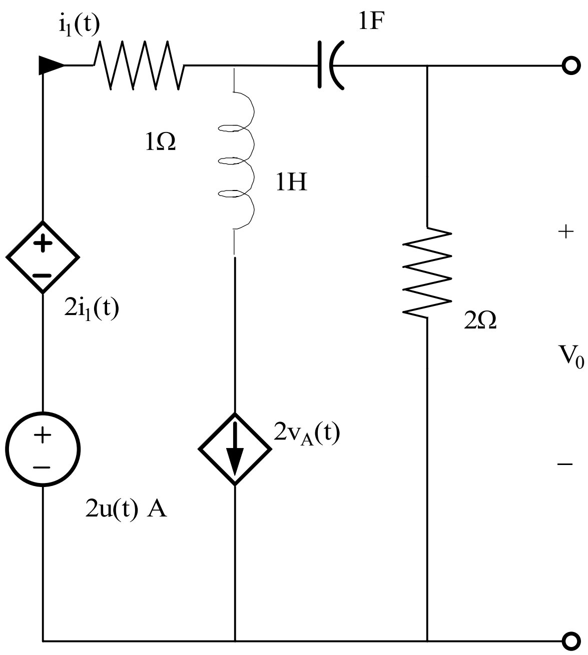
$$S\left[I_3 - 0.5(I_3 - 4/S)\right] + 1\left[I_3 - 4/S\right] + I_3 = 0$$

$$\therefore I_3 = \frac{-4(S-2)}{S(S+4)} \& V_O = I_3(1) = \frac{2}{3} + \frac{-6}{S+4}$$

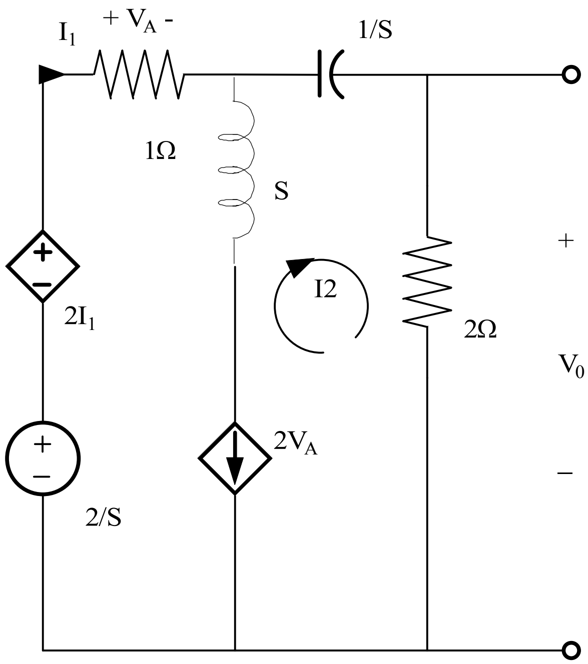
$$V_O(t) = (2 - 6e^{-4t} - \frac{20}{3}e^{-3t})u(t)V$$

Problem 13.18

Use loop equations to find Vo(t),t>0 in the network shown in fig.



Suggested Solution



$$\frac{2}{S} + 2I_1 = I_1(1) + I_2(2 + \frac{1}{S})$$

OR

$$2 = -SI_1 + I_2(2S + 1)$$

$ALSO$

$$I_1 - I_2 = 2VA = 2I_1(1) \Rightarrow I_1 = -I_2$$

$NOW,$

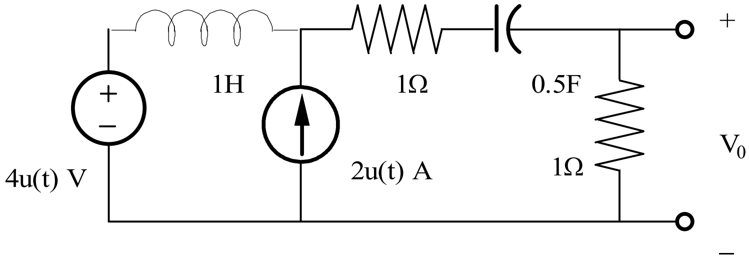
$$2 = I_2[S + 2S + 1] \Rightarrow I_2 = \frac{2}{3S + 1} = \frac{2/3}{S + 1/3}$$

$$V_O = 2I_1 = \frac{2/3}{S + 1/3}$$

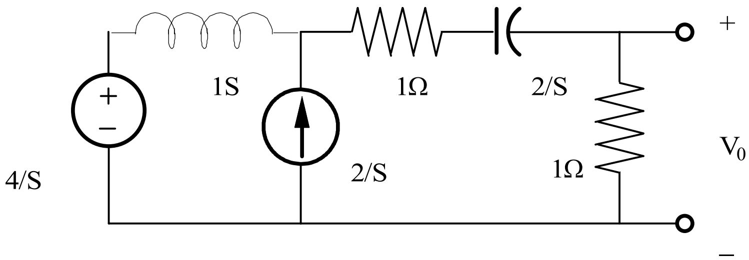
$$V_O(t) = \frac{4}{3}e^{-t/3}u(t)V$$

Problem 13.19

Use superposition to find Vo(t),t>0 in the network shown in fig.



Suggested Solution



FOR THE 4V SOURCE,

$$V_O(S) = \frac{4}{S}\frac{1}{S+2+2/S} = \frac{4}{S^2+2S+2}$$

FOR THE 2A SOURCE:

$$V_O(S) = \frac{2}{S}.S \,||\, (2+2/S) = \frac{2S}{S^2+2S+2}$$

FINALLY, $V_O(S) = \frac{4}{S^2+2S+2} + \frac{2S}{S^2+2S+2} = \frac{2S+4}{S^2+2S+2}$

AS IN P 16.5,

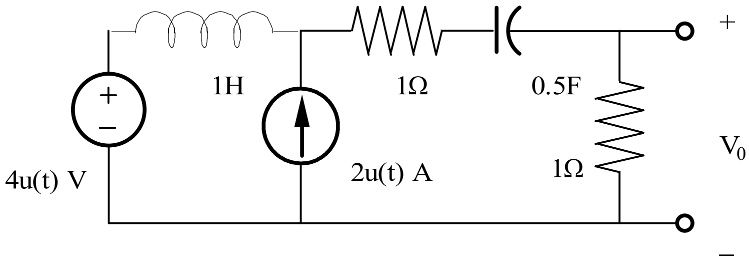
$$V_O(S) = \frac{2S+4}{S^2+2S+2}$$

YIELDS

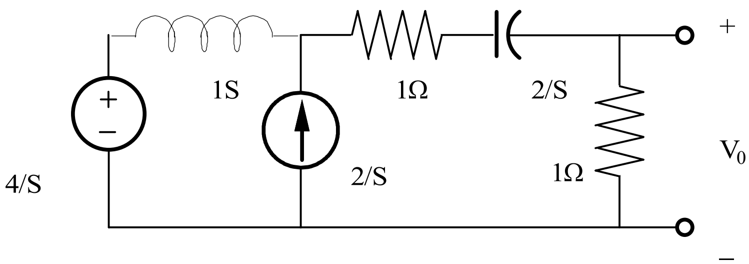
$$V_O(t) = 2\sqrt{2}e^{-t}COS(+45^0)u(t)V$$

Problem 13.20

Use source transformation to solve problem 13.19.



Suggested Solution



$$V_I(S)\left[\frac{1}{S}+\frac{1}{2+2/S}\right]=\frac{4}{S^2}+\frac{2}{S}$$

$$V_I(S)=\frac{S(2S+2)(2S+4)}{S^2(S^2+2S+2)}$$

$$V_O(S)=V_I(S)\left[\frac{1}{2+2/S}\right]$$

$$V_O(S)=\frac{(2S+4)}{(S^2+2S+2)}=\frac{K_1}{S+1-r}+\frac{K_1}{S+1+r}$$

$$K_1=\left.\frac{(2S+4)}{(S+1-r)}\right|_{S=-1+r}=\sqrt{2}\angle{-45^0}$$

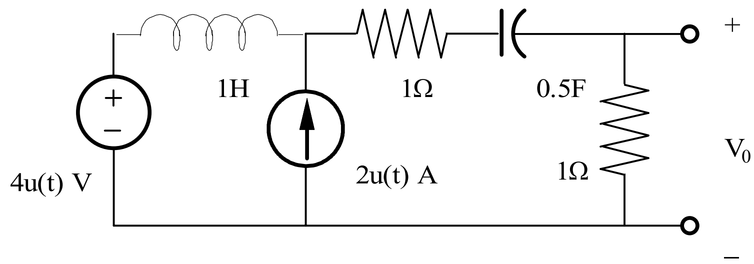
$$K_1=\left.\frac{(2S+4)}{(S+1-r)}\right|_{S=-1+r}=\sqrt{2}\angle{-45^0}$$

$$V_O(S)=\frac{\sqrt{2}\angle{-45^0}}{(S+1-r)}+\frac{\sqrt{2}\angle{-45^0}}{(S+1+r)}$$

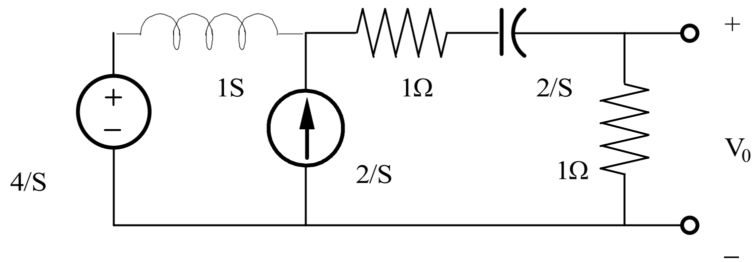
$$V_O(t)=2\sqrt{2}e^{-t}COS(t-45^0)u(t)V$$

Problem 13.21

Use Thevenin's theorem to solve problem 13.19.



Suggested Solution



WITH THE OUTPUT RESISTOR AS A LOAD,

$$V_{OC} = \frac{4}{S} + \frac{2}{S}S = \frac{2S+4}{S}$$

$$Z_{TH} = \frac{2}{S} + 1 + S = \frac{S^2+S+2}{S}$$

BY VOLTAGE DIVISION,

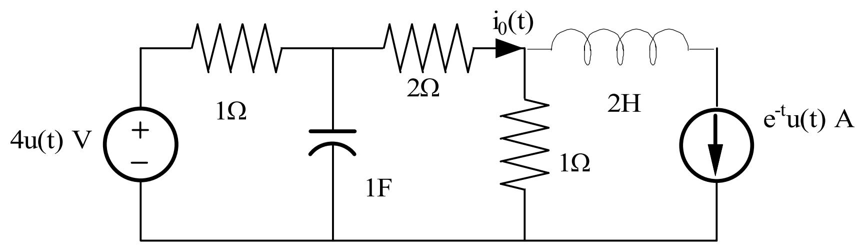
$$V_O(S) = \frac{\frac{2S+4}{S}1}{\frac{S^2+S+2}{S}1} = \frac{2S+4}{S^2+S+2}$$

$$|FV_O(S)| = \frac{2S+4}{S^2+S+2}$$

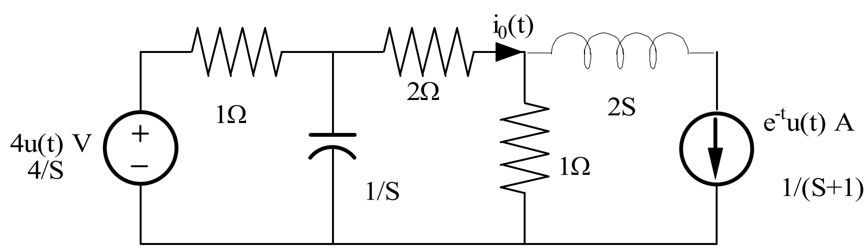
$$V_O(t) = 2\sqrt{2}e^{-t}COS(t-45^0)u(t)$$

Problem 13.22

Use Thevenin's theorem to solve problem 13.11.



Suggested Solution



$$V_{OC} = \frac{4}{S} \cdot \frac{1/S}{1+1/S} + \frac{1}{S+1} = \frac{S+4}{S(S+1)}$$

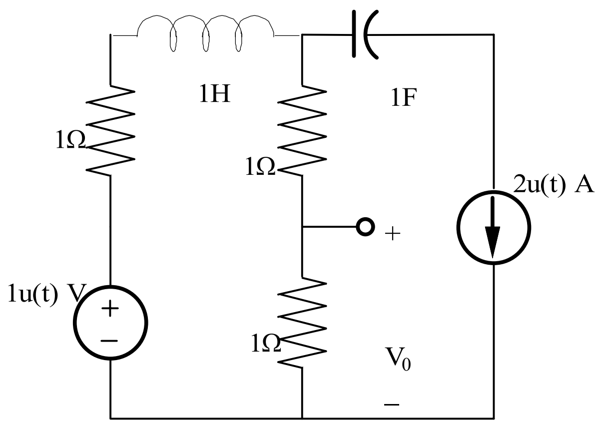
$$Z_{TH} = 1 + 1 \| 1/S = \frac{1/S}{1+1/S} + 1 = \frac{S+2}{S+1}$$

$$I_O(S) = \frac{V_{OC}}{Z_{TH}+2} = \frac{S+4}{S(S+1)} \frac{S+1}{3S+4} = \frac{S+4}{S(3S+4)}$$

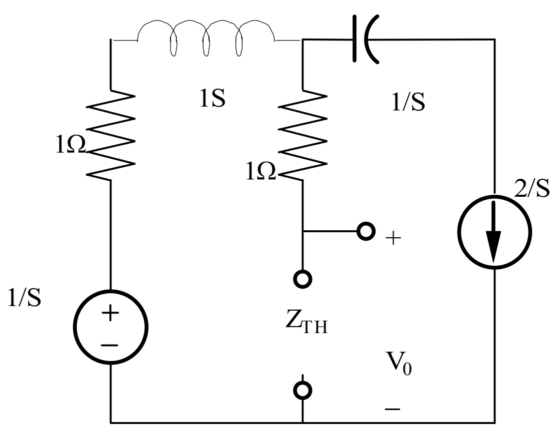
$$i_O(t) = [1 - \frac{2}{3} e^{-4/3t}]u(t)$$

Problem 13.23

Use Thevenin's theorem to find Vo(t),t>0 in the network.



Suggested Solution



Using superposition

$$V_{OC} = \frac{1}{S} - \frac{2}{S}(2s+1) = -\frac{(4s+1)}{s}$$

$$Z_{TH} = 2s + 2$$

$$V_O = V_{OC}\left(\frac{1}{Z_{TH}+1}\right) = \frac{-(4S+1)}{(2S+3)S} = -\frac{(2S+1/2)}{S(S+3/2)}$$

$$V_O = \frac{A}{S} + \frac{B}{S+3/2}$$

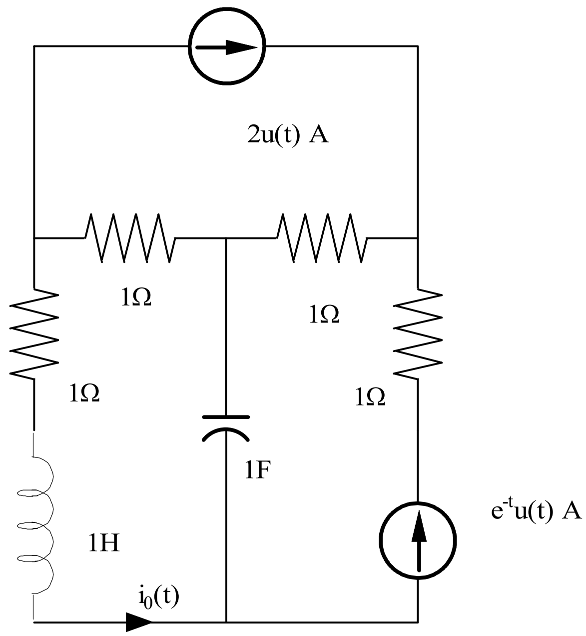
$$A = -1/3$$

$$B = -5/3$$

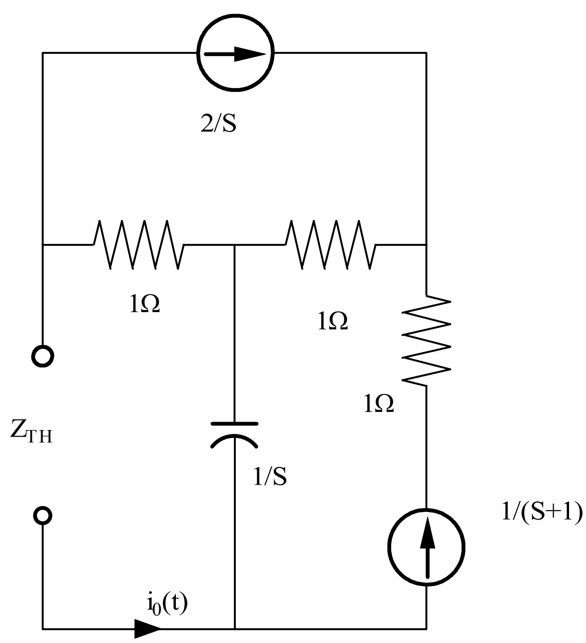
$$V_O(t) = [-\frac{1}{3} - \frac{S}{3}e^{-3/2t}]u(t)V$$

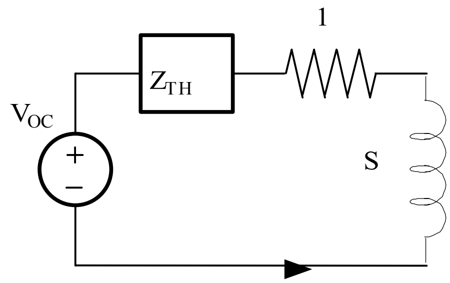
Problem 13.24

Use Thevenin's theorem to find Io(t),t>0 in the circuit shown.



Suggested Solution





$$V_{OC} = \left(\frac{1}{S+1}\right)\left(\frac{1}{S}\right) - \left(\frac{2}{S}\right)(1) = \frac{-(2S+1)}{S(S+1)}$$

$$Z_{TH} = 1 + \frac{1}{S} = \frac{S+1}{S}$$

$$I_O = \frac{-V_{OC}}{1+S+Z_{TH}} = \frac{\frac{(2S+1)}{S(S+1)}}{1+S+\frac{S+1}{S}} = \frac{(2S+1)}{(S+1)(S+1+S(S+1))}$$

$$I_O = \frac{(2S+1)}{(S+1)^3} = \frac{A}{(S+1)^3} + \frac{B}{(S+1)^2} + \frac{C}{(S+1)}$$

$A = -1$

$LET, S = 0, I_O(O) = 1 = A + B + C \Rightarrow B + C = 2$

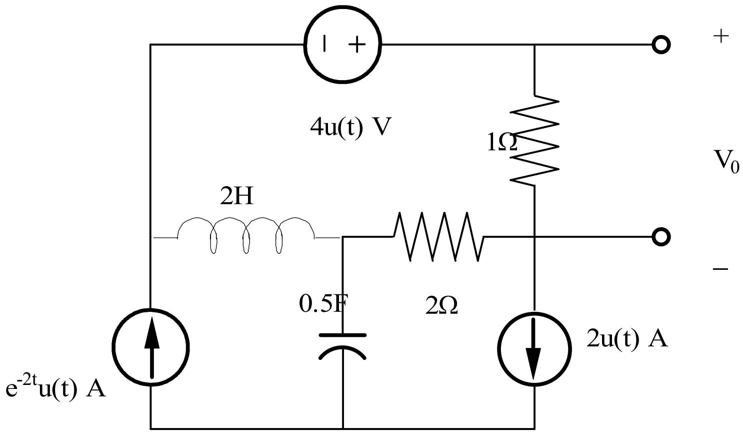
$LET, S = -0.5, I_O(-0.5) = 0 = 8A + 4B + 2C \Rightarrow 4B + 2C = 8$

SOLUTIONS ARE B=Z AND C=0 SO, $I_O = -\frac{1}{(S+1)^3} + \frac{2}{(S+1)^2}$

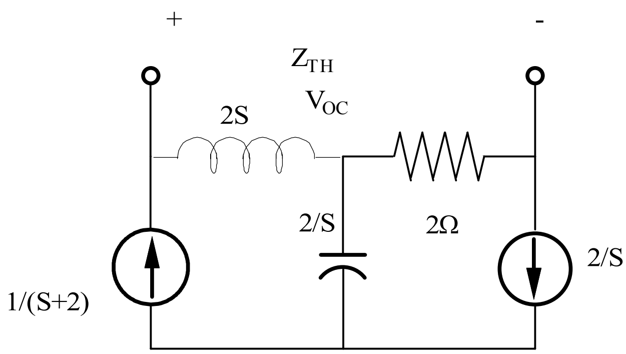
$$i_O(t) = (2te^{-t} - \frac{t^2}{2}e^{-t})u(t)A$$

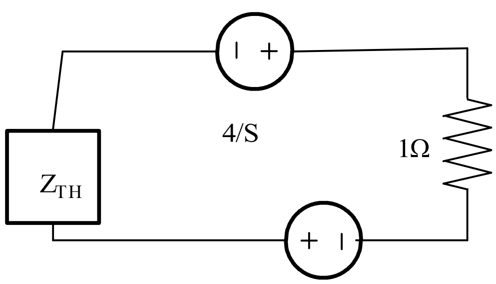
Problem 13.25

Use Thevenin's theorem to find Vo(t),t>0 in the network.



Suggested Solution





USING SUPERPOSITION,

$$\mathrm{V_{OC}} = \left(\frac{1}{\mathrm{S+2}}\right)2S + \left(\frac{2}{S}\right)^2 = \left(\frac{\mathrm{2S}}{\mathrm{S+2}}\right) + \frac{4}{S} = \frac{2S^2 + 4S + 8}{S(S+2)}$$

$$Z_{TH} = 2S + 2$$

$$V_O = \frac{\mathrm{V_{OC}} + 4/S}{1 + Z_{TH}} = \frac{2S^2 + 8S + 16}{S(S+2)(2S+3)}$$

$$V_O = \frac{2S^2 + 8S + 16}{S(S+2)(2S+3)} = \frac{A}{S} + \frac{B}{S+2} + \frac{C}{C+1.5}$$

$$A = 8/3$$

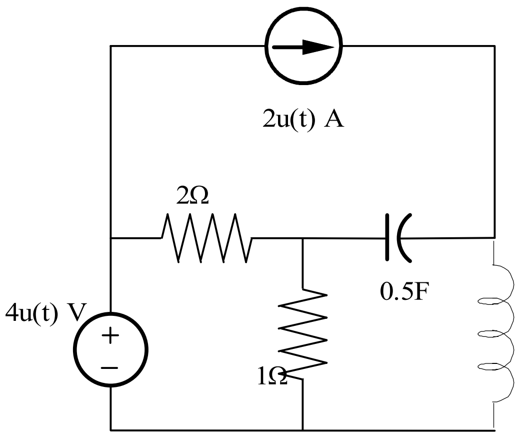
$$B = 4$$

$$C = -17/3$$

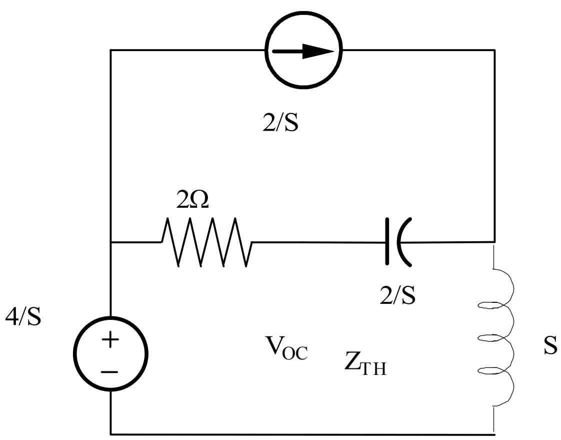
$$V_O(t) = (\frac{8}{3} + 4e^{-2t} - \frac{17}{3}e^{-3t/2})u(t)V$$

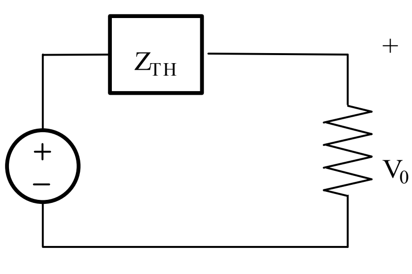
Problem 13.26

Use Thevenin's theorem to find Vo(t),t>0 in the network.



Suggested Solution





USING SUPERPOSITION,

$$V_{OC} = \frac{4}{S}\left[\frac{S+2/S}{S+2+2/S}\right] + \frac{2}{S}\left[\frac{S}{S+2+2/S}\right]2$$

$$V_{OC} = \frac{8S^2+8}{S(S+1-j1)(S+1+j1)}$$

$$V_O = \frac{V_{OC}(1)}{1+Z_{TH}} = \frac{8S+8/S}{S^2+2S+2+2S^2+4}$$

$$V_O = \frac{(8/3)(S^2+1)}{S(S^2+(2/3)S+2)}$$

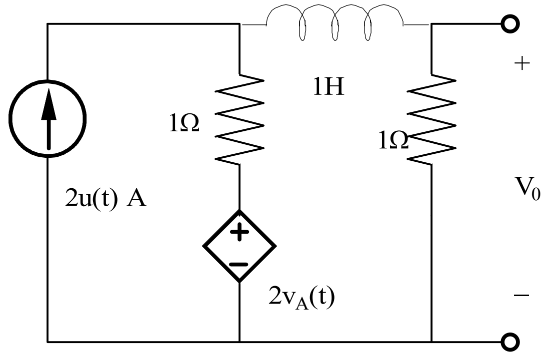
$$A = 4/3$$

$$K\underline{|\theta} = 1.273\underline{|10.05^0}$$

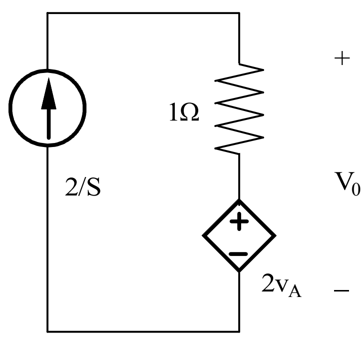
$$V_O(t) = (\frac{4}{3} + 2.546e^{-t/3}COS(\sqrt{17t} + 10.05^0))V$$

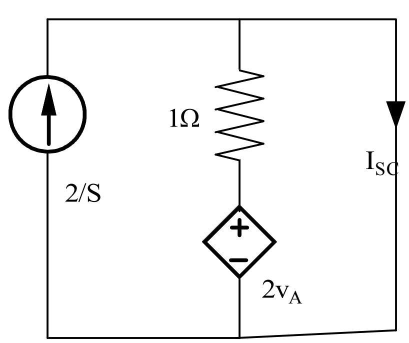
Problem 13.27

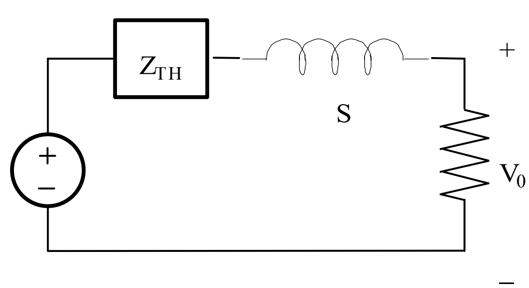
Use Thevenin's theorem to find Vo(t),t>0 in the network.



Suggested Solution



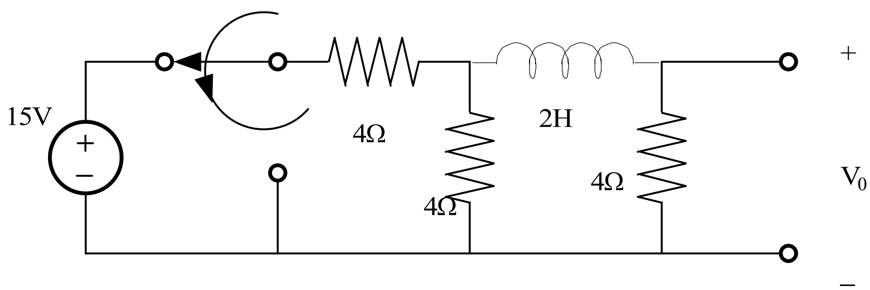




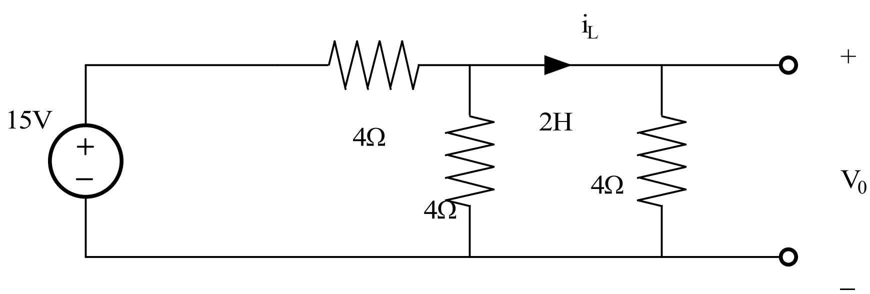
$$V_{OC} = 3V_A$$
$$2/S = V_A$$
$SO,$
$$V_{OC} = 6/S$$
$$I_{SC} = 2/S$$
$$Z_{TH} = V_{OC}/I_{SC} = 3\Omega$$
$$V_O = \frac{V_{OC}}{S+1+Z_{TH}} = \frac{6/S}{S+4} = \frac{A}{S} + \frac{B}{S+4}$$
$$A = 1.5$$
$$B = -1.5$$
$$V_O(t) = 1.5(1-e^{-4t})u(t)V$$

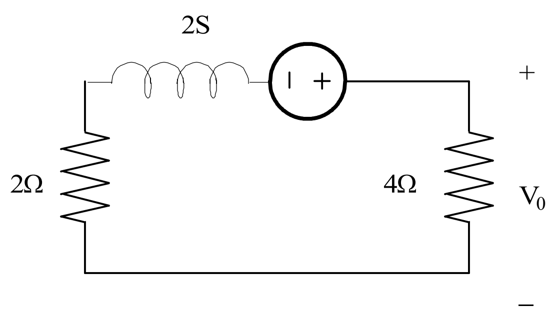
Problem 13.28

Use laplace transform to find Vo(t),t>0 in the network.assume that the circuit has reached steady state at t=0-.



Suggested Solution





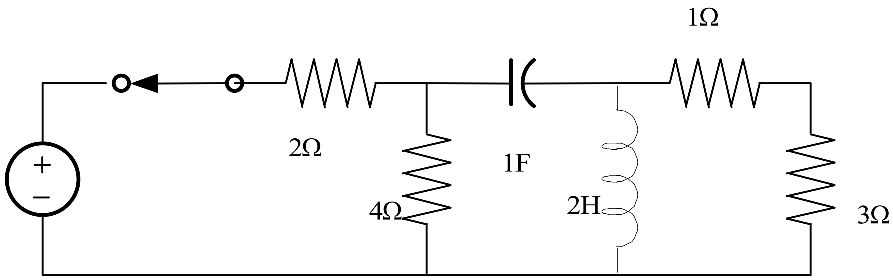
$$V_O = 2.5\left(\frac{4}{2S+6}\right)$$

$$V_O = \left(\frac{5}{S+3}\right)$$

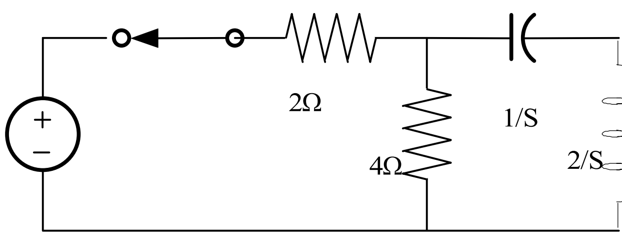
$$V_O(t) = 5(e^{-3t})u(t)V$$

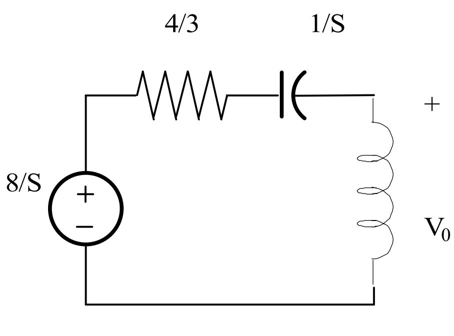
Problem 13.29

Find Io, (t),t>0 in the network.



Suggested Solution





SOURCE TRANSFORMATIONS:

$$i_2(0^-) = 0$$

$$V_C(0^-) = 0$$

$$V_{OC} = \frac{8}{S}\left[\frac{2S}{2S+4/3+1/S}\right] = \frac{16S}{2S^2+4S/3+1} = \frac{8S}{S^2+2S/3+1/2}$$

$$Z_{TH} = 2S \,||\, (4/3+1/S) = \frac{(8)S^2}{6S^2+4S+3}\Omega$$

$$I_{OC} = \frac{V_{OC}}{Z_{TH}+} = \frac{\frac{(48)S^2}{6S^2+4S+3}}{\frac{(8)S^2+6S}{6S^2+4S+3}+4} = \frac{48S^2}{8S^2+6S+24S^2+16S+12}$$

$$I_O = \frac{(48)S}{52S^2+22S+12} = \frac{1.5S}{S^2+\frac{11}{16}S+\frac{6}{16}}$$

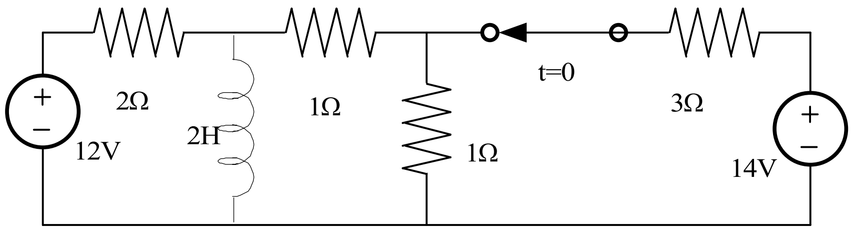
$$I_O = \frac{K\underline{|\theta}}{S+\frac{11}{32}-j\frac{\sqrt{263}}{32}} + \frac{K\underline{|-\theta}}{S+\frac{11}{32}+j\frac{\sqrt{263}}{32}}$$

$$K\underline{|\theta} = 0.906\underline{|34.15^0}$$

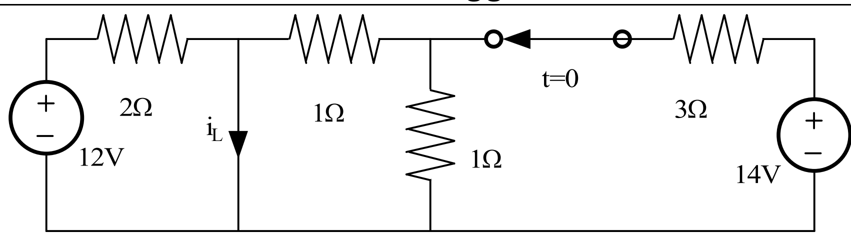
$$i_O(t) = 1.812e^{-0.34t}COS(0.51t+34.15^0)u(t)A$$

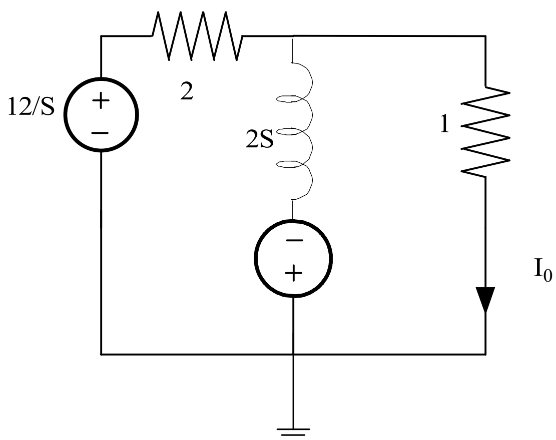
Problem 13.30

find Io(t),t>0 in the network.



Suggested Solution





FOR t= 0^-

$$i_L = \frac{12}{Z} + \frac{14}{3.5}\left(\frac{1}{1+1}\right) = 8A$$

$$\frac{\frac{12}{S} - V}{Z} = \frac{V+16}{2S} + \frac{V}{2} \Rightarrow 12 - SV = V + 16 + S$$

SO,

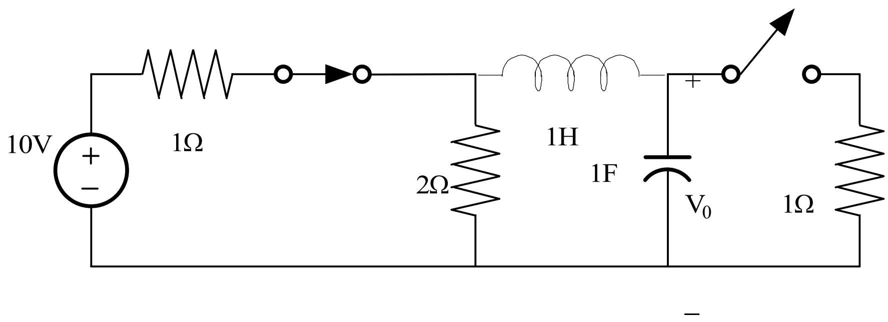
$$V = \frac{-4}{2S+1}$$

$$I_O = \frac{V}{2} = \frac{-1}{S+0.5}$$

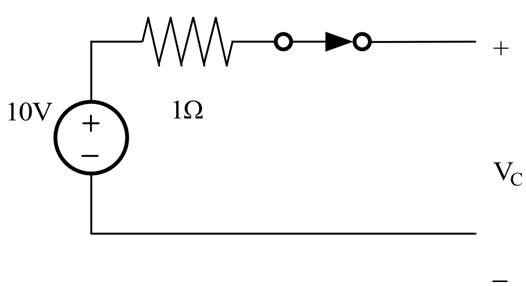
$$i_O(t) = -e^{-0.5t}u(t)A$$

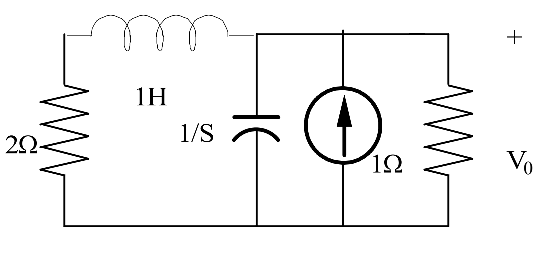
Problem 13.31

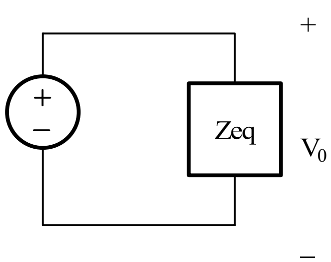
Use Thevenin's theorem to find Vo(t),t>0 in the network.



Suggested Solution







for t=0^-

$i_L=0$

$V_C=\frac{20}{3}V$

$$Z_{eq}=1\,||\,\frac{1}{S}\,||\,(S+2)=\frac{1}{S+1}\,||\,S+2=\frac{(S+2)/(S+1)}{S+2+\frac{1}{S+1}}$$

$$Z_{eq}=\frac{(S+2)}{S^2+3S+3}$$

$$V_O=\frac{20}{3}\frac{(S+2)}{\left(S+\frac{3}{2}S-j\frac{\sqrt{3}}{2}\right)\left(S+\frac{3}{2}S+j\frac{\sqrt{3}}{2}\right)}$$

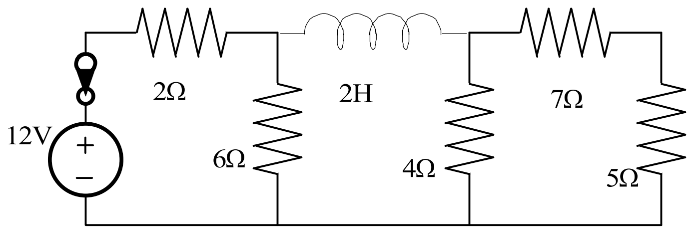
$$V_O=\frac{k\underline{|\theta}}{s+1.5-j\frac{\sqrt{3}}{2}}+\frac{k\underline{|-\theta}}{s+1.5+j\frac{\sqrt{3}}{2}}$$

$$k\underline{|\theta}=\frac{20}{3}\frac{-1.5+j\frac{\sqrt{3}}{2}+2}{j\sqrt{3}}=3.85\underline{|-30^0}$$

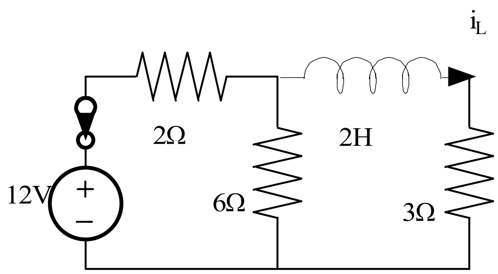
$$V_O(t)=7.7e^{-3t/2}COS(\frac{\sqrt{3}t}{2}-30^0)V$$

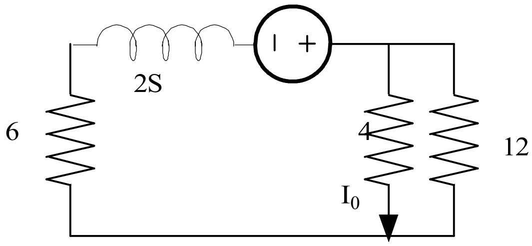
Problem 13.32

find Io(t),t>0 in the network.



Suggested Solution





$$i_L(0^-) = 2A$$

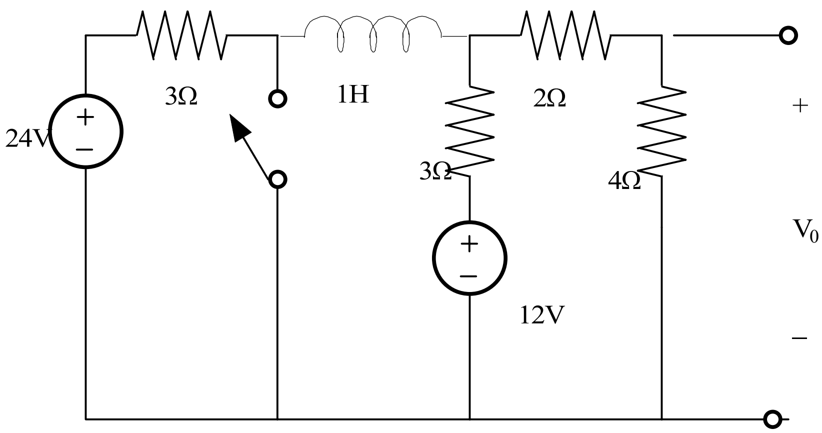
THE TRANSFORMED NETWORK FOR t>0 IS

$$I_O = \frac{4}{2S+9}\left(\frac{12}{4+12}\right) = \frac{1.5}{S+9/2}$$

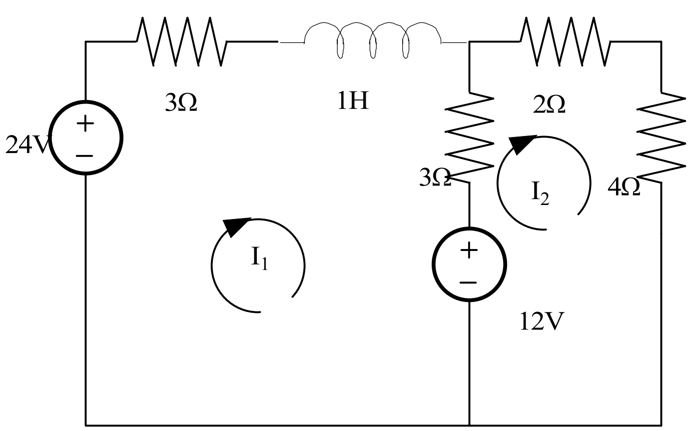
$$i_O(t) = 1.5e^{-9t/2}u(t)A$$

Problem 13.33

find Vo(t),t>0 in the network.



Suggested Solution



$$6I_1 - 3I_2 = 12$$

$$-3I_1 + 9I_2 = 12$$

$$I_1 = \frac{\begin{vmatrix} 12 & -3 \\ 12 & 9 \end{vmatrix}}{\begin{vmatrix} 6 & -3 \\ -3 & 9 \end{vmatrix}} = \frac{12(9+3)}{54-9} = 144/45$$

$$i_L(O^-) = 3.2A$$

$$\frac{V_1 - 6.4}{2S} + \frac{V_1 + 12/S}{3} + \frac{V_1}{6} = 0$$

SOLVING FOR V_1 YIELDS:

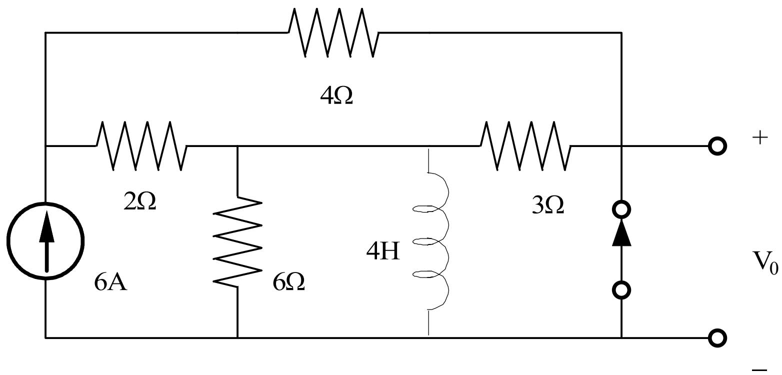
$$V_1 = \frac{14.4}{S+1}$$

$$V_0 = \frac{9.6}{S+1}$$

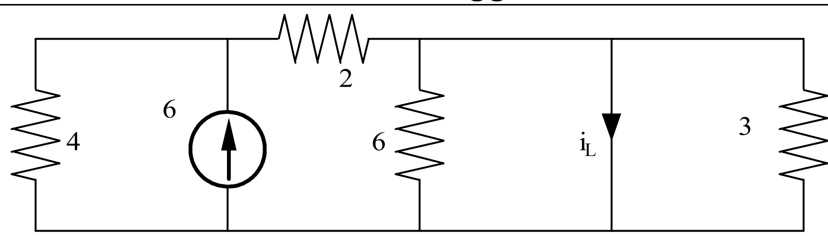
$$V_O(t) = 9.6e^{-t}u(t)V$$

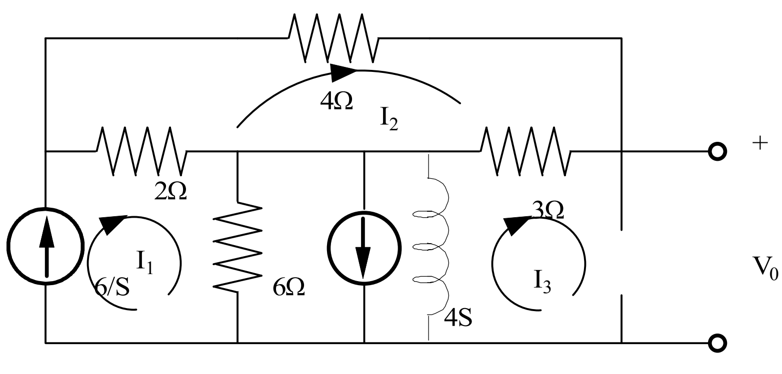
Problem 13.34

find Vo(t),t>0 in the network.



Suggested Solution





$$\frac{24s}{4s+6}=\frac{12s}{2s+3}$$

loop analysis: I_1=6/S, I_3=4/S, $9I_2$-$2I_1$=0 $\Rightarrow$ $I_2=\frac{4/3}{S}$

$$V_O=3I_2+(I_1-I_3)\left[\frac{12S}{2S+3}\right]=\frac{4}{S}+\frac{2}{S}\left(\frac{12S}{2S+3}\right)=\frac{16S+6}{S(S+1.5)}$$

$$V_O=\frac{A}{S}+\frac{B}{S+1.5}$$

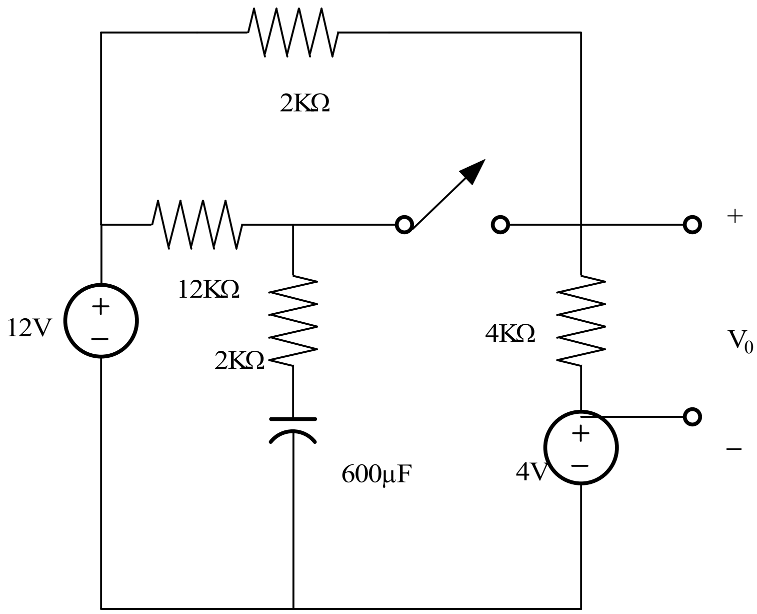
$$A=4$$

$$B=12$$

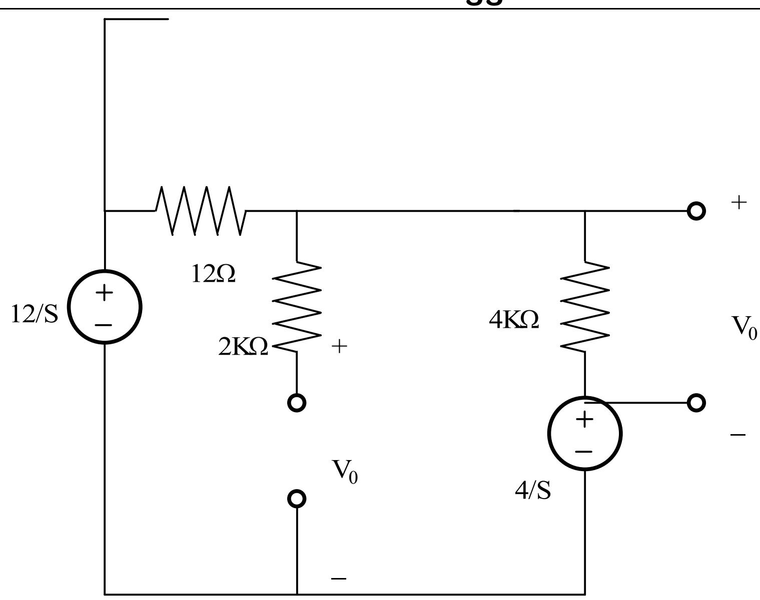
$$V_O(t)=(4+12e^{-3t/2})u(t)V$$

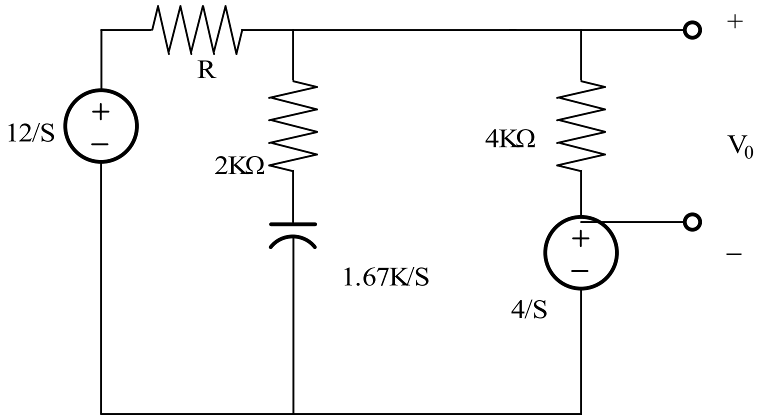
Problem 13.35

find Vo(t),t>0 in the network.



Suggested Solution





FOR t>0

R=6K || 12K=4KΩ

$V_1 = V_O + 4/S$

KCL:

$$\frac{\frac{12}{S} - V_1}{R} = \frac{V_1 - 12/S}{2K + \frac{1.67K}{S}} + \frac{V_O}{4K}$$

$$\frac{\frac{8}{S} - V_O}{4K} = \frac{V_O - 8/S}{2K + \frac{1.67K}{S}} + \frac{V_O}{4K} \Rightarrow 8 = 2SV_O + \frac{4S^2V_O - 32S}{2 + \frac{5}{3S}}$$

OR,

$$8 = 2SV_O + \frac{4S^2V_O - 32S}{2S + \frac{5}{3}} \Rightarrow 8 = 2SV_O + \frac{12SV_O - 96S}{6S + 5}$$

$$V_O = \frac{6S + 5/3}{S(S + 5/12)} = \frac{A}{S} + \frac{B}{S + 5/12}$$

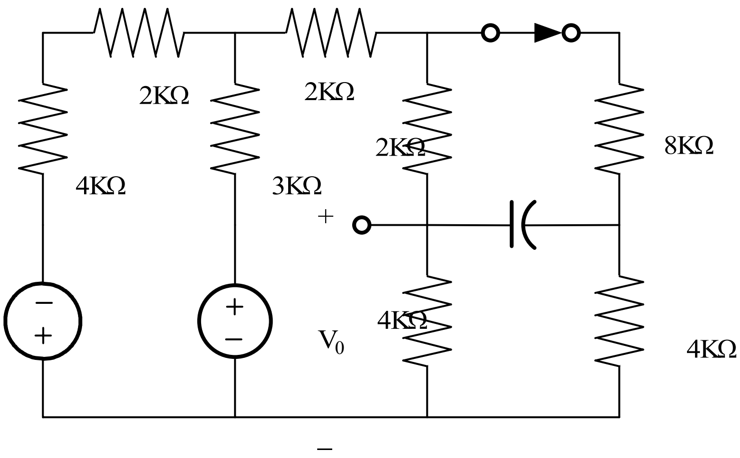
$A = 4$

$B = 2$

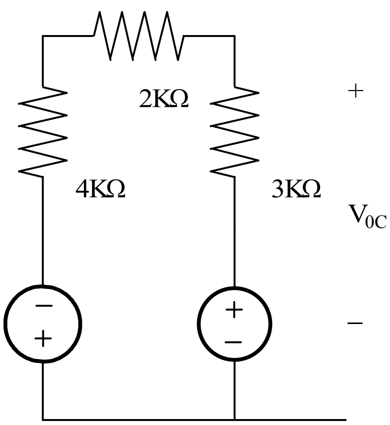
$V_O(t) = (4 + 2e^{-5t/12})u(t)V$

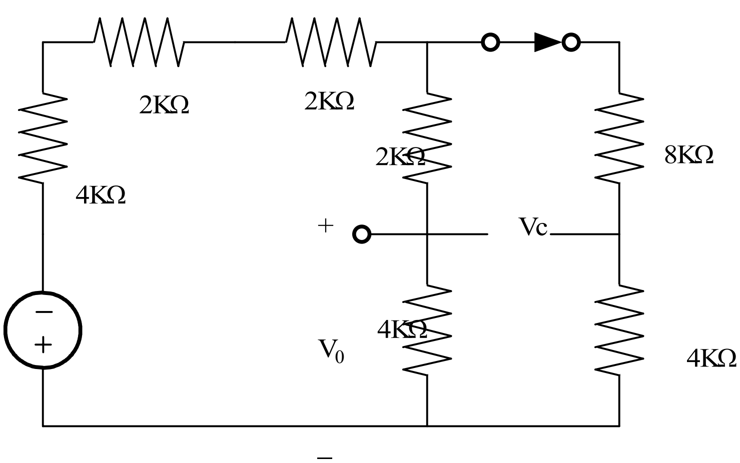
Problem 13.36

find Vo(t),t>0 in the network.



Suggested Solution





$$V_{OC} = 24\frac{6K}{9K} - 12\frac{3K}{9}$$

$$V_{OC} = 12V$$

$$R_{TH} = 3K \,||\, 6K = 2K$$

$$V = 12(4K/8K) = 6V$$

$$V_C = V\{(4/6) - (4/12)\}$$

$$V_C = 2V$$

NODAL ANALYSIS:

$$\frac{\frac{12}{\text{S}} - V_O}{6K} = \frac{V_O}{4K} + \frac{V_O - 2/S}{4K + 10K/S}$$

$$\frac{12 + SV_O}{6} = \frac{SV_O}{4} + \frac{S^2V_O - 2S}{4S + 10}$$

AND

$$24 - 2SV_O = \frac{6S^2V_O - 12S}{2S + S} + 3SV_O \Rightarrow -5SV_O + 24$$

$$6S^2V_O - 12S = -10S^2V_O - 25SV_O + 48S + 120$$

$$V_O[16S2 + 25S] = 60S \Rightarrow V_O = \frac{A}{S} + \frac{B}{S + 24/16}$$

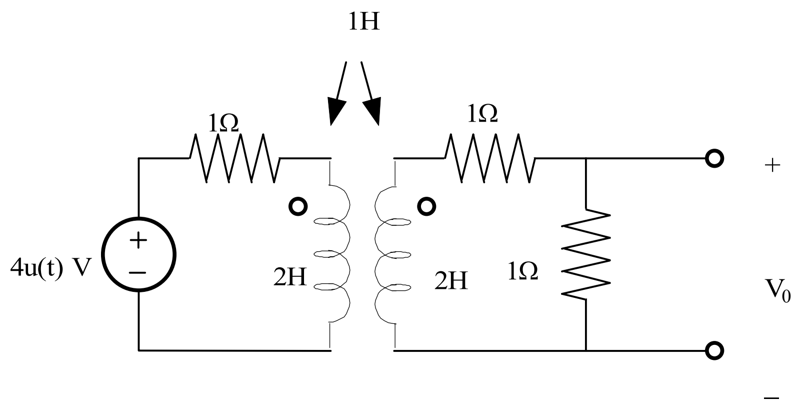
$$A = 24/5$$

$$B = -21/20$$

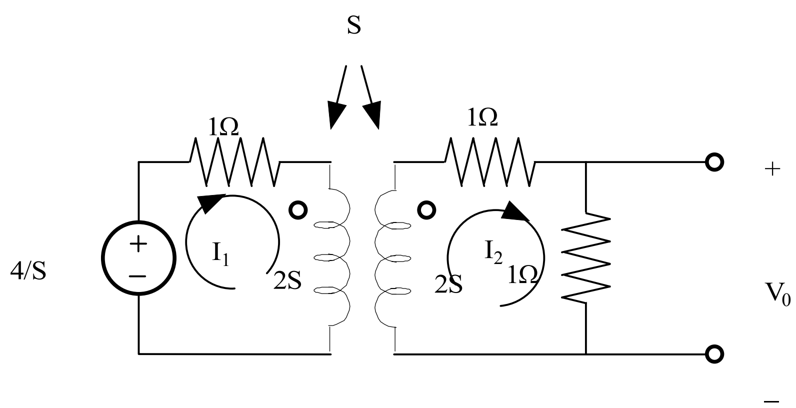
$$V_O(t) = (\frac{24}{5} - \frac{21}{20}e^{-25t/16})u(t)V$$

Problem 13.37

find Vo(t),t>0 in the network.



Suggested Solution



$$\frac{4}{S} = I_1(2S+1) - SI_1$$

$$0 = -SI_1 + I_2(2S+2) \Rightarrow I_1 = I_2((2S+2)/S)$$

SOLVING THE ABOVE TWO EQUATIONS WE GET:

$$\frac{4}{S} = I_2\left[\frac{(2S+1)(2S+2)}{S} - S\right] \Rightarrow I_2 = \frac{4}{3S^2+6S+2} \,\&\, V_O = (1)I_2$$

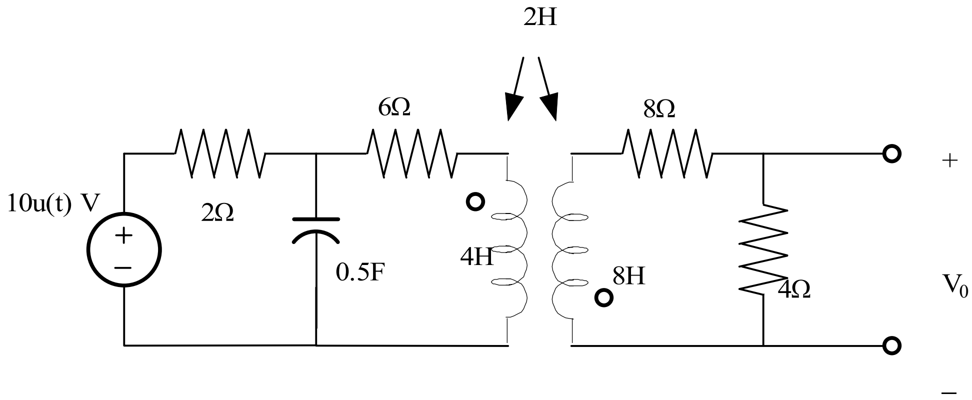
$$V_O = \frac{4}{3S^2+6S+2} = \frac{A}{S+0.42} + \frac{B}{S+1.58}$$

$$A = 1.15$$

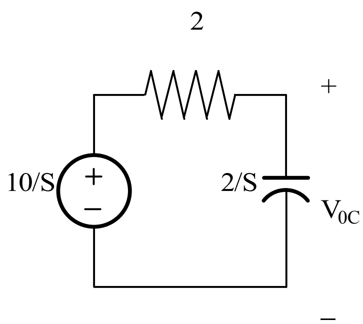
$$B = -1.15, VO(t) = 1.15[e^{-0.42t} - e^{-1.58t}]u(t)V$$

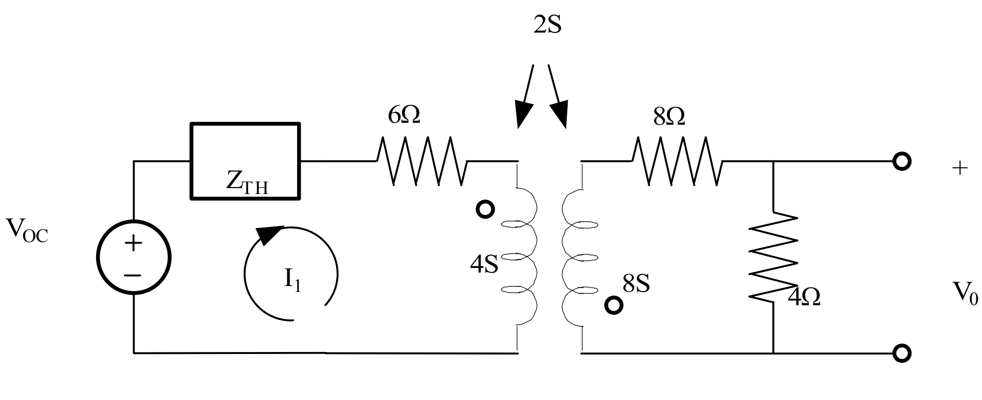
Problem 13.38

find Vo(t),t>0 in the network.



Suggested Solution





$VOC = I_1(Z_{TH} + 6 + 4S) + 2SI_2$ & $0 = 2SI_1 + I_2(8S + 12)$

FROM THE SECOND LOOP EQUATION, $I_1 = \frac{(-4S+6)}{S} I_2$, AND,

$$V_{OC} = \frac{10}{S(S+1)} = I_1[2S - \frac{4S+6}{S}(\frac{2}{S+1} + 6 + 4S)]$$

$$10 = I_2[2S^3 + 2S^2 - (4S+6)(2 + 6S + 6 + 4S^2 + 4S)]$$

$$I_2 = \frac{-5/7}{S^3 + 31/7S^2 + 46/7S + 24}$$

$$V_O = 4I_2$$

FROM A CRC MATH HANDBOOK,

$$V_O = \frac{A}{S+0.513} + \frac{K\underline{|\theta}}{S + 0.257 - j1/2} + \frac{K\underline{|-\theta}}{S + 0.257 + j1/2}$$

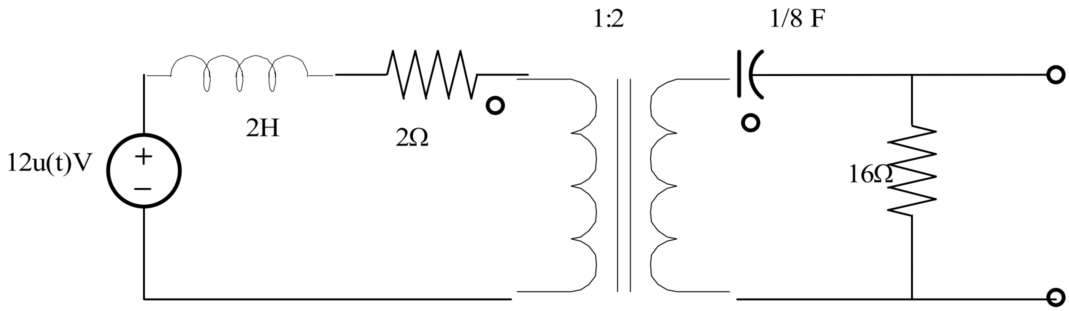
$$A = -9.05$$

$$K\underline{|\theta} = 5.08\underline{|27.20^0}$$

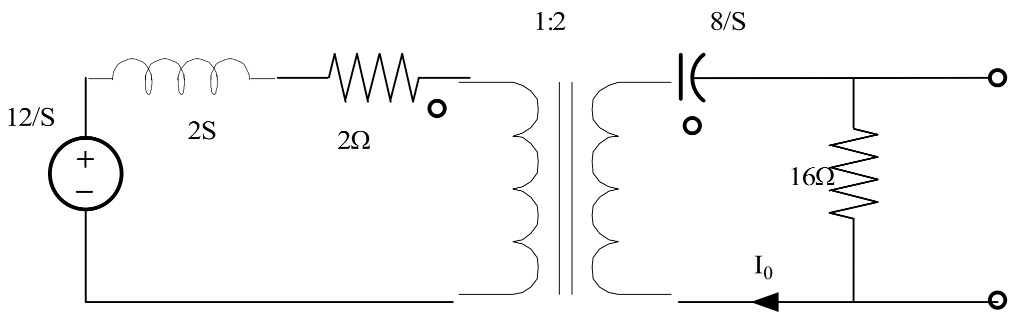
$$V_O(t) = -9.05e^{-0.513t} + 10.16e^{-0.257t}COS(0.5t + 27.20)V$$

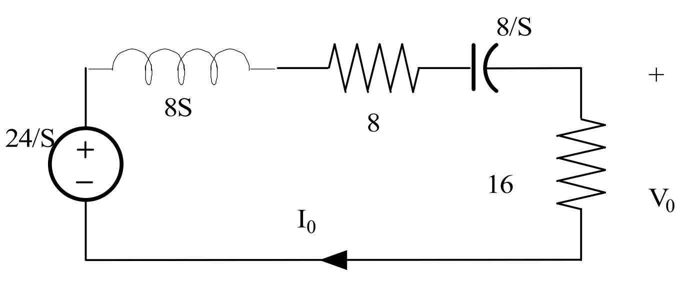
Problem 13.39

find Vo(t),t>0 in the network.



Suggested Solution





REFER PRIMARY TO SECONDARY , N=2

$$\frac{12}{\mathrm{S}} \rightarrow N(\frac{12}{\mathrm{S}}) = \frac{24}{\mathrm{S}}$$

$$2S + 2 \rightarrow (2S + 2)N^2 = 8S + 8$$

$$I_O = \frac{\frac{24}{\mathrm{S}}}{8S + 24 + 8/S} = \frac{3}{S^2 + 3S + 1}$$

$$I_O = \frac{A}{S + 0.38} + \frac{B}{S + 2.62}$$

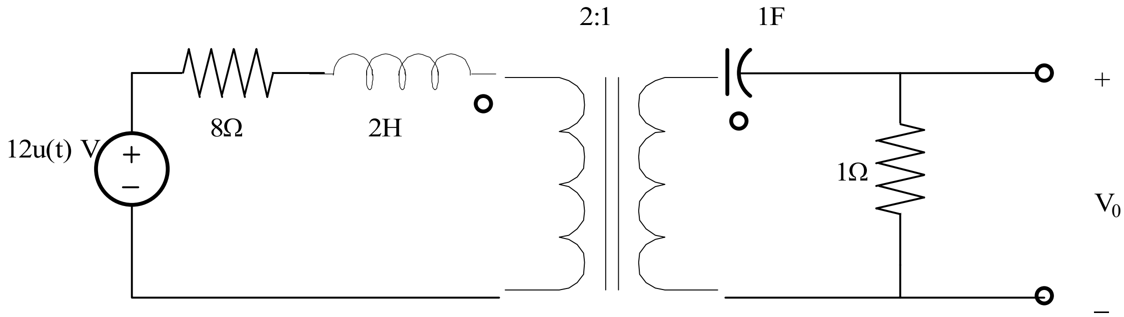
$$A = 1.34$$

$$B = -1.34$$

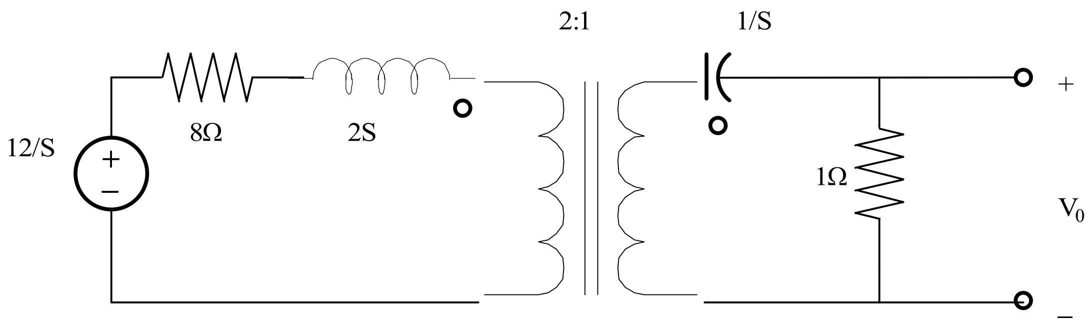
$$i_O(t) = 1.34[e^{-0.38t} - e^{-2.62t}]u(t)V$$

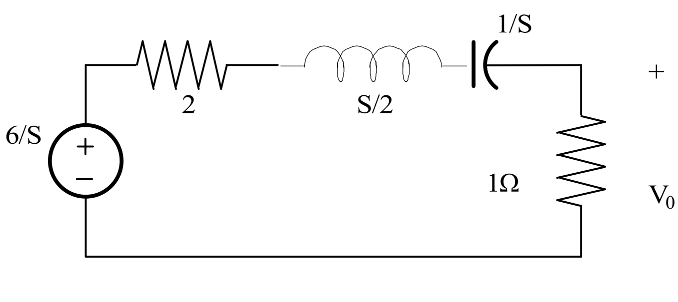
Problem 13.40

find Vo(t),t>0 in the network.



Suggested Solution





REFER PRIMARY TO SECONDARY , N=2

$$\frac{12}{\mathrm{S}} \rightarrow N(\frac{12}{\mathrm{S}}) = \frac{6}{\mathrm{S}}$$

$$2S + B \rightarrow (2S + 8)N^2 = 0.5S + 2$$

$$V_O = \frac{\frac{6}{\mathrm{S}}}{0.5S + 3 + 1/S} = \frac{12}{S^2 + 6S + 2}$$

$$V_O = \frac{A}{S + 0.35} + \frac{B}{S + 5.65}$$

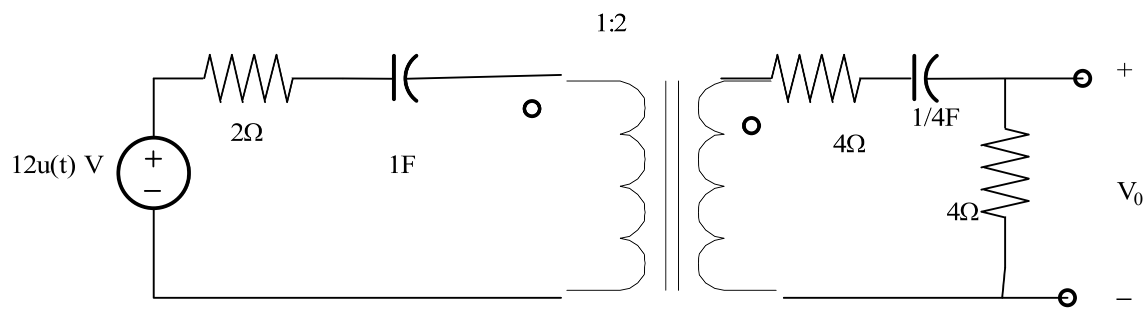
$$A = 2.31$$

$$B = -2.31$$

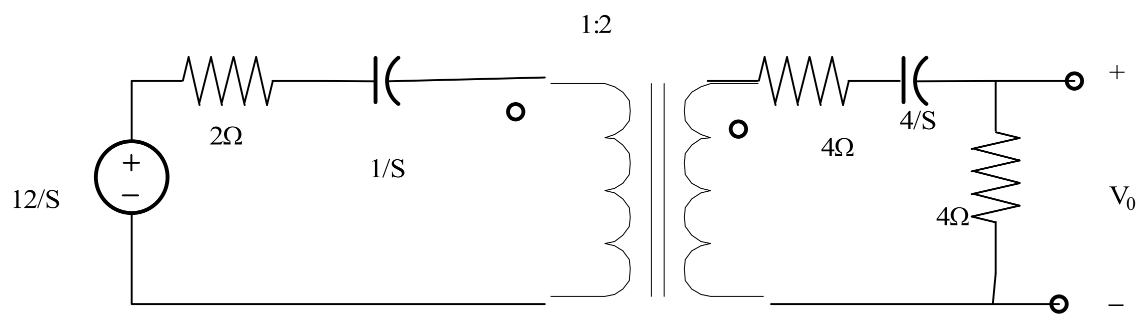
$$V_O(t) = 2.31[e^{-0.35t} - e^{-5.65t}]u(t)V$$

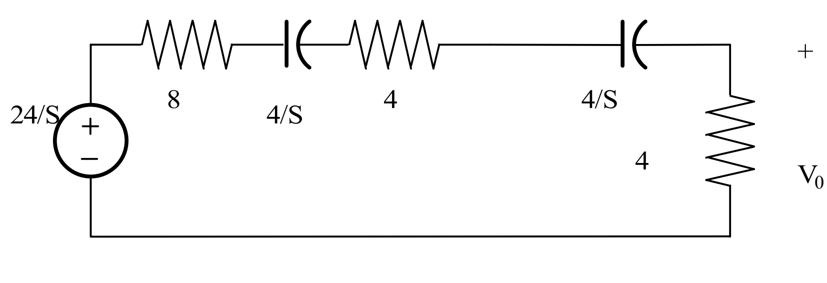
Problem 13.41

find Vo(t),t>0 in the network.



Suggested Solution





REFER PRIMARY TO SECONDARY , N=2

$$\frac{12}{S} \to N(\frac{12}{S}) = \frac{24}{S}$$

$$1/S + 2 \to (1/S + 2)N^2 = 4/S + 8$$

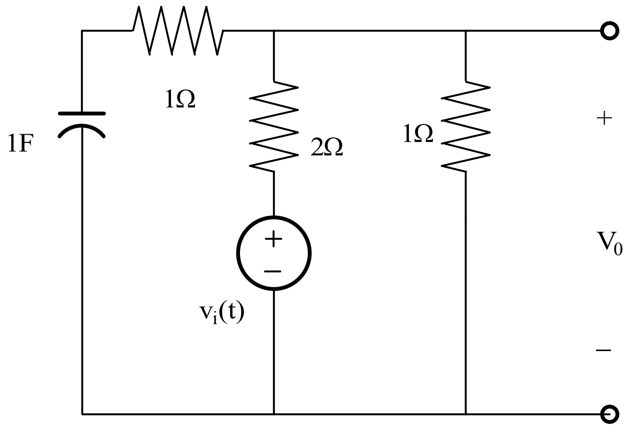
$$V_O = \frac{24}{S}(\frac{4}{16 + 8/S})$$

$$V_O = \frac{96}{16S + 8} + \frac{6}{S + 0.5}$$

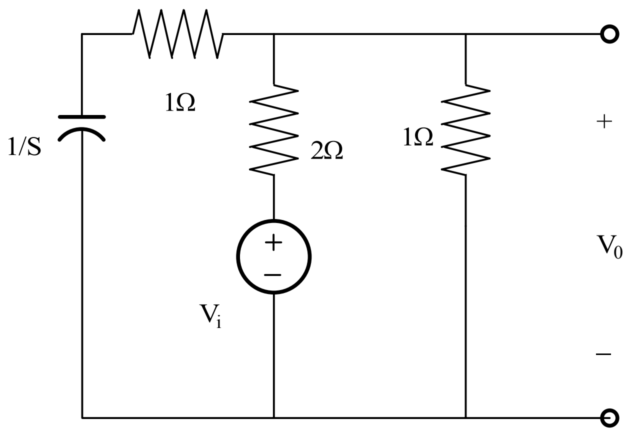
$$V_O(t) = e^{-t/2}V$$

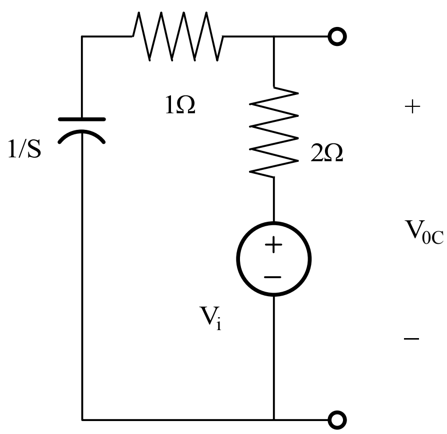
Problem 13.42

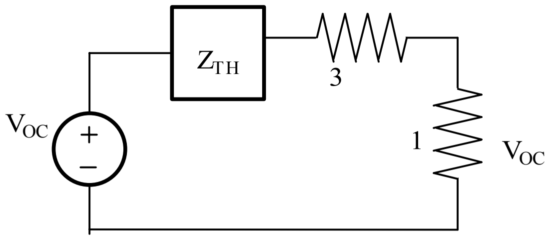
Determine the output voltage Vo(t) in the network in fig if the input is given by the source in fig.



Suggested Solution







Therevin's EQ:

$$V_{OC} = V_i\left[1 - \frac{2}{3+1/S}\right] = V_i\left(\frac{S+1}{3S+1}\right)$$

NEW CIRCUIT:

$$V_O = V_{OC}\,\frac{1}{4+Z_{TH}}$$

$$V_O = V_i\left(\frac{S+1}{3S+1}\right)\left(\frac{3S+1}{12S+4+2S+2}\right)$$

$$V_O = \frac{(S+1)}{(14S+6)} \Rightarrow V_O = \frac{6(S+1)}{S(14S+6)}(1-e^{-S})$$

$$V_O = \left[\frac{A}{S} + \frac{B}{S+3/7}\right](1-e^{-S})$$

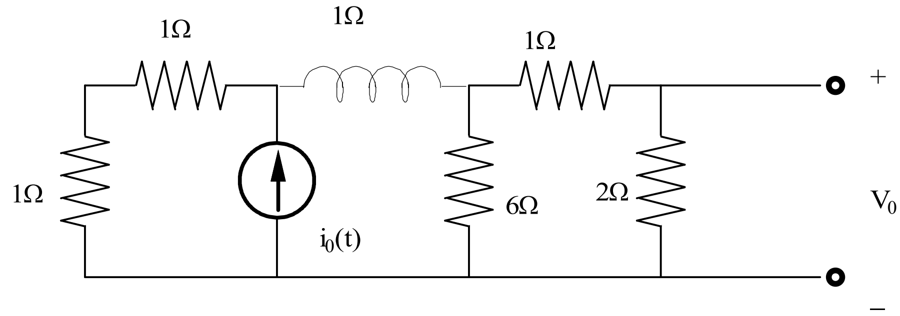
$$A = 1$$

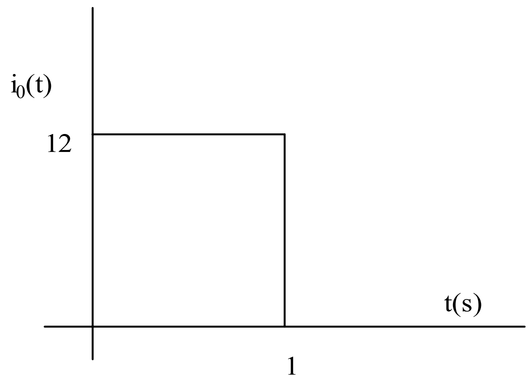
$$B = -4/7$$

$$V_O(t) = \left[1 - \frac{4}{7}e^{-3t/7}\right]u(t)V$$

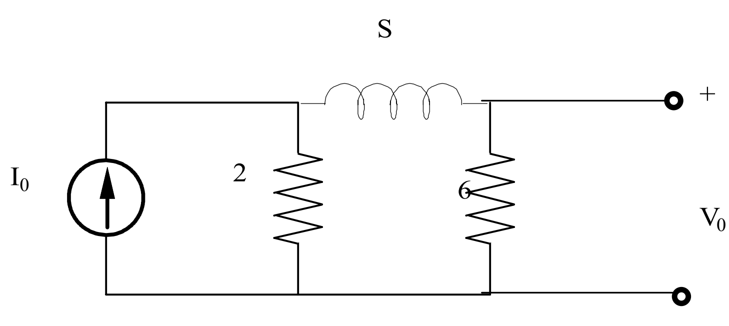
Problem 13.43

find Vo(t),t>0 in the network in fig if the input is represented by the waveform shown in fig.





Suggested Solution



$$V_{OC} = 2I_O\left[\frac{6}{S+8}\right] = \frac{24}{S} + \left(\frac{6}{S+8}\right) = \left[\frac{144}{S(S+8)}\right](1-e^{-S})$$

$$Z_{TH} = 6 \,||\, (S+2) = \frac{6S+12}{S+8}$$

NEW CIRCUIT:

$$V_O = V_{OC}\,\frac{2}{3+Z_{TH}} = V_{OC}\left[\frac{(S+8)2}{9S+36}\right]$$

$$V_O = \frac{32}{S(S+4)}$$

ADD THE TIMESHIFT TERM:

$$V_O = \frac{32}{S(S+4)}(1-e^{-S}) = \left[\frac{A}{S} + \frac{B}{S+4}\right](1-e^{-S})$$

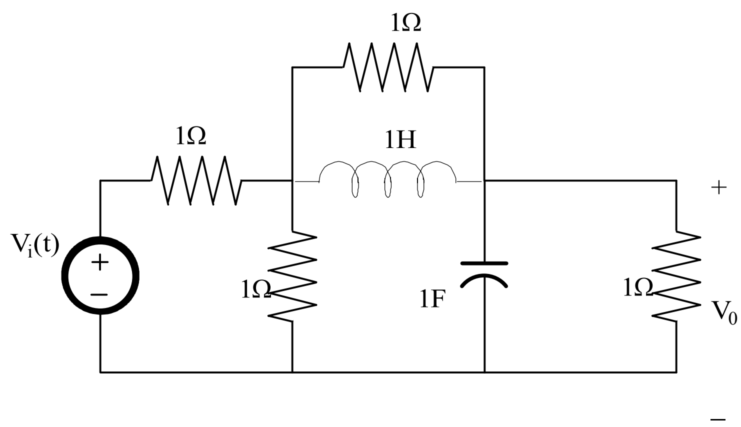
$$A = 8$$

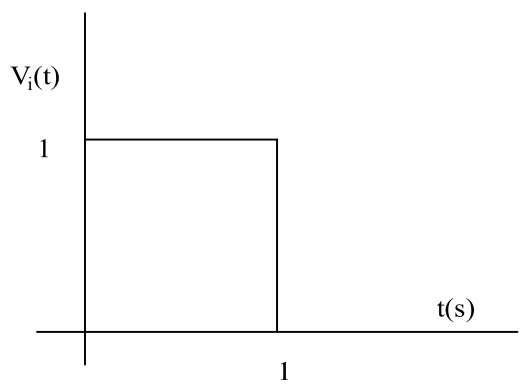
$$B = -8$$

$$V_O(t) = \left[8 - 8e^{-4(t-1)}\right]u(t-1)V$$

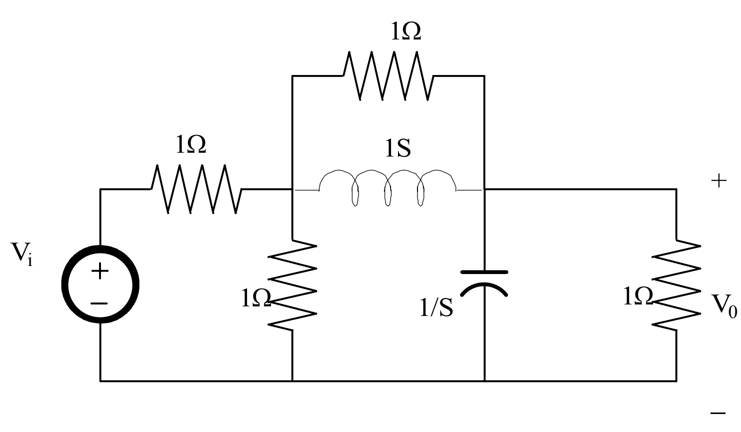
Problem 13.44

Determine the output voltage Vo(t) in the network in fig if the input is given by the source in fig.





Suggested Solution



$$V_i(t) = 1[u(t) - u(t-1)]$$

$$V_i(S) = \frac{1}{S}(1 - e^{-S})$$

$$\mathrm{V_O} = \frac{V_1}{2}\left[\frac{\frac{1}{S+1}}{0.5 + \frac{1}{S+1} + \frac{S}{S+1}}\right] = \frac{V_1}{2}\left[\frac{1}{S+1+\frac{S}{2}+\frac{1}{2}}\right] = \frac{V_1/3}{S+1}$$

$$\mathrm{V_O} = \frac{1}{3}\left(\frac{1}{S(S+1)}\right)(1 - e^{-S}) = \left[\frac{\mathrm{A}}{\mathrm{S}} + \frac{B}{S+1}\right](1 - e^{-S})$$

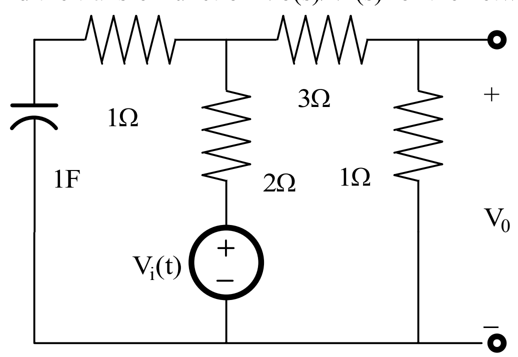
$$A = 1/3$$

$$B = -1/3$$

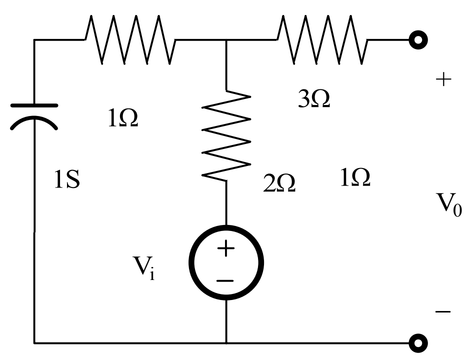
$$V_O(t) = \left[\frac{1}{3} - \frac{1}{3}e^{-(t)}\right]u(t) - \left[\frac{1}{3} - \frac{1}{3}e^{-(t-1)}\right]u(t-1)V$$

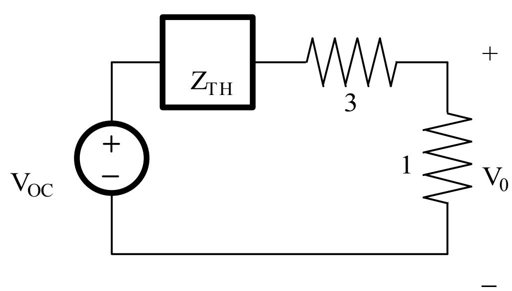
Problem 13.45

Find the transfer function Vo(s)/Vi(s) for the network shown in fig.



Suggested Solution





$$V_{OC} = V_i\left[1 - \frac{2}{3+1/S}\right] = V_i\left(\frac{S+1}{3S+1}\right)$$

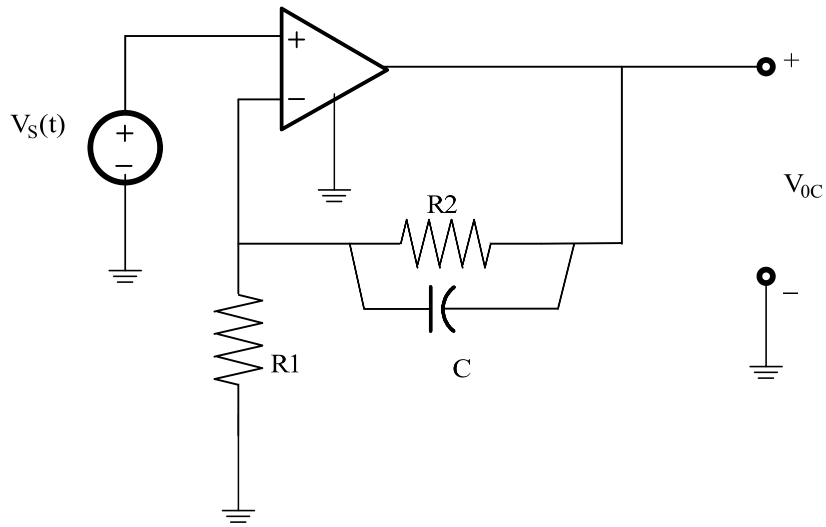
$$Z_{TH} = 2 \,||\, \left(\frac{1}{S}+1\right) = \frac{2S+2}{3S+1}$$

$$V_O = V_{OC}\,\frac{1}{4+Z_{TH}}$$

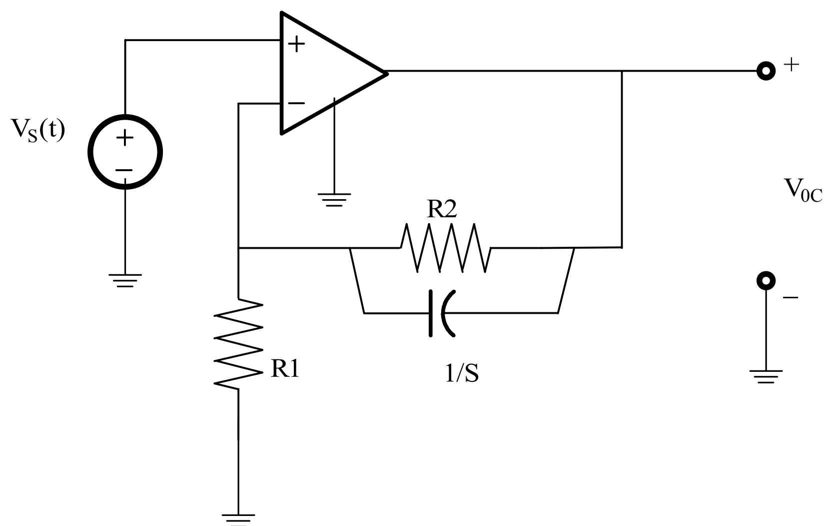
$$\frac{V_O}{V_i} = \frac{S+1}{14S+6}$$

Problem 13.46

Find the transfer function Vo(s)/Vi(s) for the network shown in fig.



Suggested Solution



$$\frac{V_o - V_S}{Z} = \frac{V_S}{R_2} \Rightarrow \frac{V_S}{V_o} = 1 + \frac{Z}{R_2}$$

$$Z = R_1 \parallel \frac{1}{SC} = \frac{R_1}{1 + SCR_1}$$

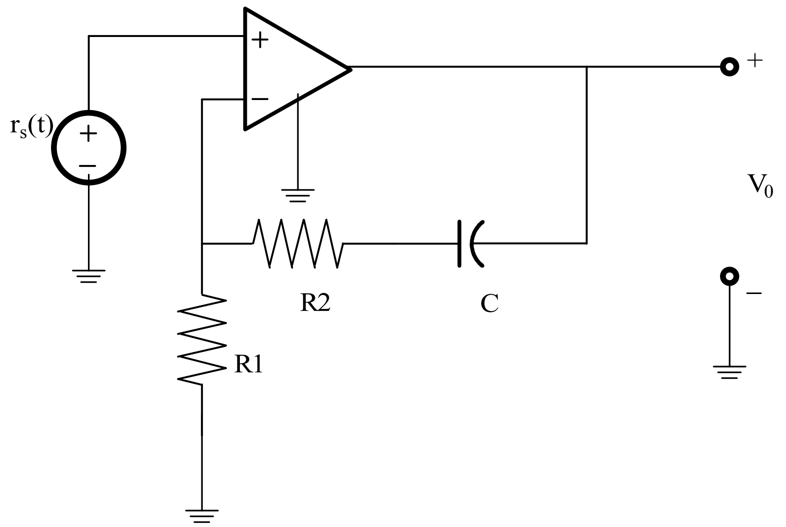
$$\frac{V_o}{V_S} = \left(1 + \frac{R_1}{R_2}\right)\left(\frac{1 + SCR}{1 + SCR_1}\right);$$

$$R = R_1 \parallel R_2$$

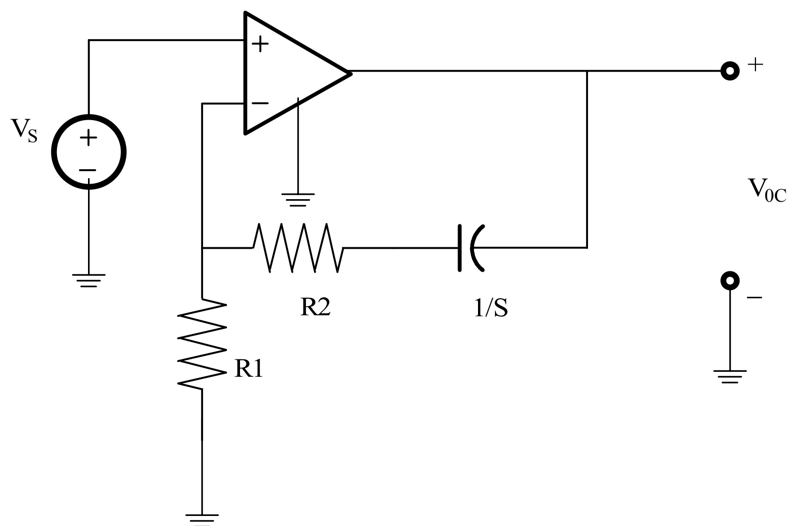
NOTE THAT THE $1+R_1/R_2$ = dC GAIN.

Problem 13.47

Find the transfer function Vo(s)/Vi(s) for the network shown in fig.



Suggested Solution



$$\frac{V_o - V_S}{R_1 + 1/SC} = \frac{V_S}{R_2} \Rightarrow \frac{V_O}{V_S} = 1 + \frac{R_1 + 1/SC}{R_2}$$

OR,

$$\frac{V_o}{V_S} = \left(1 + \frac{R_1}{R_2}\right) + \frac{1}{SCR_2} = \left(1 + \frac{R_1}{R_2}\right)\left(1 + \frac{1}{SCR}\right);$$

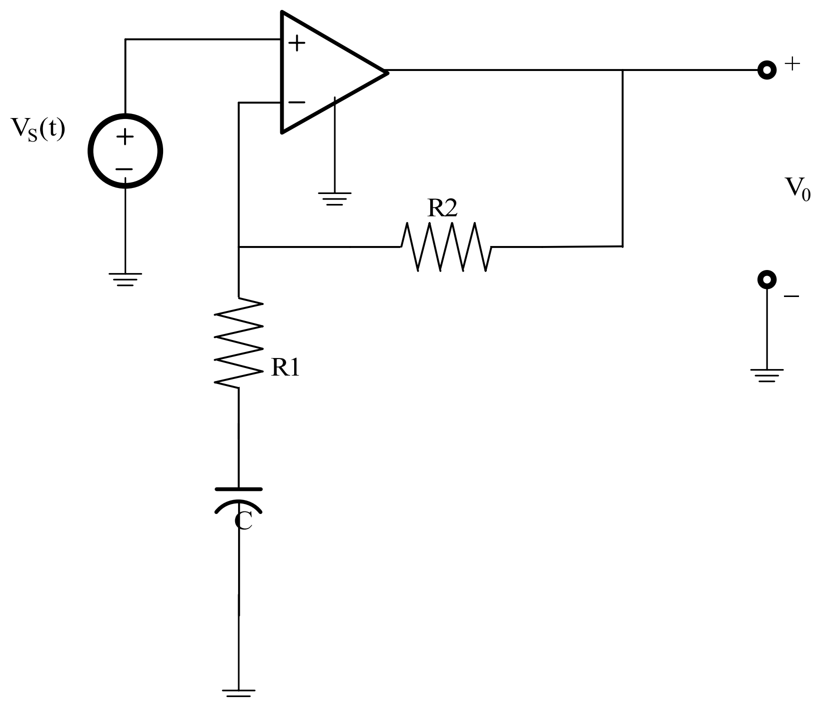
$$R = R_1 + R_2$$

$$\frac{V_o}{V_S} = \left(1 + \frac{R_1}{R_2}\right)\left(1 + \frac{1}{SCR}\right)$$

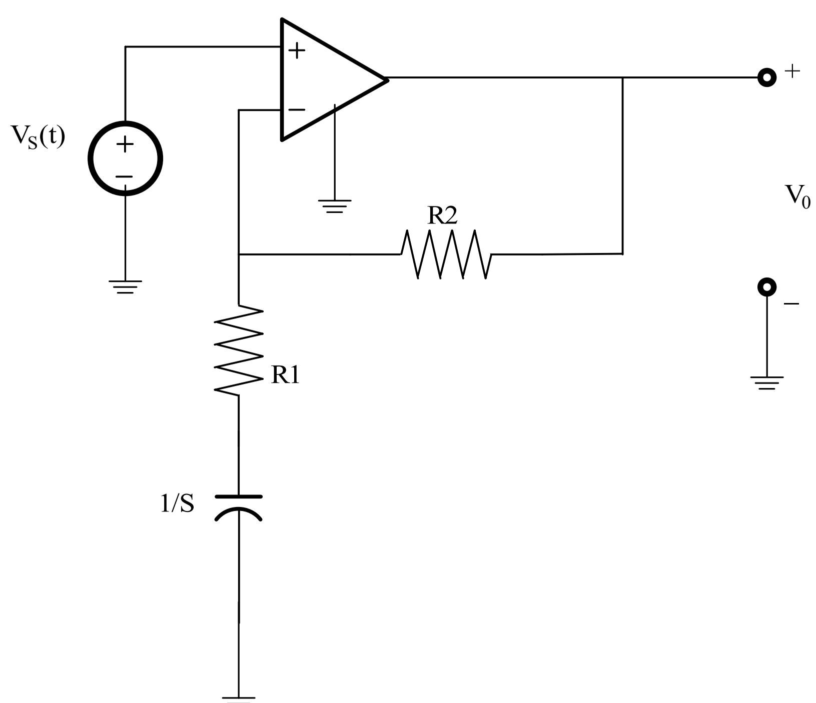
NOTE THAT THE $1+R_1/R_2$ = GAIN AT $S \rightarrow \infty$

Problem 13.48

Find the transfer function Vo(s)/Vi(s) for the network shown in fig.



Suggested Solution



$$\frac{V_o - V_S}{R_1} = \frac{V_S}{R_2 + 1/SC} \Rightarrow \frac{V_O}{V_S} = 1 + \frac{R_1}{R_2 + 1/SC}$$

$OR,$

$$\frac{V_o}{V_S} = 1 + \left(\frac{SCR_1}{1 + SCR_2}\right);$$

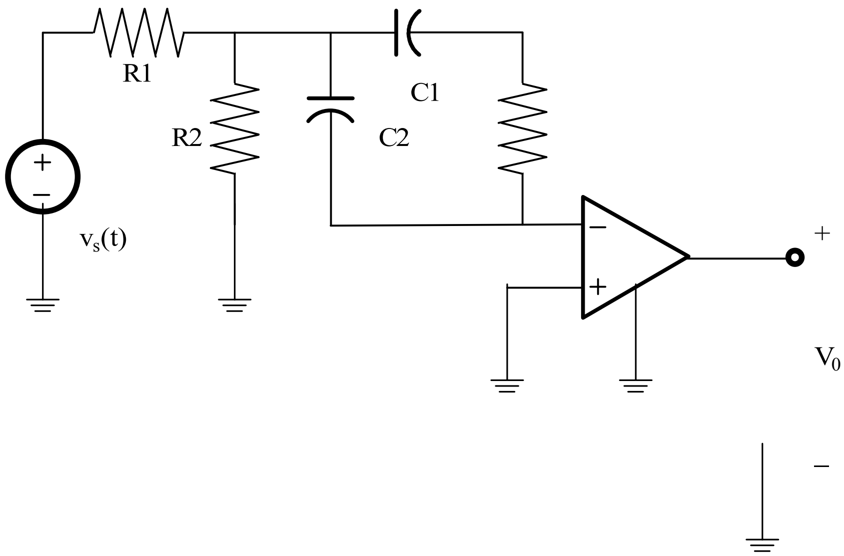
$FINALLY,$

$$\frac{V_o}{V_S} = \left(1 + \frac{R_1}{R_2}\right)\left(\frac{S + 1/(C(R_1 + R_2))}{S + \frac{1}{CR_2}}\right)$$

NOTE THAT THE $1+R_1/R_2$ = HIGH FREQUENCY GAIN

Problem 13.49

Find the transfer function Vo(s)/Vi(s) for the network shown in fig.



Suggested Solution

KCL AT NODE V_1 IS

$$\frac{V_S - V_1}{R_1} = \frac{V_1}{R_2} + SC_2V_1 + C_1S(V_1 - V_O)$$

AT THE -VE TERMINAL OF THE OP-AMP SINCE

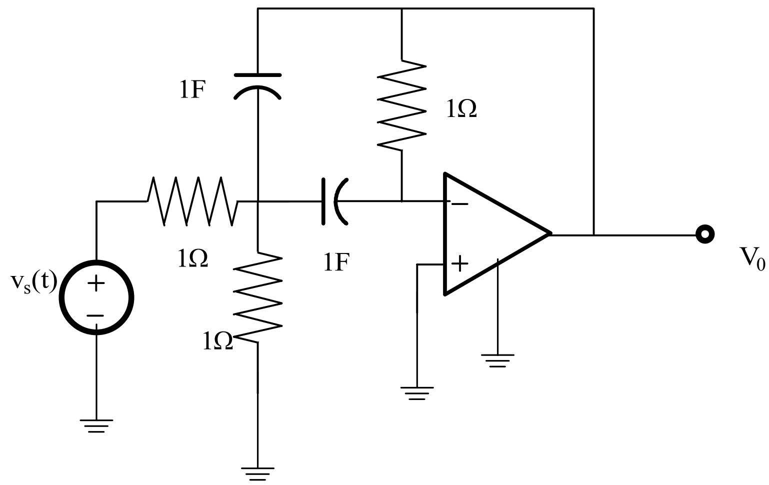
$$i_- = 0 \;\therefore SC_2V_1 + \frac{V_O}{R_3} = 0$$

SOLVING THE TWO EQUATIONS YIELDS:

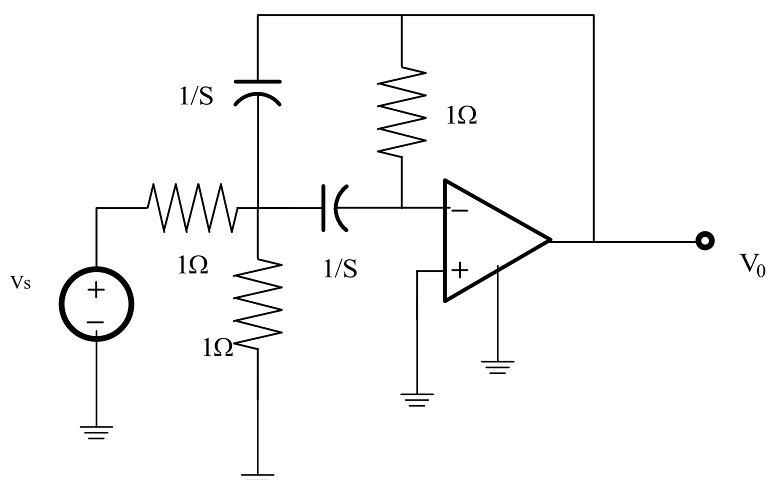
$$\frac{V_O}{V_S} = \frac{\dfrac{-S}{C_1R_1}}{S^2 + S\left(\dfrac{C_1}{C_1C_2R_3} + \dfrac{C_2}{C_1C_2R_3}\right) + \dfrac{R_1 + R_2}{C_1C_2R_1R_2R_3}}$$

Problem 13.50

Determine the transfer function for the network shown in fig. If a step function is applied to the network What type of damping will the network exhibit.



Suggested Solution



USING KCL:

AT V_1: V_S-V_1= V_1+(V_1-V_O)S+SV_1

OR, V_S=V_1(2S+2)-V_O(S)

AT (-) INPUT: SV1=-V_O $\Rightarrow$ V_1=-V_O/S

NOW,

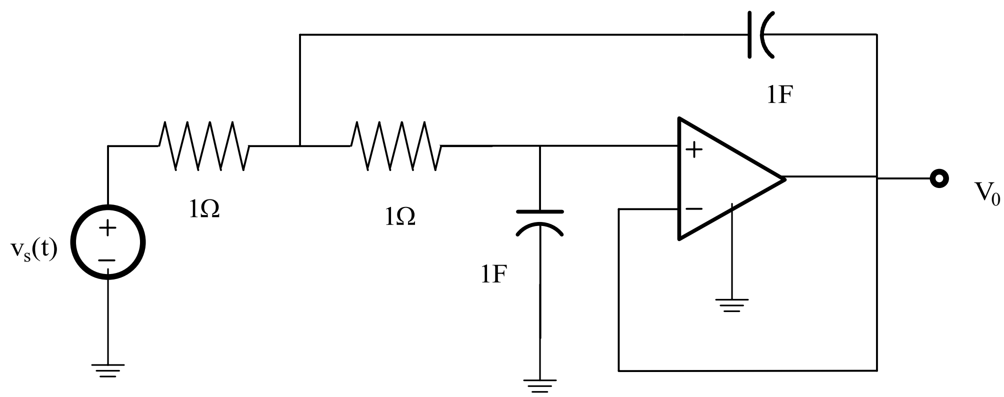
$$V_S = \frac{V_O}{S}(2S+2) - SV_O = V_O(S+2+2/S) \Rightarrow -\frac{V_O}{V_S} = \frac{S}{S^2+2S+2}$$

CHARACTERISTIC EQ: S^2+2S+2

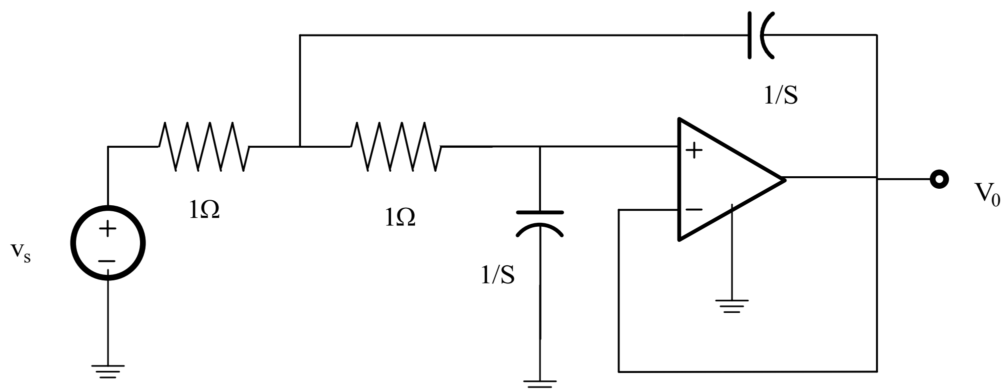
ROOTS ARE S=-1$\pm$ j1

Problem 13.51

Find the transfer function Vo(s)/Vi(s) for the network shown in fig. If the step function is applied to the network, will the response be overdamped underdamped, or critical damped.



Suggested Solution



NODAL ANALYSIS,

$$@V_1: \frac{V_S - V_1}{1} = (V_1\text{-}V_O)S + \frac{(V_1\text{-}V_O)}{1}$$

$$V_S = V_1(S+2) - V_O(S+1)$$

$$@(+)\text{ INPUT: } \frac{V_1 - V_0}{1} = (V_O)S \Rightarrow V_1 = V_O(S+1)$$

$$V_S = V_O\left[(S+1)(S+2)-1\right] = V_O(S+1)(S+1) = V_O(S+1)^2$$

$$\frac{V_O}{V_S} = \frac{1}{(S+1)^2}$$

CRITICALLY DAMPED.

Problem 13.52

The voltage response of the network to a unit step input is.

$$V_0(s) = \frac{2(s+1)}{s(s^2+12s+27)}$$

Is the response overdamped.

Suggested Solution

A Network has a step responce of

$$V_0(s) = \frac{2(s+1)}{s(s^2+12s+27)}$$

is it overdamped?

completing the square

$s^2 + 12s + 27 = (s+3)(s+4)$

$\therefore s = -3, s = -9$

system is overdamped.

Problem 13.53

The transfer function of the network is given by the expression

$$G(s) = \frac{100s}{s2 + 22s + 40}$$

Determine the damping ratio, the underdamped natural frequency and the type of response that will be exhibited by the network.

Suggested Solution

Asystem has teh transfer function

$$G(s) = \frac{100s}{s2 + 22s + 40}$$

find

ξ, ω_0

The characteristic equation

$$s^2 + 22s + 40 = 0$$

with analogy to

$$s2 + 2\xi\omega_0 s + \omega_0{}^2$$

$$\omega_0 = \sqrt{40}, 2\xi\omega_0 = 22$$

$$\xi = \frac{11}{2\sqrt{10}}$$

overdamped

Problem 13.54

The current response of a network to a unit step input is

$$I_0(s) = \frac{10(s+2)}{s^2(s^2+11s+30)}$$

Is the response underdamped?

Suggested Solution

If step responce is

$$I_0(s) = \frac{10(s+2)}{s^2(s^2+11s+30)}$$

How is it Damped?

The characteristic equation is

$$s^2(s^2+11s+30) = 0$$

$$s^2(s+6)(s+5)$$

Roots

$$s = -6, s = -5$$

Being real and unequal it is overdamped.

Problem 13.55

The voltage response of a network to a unit step input is

$$V_0(s) = \frac{10}{s(s^2 + 8s + 16)}$$

Is the response critically damped.

Suggested Solution

If step responce is

$$V_0(s) = \frac{10}{s(s^2 + 8s + 16)}$$

The characteristic equation is

$$(s^2 + 8s + 16) = 0$$

$$(s + 4)^2 = 0$$

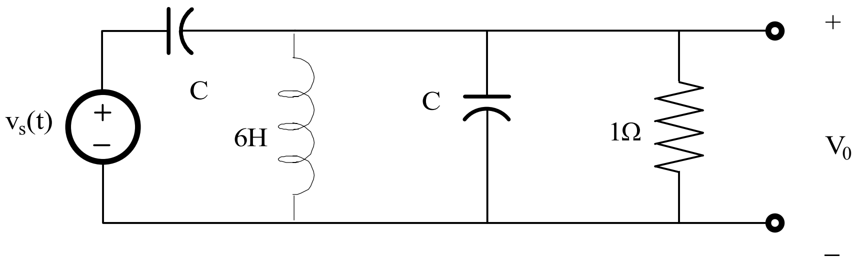
Roots

$$s = -4$$

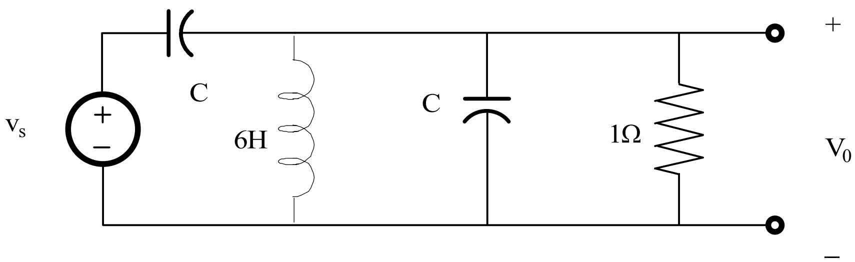
Critically Damped.

Problem 13.56

For the network in fig choose the value of C for critical damping.



Suggested Solution



Parallel RLC Circuit

$R = 1\Omega, L = 6h, C_{eq} = C + 1F;$

Characteristic Equation is

$$s^2 + \frac{s}{RC_{eq}} + \frac{1}{LC_{eq}} = 0$$

$$s^2 + 2\xi\omega_n s + \omega_n^{\ 2} = 0$$

For critical damping, $\xi = 1$

$$\omega_n = \frac{1}{\sqrt{LC_{eq}}} = \frac{1}{\sqrt{6C_{eq}}}$$

$$RC_{eq} = C_{eq} = \frac{1}{2\omega_n} = \frac{\sqrt{6}}{2} C_{eq}$$

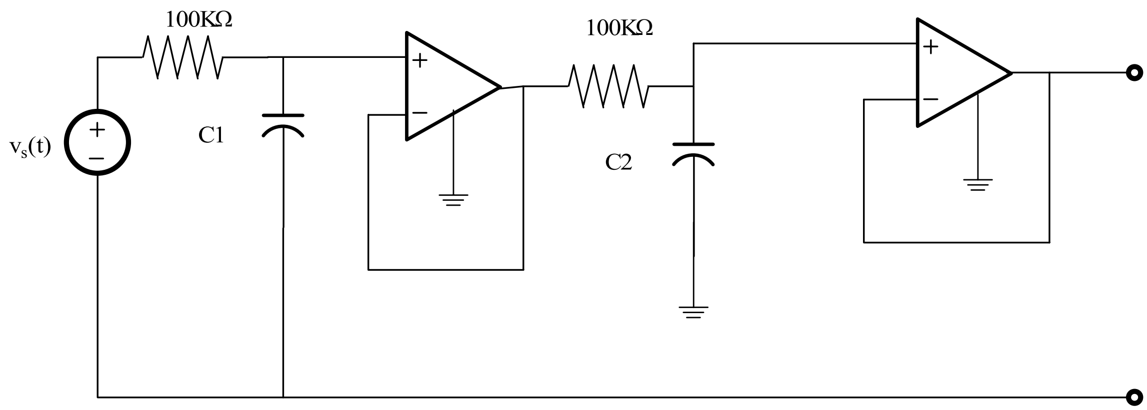
$$\sqrt{C_{eq}} = \frac{\sqrt{6}}{2}$$

$$C_{eq} = \frac{6}{4} = 1.5 = C + 1$$

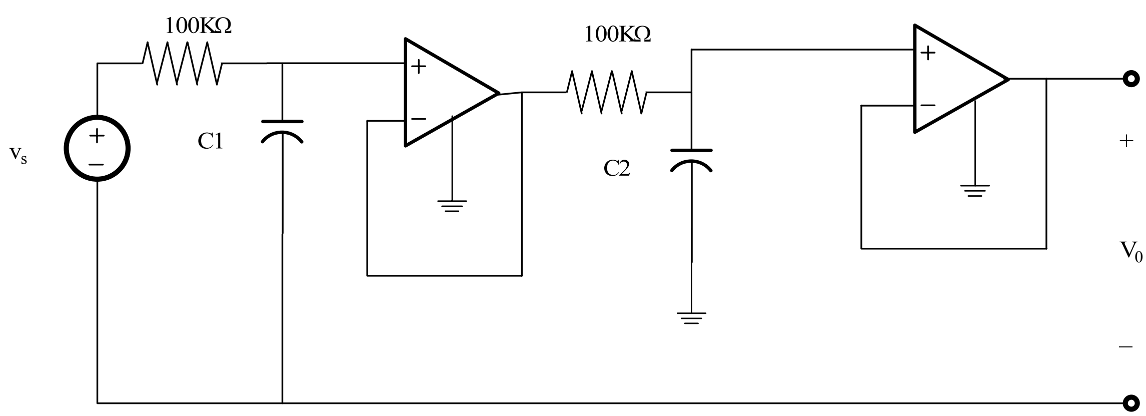
$$C = 0.5F$$

Problem 13.57

For the filter in fig choose the values of c1 and c2 to place poles at s=-2 and s=-5 rad/s.



Suggested Solution



Both opamps are connected as unity gain bufffers

$R = 100k\Omega$

$V_1 = V_s(\frac{1/sc_1}{1/sc_1 + R})$

also

$V_0 = V_1(\frac{1/sc_2}{R + 1/sc_2})$

or

$\frac{V_0}{V_s} = \frac{1}{R^2c_1c_2}[\frac{1}{(s+\frac{1}{Rc_1})(s+\frac{1}{Rc_2})}]$

For poles at -2 and -5

the transfer function is

$\frac{V_o}{V_s} = \frac{1}{R^2c_1c_2}[\frac{1}{(s+2)(s+5)}]$

so

$Rc_1 = 0.5$

$Rc_2 = 0.2$

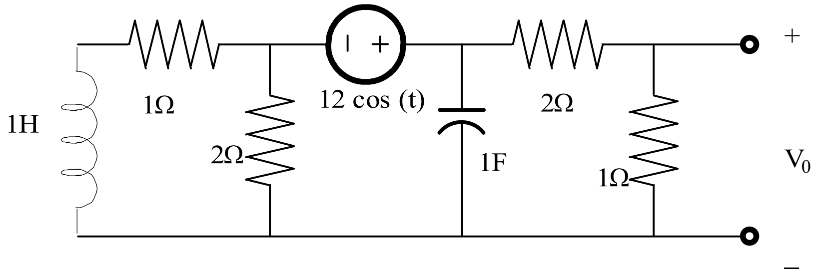
$c_1 = 5\mu F$

$c_2 = 2\mu F$

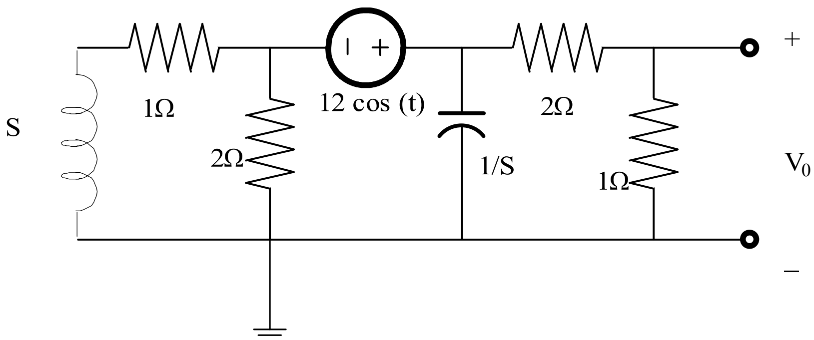
c_1 and c_2 interchagable

Problem 13.58

Find the steady state response Vo(t) for the network in fig.



Suggested Solution



Using KCL

$$\frac{V_1}{s+1}+\frac{V_2}{2}+\frac{V_2}{1/s}+\frac{V_2}{3}=0$$

$$V_1 = V_2 - V_i$$

$$V_i(t) = 12\cos(t)V$$

$$V_z\left(\frac{1}{s+1}+\frac{1}{2}+s+\frac{1}{3}\right)=V_i\left(\frac{1}{s+1}+\frac{1}{2}\right)$$

Solve fo rthe Transfer Function

$$\frac{V_2}{V_i}=\frac{3s+9}{6s^2+11s+11}$$

$$\frac{V_0}{V_i}=\frac{V_2}{3/V_i}=\frac{s+3}{6s^2+11s+11}$$

For Steady State s = j1

$$\left.\frac{V_0}{V_i}\right|_{j1}=\frac{3+j1}{-6+11+j11}=0.26\angle-47.2^o$$

Also

$|V_{in}| = 12$

so

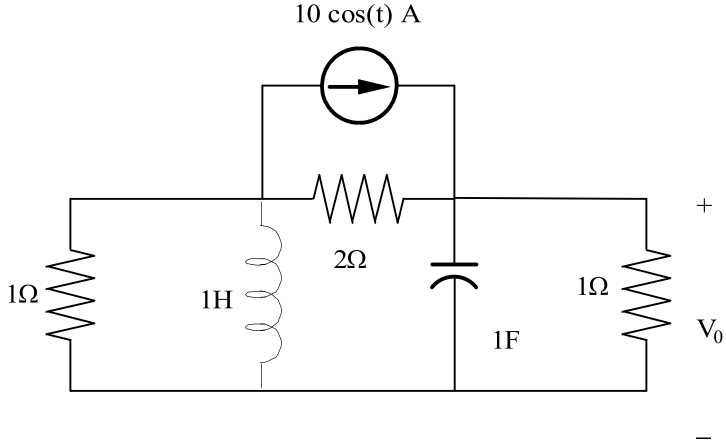
$V_0 = 12(0.26) - 47.2^o$

Steady Staet Phasor

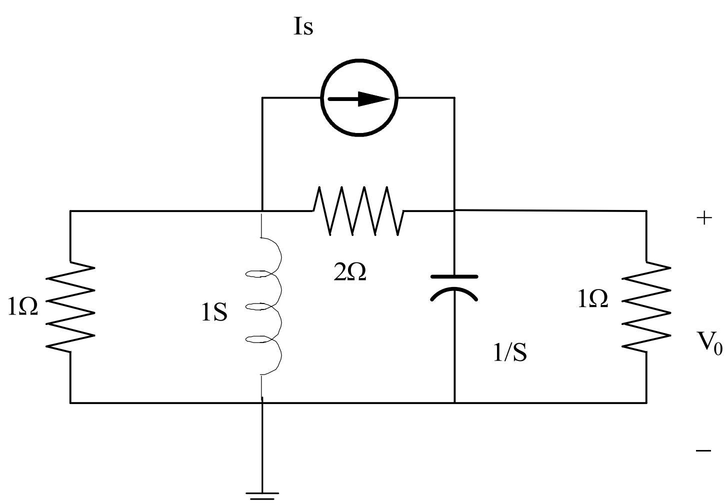
$V_0(t) = 3.13\cos(t - 47.2^o)V$

Problem 13.59

Find the steady state response Vo(t) for the network in fig.



Suggested Solution



Nodal Analysis

$$\frac{V_1}{1}+\frac{V_1}{s}+V_2(s)+\frac{V_2}{1}=0$$

$$V1(1+\frac{1}{s})=-V2(s+1)$$

or

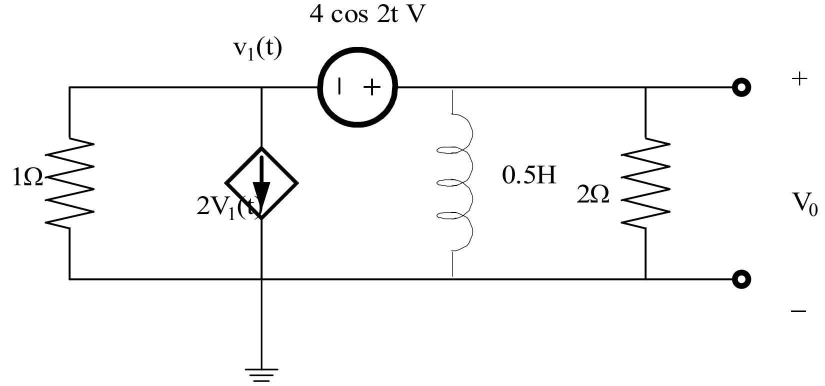
$$V_1=-sV_2$$

$$I_s+\frac{V_1-V_2}{2}=V_2s+V_2$$

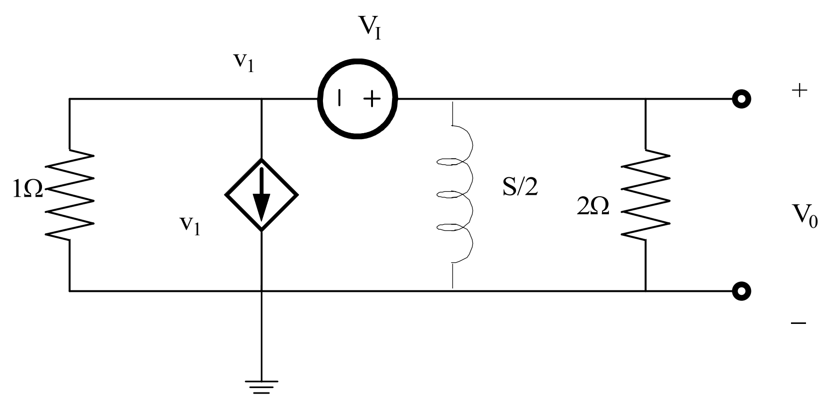
$$V_1 = V_2(2s+3) - 2I_s$$
$$-sV_2 = V_2(2s+3) - 2I_s$$
$$V_2 = V_0 = I_s(\frac{2}{3s+3})$$
$$Transfer\_Function$$
$$\frac{V_0}{I_s} = \frac{2/3}{s+1}$$
For steady state s=j1 and
$$\frac{V_0}{I_s} = \frac{2/3}{1+j1} = \frac{\sqrt{2}}{3}\angle -45^o$$
$$|I_s| = 10$$
$$so,$$
$$V_0 = \frac{10\sqrt{2}}{3}\angle -45^o$$
$$V_0(t) = 4.7\cos(t - 45^o)V$$

Problem 13.60

Find the steady state response Vo(t) for the network in fig.



Suggested Solution



Nodal _ Analysis

$$V_1 + 2V_1 + V_0(\frac{2}{s}) + \frac{V_0}{2} = 0$$

$$V_1 = V_0 - V_I$$

$$3V_0 - 3V_I + \frac{2V_0}{s} + \frac{V_0}{2} = 0$$

so,

$$6V_I = V_0(6 + \frac{4}{s} + 1)$$

$$\frac{V_0}{V_I} = \frac{6s}{7s+4}$$

at Steady State s= j2 since

$$V_i(t) = 4\cos(2t)$$

$$\left.\frac{V_0}{V_f}\right|_{j2} = \frac{j12}{4+j14} = 0.82\angle 15.95^o$$

$\sin ce$

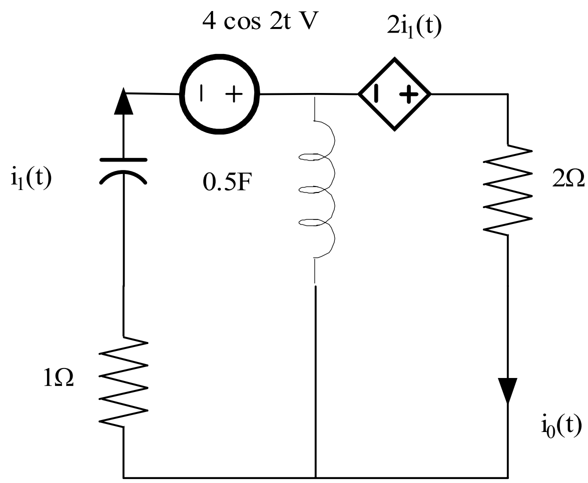
$|V_i| = 4$

$V_0 = 4(0.82)\angle 15.95^o = 3.3\angle 15.95^o$

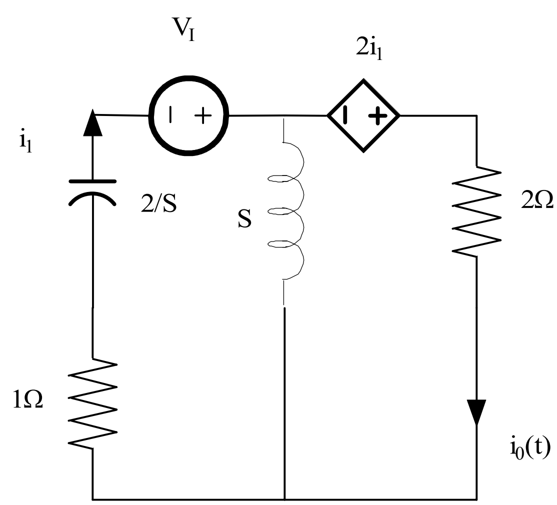
$V_0(t) = 3.3\cos(2t + 15.95^o)V$

Problem 13.61

Find the steady state response Io(t) for the network shown the fig.



Suggested Solution



$$Loop\_Equations$$

$$I_0 Loop: 2I_1 = (s+2)I_0 - sI_1$$

$$I_1 = I_0$$

$$I_1 Loop: V_I = s(I_1 - I_0) + I_1 + \frac{2I_1}{s}$$

$$V_I = I_0(1 + \frac{2}{s})$$

$$\frac{I_0}{V_I} = \frac{s}{s+2}$$

$\sin ce$

$V_i(t) = 4\cos(2t)$

at Steady State j = j2

$$\frac{I_0}{V_i} = \frac{j2}{s+j2} = \frac{1}{\sqrt{2}}\angle 45^o$$

$\sin ce$

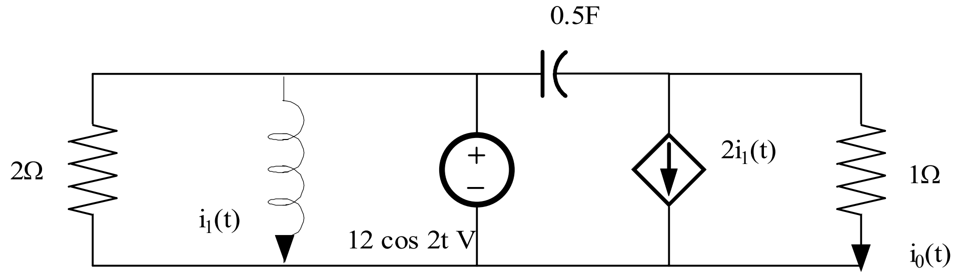
$|V_I| = 4$

$$I_0 = \frac{4}{\sqrt{2}}\angle 45^o$$

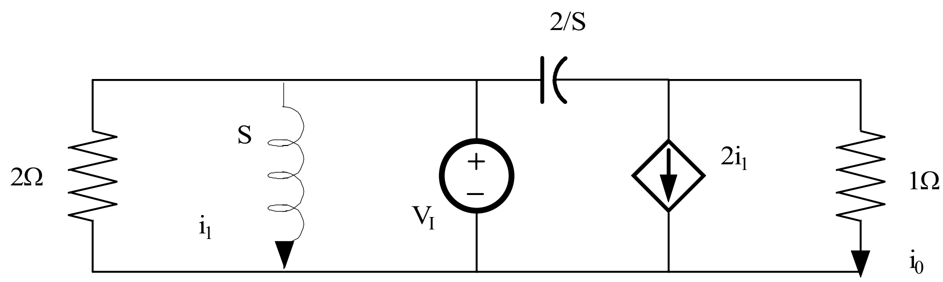
$I_0(t) = 2\sqrt{2}\cos(2t + 45^o)A$

Problem 13.62

Find the steady state response Io(t) for the network shown the fig.



Suggested Solution



KCL

$$\frac{V_0}{1} + 2I_1 + (V_0 - V_I)\frac{s}{2} = 0$$

$$I_0 = V_0/1, I_1 = V_I/s$$

$$I_0 + \frac{2V_I}{s} + I_0(\frac{s}{2}) - V_I(\frac{s}{2}) = 0$$

or

$$I_0(1 + \frac{s}{2}) = V_I(\frac{s}{2} - \frac{2}{s}) = V_I(\frac{s^2 - 4}{2s})$$

$$I_0(\frac{s+2}{2}) = V_I(\frac{s^2 - 4}{2s})$$

Finally

$$\frac{I_0}{V_I} = \frac{s-2}{s}$$

at steady state s=j2

$V_i(t) = 12\cos(2t)$

$$\left.\frac{I_0}{V_I}\right|_2 = \frac{j2-2}{j2} = \sqrt{2}\angle 45^o$$

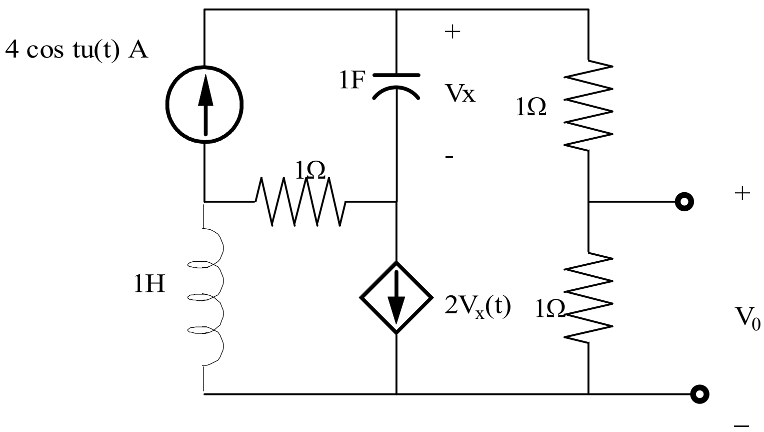
and

$$I_0 = |V_I|\sqrt{2}\angle 45^o = 12\sqrt{2}\angle 45^o$$

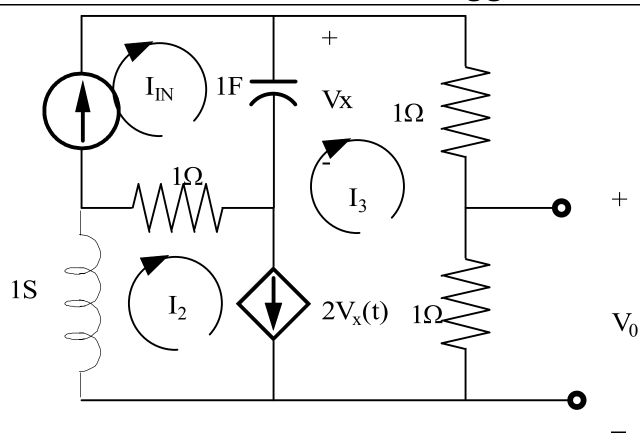
$$I_0 = 12\sqrt{2}\cos(2t + \angle 45^o)A$$

Problem 13.63

Find the steady state response Io(t) for the network shown the fig.



Suggested Solution



Loop Equations

$$I_3(2) + I_2(S) + (I_2 - I_{IN})(1) + \frac{I_3 - I_{IN}}{S} = 0$$

$$I_2(S+1) + I_3(2 + \frac{1}{S}) - I_{IN}(1 + \frac{1}{S}) = 0$$

$$V_O I_3$$

$$I_2 - I_3 = 2V_A = 2(I_{IN} - I_3)$$

$$I_{IN}(2 + \frac{2}{S} - 1 - \frac{1}{3}) + I_3[S + 1 - 2 + 2 - \frac{2}{S} + \frac{1}{S}] = 0$$

$$SI_{IN}(1+\frac{1}{S}) = V_0[-S^2 - S + 1]$$

$$\frac{V_0}{I_{IN}} = \frac{-(S+1)}{S^2 + S - 1}$$

SINCE

$$I_{IN}(t) = 4COSt(t > 0)$$

at steady state s $\rightarrow$ j1

$$\frac{V_0}{I_{IN}} = \frac{-(1+J1)}{-2+J1} = 0.63\angle 71.57^O$$

SINCE

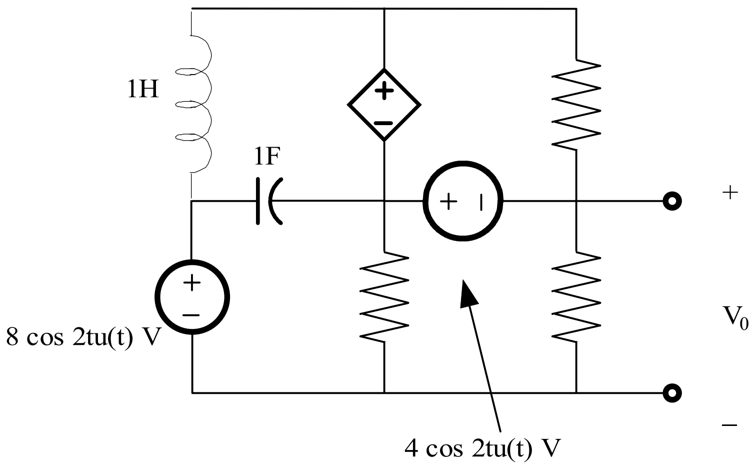
$$|i_{IN}| = 4A$$

$$V_0 = 4(0.63)\angle 71.57^O$$

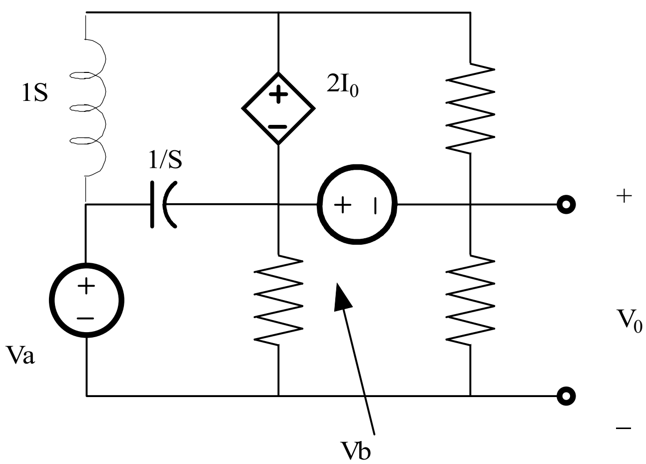
$$V_0(t) = 2.52COS(t + 71.57^O)V$$

Problem 13.64

Find the steady state response Io(t) for the network shown the fig.



Suggested Solution



$$V_A(t) = 8\cos 2t$$

$$Va(t) = 4\cos 2t$$

at steady state

$$V_A = 8\angle 0^o V$$

$$V_B = 4\angle 0^o V$$

KCL

$$\frac{V_1 - V_A}{s} + (V_2 - V_A)s = I_1$$

$$I_1 + V_2 + V_0 = 0$$

also

$V_2 = V_0 + V_B$

$V_1 = 2I_0 + V_2 = 3V_2 = 3V_0 + 3V_B$

combinig these equations yeilds

$$V_0 = V_A(\frac{s^2+1}{s^2+2s+3}) - V_B(\frac{s^2+s+3}{s^2+2s+3})$$

at steady state

$$V_0 = \frac{8(-4+1)-4(-4+3+j2)}{-1+3+j4} = 5.22\angle 97.77^o$$

$V_0(t) = 5.22\cos(2t + 97.77^o)u(t)V$

Problem 13FE-1

A single loop second order circuit is described by the following differential equation

What is the correct form of the total (natural plus forced) response?

Suggested Solution

The Characteristic Eqn is

$s^2 + 2s + 2 = 0$

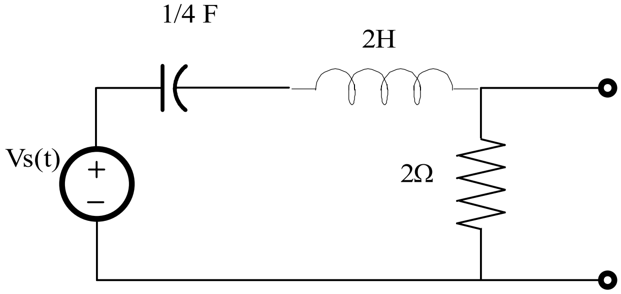
or

$(s + 1 \pm j1) = 0$

with a constant forcing function - the answer is (0)

Problem 13FE-2

If all initial conditions are zero in the network in fig, find transfer function Vo(s)/Vs(s) and determine the type of damping exhibited by the network.



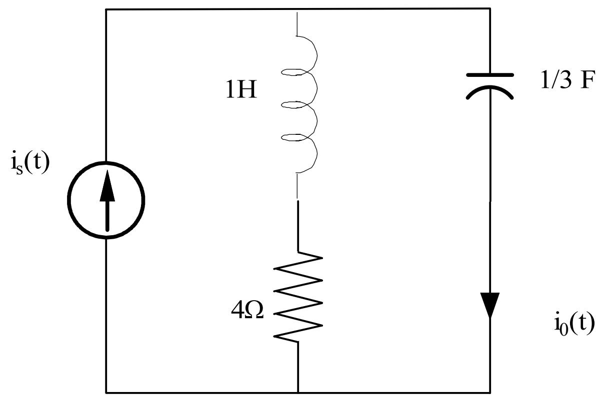
Suggested Solution

$$\frac{V_0}{V_s}(s) = \frac{2}{2 + 2s + \frac{4}{s}} = \frac{2s}{2s + s^2 + 4} = \frac{s}{s^2 + s + 2} = \frac{s}{s + \frac{1}{2}s \pm j\frac{\sqrt{7}}{4}}$$

The network is underdamped

Problem 13FE-3

The initial conditions in the circuit in fig are zeros. Find the transfer function Io(s)/Is(s) and determine the type of damping exhibited by the circuit.



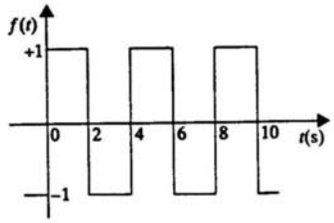
Suggested Solution

$$\frac{I_0}{I_s}(s) = \frac{s+4}{s+4+\frac{3}{s}} = \frac{s(s+4)}{s^2+4s+3} = \frac{s(s+4)}{(s+1)(s+3)}$$

The network is overdamped

Problem 14.1

Find the exponential Fourier series for the signal shown.



Suggested Solution

$$C_n = \frac{1}{T_0}\int_{\frac{-T_0}{2}}^{0} -e^{-jn\omega_0 t}dt + \frac{1}{T_0}\int_{0}^{\frac{T_0}{2}} e^{-jn\omega_0 t}dt$$

$$C_n = \frac{1}{-T_0 jn\omega_0}\left[E^{-jn\omega_0\frac{T_0}{2}} -1-1+e^{jn\omega_0\frac{T_0}{2}}\right]$$

$$\omega_0 = \frac{2\pi}{T_0}$$

$$C_n = \frac{1}{2jn\pi}\left[2-e^{jn\pi}-e^{-jn\pi}\right] = \frac{1}{2jn\pi}\left[2-2\cos(n\pi)\right]$$

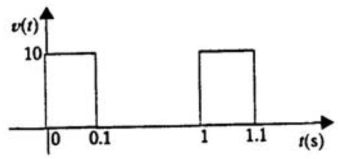
$$C_0 = 0$$

$$C_n = \frac{1-\cos(n\pi)}{jn\pi} = \begin{cases} 0 & n \quad even \\ \dfrac{2}{jn\pi} & n \quad odd \end{cases}$$

$$f(t) = \sum_{\substack{n=-\infty \\ n\neq 0 \\ n\ odd}}^{n=\infty} \frac{2}{jn\pi} e^{jn\omega_0 t}$$

Problem 14.2

Find the exponential Fourier series for the periodic pulse train shown.



Suggested Solution

$$C_0 = \frac{10(0.1)}{1} = 1 \qquad T_0 = 1 \quad \omega_0 = 2\pi$$

$$C_n = \frac{1}{T_0}\int_0^{\frac{T_0}{10}} 10 e^{-jn\omega_0 t} dt$$

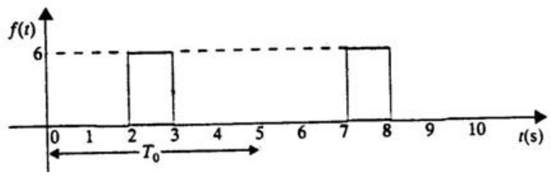
$$C_n = \frac{10}{-jn\omega_0 T_0}\left[e^{-jn\omega_0 t}\right]_0^{\frac{T_0}{10}} = \frac{5}{jn\pi}\left[1 - e^{-jn\frac{\pi}{5}}\right] = \frac{10}{-jn\omega_0 T_0}\left[\frac{e^{jn\frac{\pi}{10}} - e^{-jn\frac{\pi}{10}}}{j2}\right]$$

$$C_n = \frac{10}{n\pi} e^{-jn\frac{\pi}{10}} \sin\left(n\frac{\pi}{10}\right)$$

$$f(t) = \frac{10}{\pi}\sum_{n=-\infty}^{\infty} \frac{1}{n} e^{-jn\frac{\pi}{10}} \sin\left(n\frac{\pi}{10}\right) e^{jn\omega_0 t}$$

Problem 14.3

Find the exponential Fourier series for the periodic signal shown.



Suggested Solution

$T_0 = 5 \qquad \omega_0 T_0 = 2\pi$

use time shift theorem with $t_0 = 2\sec$

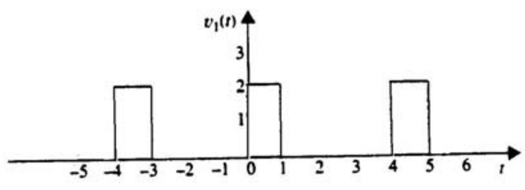
$$C_n = \frac{1}{T_0}\int_0^{\frac{T_0}{5}} 6e^{-jn\omega_0 t}dt = \frac{6}{jn2\pi}\left[e^{jn\frac{\pi}{5}} - e^{-jn\frac{\pi}{5}}\right]$$

$$C_n = \frac{6e^{-jn\frac{\pi}{5}}}{n\pi}\sin\left(n\frac{\pi}{5}\right) \quad and \quad f(t) = \sum_{n=-\infty}^{\infty} C_n e^{-jn\omega_0 t} e^{jn\omega_0 t}$$

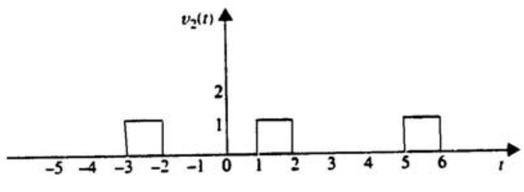
$$f(t) = \sum_{n=-\infty}^{\infty} \frac{6}{n\pi} e^{-jn\pi} \sin\left(n\frac{\pi}{5}\right) e^{j0.4\pi t}$$

Problem 14.4

Compute the exponential Fourier series for the waveform that is the sum of the two waveforms shown by computing the exponential Fourier series of the two waveforms and adding them.



(a)



(b)

Suggested Solution

$$For\ V_1(t), \qquad T_0 = 4 \quad \omega_0 T_0 = 2\pi$$

$$C_{n_1} = \frac{2}{T_0}\int_0^{\frac{T_0}{4}} e^{-jn\omega_0 t}dt = \frac{2}{jn2\pi}\left[1 - e^{-jn\frac{\pi}{2}}\right]$$

$$For V_2(t),\ note\ V_2(t)\ is\ half\ as\ large\ and\ time\ shifted\ 1\text{sec},\ or \frac{T_0}{4}$$

$$C_{n_2} = \frac{C_{n_1}}{2}e^{-jn\omega_0(1)} = \frac{C_{n_1}}{2}e^{-jn\frac{\pi}{2}}$$

$$C_n = C_{n_1} + C_{n_2}$$

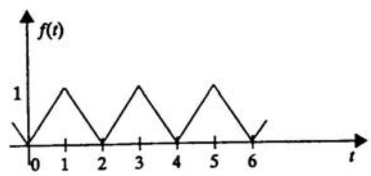
$$C_n = \frac{1}{jn2\pi}\left\{\left[2 - 2e^{-jn\frac{\pi}{2}}\right] + e^{-jn\frac{\pi}{2}} - e^{-jn\pi}\right\} = \frac{1}{jn\pi}\left[1 - \left(\frac{e^{jn\frac{\pi}{4}} + e^{-jn\frac{\pi}{4}}}{2}\right)e^{-jn\frac{3\pi}{4}}\right]$$

$$C_n = \frac{1}{jn\pi}\left(1 - e^{-jn\frac{3\pi}{4}}\cos\left(n\frac{\pi}{4}\right)\right)$$

$$V(t) = \sum_{n=-\infty}^{\infty}\left[\frac{i}{jn\pi}\left(1 - e^{-jn\frac{3\pi}{4}}\cos\left(n\frac{\pi}{4}\right)\right)\right]e^{jn\frac{\pi}{2}t}\ V$$

Problem 14.5

Find the exponential Fourier series for the signal shown.



Suggested Solution

Comparing $f(t)$ *to the waveforms in table* 14.1 *notice that* $f\left(t\right)$ *matches the even function symmetry triangular wave where*

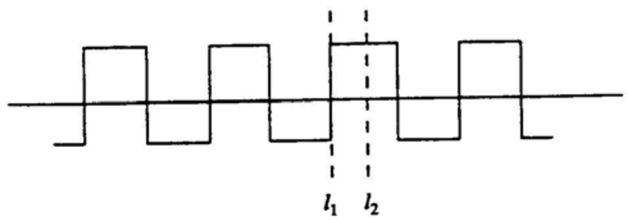
$A = 1 \quad T_0 = 2 \;\; \& \;\; \omega_0 = \pi$

$$C_0 = \frac{A}{2} = \frac{1}{2} \qquad\qquad C_n = \frac{-2A}{n^2\pi^2} = \frac{-2}{n^2\pi^2} \quad \begin{cases} n \neq 0 \\ n \;\; odd \end{cases}$$

$$f\left(t\right) = \frac{1}{2} + \sum_{\substack{n=-\infty \\ n\neq 0 \\ n\ odd}}^{\infty} \frac{-2}{n^2\pi^2} e^{jn\pi t}$$

Problem 14.6

Given the waveform shown, determine the type of symmetry that exists if the origin is selected at: (a) l_1 and (b) l_2.



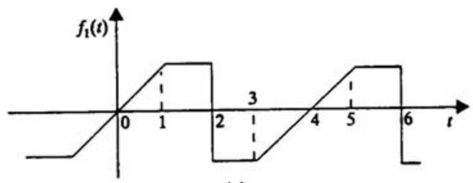
Suggested Solution

If origin is at l_1, then $v(t) = -v(-t)$ odd symmetry

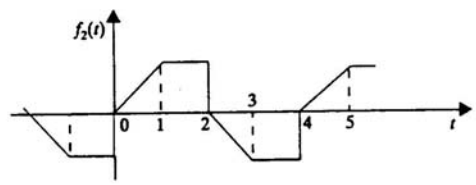
If origin is at l_2, then $v(t) = v(-t)$ even symmetry

Problem 14.7

What type of symmetry is exhibited by the two waveforms shown?



(a)



(b)

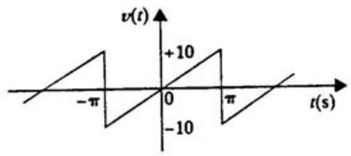
Suggested Solution

In part a) $f_1(t) = -f_1(t)$ odd symmetry

In part b) $f_2(t) = -f_2(t - T_0/2)$ half wave symmetry

Problem 14.8

Derive the trigonometric Fourier series for the waveform shown.



Suggested Solution

$$b_n = \frac{2}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \frac{20}{T_0} t\sin(n\omega_0 t)dt = \frac{40}{T_0^{\,2}}\left[\frac{\sin(n\omega_0 t)}{(n\omega_0)^2} - \frac{t\cos(n\omega_0 t)}{n\omega_0}\right]_{-\frac{T_0}{2}}^{\frac{T_0}{2}}$$

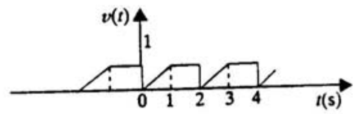
$$b_n = \frac{40}{T_0^{\,2}}\left[\frac{\sin(n\pi)}{(n\omega_0)^2} - \frac{T_0\cos(n\pi)}{2n\omega_0} + \frac{\sin(n\pi)}{(n\omega_0)^2} - \frac{T_0\cos(n\pi)}{2n\omega_0}\right]$$

$$b_n = \frac{40}{T_0^{\,2}}\frac{T_0}{n\omega_0}\cos(n\pi) = (-1)^{n+1}\frac{20}{n\pi}$$

$$V(t) = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{20}{n\pi}\sin(nt)$$

Problem 14.9

Find the trigonometric Fourier series coefficients for the waveform shown.



Suggested Solution

$$a_0 = \left[\frac{1}{2}(1)(1) + (1)(1)\right] = \frac{3}{4} \qquad T_0 = 2\sec$$

$$a_n = \frac{2}{T_0}\int_0^1 t\cos(n\omega_0 t)\,dt + \frac{2}{T_0}\int_1^2 \cos(n\omega_0 t)\,dt$$

$$a_n = \frac{2}{T_0}\left[\frac{t\sin(n\omega_0 t)}{n\omega_0} + \frac{\cos(n\omega_0 t)}{(n\omega_0)^2}\right]_0^1 + \frac{2}{n\omega_0 T_0}\sin(n\omega_0 t)\Big|_1^2$$

$$a_n = \frac{\sin(n\pi)}{n\pi} + \frac{\cos(n\pi)}{(n\pi)^2} - \frac{1}{(n\pi)^2} + \frac{\sin(2n\pi)}{n\pi} - \frac{\sin(2n\pi)}{n\pi} = \frac{\cos(n\pi)-1}{(n\pi)^2}$$

$$b_n = \frac{2}{T_0}\int_0^1 t\sin(n\omega_0 t)\,dt + \frac{2}{T_0}\int_1^2 \sin(n\omega_0 t)\,dt$$

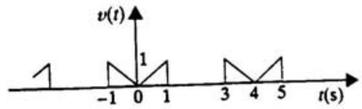
$$b_n = \left[\frac{\sin(n\omega_0 t)}{(n\omega_0)^2} - \frac{t\cos(n\omega_0 t)}{n\omega_0}\right]_0^1 + \frac{\cos(n\omega_0 t)}{n\omega_0}\Big|_2^1$$

$$b_n = \frac{\sin(n\pi)}{(n\pi)^2} - \frac{\cos(n\pi)}{n\pi} + \frac{\cos(n\pi)}{n\pi} - \frac{\cos(2n\pi)}{n\pi} = -\frac{1}{n\pi}$$

$$a_0 = \frac{3}{4} \qquad a_n = \frac{\cos(n\pi)-1}{(n\pi)^2} \qquad b_n = -\frac{1}{n\pi}$$

Problem 14.10

Find the trigonometric Fourier series coefficients for the waveform shown.



Suggested Solution

Even symmetry, $T_0 = 4 \quad \omega_0 = \dfrac{\pi}{2}$

$$a_0 = \frac{\left(\frac{1}{2}\right)(1)(1)(2)}{4} = \frac{1}{4} \qquad b_n = 0$$

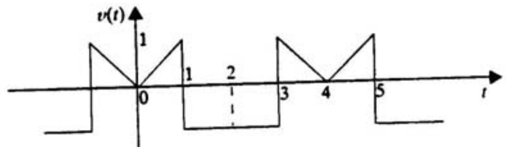
$$a_n = \int_0^1 t\cos\left(\frac{n\pi}{2}t\right)dt = \left[\frac{\cos(n\omega_0 t)}{(n\omega_0)^2} + \frac{t\sin(n\omega_0 t)}{n\omega_0}\right]_0^1$$

$$a_n = \frac{\cos\left(n\frac{\pi}{2}\right)}{(n\omega_0)^2} + \frac{\sin\left(n\frac{\pi}{2}\right)}{n\omega_0} - \frac{1}{(n\omega_0)^2} = \frac{4}{(n\pi)^2}\left(\cos\left(n\frac{\pi}{2}\right) - 1\right) + \frac{2}{n\pi}\sin\left(n\frac{\pi}{2}\right)$$

$$a_0 = \frac{1}{4} \qquad a_n = \frac{4}{(n\pi)^2}\left(\cos\left(n\frac{\pi}{2}\right) - 1\right) + \frac{2}{n\pi}\sin\left(n\frac{\pi}{2}\right)$$

Problem 14.11

Find the trigonometric Fourier series coefficients for the waveform shown.



Suggested Solution

$V(t)$ can be expressed as the sum of the two wave forms shown below where
$V_1(t)$ is the wave form in Problem 14.10
Since $V_2(t)$ is of even form

$$a_{0_2} = -\frac{1}{2} \qquad b_{n_2} = 0 \qquad T_0 = 4$$

$$a_{n_2} = \frac{4}{T_0}\int_0^{\frac{T_0}{2}} V_2(t)\cos(n\omega_0 t)\,dt = \int_1^2 -\cos(n\omega_0 t)\,dt = \frac{-1}{n\omega_0}\left[\sin(n\pi) - \sin\left(n\frac{\pi}{2}\right)\right]$$

$$a_{n_2} = \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

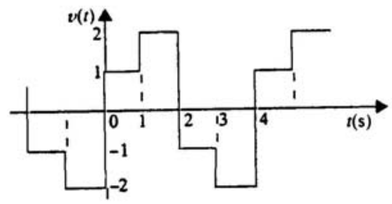
From problem 14.10: $a_{0_1} = \frac{1}{4}$ *and* $a_{n_1} = \frac{4}{(n\pi)^2}\left(\cos\left(n\frac{\pi}{2}\right) - 1\right) + \frac{2}{n\pi}\sin\left(n\frac{\pi}{2}\right)$

$$a_0 = a_{0_1} + a_{0_2} = -\frac{1}{4} \qquad b_n = 0$$

$$a_n = \frac{4}{(n\pi)^2}\left(\cos\left(n\frac{\pi}{2}\right) - 1\right) + \frac{4}{n\pi}\sin\left(n\frac{\pi}{2}\right)$$

Problem 14.12

Find the trigonometric Fourier series for the waveform shown.



Suggested Solution

Half wave symmetry

$$a_0 = 0 \qquad T_0 = 4\sec \qquad \omega_0 = \frac{\pi}{2}$$

$$a_n = b_n = 0 \qquad \textit{for n even}$$

$$a_n = \frac{4}{T_0}\int_0^{\frac{T_0}{2}} f(t)\cos(n\omega_0 t)\,dt \ \ \textit{for n odd}$$

$$a_n = \int_0^1 \cos(n\omega_0 t dt) + \int_1^2 2\cos(n\omega_0 t\, dt)$$

$$a_n = \left.\frac{\sin n\omega_0 t}{n\omega_0}\right|_0^1 + \left.\frac{2\sin n\omega_0 t}{n\omega_0}\right|_1^2$$

$$a_n = \frac{\sin\left(\frac{n\pi}{2}\right) + 2\sin(n\pi) - 2\sin\left(\frac{n\pi}{2}\right)}{\frac{n\pi}{2}}$$

$$a_n = \frac{-2}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

$$b_n = \frac{4}{T_0}\int_0^{\frac{T_o}{2}} f(t)\sin(n\omega_0 t)\,dt = \int_0^1 \sin(n\omega_0 t)\,dt + 2\int_1^2 \sin(n\omega_o t)\,dt$$

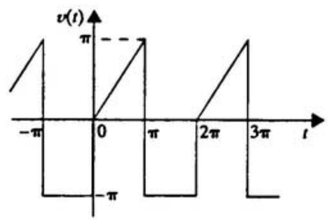
$$b_n = \left.\frac{\cos(n\omega_o t)}{n\omega_0}\right|_1^0 + \left.\frac{\cos(n\omega_o t)}{n\omega_o}\right|_2^1 = \frac{1 - \cos\left(\frac{n\pi}{2}\right) + 2\cos\left(\frac{n\pi}{2}\right) - 2\cos(n\pi)}{\frac{n\pi}{2}}$$

Since b_n *holds only for n odd*, $b_n = \frac{6}{n\pi}$

$$v1 + 1 = \frac{2}{\pi}\sum_{\substack{n=1 \\ n\ odd}}^{\infty} \frac{3}{n}\sin n\omega_o t - \frac{1}{n}\sin\left(\frac{n\pi}{2}\right)\cos n\omega_o t$$

Problem 14.13

Find the trigonometric Fourier series for the waveform shown.



Suggested Solution

$T = 2\pi \text{ sec} \qquad \omega_0 = 1 r/s \qquad a_0 = \dfrac{\dfrac{\pi^2}{2} - \pi^2}{2\pi} = \dfrac{-\pi}{4}$

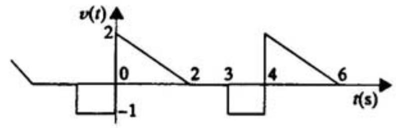
This waveform is similar to that of prob. 14.15. *If one multiplis* $\omega(t)$ *in* 14.15 *by negative* π, *one gets this waveform here. From* 14.15.

$a_0 = \dfrac{1}{4} \qquad a_n = \dfrac{1}{n^2\pi^2}\left(1 - \cos(n\pi)\right) \qquad b_n = \dfrac{2\cos(n\pi) - 1}{n\pi}$

$a_0 = \dfrac{-1}{4} \qquad a_n = \dfrac{1}{\pi n^2}\left(\cos(n\pi) - 1\right) \qquad b_n = \dfrac{1}{n}\left(1 - 2\cos(n\pi)\right)$

Problem 14.14

Find the trigonometric Fourier series coefficients for the waveform shown.



Suggested Solution

$$T_0 = 4 \text{ sec} \qquad \omega_0 = \frac{\pi}{2} \qquad a_0 = \frac{1}{T_0}\left[\frac{1}{2}(2)(2) - (1)(1)\right] = \frac{1}{4}$$

$$a_n = \frac{2}{T_0}\int_0^2 (2-t)\cos(n\omega_0 t)\,dt + \frac{2}{T_0}\int_3^4 -\cos(n\omega_0 t)\,dt$$

$$a_n = \left.\frac{\sin(n\omega_0 t)}{n\omega_0}\right|_0^2 + \left.\left(\frac{t\sin(n\omega_0 t)}{2n\omega_0} + \frac{\cos(n\omega_0 t)}{2(n\omega_0)^2}\right)\right|_2^0 + \left.\frac{\sin(n\omega_0 t)}{2n\omega_0}\right|_4^3$$

so,

$$a_n = \frac{\sin(n\pi)}{n\omega_0} + \frac{1}{2(n\omega_0)^2} - \frac{\sin(n\pi)}{n\omega_0} - \frac{\cos(n\pi)}{2(n\omega_0)^2} + \frac{\sin\left(\frac{n3\pi}{2}\right) - \sin(n2\pi)}{2n\omega_0}$$

$$a_n = \frac{2}{n^2\pi^2}(1-\cos(n\pi)) - \frac{\sin\left(\frac{n\pi}{2}\right)}{n\pi}$$

$$b_n = \frac{2}{T_0}\int_0^2 (2-t)\sin(n\omega_0 t)\,dt - \frac{2}{T_0}\int_{-1}^0 \sin(n\omega_0 t)\,dt$$

$$b_n = \left.\frac{\cos(n\omega_0 t)}{n\omega_0}\right|_2^0 + \left.\left(\frac{t\cos(n\omega_0 t)}{2n\omega_0} - \frac{\sin(n\omega_0 t)}{2(n\omega_0)^2}\right)\right|_0^2 + \left.\frac{\cos(n\omega_0 t)}{2n\omega_0}\right|_{-1}^0$$

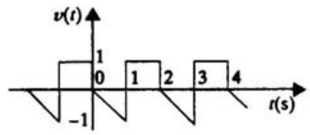
$$b_n = \frac{1-\cos(n\pi)}{n\omega_0} + \frac{\cos(n\pi)}{n\omega_0} - \frac{\sin(n\pi)}{2(n\omega_0)^2} + \frac{1-\cos(-n\omega_0)}{2n\omega_0}$$

$$b_n = \frac{1}{2n\omega_0}\left[3 - \cos(n\omega_0)\right] = \frac{1}{n\pi}\left[3 - \cos\left(\frac{n\pi}{2}\right)\right]$$

$$a_0 = \frac{1}{4} \qquad a_n = \frac{2}{n^2\pi^2}(1-\cos(n\pi)) - \frac{\sin\left(\frac{n\pi}{2}\right)}{n\pi} \qquad b_n = \frac{1}{n\pi}\left[3 - \cos\left(\frac{n\pi}{2}\right)\right]$$

Problem 14.15

Find the trigonometric Fourier series coefficients for the waveform shown.



Suggested Solution

$$T_0 = 2 \text{ sec} \qquad \omega_0 = \pi \ r/s \qquad a_0 = \frac{2}{T_0}\left[(-1)(1)\left(\frac{1}{2}\right) = (1)(1)\right] = \frac{1}{4}$$

$$a_n = \frac{2}{T_0}\int_0^1 -t\cos(n\omega_0 t)\,dt + \frac{2}{T_0}\int_1^2 \cos(n\omega_0 t)\,dt$$

$$a_n = \left(\frac{\cos(n\pi t)}{(n\pi)^2} + \frac{t\sin(n\pi t)}{n\pi}\right)\Bigg|_1^0 + \frac{\sin(n\pi t)}{n\pi}\Bigg|_1^2$$

$$a_n = \frac{1}{(n\pi)^2} - \frac{\cos(n\pi)}{(n\pi)^2} - \frac{\sin(n\pi)}{n\pi} + \frac{\sin(n2\pi) - \sin(n\pi)}{n\pi}$$

$$a_n = \frac{1}{n^2\pi^2}(1 - \cos(n\pi))$$

$$b_n = \frac{2}{T_0}\int_0^1 -t\sin(n\omega_0 t)\,dt + \frac{2}{T_0}\int_1^2 \sin(n\omega_0 t)\,dt$$

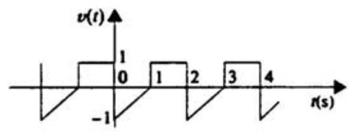
$$b_n = \left(\frac{t\cos(n\omega_0 t)}{n\omega_0} - \frac{\sin(n\omega_0 t)}{(n\omega_0)^2}\right)\Bigg|_0^1 + \frac{\cos(n\omega_0 t)}{n\omega_0}\Bigg|_2^1$$

$$b_n = \frac{\cos(n\pi)}{n\pi} - \frac{\sin(n\pi)}{(n\pi)^2} - \frac{\cos(n\pi)}{n\pi} - \frac{\cos(2\pi n)}{n\pi} = \frac{2}{n\pi}\cos(n\pi) - \frac{1}{n\pi}$$

$$a_0 = \frac{1}{4} \qquad a_n = \frac{1}{n^2\pi^2}(1 - \cos(n\pi)) \qquad b_n = \frac{1}{n\pi}\left[2\cos(n\pi) - 1\right]$$

Problem 14.16

Find the trigonometric Fourier series coefficients for the waveform shown.



Suggested Solution

$T_0 = 2 \text{ sec} \qquad \omega_0 = \pi \ r/s \qquad a_0 = \frac{1}{T_0}\left[\left(\frac{1}{2}\right)(1)(-1)+(1)(1)\right] = \frac{1}{4}$

$$a_n = \frac{2}{T_0}\left[\int_0^1 (t-1)\cos(n\omega_0 t)\,dt + \int_1^2 \cos(n\omega_0 t)\,dt\right]$$

$$a_n = \left(\frac{\cos(n\pi t)}{(n\pi)^2} + \frac{t\sin(n\pi t)}{n\pi}\right)\Bigg|_0^1 - \frac{\sin(n\pi t)}{n\pi}\Bigg|_0^1 + \frac{\sin(n\pi t)}{n\pi}\Bigg|_1^2$$

$so,$

$$a_n = \frac{\cos(n\pi)}{(n\pi)^2} + \frac{\sin(n\pi)}{n\pi} - \frac{1}{(n\pi)^2} - \frac{\sin(n\pi)}{n\pi} + \frac{\sin(2\pi n)}{n\pi} - \frac{\sin(n\pi)}{n\pi}$$

$$a_n = \frac{1}{n^2\pi^2}\left[\cos(n\pi) - 1\right]$$

$$b_n = \frac{2}{T_0}\int_0^1 (t-1)\sin(n\omega_0 t)\,dt + \frac{2}{T_0}\int_1^2 \sin(n\omega_0 t)dt$$

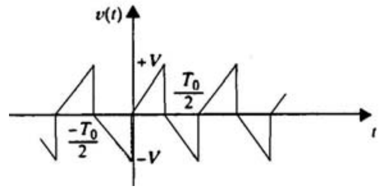
$$b_n = \left(\frac{\sin(n\omega_0 t)}{(n\omega_0)^2} - \frac{t\cos(n\omega_0 t)}{n\omega_0}\right)\Bigg|_0^1 + \frac{\cos(n\omega_0 t)}{n\omega_0}\Bigg|_2^1 + \frac{\cos(n\omega_0 t)}{n\omega_0}\Bigg|_0^1$$

$$b_n = \frac{\sin(n\pi)}{(n\pi)^2} - \frac{\cos(n\pi)}{n\pi} + \frac{\cos(n\pi)}{n\pi} - \frac{\cos(2\pi n)}{n\pi} + \frac{\cos(n\pi)}{n\pi} - \frac{1}{n\pi}$$

$b_n = \frac{1}{n\pi}\left[\cos(n\pi) - 2\right] \qquad a_n = \frac{1}{n^2\pi^2}\left[\cos(n\pi) - 1\right] \qquad a_0 = \frac{1}{4}$

Problem 14.17

Derive the trigonometric Fourier series for the function shown.



Suggested Solution

Half wave symmetry $\quad a_0 = 0 \quad a_n = b_n = 0$ *for n even*

$$a_n = \frac{4}{T_0}\int_0^{\frac{T_0}{2}} \frac{2vt}{T_0}\text{co}(nw_0 t)\,dt = \frac{8v}{T_0^2}\int_0^{\frac{T_0}{2}} t\cos(n\omega_0 t)\,dt$$

$$a_n = \frac{8v}{T_0^2}\left[\frac{t\sin(n\omega_0 t)}{n\omega_0} + \frac{\text{co}(n\omega_0 t)}{(n\omega_0)^2}\right]\Bigg|_0^{\frac{T_0}{2}}$$

$$a_n = \frac{8v}{T_0^2}\left[\frac{T_0}{2}\left(\frac{\sin n\pi}{n\omega_0} + \frac{\cos(n\pi)-1}{(nw_0)^2}\right)\right]$$

$$\text{For } n \text{ odd}, \quad a_n = \frac{8v}{T_0^2}\left[\frac{-2T_0^2}{4n^2\pi^2}\right] = \frac{-4v}{n^2\pi^2}$$

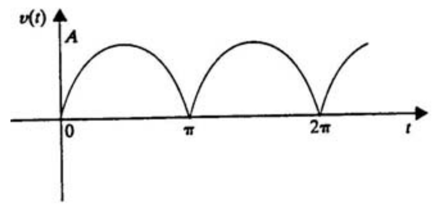
$$b_n = \frac{4}{T_o}\int_0^{\frac{T_0}{2}} \frac{2v}{T_0} t\sin(n\omega_0 t)\,dt = \frac{8v}{T_0^2}\left[\frac{\sin(n\omega_0 t)}{(n\omega_0)^2} - \frac{t\cos(n\omega_0 t)}{n\omega_0}\right]\Bigg|_0^{\frac{T_0}{2}}$$

$$b_n = \frac{8v}{T_0^2}\left[\frac{\sin(n\pi)}{(n\omega_0)^2} - \frac{T_0}{2}\left(\frac{\cos(n\pi)}{n\omega_0} = 0\right)\right] \qquad \text{for } n \text{ odd}, \quad b_n = \frac{2v}{n\pi}$$

$$v(t) = \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{2v}{n\pi}\text{si}(n\omega_0 t) - \frac{4v}{n^2\pi^2}\cos(n\omega_0 t)V$$

Problem 14.18

Derive the trigonometric Fourier series for the function *v(t)* = A|sin *t*| as shown.



Suggested Solution

$$T_o = \pi \qquad \omega_0 = 2 \qquad v(t) = A\sin|t|$$

$$even\ \ function, \qquad b_n = 0$$

$$a_0 = \frac{1}{T_0}\int_0^{T_0} A\sin(t)\,dt = \frac{A}{T_0}\cos t\Big|_{T_0}^{0} = \frac{2A}{T_0}$$

$$a_n = \frac{4}{T_0}\int_0^{\frac{T_0}{2}} A\sin(t)\cos(n\omega_0 t)\,dt$$

Use trig. identity,

$$\sin\phi\cos\theta = \frac{\sin(\phi+\theta)}{2} + \frac{\sin(\phi+\theta)}{2}$$

Now,

$$a_n = \frac{4A}{2T_0}\int_0^{\frac{T_0}{2}}\left[\sin(2n+1)t + \sin(1-2n)t\right]dt$$

$$a_n = \frac{2A}{\pi}\left[\frac{\cos(1+2n)t}{1=2n} + \frac{\cos(1+2n)t}{1-2n}\right]\Bigg|_{\frac{T_0}{2}}^{0} = \frac{1-\cos\left((1=2n)\frac{\pi}{2}\right)}{1+2n} + \frac{1-\cos\left((1-2n)\frac{\pi}{2}\right)}{1-2n}$$

$$Since \qquad \cos\left[(1+2n)\frac{\pi}{2}\right] = \cos\left[(1-2n)\frac{\pi}{2}\right] = 0\ \ for\ all\ n,$$

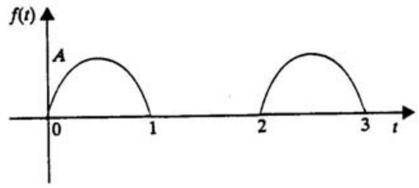
$$a_n = \frac{2A}{\pi}\left[\frac{1-2n+1+2n}{(1-2n)(1+2n)}\right] = \frac{4A}{\pi(1-4n^2)}$$

$$f(t) = \frac{2A}{\pi}\left[1 + 2\sum_{n=1}^{\infty}\frac{\cos(2nt)}{1-4n^2}\right]$$

$$a_0 = \frac{2A}{\pi} \qquad a_n = \frac{4A}{\pi(1-4n^2)} \qquad b_n = 0$$

Problem 14.19

Derive the trigonometric Fourier series for the waveform shown.



Suggested Solution

$$T_0 = 2 \qquad \omega_0 = \pi \qquad a_0 = \frac{1}{T_0}\int_0^1 A\sin(\pi t)\,dt = \frac{A}{2\pi}\cos(\pi t)\Big|_1^0 = \frac{A}{\pi}$$

$$a_n = \frac{2}{T_0}\int_0^1 A\sin(\pi t)\cos(n\pi t)\,dt$$

$$Trig.\ identity \quad \sin\phi\cos\theta = \frac{\sin(\phi-\theta)+\sin(\phi+\theta)}{2}$$

$$a_n = \frac{A}{2}\int_0^1 \left(\sin(1-n)\pi t + \sin(1+n)\pi t\right)dt$$

$$a_n = \frac{A}{2}\left[\frac{\cos(1-n)\pi t}{(1-n)\pi} + \frac{\cos(1+n)\pi t}{(1+n)\pi}\right]\Bigg|_1^0$$

so,

$$a_n = \frac{A}{2}\left[\frac{1-\cos(1-n)\pi}{(1-n)\pi} + \frac{1-\cos(1+n)\pi}{(1+n)\pi}\right] \quad if\ n\ is\ odd\ \ a_n = 0$$

$$If\ n\ is\ even, \quad a_n = \frac{A}{2\pi}\left[\frac{2}{1-n} + \frac{2}{1+n}\right] = \frac{2A}{\pi(1-n^2)}$$

$$b_n = \frac{2}{T_0}\int_0^1 A\sin(\pi t)\sin(n\pi t)\,dt = \begin{cases} 0, \ for n \neq 1 \\ A\int_0^1 \sin^2(\pi t)\,dt \quad for\ n = 1 \end{cases}$$

$$Since\ \sin^2\theta = \frac{1}{2}(1-\cos 2\theta),$$

$$b_n = \frac{A}{2}\int_0^1 (1-\cos(2\pi t))\,dt = \frac{A}{2}\left[t - \frac{\sin(2\pi t)}{2\pi}\right]\Bigg|_0^1 = \frac{A}{2}[1]$$

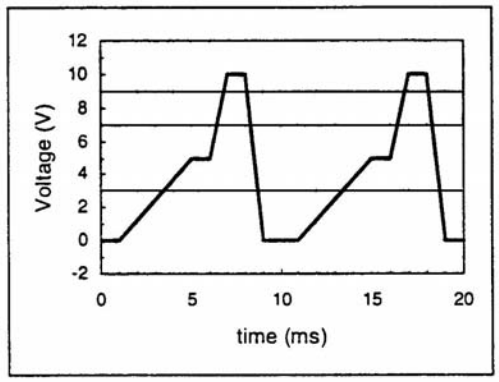
so,

$$f(t) = \frac{A}{\pi} + \frac{A}{2}\sin(\pi t) + \sum_{\substack{n=2 \\ n\ even}}^{\infty} \frac{2A}{\pi(1-n^2)}\cos(n\omega_0 t)$$

Problem 14.20

Use PSpice to determine the Fourier series of the waveform shown in the form.

$$V_S(t) = a_0 + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t + \theta_n)$$



Suggested Solution

Transient specifics were Final Time=10ms, Step Ceiling=10us,
Center Frequency=100 Hz and Number of Harmonics=10

FOURIER COMPONENTS OF TRANSIENT RESPONSE V(Vs)
DC COMPONENT= 3.750000E+00

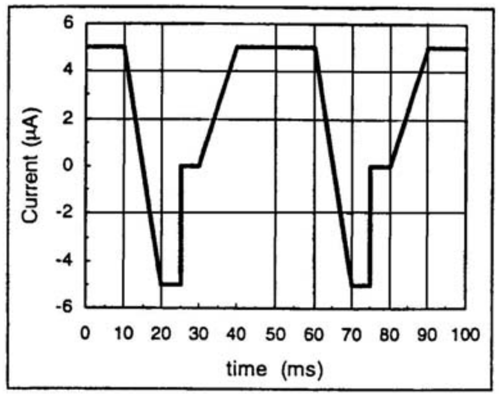
HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE(DEG)
1	1.000E+02	3.954E+00	1.000E+00	-1.424E+02	0.000E+00
2	2.000E+02	2.016E+00	5.099E-01	-8.774E+01	1.971E+02
3	3.000E+02	1.247E+00	3.154E-01	-8.735E+00	4.185E+02
4	4.000E+02	6.417E-01	1.623E-01	6.991E+01	6.395E+02
5	5.000E+02	2.027E-01	5.126E-02	9.000E+01	8.020E+02
6	6.000E+02	2.852E-01	7.213E-02	1.101E+02	9.645E+02
7	7.000E+02	2.291E-01	5.793E-02	-1.713E+02	8.256E+02
8	8.000E+02	1.260E-01	3.188E-02	-9.226E+01	1.047E+03
9	9.000E+02	4.883E-02	1.235E-02	-3.759E+01	1.244E+03
10	1.000E+03	1.993E-08	5.040E-09	-1.206E+02	1.303E+03

TOTAL HARMONIC DISTORTION = 6.310255E+01 PERCENT

Problem 14.21

Use PSpice to determine the Fourier series of the waveform shown in the form

$$v_s(t) = a_0 + \sum_{n=1}^{\infty} b_n \sin\left(n\omega_0 t = \theta_n\right)$$



Suggested Solution

Transient specifics were Final Time=10ms, Step Ceiling=10us,
Center Frequency=100 Hz and Number of Harmonics=10

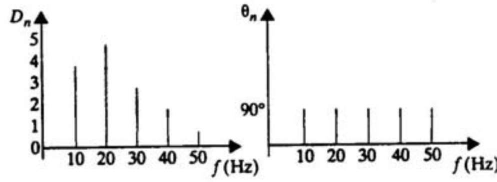
FOURIER COMPONENTS OF TRANSIENT RESPONSE V(Vs)
DC COMPONENT= 1.997500E+00

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE(DEG)
1	2.000E+01	4.401E+00	1.000E+00	1.049E+02	0.000E+00
2	4.000E+01	1.669E+00	3.792E-01	-3.975E+01	-2.495E+02
3	6.000E+01	8.810E-01	2.002E-01	-1.480E+02	-4.627E+02
4	8.000E+01	4.518E-01	1.027E-01	3.521E+01	-3.844E+02
5	1.000E+02	3.183E-01	7.233E-02	1.791E+02	-3.454E+02
6	1.200E+02	3.001E-01	6.819E-02	2.212E+01	-6.073E+02
7	1.400E+02	2.046E-01	4.648E-02	-1.565E+02	-8.907E+02
8	1.600E+02	1.829E-01	4.157E-02	-2.297E+01	-8.621E+02
9	1.800E+02	1.913E-01	4.347E-02	1.625E+02	-7.816E+02
10	2.000E+02	1.592E-01	3.617E-02	-1.800E+00	-1.051E+03

TOTAL HARMONIC DISTORTION= 4.597325E+O1 PERCENT

Problem 14.22

The discrete line spectrum for a periodic function *f*(*t*) is shown. Determine the expression for *f*(*t*).



Suggested Solution

$f_0 = 10HZ$ $\qquad \omega_0 = 20\pi \ r/s$ $\qquad D_n = 2|Cn|$

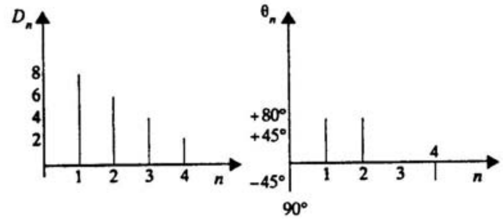
$D_n = \underline{|\theta n} = a_n - jb_n$

$all \ \theta n = 90°$ $\qquad a_n = 0$ $\qquad and \ b_n = -D_n$

$$f(t) = -4\sin(20\pi t) - 5\sin(40\pi t) - 3\sin(60\pi t) - 2\sin(80\pi t) - \sin(100\pi t)$$

Problem 14.23

The amplitude and phase spectra for a periodic function $v(t)$ that has only a small number of terms is shown. Determine the expression for $v(t)$ if $T_0 = 0.1$ s.



Suggested Solution

$T_0 = 0.1\ \text{sec} \qquad \omega_0 = 20\pi\ r/s$

n	a_n	θ_n
1	8	80°
2	6	80°
3	4	0°
4	2	−45°

$$v(t) = 8\cos(20\pi t + 80°) + 6\cos(40\pi t + 80°) + 4\cos(60\pi t) + 2\cos(80\pi t - 45°)$$

Problem 14.24

Plot the first four terms of the amplitude and phase spectra for the signal

$$f(t) = \sum_{\substack{n=1 \\ n\,odd}}^{\infty} \frac{-2}{n\pi} \sin\left(\frac{n\pi}{2}\right) \cos(n\omega_0 t) + \frac{6}{n\pi} \sin(n\omega_0 t)$$

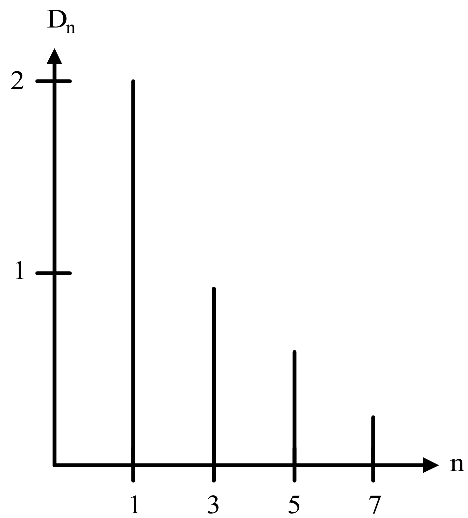
Suggested Solution

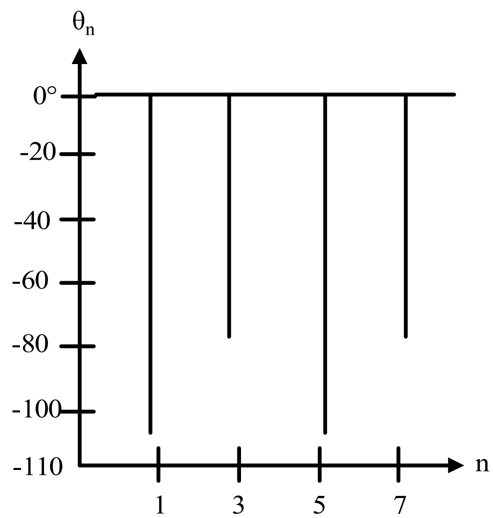
$$f(t) = \sum_{\substack{n=1 \\ n\ odd}}^{\infty} \frac{-2}{n\pi} \sin\left(\frac{n\pi}{2}\right) \cos\left(n\omega_0 t + \frac{6}{n\pi}\right) \sin(n\omega_0 t)$$

$$\left.\begin{aligned} a_n &= \frac{-2}{n\pi} \sin\left(\frac{n\pi}{2}\right) \\ b_n &= \frac{6}{n\pi} \end{aligned}\right\} n\ odd$$

$$D_n = \sqrt{a_n^{\ 2} + b_n^{\ 2}} \qquad \theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)$$

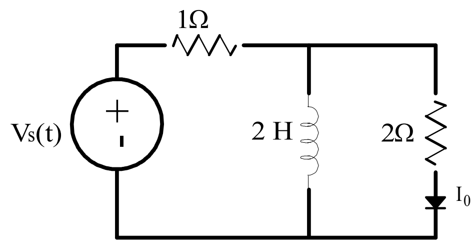
n	D_n	$\theta_n\ (^\circ)$
1	2.01	−108
2	0	0
3	0.67	−72
4	0	0
5	0.40	−108
6	0	0
7	0.29	−72





Problem 14.25

Determine the steady-state response of the current $i_o(t)$ in the circuit shown if the input voltage is described by the waveform shown in Problem 14.8.



Suggested Solution

$$Z = jzn \parallel 1 = \frac{jzn}{1 + jzn}$$

$$From\ 14.8,\ T_0 = 2\pi,\ \omega_0 = 1\ r/s$$

$$v_s(t) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{20}{n\pi} \cos(nt - 90°) V$$

$$\frac{I_0}{V_s} = \frac{Z}{Z+2} = \frac{jzn}{jzn + z + j4n} = \frac{jn}{1 + j3n}$$

$$let\ G(n) = \frac{jn}{1 + j3n}\ \ and\ \ \theta_n = \angle G(n)$$

$$I_{0(n)} = V_s(n) G(n)$$

$$i_0(t) = \frac{(-1)^{n+1} 20}{n\pi} |G(n)| \cos(nt - 90° + \theta_n)$$

Problem 14.26

If the input voltage in Problem 14.25 is

$$v_s(t) = 1 - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(0.2\pi \mathrm{nt})V$$

find the expression for the steady-state current $i_o(t)$.

Suggested Solution

$$prob.\ 14.25\ circuit\ w/\ v_s(t) = 1 - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{\pi nt}{5}\right)V$$

$$Now,\quad \omega_0 = \frac{\pi}{s},\qquad \frac{I_0}{V_s} = \frac{j2n\omega_0}{2 + j6n\omega_0} = \frac{jn\frac{\pi}{s}}{1 + j3n\frac{\pi}{s}} = \frac{jn\pi}{5 + j3n\pi}$$

$$Now,\quad G(n) = \frac{jn\pi}{5 + j3n\pi}\ and\ \theta_n = \underline{|G(n)}$$

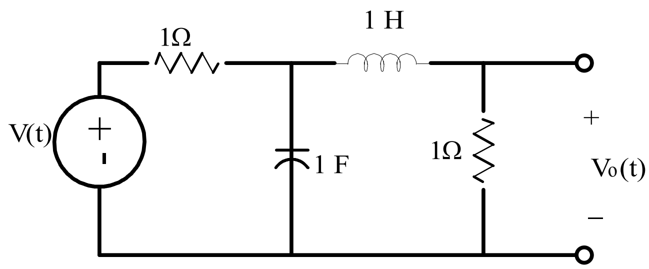
$$From\ the\ circuit,\ i_0\ dc\ component\ is\ zero,\ 50,$$

$$i_0(t) = \sum_{n=1}^{\infty}\frac{-2}{\pi n}|G(n)|\cos\left(\frac{\pi nt}{s} - 90^\circ + \theta_n\right)A$$

Problem 14.27

Determine the first three terms of the steady-state voltage $v_o(t)$ if the input voltage is a periodic signal of the form

$$v_s(t)=\frac{1}{2}+\sum_{n=1}^{\infty}\frac{1}{n\pi}\left(\cos(n\pi-1)\right)\sin(nt)V$$



Suggested Solution

$$V_{TH}=\frac{V}{1+jn} \qquad Z_{TH}=\frac{1}{1+jn}$$

$$\frac{V_0}{V_{TH}}=\frac{1}{1+jn+Z_{TH}}=\frac{1+jn}{(1+jn)^2+1}$$

$$V_0=\frac{V(n)}{2-n^2+j2n} \qquad let \qquad H(n)=\frac{1}{2-n^2+j2n} \quad and \quad \theta_n=\angle H(n)$$

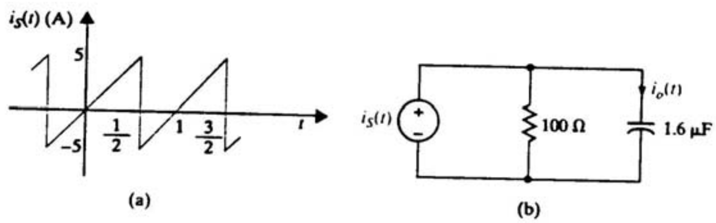
$$V_0=\frac{1}{4} \qquad V_0(1)=\frac{\frac{+2}{\pi}\angle 90^\circ}{1+j2}=0.28\angle 26.6^\circ$$

$$V_0(2)=0 \qquad V_0(3)=\frac{\frac{2}{3\pi}\angle -49^\circ}{-7+j6}=0.023\angle -49^\circ$$

$$V_0=\frac{1}{4} \qquad V_0(1)=0.28\angle 26.6 \qquad V_0(2)=0 \qquad V_0(3)=0.023\angle -49.4^\circ$$

Problem 14.28

The current I_s (t) shown is applied to the circuit shown. Determine the expression for the steady-state current i_o (t) using the first four harmonics.



Suggested Solution

$T_0 = 1 \qquad \omega_0 = 2\pi$

From Table 15.1, $\quad i_s(t) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{10}{n\pi} \sin(n\omega_0 t) \backslash$

$$I_0 = I_s \left[\frac{100}{100 - j\frac{100k}{n}} \right] = \left[\frac{n}{n - j100} \right] \quad Let\ H(n) = \frac{n}{n - j100}$$

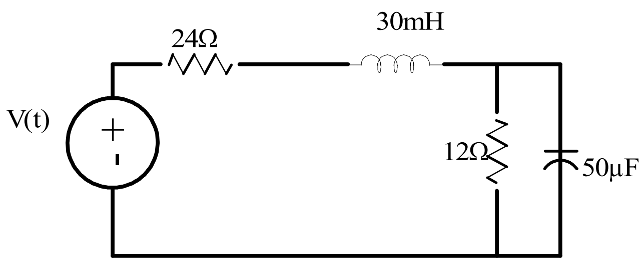
n	$\lvert H(n) \rvert$	$\underline{\lvert H(n)}$	$b_n\ (for\ i_s\ lt)$
1	10^{-3}	89.9	$\frac{10}{\pi}$
2	2×10^{-3}	89.9	$\frac{-5}{\pi}$
3	3×10^{-3}	89.8	$\frac{10}{3\pi}$
4	4×10^{-3}	89.8	$\frac{-5}{2\pi}$

$$i_0 = 3.18\sin(2\pi t + 89.9°) - 3.18\sin(4\pi t + 89.9°) + 3.18\sin(6\pi t + 89.8°)$$
$$-3.18\sin(8\pi t + 89.8°)\ mA$$

Problem 14.29

Find the average power absorbed by the network shown if

$$V(t) = 50 + 25\cos(377t + 45°) + 24\cos(754t - 60°) \text{ V.}$$



Suggested Solution

$$v(t) = 60 + 36\cos(\omega_0 t + 45°) + 24(2\omega_0 t - 60°)V$$

$$\omega_0 = 377\, r/s$$

$$\frac{I}{V} = \frac{1}{24 + jn11.31 + 12 \parallel \frac{53.1}{jn}} = \frac{1}{24 - jn11.31 + \frac{636.6}{53.1 + jn12}} = \frac{53.1 + jn12}{1911 - n^2 135.7 + jn600}$$

$$V_{dc} = 60V \qquad I_{dc} 60\left(\frac{53.1}{1911}\right) = 1.67A$$

$$V(1) = 36V \qquad \theta_{v_1} = 45° \qquad I(1) = 1.05\underline{|39.1°}$$

$$V(2) = 24V \qquad \theta_{v_2} = -60° \qquad I(2) = 0.77\underline{|-76.9°}$$

$$P = 60(1.67) + \frac{36(1.05)}{2}\cos(5.9) + \frac{24(0.77)}{2}\cos(16.9°)$$

$$P = 127.8W$$

Problem 14.30

Find the average power absorbed by the 12-Ω resistor in the network of Problem 14.29 if

$$V(t) = 50 = 25\cos(377t - 45°) + 12.5\cos(754t + 45°) \text{ V.}$$

Suggested Solution

$$V(t) = 50 + 25\cos(\omega_0 - 45°) + 12.5\cos(2\omega_0 t + 45°)$$

$$\omega_0 = 377\frac{rad}{s}$$

$$Z = 12 \parallel \frac{53.1}{jn} = \frac{637.2}{53.1 + j12n}$$

$$\frac{V_0}{V} = \frac{Z}{Z + 24 + jn11.31} = \frac{637.2}{637.2 + 1274.4 - n^2 137.7 + j888.6n}$$

$$V_0(n) = V(n) = \frac{1}{3 - n^2(0.22) + j1.39n} \qquad I_0(n) = \frac{V_0(n)}{12}$$

$$V_{DC} = 50 \qquad V_{0_{DC}} = \frac{50}{3} \qquad I_{0_{DC}} = \frac{50}{36} = 1.39$$

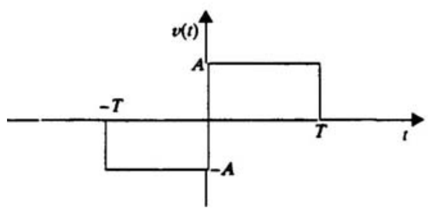
$$V(1) = 25 \qquad \theta_1 = -45° \qquad V_0(1) = 8.04\angle -71.6° \qquad I_0(1) = 0.67\angle -71.6°$$

$$V(2) = 12.5 \qquad \theta_2 = 45° \qquad V_0(2) = 3.58\angle -7.67° \qquad I_0(2) = 0.30\angle -7.67°$$

$$P = \left(\frac{50}{3}\right)1.39 + \frac{(8.04)0.67}{2} + \frac{(3.58)0.30}{2} = 26.40\ W$$

Problem 14.31

Determine the Fourier transform of the waveform shown.



Suggested Solution

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

$$V(\omega) = \int_{-T}^{0} -Ae^{-j\omega t}dt + \int_{0}^{T} Ae^{-j\omega t}dt = \frac{A}{j\omega}\left\{ e^{-j\omega t}\Big|_{-T}^{0} + e^{-j\omega t}\Big|_{0}^{T} \right\}$$

$$V(\omega) = \frac{A}{j\omega}\left\{1 - e^{-j\omega t} + 1 - e^{-j\omega t}\right\} = \frac{2A}{j\omega}\left\{1 - \cos(\omega t)\right\}$$

Problem 14.32

Derive the Fourier transform for the following functions:

(a) $f(t) = e^{-2t} \cos 4t u(t)$.

(b) $f(t) = e^{-2t} \sin 4t u(t)$.

Suggested Solution

PART A

$$f(t) = e^{-2t}\cos(4t)u(t)$$

$$F(\omega) = \int_0^\infty e^{-2t}\cos(4t)e^{-j\omega t}dt = \frac{1}{2}\int_0^\infty \left(e^{-(2-j4+j\omega)t} + e^{-(2+j4+j\omega)t}\right)dt$$

$$F(\omega) = \frac{1}{2}\left[\left.\frac{e^{-(2-j4+j\omega)t}}{2-j4+j\omega}\right|_\infty^0 + \left.\frac{e^{-(2+j4+j\omega)t}}{2+j4+j\omega}\right|_\infty^0\right] = \frac{2+j\omega}{(2+j\omega-j4)(2+jw+j4)}$$

$$F(\omega) = \frac{2+j\omega}{(2+j\omega)^2+16}$$

PART B

$$f(t) = e^{-2t}\sin(4t)u(t) = \frac{1}{j2}\left[e^{-(2-j4)t} - e^{-(2+j4)t}\right]u(t)$$

$$F(\omega) = \frac{1}{j2}\int_0^\infty \left(e^{-(2-j4+j\omega)t} + e^{-(2+j4+j\omega)t}\right)dt$$

$$F(\omega) = \frac{1}{j2}\left[\left.\frac{e^{-(2-j4+j\omega)t}}{2-j4+j\omega}\right|_\infty^0 + \left.\frac{e^{-(2+j4+j\omega)t}}{2+j4+j\omega}\right|_0^\infty\right] = \frac{1}{j2}\left[\frac{j8}{(2+j\omega-j4)(2+jw+j4)}\right]$$

$$F(\omega) = \frac{4}{(2+j\omega)^2+16}$$

Problem 14.33

Show that

$$F\left[f_1(t)\,f_2(t)\right]=\frac{1}{2\pi}\int_{-\infty}^{\infty}F_1(x)F_2(\omega-x)\,dx$$

Suggested Solution

$$Let\ G=\frac{1}{2\pi}\int_{-\infty}^{\infty}F_1(x)F_2(\omega-x)\ dx$$

$$F^{-1}[G]=\frac{1}{(2\pi)^2}\int_{x=-\infty}^{\infty}F_1(x)\int_{\omega=-\infty}^{\infty}F_2(\omega-x)e^{j\omega t}d\omega dx\ \ let\ u=\omega-x\ and\ du=d\omega$$

$$F^{-1}[G]=\frac{1}{(2\pi)^2}\int_{x=-\infty}^{\infty}F_1(x)\int_{u=-\infty}^{\infty}F_2(u)e^{jut}e^{jxt}dudx$$

$$F^{-1}[G]=\frac{1}{(2\pi)^2}\int_{x=-\infty}^{\infty}F_1(x)e^{jxt}dx\int_{u=-\infty}^{\infty}F_2(u)e^{jut}du=f_1(t)\,f_2(t)$$

$$F\left[f_1(t)\,f_2(t)\right]=G=\frac{1}{2\pi}\int_{-\infty}^{\infty}F_1(x)F_2(\omega-x)\ dx$$

Problem 14.34

Find the Fourier transform of the function $f(t) = e^{-a|t|}$.

Suggested Solution

$$f(t) = e^{-a|t|} \qquad F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt = \int_{-\infty}^{0} e^{(a-j\omega)t} dt + \int_{0}^{\infty} e^{-(a-j\omega)t} dt$$

$$F(\omega) = \left. \frac{e^{(a-j\omega)t}}{a-j\omega} \right|_{-\infty}^{0} + \left. \frac{e^{-(a-j\omega)t}}{a+j\omega} \right|_{\infty}^{0} = \frac{1}{a-j\omega} + \frac{1}{a+j\omega} = \frac{2a}{a^2+\omega^2}$$

Problem 14.35

Find the Fourier transform of the function $f(t0 = 12e^{-2|t|}\cos 4t$.

Suggested Solution

$f(t) = 12e^{-2|t|}\cos 4t \qquad Let\ g(t) = 12e^{-2|t|}$

$From\ table\ 14.3, \qquad F\left[g(t)\cos\omega_0 t\right] = \frac{1}{2}\left[G(\omega-\omega_0) + G(\omega+\omega_0)\right]$

$AND, \qquad G(\omega) = \frac{48}{4+\omega^2}$

$$F(\omega) = \frac{1}{2}\left[\frac{48}{4+(\omega-4)^2} + \frac{48}{4+(\omega+4)^2}\right] = \frac{24}{4+(\omega-4)^2} + \frac{24}{4+(\omega+4)^2}$$

Problem 14.36

Determine the output signal $v_o(t)$ of a network with input signal $v_i(t) = 3e^{-t}u(t)$ and network impulse response $h(t) = e^{-2t} u(t)$. Assume that all initial conditions are zero.

Suggested Solution

$$v_i(t) = 3e^{-t}u(t) \qquad V_i(\omega) = \frac{3}{1 + j\omega}$$

$$h(t) = e^{-2t}u(t) \qquad H(\omega) = \frac{1}{2 + j\omega}$$

$$V_0(\omega) = V_i(\omega)H(\omega) = \frac{3}{(1 + j\omega)(2 + j\omega)} = \frac{3}{1 + jw} - \frac{3}{2 + j\omega}$$

$$v_0(t) = 3\left(e^{-t} - e^{-2t}\right)u(t)\ V$$

Problem 14.37

The input signal to a network is $v_i(t) = e^{-3t}\, u(t)$. The transfer function of the network is $\mathbf{H}(j\omega) = 1/(j\omega + 4)$. Find the output of the network $v_o(t)$ if the initial conditions are zero.

Suggested Solution

$$v_i(t) = e^{-3t} u(t) \Rightarrow V_i(\omega) = \frac{1}{3 + j\omega}$$

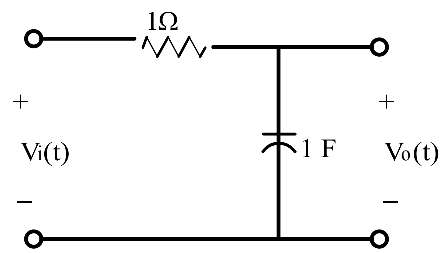
$$H(j\omega) = \frac{1}{4 + j\omega}$$

$$V_0(\omega) = V_i(\omega) H(\omega) = \frac{3}{(3 + j\omega)(4 + j\omega)} = \frac{1}{3 + jw} - \frac{1}{4 + j\omega}$$

$$v_0(t) = \left(e^{-3t} - e^{-4t}\right) u(t)\ V$$

Problem 14.38

The input signal for the network shown is $v_i(t) = 10e^{-5t}(t)$ V. Determine the total 1-Ω energy content of the output $v_o(t)$.



Suggested Solution

$$v_i(t) = 10e^{-5t}u(t) \Rightarrow V_i(\omega) = \frac{10}{5 + j\omega}$$

$$H(\omega) = \frac{\frac{1}{j\omega}}{1 + \frac{1}{j\omega}} \qquad V_0 = \frac{10}{(1 + j\omega)(5 + j\omega)}$$

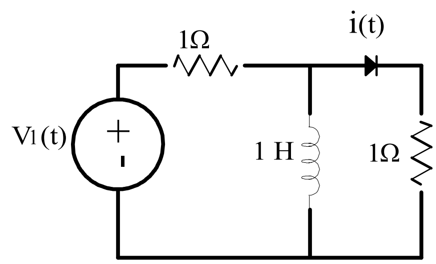
$$|V_0(\omega)|^2 = \frac{100}{(1+\omega^2)(25+\omega^2)} = \frac{\frac{100}{24}}{1+w^2} - \frac{\frac{100}{24}}{25+\omega^2}$$

$$W = \frac{1}{2\pi}\int_{-\infty}^{\infty} |V_0(\omega)|^2\, d\omega = \frac{25}{12\pi}\left(\int_{-\infty}^{\infty}\frac{d\omega}{1+w^2} - \int_{-\infty}^{\infty}\frac{d\omega}{25+w^2}\right)$$

$$W = \frac{25}{12\pi}\left\{\tan^{-1}(\omega)\Big|_{-\infty}^{\infty} - \frac{1}{5}\tan^{-1}\left(\frac{\omega}{5}\right)\Big|_{-\infty}^{\infty}\right\} = \frac{25}{12\pi}\left[\pi - \frac{\pi}{5}\right] = \frac{5}{3}J$$

Problem 14.39

Use the Fourier transform to find $i(t)$ in the network shown if $v_i(t) = 2e^{-t}u(t)$.



Suggested Solution

$$Z = j\omega \parallel 1 = \frac{j\omega}{1+j\omega} \qquad V = V_i\frac{Z}{Z+1} = V_i\left(\frac{j\omega}{1+2j\omega}\right)$$

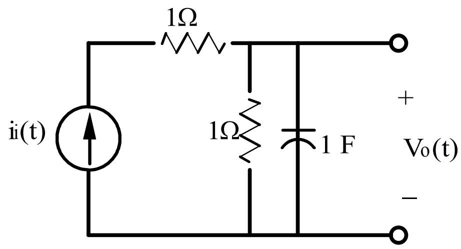
$$V_i = \frac{2}{1+j\omega} \qquad I = \frac{V}{1} = \frac{j\omega}{(1+j\omega)(1/2+j\omega)}$$

$$I = \frac{2}{1+j\omega} - \frac{1}{\frac{1}{2}+j\omega}$$

$$i(t) = 2e^{-t} - e^{-\frac{t}{2}}u(t)\ A$$

Problem 14.40

Determine the voltage $v_o(t)$ in the circuit shown, using the Fourier transform if $v_i(t) = 2e^{-4t}\, u(t)$.



Suggested Solution

$$\frac{V_0}{I_i} = \frac{1}{j\omega} \| 1 = \frac{1}{j\omega + 1} = H(\omega) \qquad V_0 = \frac{2}{(j\omega+1)(j\omega+4)}$$

$$V_0 = \frac{\frac{2}{3}}{j\omega+1} - \frac{\frac{2}{3}}{j\omega+4}$$

$$v_0(t) = \frac{2}{3}\left[e^{-t} - e^{-4t}\right]u(t)\ V$$

Problem 14.41

Compute the 1-Ω energy content of the signal $v_o(t)$ in Problem 14.38 in the frequency range from ω = 2 to ω = 4 rad/s.

Suggested Solution

FROM 14.38

$$W = \frac{25}{12\pi}\left\{ \tan^{-1}(\omega)\Big|_2^4 - \frac{1}{5}\tan^{-1}\left(\frac{\omega}{5}\right)\Big|_2^4 \right\} = 0.106J$$

Problem 14.42

Determine the 1-Ω energy content of the signal $v_o(t)$ in Problem 14.38 in the frequency range from 0 to 1 rad/s.

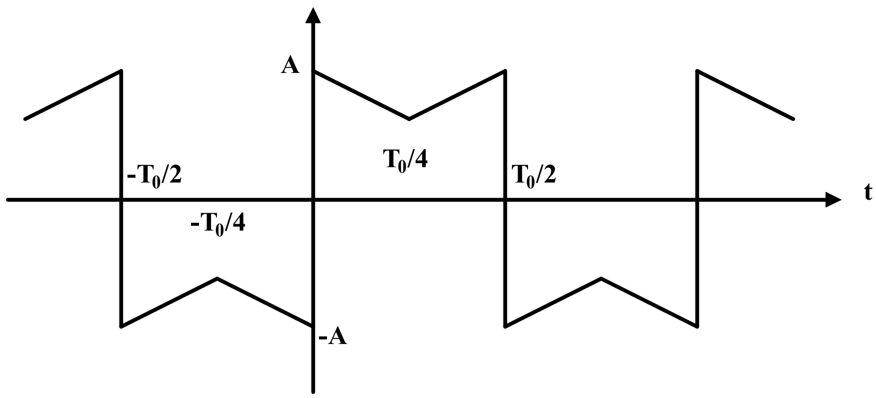
Suggested Solution

FROM 14.38

$$W = \frac{25}{12\pi}\left\{ \tan^{-1}(\omega)\Big|_0^1 - \frac{1}{5}\tan^{-1}\left(\frac{\omega}{5}\right)\Big|_0^1 \right\} = 0.50J$$

Problem 14FE-1

Given the waveform shown, determine which of the trigonometric Fourier coefficients have zero value, which have nonzero value and why.



Suggested Solution

$a_0=0$ Average value is zero

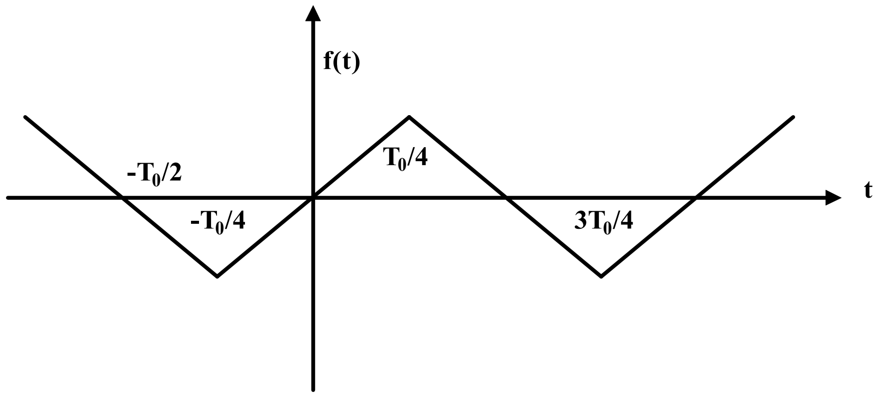
$a_n=0$ for all n since this is an odd function

$b_n=0$ n-even because of halfwave symmetry

b_n=finite number for n-odd

Problem 14FE-2

Given the waveform shown, describe the type of symmetry and its impact on the trigonometric coefficients in the Fourier series, i.e. a_0, a_n, b_n.

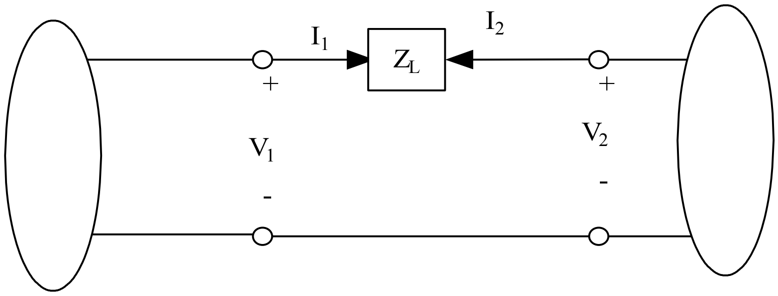


Suggested Solution

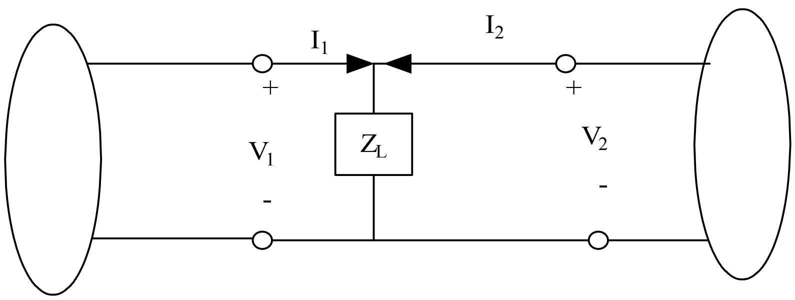
$a_0=0$ Average value is zero
$a_n=0$ for all n since this is an odd function
$b_n=0$ n-even because of halfwave symmetry
b_n=finite number for n-odd

Problem 15.1

Given the two networks shown, find the Y parameters for the circuit in (a) and Z parameters for the circuit in (b).



(a)



(b)

Suggested Solution

(a)

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V2=0} = \frac{1}{Z_L} \qquad Y_{21} = \left.\frac{I_2}{V_1}\right|_{V2=0} = -\frac{1}{Z_L}$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V1} = 0 = -\frac{1}{Z_L} \qquad Y_{22} = \left.\frac{I_2}{V_\infty}\right|_{V1=0} = \frac{1}{Z_L}$$

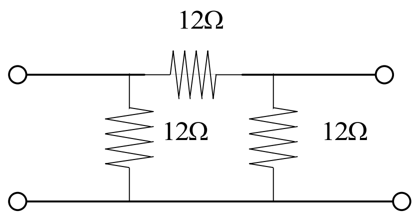
(b)

$$Z_{11} = \left.\frac{V_1}{I1}\right|_{I2=0} = Z_L \qquad Z_{21} = \left.\frac{V_2}{I1}\right|_{I2=0} = Z_L$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I1=0} = Z_L \qquad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I1=0} = Z_L$$

Problem 15.2

Find the Y parameters for the two-port network shown .



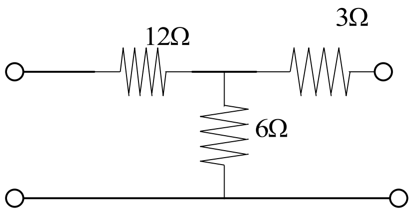
Suggested Solution

$$Y_{11} = \left.\frac{I_1}{V_2}\right|_{V2=0} = \frac{S}{6} \qquad Y_{21} = \left.\frac{I_2}{V_1}\right|_{V2=0} = -\frac{S}{12}$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V1=0} = -\frac{S}{12} \qquad Y_{12} = \left.\frac{I_2}{V_2}\right|_{V1=0} = \frac{S}{6}$$

Problem 15.3

Find the Y parameters for the two-port network shown.



Suggested Solution

$$Y_{11} = \left.\frac{I_1}{V_2}\right|_{V2=0} = \frac{1}{12+3\,||\,6} = \frac{S}{14}$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V1=0} = I_1, when, V_2 = 1, \frac{1-V}{3} = \frac{V}{6\,||\,12}, V = \frac{4}{7}$$

$$So, I1 = \frac{-4/7}{12\Omega} = \frac{-1}{21}A$$

$$Y_{12} = \frac{-1}{21}S$$

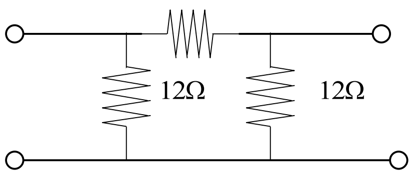
$$C = \left.\frac{I_2}{V_1}\right|_{V2=0} = I_2, When, V1 = 1, \frac{1-V}{12} = \frac{V}{3\,||\,6}, V = \frac{1}{7}$$

$$So, I_2 = \frac{-V/7}{3\Omega} = \frac{-A}{21}, Y_{21} = \frac{-S}{21}$$

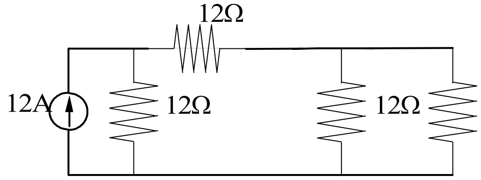
$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V1=0} = \frac{1}{3+6\,||\,12} = \frac{S}{7}$$

Problem 15.4

If a 12-A source is connected at the input port of the network shown, find the current in a 4-ohm load resistor.



Suggested Solution



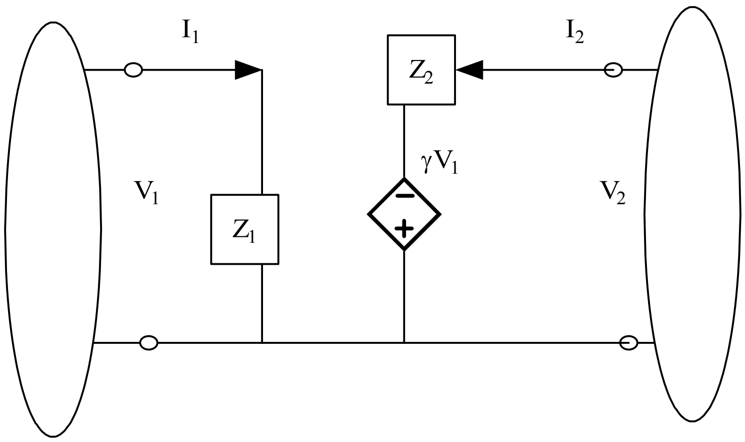
The equations for the two-port are: $\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1/6 & -1/12 \\ -1/12 & 1/6 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$,But the terminal conditions are

$$I_1 = 12A, I_2 = \frac{-V_2}{4}, \ \therefore \ \begin{aligned} 12 &= \frac{1}{6}V_1 - \frac{1}{12}V_2 \Rightarrow V_2 = 16V, I_2 = -4A \\ 0 &= -\frac{1}{12}V_1 + \frac{5}{12}V_2 \end{aligned}$$

$V_2 = 16V, I_2 = -4A$

Problem 15.5

Find the Y parameters for the two-port network shown.



Suggested Solution

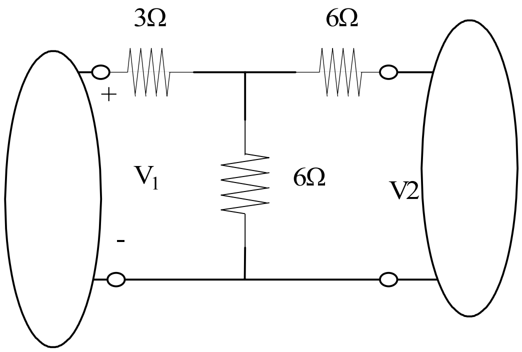
$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V2=0} = \frac{1}{21}$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V1=0} = 0$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V1=0} = \frac{1}{Z_2} \qquad Y_{21} = \left.\frac{I_2}{V_1}\right|_{V2=0} = \frac{8}{Z_2}$$

Problem 15.6

Determine the Y parameters for the network shown.



Suggested Solution

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V2=0} = \frac{1}{3+6 \| 6} = \frac{1}{6} S$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V1=0} = I_1, WHEN, V_2 = 1, \frac{V_2 - V}{6} = \frac{V}{6 \| 3}, V = \frac{1}{4}$$

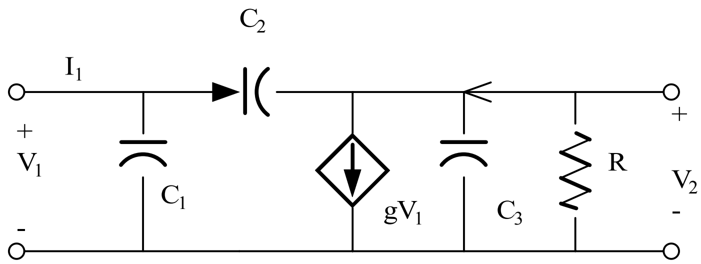
THEN, $I_1 = \frac{-V/4}{3\Omega} = \frac{-1}{12} A$ and $Y_{12} = \frac{-1}{12} S$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V2=0} = I_2, WHEN, V_1 = 1, \frac{1-V}{3} = \frac{V}{6 \| 6}, V = \frac{1}{2} \quad \text{THEN, } I_2 = \frac{-V/2}{6\Omega} = \frac{-A}{21}, Y_{21} = \frac{-S}{12}$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V1=0} = \frac{1}{6+3 \| 6} = \frac{S}{8}$$

Problem 15.7

Determine the admittance parameters for the network shown.



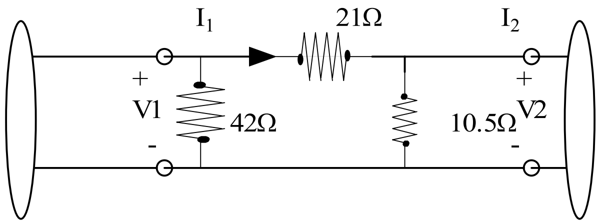
Suggested Solution

$Y_{11} = jw(C_1 + C_2), Y_{21} = g - jwC_2$

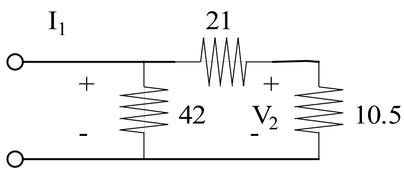
$Y_{12} = -jwC_2, Y_{22} = \dfrac{1}{R} + jw(C_2 + C_3)$

Problem 15.8

Find the Z parameters of the two port network shown.



Suggested Solution



$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{(42)(31.5)}{42+31.5} = 18\Omega$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$V_2 = \frac{10.5V_1}{21+10.5} = \frac{10.5}{31.5}(18I_1)$$

$$V_2 = 6I_1 \Rightarrow Z_{21} = 6\Omega$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{(10.5)(63)}{42+31.5} = 9\Omega$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{(42/63)(V_2)}{I_2} = \frac{2}{3}Z_{22} = 6\Omega$$

Problem 15.9

Find Z parameters of the network in Problem 15.5.

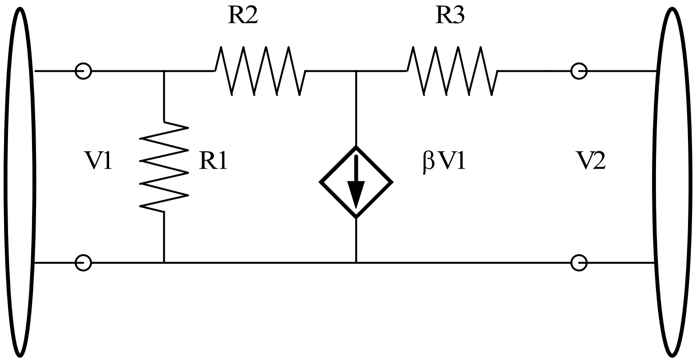
Suggested Solution

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = Z_1, Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 0$$

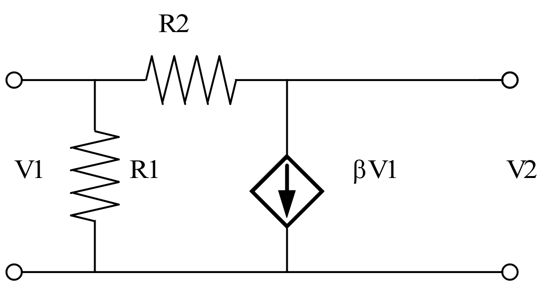
$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = -\gamma Z_1, Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = Z_2$$

Problem 15.10

Determine the Z parameters for the two port network shown.



Suggested Solution



$$I_2 = 0$$

$$I_1 = \frac{V_1}{R_1} + \beta\, V_1$$

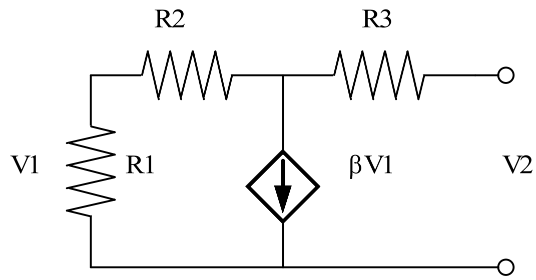
$$I_1 = (\frac{1}{R_1} + \beta)V_1$$

$$I_1 = (\frac{1+\beta R_1}{R_1})V_1 \Rightarrow Z_{11} = \frac{R_1}{1+\beta R_1}$$

$$V_2 = V_1 - \beta\, V_1 R_2 \Rightarrow V_2 = V_1(1-\beta\, R_2) = (1-\beta\, R_2)(\frac{R_1}{1+\beta R_1})I_1$$

$$Z_{21} = \frac{R_1(1-\beta\, R_2)}{1+\beta R_1}$$

$$Z_{21} = \frac{R_1(1-\beta\, R_2)}{1+\beta R_1}$$



$$I_1 = 0$$

$$I_2 = \frac{V_1}{R_1} + \beta V_1$$

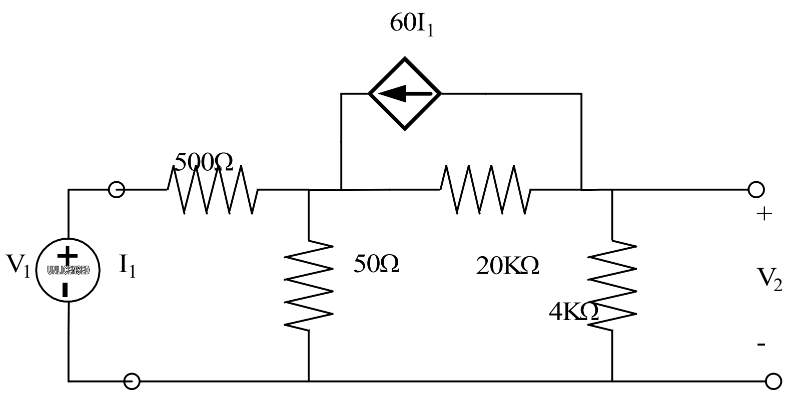
$$I_2 = (\frac{1}{R_1} + \beta)V_1 = (\frac{1+\beta R_1}{R_1})V_1 \Rightarrow V_1 = \frac{R_1}{1+\beta R_1} I_2$$

$$Z_{12} = \frac{R_1}{1+\beta R_1}$$

$$V_2 = I_2 R_3 + (\frac{V_1}{R_1})R_2 + V_1 = I_2 R_3 + V_1(\frac{R_2}{R_1} + 1)$$

Problem 15.11

Find the Z parameters for the two port shown. Determine the voltage gain of the entire circuit with a 4k-ohm load attached.



Suggested Solution

$$Z_{11} = 550, Z_{21} \Rightarrow V_2 = 50I - 60(20K)I_1 \Rightarrow Z_{21} = 1200K$$

$$Z_{12} = 50, Z_{22} = 20K + 50 = 20K$$

$$I_2 = \frac{-V0}{4K}, V_2 = V0$$

$$V_1 = 550I_1 + 50I_2$$

$$V_2 = -1200KI_1 + 20KI_2$$

$$V_1 = 550 - \frac{50}{4K}V_0$$

$$V_0 = -1200KI_1 - \frac{20K}{4K}V_C$$

$$550I_1 = V_1 + 0.0125V_0$$

$$1200KI_1 = -V_0 - 5V_C$$

$$1200KI_1 = 2181.82V_1 + 2727V_0$$

$$1200KI_1 = -6V_0$$

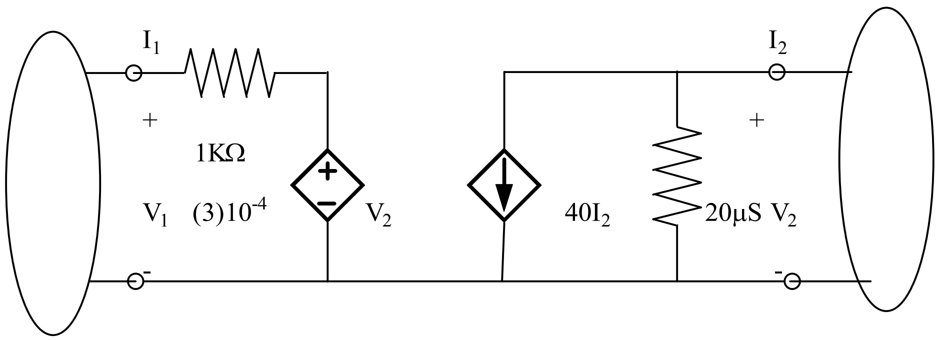
$$\therefore -6V_0 = 2181.82V_1 + 27.27V_0 \Rightarrow -33.27V_0 = 2181.82V_1$$

$$\frac{V_0}{V_1} = -65.6$$

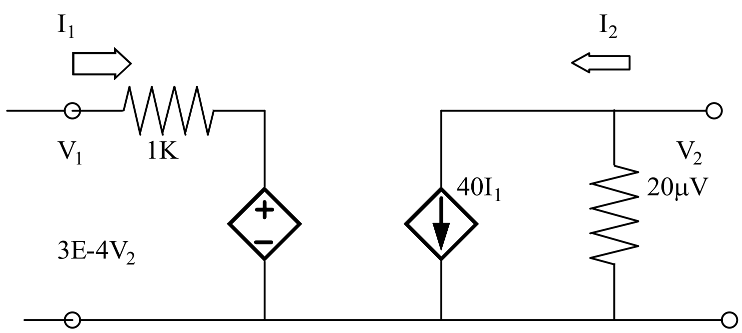
$$\frac{V_0}{V_1} = -65.6$$

Problem 15.12

Find the Z parameters for the two port network shown.



Suggested Solution



$h_{11} = 1K, h_{21} = 40, h_{12} = 3E - v, h_{22} = 20\mu$

$\Delta H = h_{11}h_{22} - h_{12}h_{21} = 8E - 3$

$$Z_{11} = \frac{\Delta h}{h_{22}} = \frac{8E-3}{2E-5} = 400\Omega$$

$$Z_{12} = \frac{h_{12}}{h_{22}} = \frac{3E-4}{2E-5} = 15\Omega$$

$$Z_{21} = \frac{-h_{21}}{h_{22}} = \frac{-40}{20\mu} = -2E-6\Omega$$

$$Z_{22} = \frac{1}{h_{22}} = 50K\Omega$$

Problem 15.13

Find the voltage gain of the two port network in Problem 15.12 with a 12k-ohm load.

Suggested Solution

$$V_1 = 400I_1 + 15I_2$$

$$V_2 = -2X10^{-6} I_1 + 50X10^3 I_2$$

$$V_1 = 400I_1 - \frac{15V_2}{12K}$$

$$V_2 = -2X10^6 I_1 - \frac{50X10^3 V_2}{12X10^3}$$

$$\frac{V_2}{V_1} = -438$$

$$5000V_1 = 2X10^6 - 6.25V_2$$

$$V_2 = -2X10^6 I_1 - 4.17V_2$$

$$5000V_1 + V_2 = -10.42V_2 \Rightarrow -11.42 = 5000V_1 \Rightarrow \frac{V_2}{V_1} = -438$$

Problem 15.14

Find the input impendence of the network in Problem 15.12

Suggested Solution

From problem 15.13 solution:

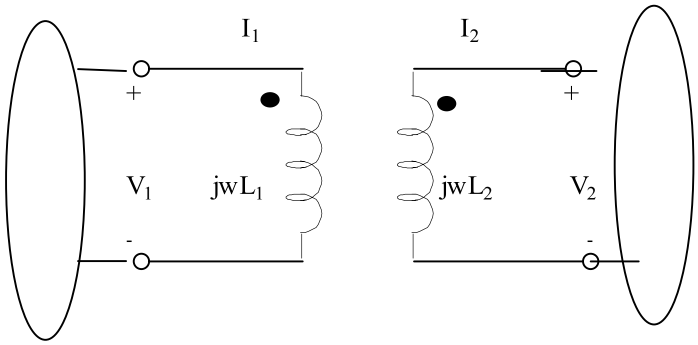
$$5000V_1 = 2X10^6 I_1 - 6.25(-4.38V_1)$$

$$(5000 - 2737.5)V_1 = 2X10^6 I_1 \Rightarrow Z_{in} = \frac{V_1}{I_1} = \frac{2X10^6}{2262.5} = 884\Omega$$

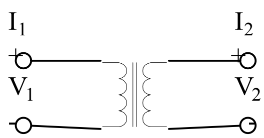
$$Z_{in} = 884\Omega$$

Problem 15.15

Find the Z parameters of the two port network shown.



Suggested Solution

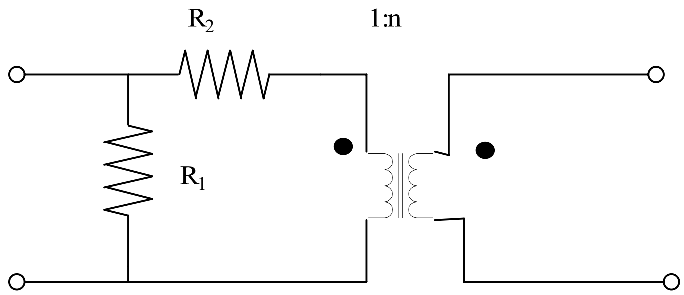


$$V_1 = I_1 j\omega L_1 + I_1 nj\omega$$
$$V_2 = I_1 nj\omega + I_2 j\omega L_2$$
$$Z_{11} = j\omega L_1, Z_{12} = j\omega n, Z_{21} = j\omega n, Z_{22} = j\omega L_2$$

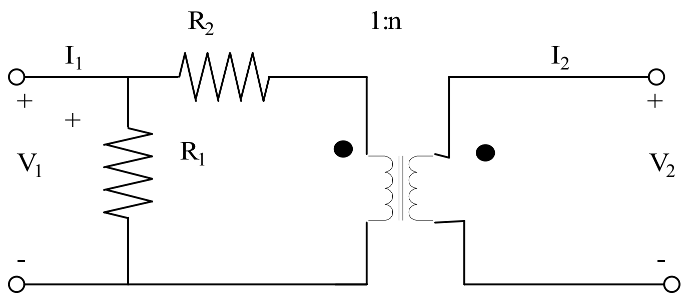
$$Z_{11} = j\omega L_1, Z_{12} = j\omega n, Z_{21} = j\omega n, Z_{22} = j\omega L_2$$

Problem 15.16

Determine the Z parameters of the two port network shown.



Suggested Solution



$V_1 = nI_2R_1$

$\therefore Z_{12} = nR_1$

$Z_{11} = R_1$

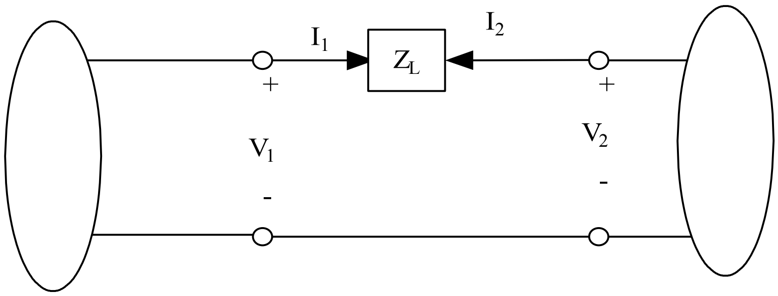
$V_2 = nR_1I_1$

$\therefore Z_{21} = nR_1$

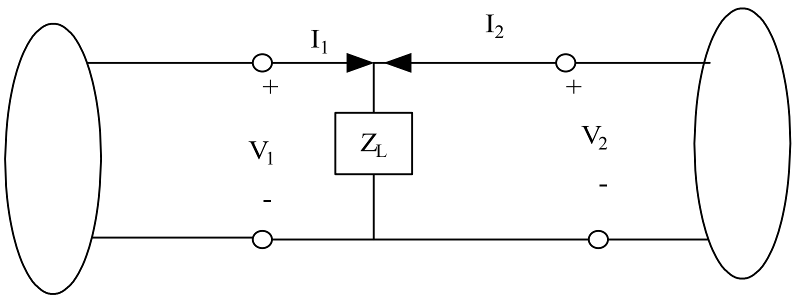
$Z_{22} = n^2(R_1 + R_2)$

Problem 15.17

Compute the hybrid parameters for the network shown.

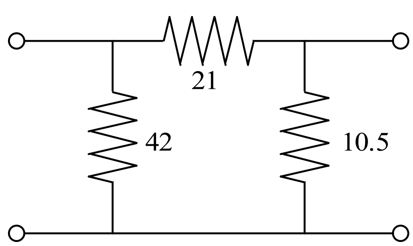


(a)



(b)

Suggested Solution



$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V2=0} = 21 \,\|\, 42 = 14\Omega$$

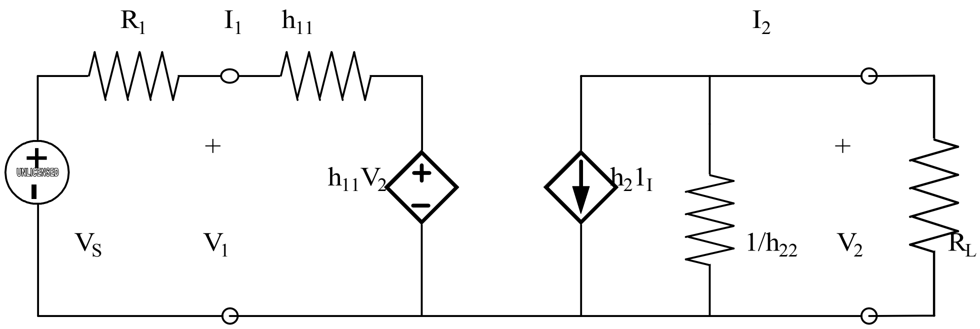
$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I1=0} = \frac{42}{42+21} = \frac{2}{3}$$

$$h_{21} = \left.\frac{I_1}{I_2}\right|_{V2=0} = \frac{-42}{42+21} = \frac{-2}{3}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I1=0} = \frac{1}{10.5 \,\|\, (21+42)} = \frac{1}{9}S$$

Problem 15.18

Consider the network shown. The two port network is a hybrid model of a basic transistor . Determine the voltage gain of entire network, V2/Vs, if a source Vs with internal resistance R1 is applied at the input to the two port and a load RL is connected.



Suggested Solution

$$V_{11} = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$
Also
$$V_2 = -R_L I_2 or I_2 = \frac{-V_2}{R_L} and V_1 = V_S - R_1 I_1$$

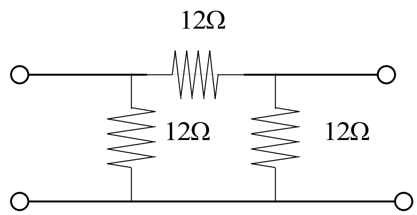
$$V_S - R_1 I_1 = h_{11}I_1 + h_{12}V_2$$
$$\frac{-V_2}{R_L} = h_{21}I_1 + h_{22}V_2$$

$$V_S = (R_1 + h_{11})I_1 + h_{12}V_2$$
$$h_{21}I_1 = -(h_{22} + \frac{1}{R_L})V_2$$

$$I_1 = -(h_{22} + \frac{1}{R_L})V_2 \frac{1}{h_{21}}$$
$$\therefore V_S = [h_{12} - \frac{1}{h_{21}}(h_{22} + \frac{1}{R_L})(R_1 + h_{11})]V_2$$
$$V_S = [h_{12} - \frac{(1 + h_{22}R_L)(R_1 + h_{11})}{h_{21}R_L}]V_2$$
$$V_S = [\frac{h_{12}h_{21}R_L - (1 + h_{22}R_L)(R_1 + h_{11})}{h_{21}R_L}]V_2$$
$$\frac{V_2}{V_S} = \frac{h21RL}{h_{12}h_{21}R_L - (1 + h_{22}R_L)(R_1 + h_{11})}$$

Problem 15.19

Find the hybrid parameters for the network shown.



Suggested Solution

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V2=0} = 12 \parallel 12 = 6\Omega$$

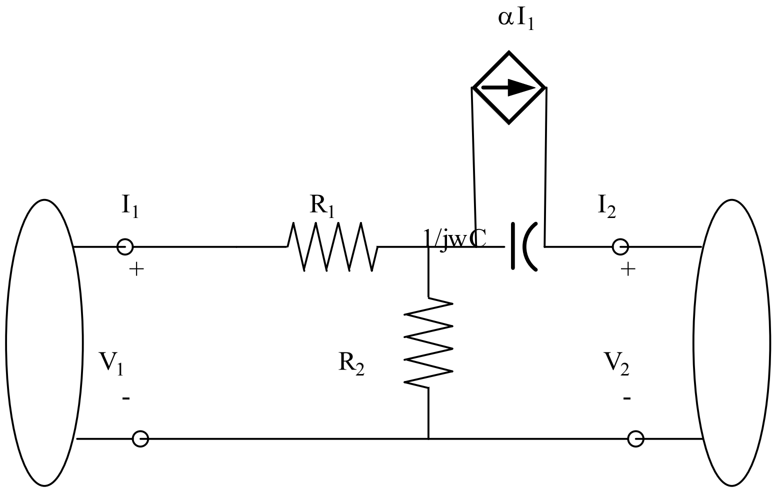
$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I1=0} = \frac{12}{12+12} = 0.5$$

$$h_{21} = \left.\frac{I_1}{I_2}\right|_{V2=0} = \frac{-12}{12+12} = -0.5$$

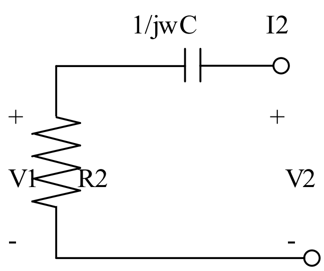
$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I1=0} = \frac{1}{12 \parallel (12+12)} = \frac{1}{8}S$$

Problem 15.20

Determine the hybrid parameters for the network shown.



Suggested Solution



$$I_1 = 0$$

$$V_1 = \frac{R_2 V_2}{R_2 + \frac{1}{j\omega C}}$$

$$h_{12} = \frac{j\omega C R_2}{1 + j\omega C R_2}$$

$$I_2 = \frac{V_2}{R_2 + \frac{1}{j\omega}} = \frac{j\omega C V_2}{1 + j\omega C R_2} \Rightarrow h_{22} = \frac{j\omega C}{1 + j\omega C R_2}$$

$$V_2 = 0$$

$$R_2(I_1 + I_2) + \frac{1}{j\omega C}(\alpha I_1 + I_2) = 0$$

$$(R_2 + \frac{\alpha}{j\omega C})I_1 + (R_2 + \frac{1}{j\omega C})I_2 = 0$$

$$(\alpha + j\omega R_2 C)I_1 = -(1 + j\omega R_2 C)I_2$$

$$I_2 = [\frac{-(\alpha + j\omega R_2 C)}{1 + j\omega R_2 C}]I_1$$

$$\therefore h_{21} = -(\frac{\alpha + j\omega R_2 C}{1 + j\omega R_2 C})$$

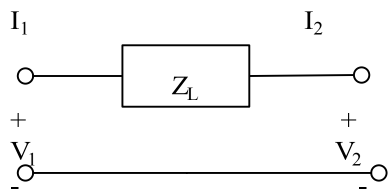
$$V_1 = (R_1 + R_2)I_1 + R_2 I_2 = [R_1 + R_2 - \frac{R_2(\alpha + j\omega R_2 C)}{1 + j\omega R_2 C}]I_1$$

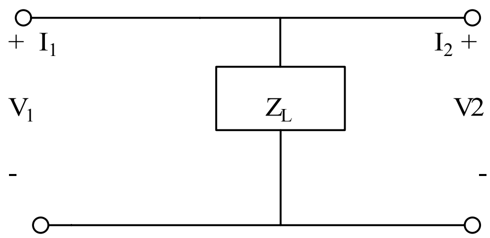
$$\therefore h_{11} = \frac{R_1 j\omega R_1 R_2 C + R_2 + j\omega R_2^{\,2} C - \alpha R_2 - j\omega R_2^{\,2} C}{1 + j\omega R_2 C}$$

$$h_{11} = \frac{R_1 + R_2(1 - \alpha) + j\omega R_1 R_2 C}{1 + j\omega R_2 C}$$

Problem 15.21

Find the ABCD parameters for the networks shown.





Suggested Solution

$$A = \left.\frac{V_1}{V_2}\right|_{I2=0} = 1$$

$$B = \left.\frac{I_1}{-I_2}\right|_{V2=0} = Z_2$$

$$C = \left.\frac{I_1}{V_2}\right|_{I2=0} = 0$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V2=0} = 1$$

$$A = \left.\frac{V_1}{V_2}\right|_{I2=0} = 1$$

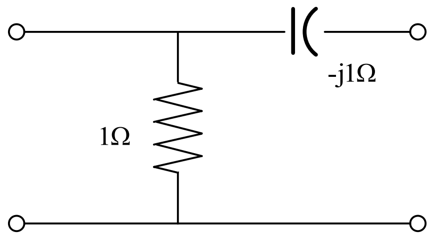
$$B = 0$$

$$C = \frac{1}{Z_2}$$

$$D = 1$$

Problem 15.22

Find the transmission parameters for the network shown.



Suggested Solution

$A = 1$

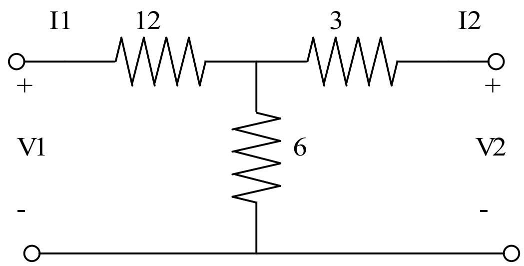
$B = -1j\Omega$

$C = 1s$

$\frac{1(I_1)}{1-j} = -I_2 \Rightarrow D = 1 - j$

Problem 15.23

Find the transmission parameters for the network shown.



Suggested Solution

$$A = \left.\frac{V_1}{I_1}\right|_{I2=0} = 3$$

$$C = \left.\frac{I_1}{V_2}\right|_{I2=0} = \frac{I1}{6I1} = \frac{1}{6}S$$

$$B = \left.\frac{-V_1}{I_2}\right|_{V2=0}$$

$$D = \left.\frac{-I_1}{I_2}\right|_{V2=0} = \frac{-I_1}{\frac{6I_1}{9}} = \frac{3}{2}$$

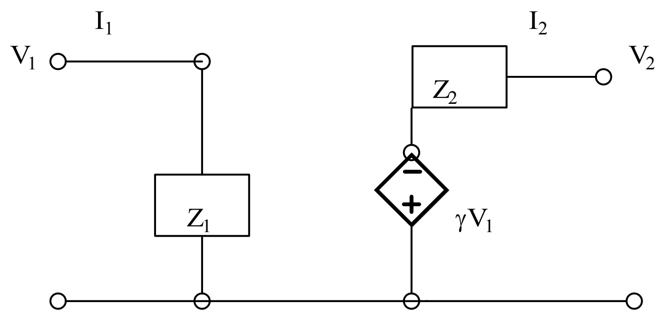
$$I_1 = 1.5I_2$$

$$V_1 = 12I_1 + 6(I_1 + I_2) = 18I_1 + 6I_2 = -21I_2$$

$$B = \frac{-21I_2}{I_2} = 21\Omega$$

Problem 15.24

Find ABCD parameters for the network shown.



Suggested Solution

$$A = \left.\frac{V_1}{V_2}\right|_{I2=0} = \frac{V_1}{\gamma V_1} = \frac{-1}{\gamma}$$

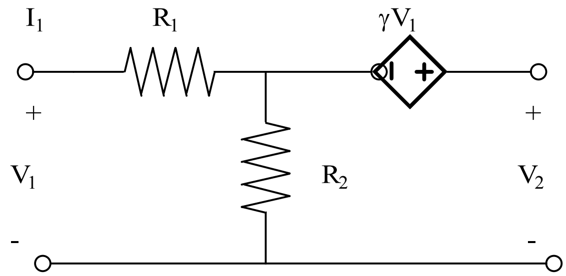
$$B = \left.\frac{V_1}{-I_2}\right|_{V2=0} = \frac{V_1}{\frac{-(V_2+\gamma V_1)}{Z_2}} = \frac{-Z_2}{\gamma}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I2=0} = \frac{I_1}{-\gamma V_2} = \frac{-1}{\gamma Z_1}$$

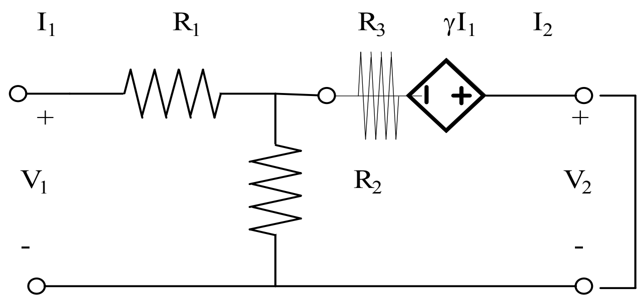
$$D = \left.\frac{I_1}{-I_2}\right|_{V2=0} = \frac{I_1}{\frac{-\gamma V_1}{Z_2}} = \frac{-Z_2}{\gamma Z_1}$$

Problem 15.25

Determine the transmission parameters for the network shown.



Suggested Solution



$$I_2 = 0$$

$$V_2 = (\gamma + R_2)I_1$$

$$C = \frac{1}{\gamma + R_2}$$

$$V_1 = (R_1 + R_2)I_1 \Rightarrow A = \frac{R_1 + R_1}{\gamma + R_2}$$

$$V_2 = 0$$

$$\gamma I_1 + (R_3 + R_2)I_2 + R_2 I_1 = 0$$

$$(\gamma + R_2)I_1 = -(R_3 + R_2)I_2$$

$$I_1 = -(\frac{R_3 + R_2}{\gamma + R_2})I_2 \Rightarrow D = \frac{R_3 + R_2}{\gamma + R_2}$$

$$V_1 = (R_1 + R_2)I_1 + R_2 I_2 = -(\frac{R_3 + R_2}{\gamma + R_2})(R_1 + R_2)I_2 + R_2 I_2$$

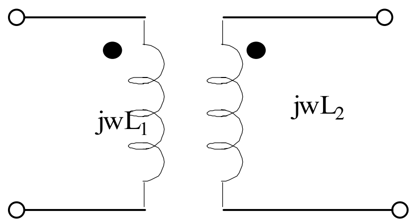
$$V1 = -[\frac{(R_3 + R_2)(R_1 + R_2)}{\gamma + R_2} - R_2]I_2$$

$$\therefore B = \frac{R_3 R_1 + R_3 R_2 + R_2 R_1 + R_2{}^2 - \gamma R_2 - R_2{}^2}{\gamma + R_2} = \frac{R_3 R_1 + R_3 R_2 + R_2 R_1 - \gamma R_2}{\gamma + R_2}$$

$$\frac{R_3 R_1 + R_3 R_2 + R_2 R_1 - \gamma R_2}{\gamma + R_2}$$

Problem 15.26

Find the transmission parameters for the circuit shown.



Suggested Solution

$$For I_2 = 0, I_1 = \frac{V_1}{j\omega L_1}$$

AND

$$V_2 = j\omega M I_1$$

$$\therefore V_2 = \frac{j\omega M V_1}{j\omega L_1}, A = \frac{V_1}{V_2} = \frac{L_1}{M}$$

$Since$

$$V_2 = 0, V_1 = j\omega L_1 I_1 + j\omega M I_2$$

$$0 = j\omega M I_1 + j\omega L_2 I_2$$

$$\therefore I_1 = \frac{L2}{M}(-I_2)$$

$$V_1 = -j\omega L_1 (\frac{L_1}{M})(-I_2) - j\omega M(-I_2)$$

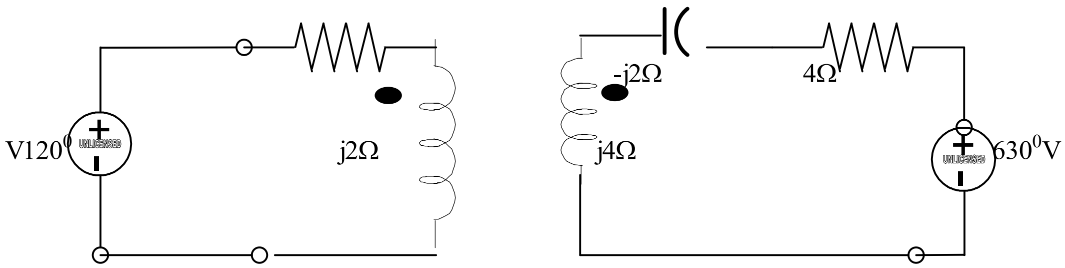
$$\therefore B = \frac{V_1}{-I_2} = j\omega[\frac{L_1 L_2 - M^2}{M}]$$

$$D = \frac{I_1}{-I_2} = \frac{L_2}{M}$$

$$\therefore \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{L_1}{M} & j\omega \frac{L_1 L_2 - M^2}{M} \\ \frac{1}{j\omega M} & \frac{L_2}{M} \end{bmatrix}$$

Problem 15.27

Find the transmission parameters for two port network and then find I_0 using the terminal conditions.



Suggested Solution

$$\Rightarrow \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & j(8-1) \\ -j1 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 4-2j \\ 0 & 1 \end{bmatrix}$$

THEN

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & j(8-1) \\ -j1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 4-2j \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2-2j & 12-5j \\ -1j & 2-4j \end{bmatrix}$$

$V_1 = 12\angle 0^0 V$

$V_2 = 6\angle 30^0 V$

$-I_2 = I_0$

$V_1 = AV_2 + B(-I_2)$

$12\angle 0^0 = (2-2j)6\angle 30^0 + (12-5j)I_0$

Solving

$I_0 = -0.48\angle -22.35^0\ A$

Problem 15.28

Following are the hybrid parameters for a network.

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{11}{5} & \frac{2}{5} \\ \frac{-2}{5} & \frac{1}{5} \end{bmatrix}$$

Determine the Y parameters of the network

Suggested Solution

Using Table 15.1

$$y_{11} = \frac{1}{h_{11}} = \frac{5s}{11}$$

$$y_{12} = \frac{-h_{12}}{h_{11}} = \frac{-2s}{11}$$

$$y_{21} = \frac{h_{21}}{h_{11}} = \frac{-2s}{11}$$

$$y_{22} = \frac{\Delta H}{h_{11}} = \frac{3s}{11}$$

Problem 15.29

If the Y parameters for a network are known to be

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} \frac{5}{11} & \frac{-2}{11} \\ \frac{-2}{11} & \frac{3}{11} \end{bmatrix}$$

Find the Z parameters.

Suggested Solution

$$Y = \begin{bmatrix} \frac{5}{11} & \frac{-2}{11} \\ \frac{-2}{11} & \frac{3}{11} \end{bmatrix}$$

$$\Delta Y = \frac{3x5}{|2|} - \frac{(-2)(-2)}{|2|} = y_{11}y_{22} - y_{21}y_{12} = \frac{1}{11}$$

Then

$$Z_{11} = \frac{y_{22}}{\Delta Y} = \frac{3/11}{1/11} = 3\Omega$$

$$Z_{12} = \frac{-y_{12}}{\Delta Y} = \frac{2/11}{1/11} = 2\Omega$$

$$Z_{12} = \frac{-y_{21}}{\Delta Y} = 2\Omega$$

$$Z_{22} = \frac{y_{11}}{\Delta Y} = 5\Omega$$

Problem 15.30

Find the Z parameters in terms of the ABCD parameters

Suggested Solution

The ABCD Parameters equations are:

$V_1 = AV_1 - BI_2$

$I_1 = CV_2 - DI_2$

$\Rightarrow$

$V_2 = \frac{1}{C}[I_1 + DI_2]$

$V_1 = A[\frac{I_1}{C} + \frac{DI_2}{C}] - BI_2 = \frac{AI_1}{C} + (\frac{AD}{C} - B)I_2$

$Z_{11} = \frac{A}{C}$

$Z_{12} = \frac{AD - BC}{C}$

$Z_{21} = \frac{1}{C}$

$Z_{22} = \frac{D}{C}$

Problem 15.31

Find the Hybrid parameters in terms of the Impendance parameters

Suggested Solution

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$
$$I_2 = \frac{-Z_{21}}{Z_{22}}I_1 + \frac{V_2}{Z_{22}}$$

Substitute this value into the first equation yields

$$V_1 = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{22}}I_1 + \frac{Z_{12}}{Z_{22}}V_2$$

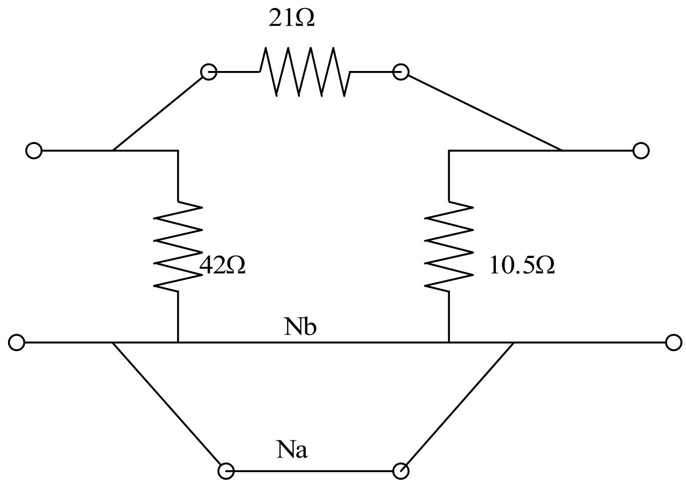
$$h_{11} = \frac{\Delta Z}{Z_{22}}$$
$$h_{12} = \frac{Z_{12}}{Z_{22}}$$
$$h_{21} = \frac{-Z_{21}}{Z_{22}}$$
$$h_{22} = \frac{1}{Z_{22}}$$

Where

$$\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21}$$

Problem 15.32

Find the Y parameters of the network shown, by considering the network to be a parallel connection of two port networks.



Suggested Solution

ForNA

$$Y_A = \begin{bmatrix} \frac{1}{21} & \frac{-1}{21} \\ \frac{-1}{21} & \frac{1}{21} \end{bmatrix}$$

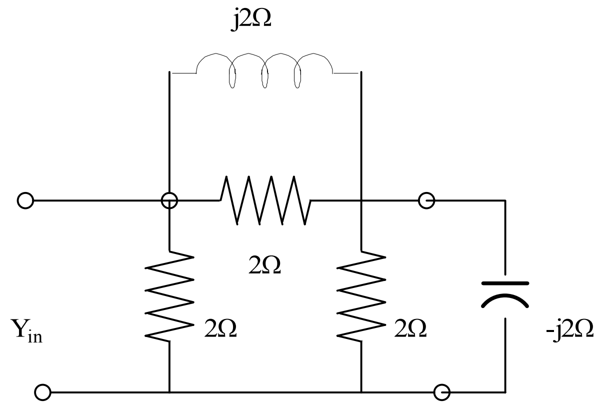
NB

$$Y_B = \begin{bmatrix} \frac{1}{42} & 0 \\ 0 & \frac{1}{10.5} \end{bmatrix}$$

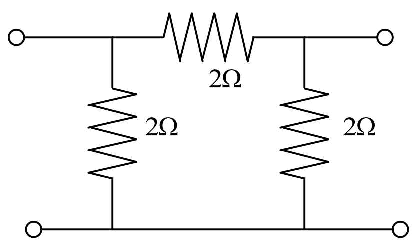
$$Y_1 = Y_A + Y_B = \begin{bmatrix} \frac{1}{14} & \frac{-1}{21} \\ \frac{-1}{21} & \frac{1}{7} \end{bmatrix}$$

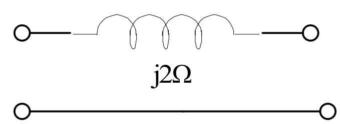
Problem 15.33

Find the Y parameters of the two port shown. Find the input admittance of the network when the capacitor is connected to the output port



Suggested Solution





$$Y_1 = \begin{bmatrix} \frac{1}{j2} & -\frac{1}{j2} \\ -\frac{1}{j2} & \frac{1}{j2} \end{bmatrix}$$

$$Y_2 = \begin{bmatrix} 1 & \frac{-1}{2} \\ \frac{-1}{2} & 1 \end{bmatrix}$$

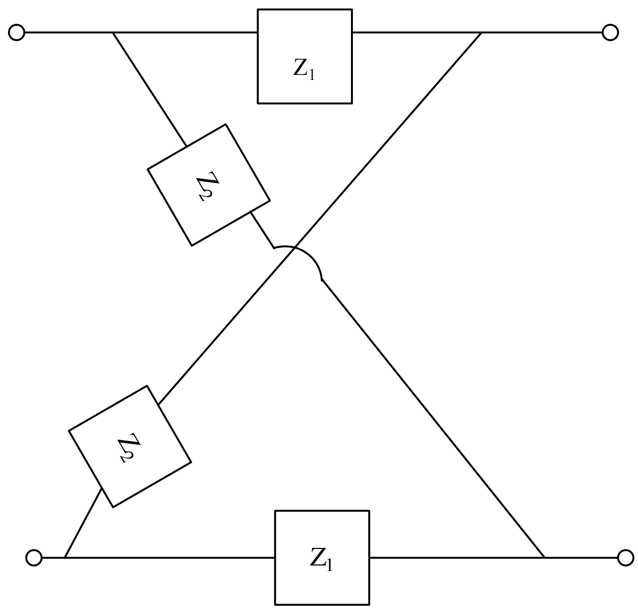
$$Y_T = \begin{bmatrix} 1+\frac{1}{j2} & \frac{-1}{2}-\frac{1}{j2} \\ \frac{-1}{2}-\frac{1}{j2} & 1+\frac{1}{j2} \end{bmatrix}$$

$$Y_{IN} = Y_{11} - \frac{Y_{12}Y_{21}}{Y_{22}Y_L} = 1 - 0.5J - [\frac{(\frac{-1}{2}-\frac{1}{2j})(-\frac{1}{2}-\frac{1}{2j})}{1-0.5j+0.5j}]$$

$$= 1s$$

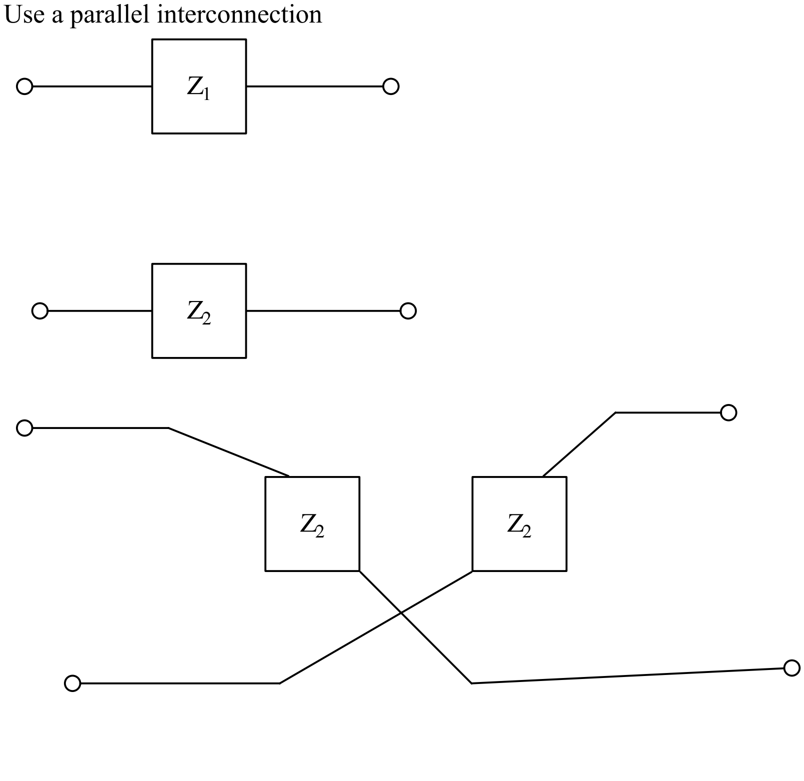
Problem 15.34

Find the Y parameters for the network shown.



Suggested Solution

Use a parallel interconnection



$$y_{11} = \frac{1}{2Z_1}$$

$$y_{12} = -\frac{1}{2Z_1}$$

$$y_{21} = -\frac{1}{2Z_1}$$

$$y_{22} = \frac{1}{2Z_1}$$

$$y_{11} = \frac{1}{2Z_2}$$

$$y_{12} = \frac{1}{2Z_2}$$

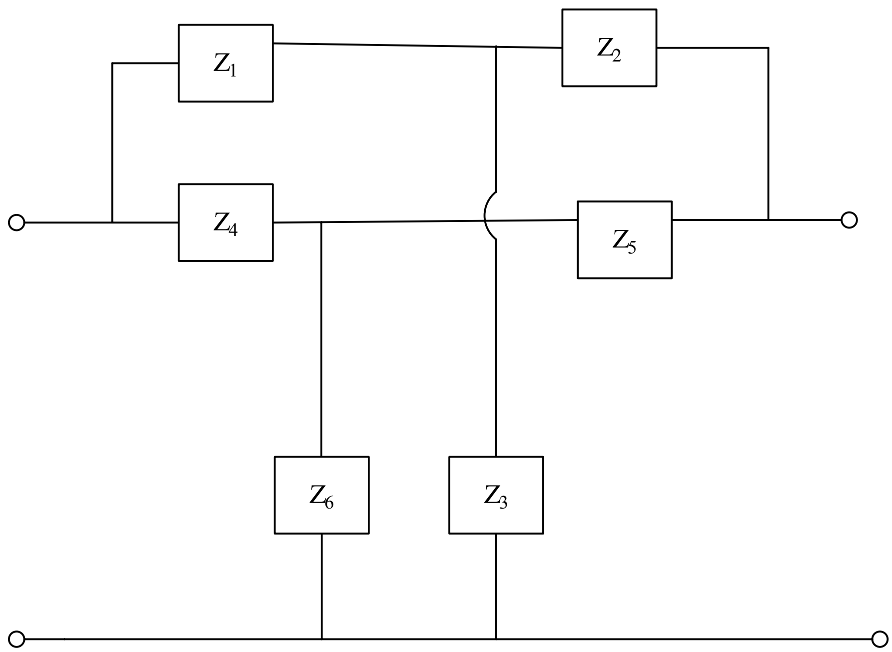
$$y_{21} = \frac{1}{2Z_2}$$

$$y_{22} = \frac{1}{2Z_2}$$

$$\begin{bmatrix} y_{11} & y12 \\ y_{21} & y_{22} \end{bmatrix}_{TOTAL} = \begin{bmatrix} \frac{1}{2Z_1} + \frac{1}{2Z_2} & \frac{-1}{2Z_1} + \frac{1}{2Z_2} \\ \frac{-1}{2Z_1} + \frac{1}{2Z_2} & \frac{1}{2Z_1} + \frac{1}{2Z_2} \end{bmatrix}$$

Problem 15.35

Determine the Y parameters for the network shown.



Suggested Solution

For one T network

$$Y_{11} = \frac{1}{Z_1 + Z_2 \parallel Z_3} = \frac{Z_2 + Z_3}{\Delta Z_{123}}$$

WHERE

$$Z_{123} = Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3$$

$$Y_{12} = Y_{21} = \frac{-Z_3}{\Delta Z_{123}}$$

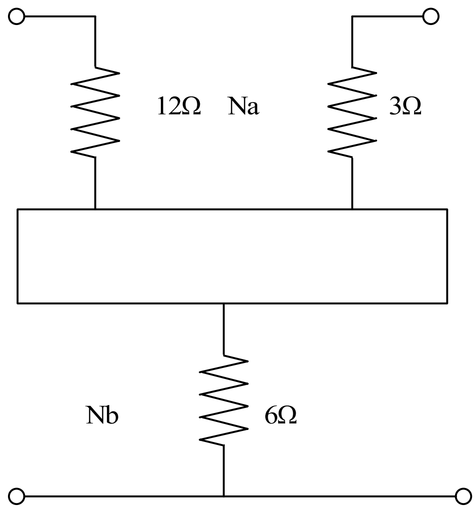
$$Y_{22} = \frac{Z_1 + Z_3}{\Delta Z_{123}}$$

Then for the total network

$$Y_T = \begin{bmatrix} \dfrac{Z_2 + Z_3}{\Delta Z_{123}} + \dfrac{Z_5 + Z_6}{\Delta Z_{456}} & \dfrac{-Z_3}{\Delta Z_{123}} - \dfrac{Z_6}{\Delta Z_{456}} \\ \dfrac{-Z_3}{\Delta Z_{123}} - \dfrac{Z_6}{\Delta Z_{456}} & \dfrac{Z_2 + Z_3}{\Delta Z_{123}} + \dfrac{Z_5 + Z_6}{\Delta Z_{456}} \end{bmatrix}$$

Problem 15.36

Find the Z parameters of the network shown by considering the circuit to be a series connection of two port networks.



Suggested Solution

ForNA

$$Z_A = \begin{bmatrix} 12 & 0 \\ 0 & 3 \end{bmatrix}$$

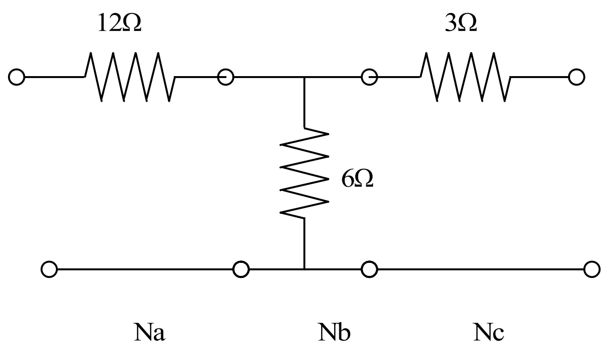
ForNB

$$Z_B = \begin{bmatrix} 6 & 6 \\ 6 & 6 \end{bmatrix}$$

$$\therefore Z_T = Z_A + Z_B = \begin{bmatrix} 18 & 6 \\ 6 & 9 \end{bmatrix}$$

Problem 15.37

Find the transmission parameters of the network shown by considering the circuit to be a cascade connection of three two port networks.



Suggested Solution

$For N_a$

$$\begin{bmatrix} 1 & 12 \\ 0 & 1 \end{bmatrix}$$

$For N_b$

$$\begin{bmatrix} 1 & 0 \\ \frac{1}{6} & 1 \end{bmatrix}$$

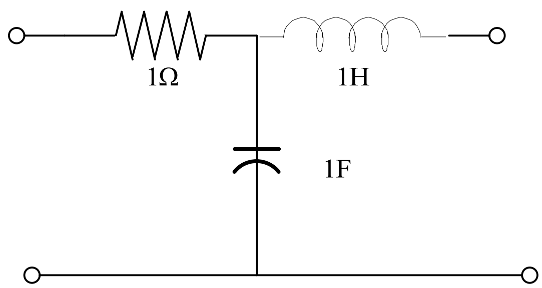
$For N_c$

$$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

$$\therefore ABCD_T = \begin{bmatrix} 1 & 12 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{6} & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 21 \\ \frac{1}{6} & \frac{3}{2} \end{bmatrix}$$

Problem 15.38

Find the ABCD parameters for the circuit shown.



Suggested Solution

For the Resistor

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

For the Inductor

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ j\omega & 1 \end{bmatrix}$$

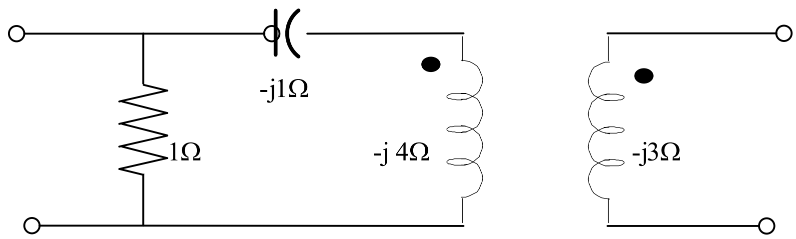
For the Capacitor

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & j\omega \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} AB \\ CD \end{bmatrix} = \begin{bmatrix} 1 + j\omega & 1 + j\omega - \omega^2 \\ j\omega & 1 - \omega^2 \end{bmatrix}$$

Problem 15.39

Find the transmission parameters for the two port network shown.



Suggested Solution

$A = 1$

$C = 1$

$B = -j$

$D = -j$

$A = -1$

$C = -j$

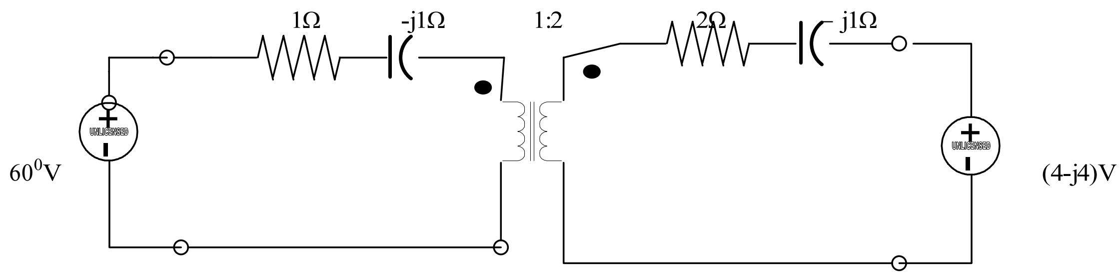
$D = 3$

$B = j^{11}$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & -j \\ 1 & 1-j \end{bmatrix} \begin{bmatrix} 4 & j^{11} \\ -j & 3 \end{bmatrix} = \begin{bmatrix} 3 & j8 \\ 3-j1 & 3+j8 \end{bmatrix}$$

Problem 15.40

Find the transmission parameters of the two port and then use the terminal conditions to find I.



Suggested Solution

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 1-j \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2-j1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.5 & 3-J2.5 \\ 0 & 2 \end{bmatrix}$$

$$\therefore V_1 = 0.5V_2 + (3-j2.5)(-I_2)$$
$$I_1 = 2(-I_2)$$
$$V_1 = 6\angle 0$$
$$V_2 - 4 - j4$$
$$I_0 = -I_2$$

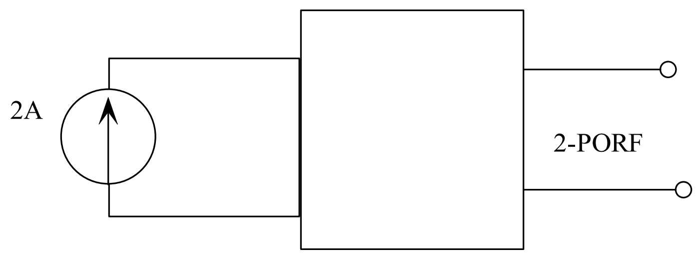
$$\therefore 6\angle 0 = 0.5(4-j4) + (3-j2.5)(I_0)$$

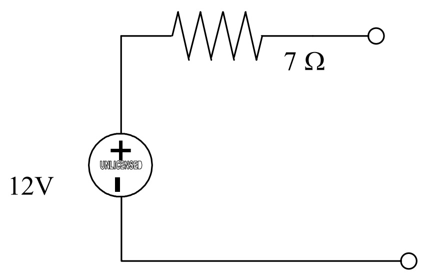
$$I_0 = \frac{4+j2}{3-j2.5} = 1.14\angle 66.38^0\,A$$

Problem 15FE-1

A two port network is know to have the following parameters

Y11 = 1/14s
Y12 = Y21=-1/21s
Y22 = 1/7s





If 2 A current source is connected to the input terminals find the voltage across this source.

Suggested Solution

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

$$Z = \frac{1}{14}V_1 - \frac{1}{21}V_2$$

$$0 = \frac{-1}{21}V_1 + \frac{1}{7}V_2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \frac{1}{\Delta Y}\begin{bmatrix} \frac{1}{7} & \frac{1}{21} \\ \frac{1}{21} & \frac{1}{14} \end{bmatrix}\begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

WHERE

$$\Delta Y = (\frac{1}{14})(\frac{1}{7}) - (\frac{1}{21})(\frac{1}{21}) = \frac{3.5}{441}$$

CHECK

$$Z_{11} = \frac{Y_{22}}{\Delta Y} = \frac{1/3}{3.5/441} = 18$$
$$V_1 = (2)(Z_{11}) = 36V$$

$$V_1 = 36V$$

Problem 15FE-2

Find the Thevenin equivalent circuit at the output terminals of the network. of Problem 15FE1.

Suggested Solution

See previous solution.

$$V_2 = V_{0C} = 12V$$

$$Z_{TA} = \frac{1}{Y_{22}} = 7\Omega$$

The Thevenin equivalent circuit is

Problem 16.1

A diode has Is = $10^{(-15)}$ A and n= 1. Using the diode equation find Id for Vd = 0V, 0.25V, 0.5V, 0.75V.

Suggested Solution

$$I_D = I_S[e^{V_D/nV_T} - 1] = 10^{-15}[e^{39V_D} - 1]$$

Vd(V)	Id(A)
0	0
0.25	$1.7x10^{-11}$
0.50	$2.9x10^{-7}$
0.75	$5.0x10^{-3}$

Problem 16.2

Plot an Id-Vd curve for the diode in Problem 16.1 over the Vd range 0 t +0.65V. On the same graph, add curves for constant voltage and piece wise linear models for current between 10 to 100 uA. What are your estimates for Von Vf and Rd.

Suggested Solution

At this level of study, selecting values for Von, Vf and Rd is essentially a matter of preference. From the plot, the following values produce reasonable fits to the diode curve.

Constant voltage model = Von = 0.625V

Piecewise linear model Vf = 0.585V and Rd = 740 ohm. Recall Rd is the reciprocal of slope of piecewise linear model I-V curve.

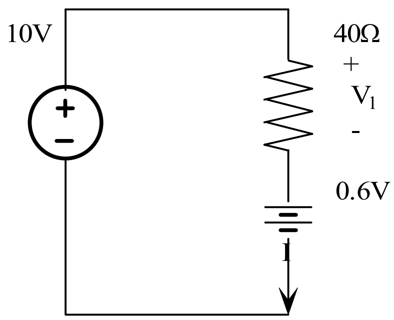
Problem 16.3

Repeat 16.1 using the constant voltage model with Von = 0.6V

Suggested Solution

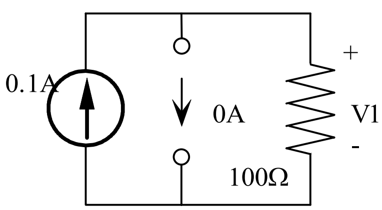
a. Diode is Forward biased

V1= 10 – 0.6 = 9.4V



b. Diode is Reverse biased

V1= (0.1)(100) = 10V



c. Diode is Forward biased

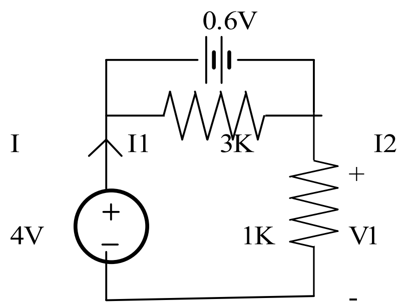
4=0.6+V1
V1=3.4V

d. Diode is Reverse biased

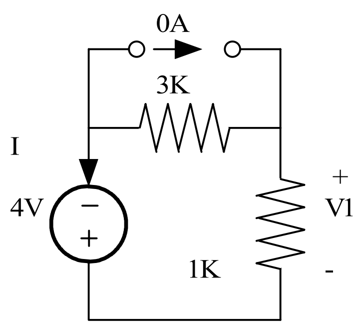
I = (4/3k+1k) = 1mA

V1=-I(1k)=-1V

C.

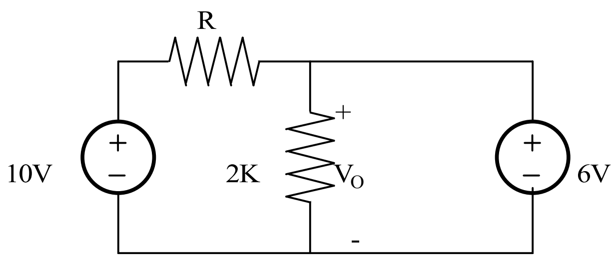


D.



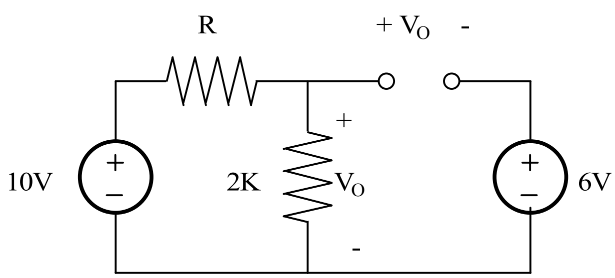
Problem 16.4

For the circuit shown find V0 using a)the ideal model of diode, b) the constant voltage model with Von = 0.7 V, and c) the piecewise linear model with Vf=0.6V and Rd=10ohm. Give your self an attaboy if you can use the mathematical diode equation with Is=1e-15A, n=1 and R = 1Kohm.



Suggested Solution

a. Assume Diode is Reverse biased



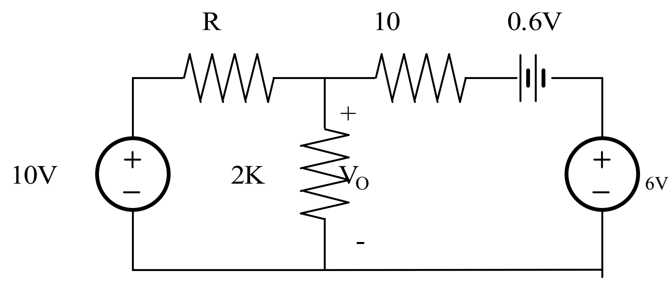
$$V_0 = 10[\frac{2R}{2R+R}] = 6.67V$$
$$V_D = V_0 - 6 = 0.67V$$

Since $V_D > 0$, is our assumption was wrong. Try again with forward bias

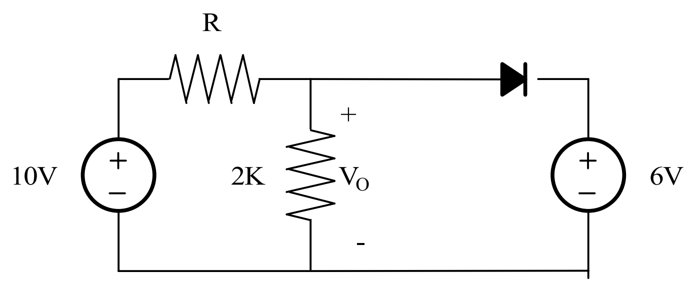
b. Assume reverse bias from a) $V_D = 0.67$V, since $V_D < 0.7$V our assumption is good.

c. Assume reverse bias from a) $V_D = 0.67$V. Since $V_D > 0.6$V, our assumption is bad. Try forward biased. By superposition

$$V_0 = 10[\frac{2000 \| 10}{(2000 \| 10) + 1000}] + 6.6[\frac{1000 \| 2000}{(1000 \| 2000) + 10}] = 6.6V$$



To use the diode equation, start by applying KCL at Vo



$$\frac{10 - V0}{R} = \frac{V_0}{2R} + I_0$$

aLSO

$$V_0 = V_D + 6$$

Thus

$$\frac{10 - V_D - 6}{R} - \frac{V_D + 6}{2R} = ID = \frac{1 - 1.5V_D}{R}$$

From the diode equation

$$V_D = nV_T \ln[\frac{I_D}{I_S} + 1]$$

and

$$I_D = \frac{1 - (3/2)nV_T \ln[\frac{I_D}{I_S} + 1]}{R}$$

can not be solved in closed form. We must iterate.

Procedure:

1. Guess a value for Id
2. Substitute in right side of equation
3. Get answer for Id
4. Put that value back in right side
5. When the difference between successive calculation is small, we have an answer.

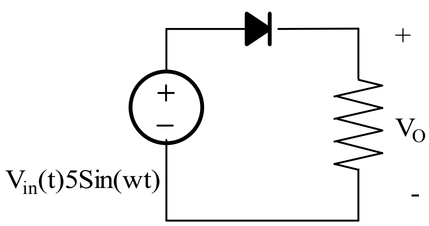
This procedure is easy to automate in Excel. The excel values are shown

Id (mA) Guess	Id (mA) Result	Vd(V)
1.00E-01	2.58E-02	0.438
2.58 E-02	7.79 E-02	0.466
7..79 E-02	3.54 E-02	0.446
3.54 E-02	6.57 E-02	0.462
6.57 E-02	4.20 E-02	0.450
4.20 E-02	5.92 E-02	0.459
5.92 E-02	4.60 E-02	0.452
4.60 E-02	5.57 E-02	0.457
5.57 E-02	4.83 E-02	0.454
4.83 E-02	5.38 E-02	0.456
5.38 E-02	4.97 E-02	0.454
4.97 E-02	5.27 E-02	0.456
5.27 E-02	5.04 E-02	0.455
5.04 E-02	5.22 E-02	0.456
5.22 E-02	5.09 E-02	0.455
5.09 E-02	5.18 E-02	0.455
5.18 E-02	5.11 E-02	0.455
5.11 E-02	5.16 E-02	0.455
5.16 E-02	5.12 E-02	0.455
5.12 E-02	5.15 E-02	0.455
5.15 E-02	5.13 E-02	0.455
5.13 E-02	5.15 E-02	0.455
5.15 E-02	5.14 E-02	0.455
5.14 E-02	5.15 E-02	0.455
5.15 E-02	5.14 E-02	0.455
5.14 E-02	5.14 E-02	0.455

These are the final results.

Problem 16.5

The circuit in fig, is called half wave rectifier. Plot Vin and Vout verses time for 3 cycles of input voltage using a)the ideal diode model, b)constant voltage model with Von = 1.0V



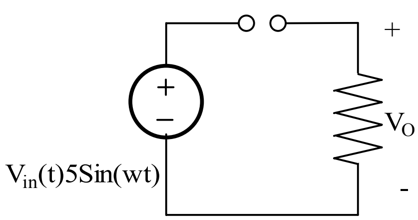
Suggested Solution

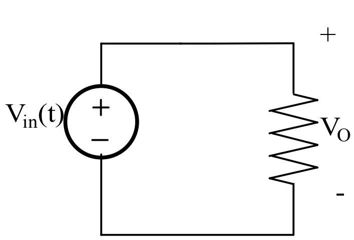
Using the ideal model procedures the circuits and b for forward and reverse bias conditions respectively. When Vin(t) is positive, the diode is forward biased and V0(t) = 5sin(wt). Alternatively, when the diode is reversed biased Vin(t) is negative and V0(t)= 0.

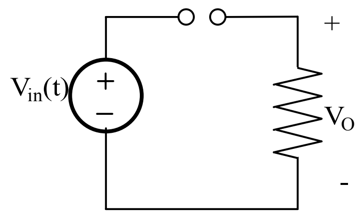
The circuits, c and d model forward and reverse bias conditions respectively for the constant voltage model. When Vin(t) is greater than Von, the diode is forward biased and

$$V_0(t) = 5\sin(\omega t) - 1$$

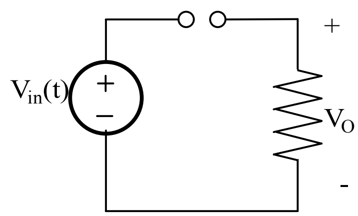
Else V0(t) = 0, Plot V0 for both models.







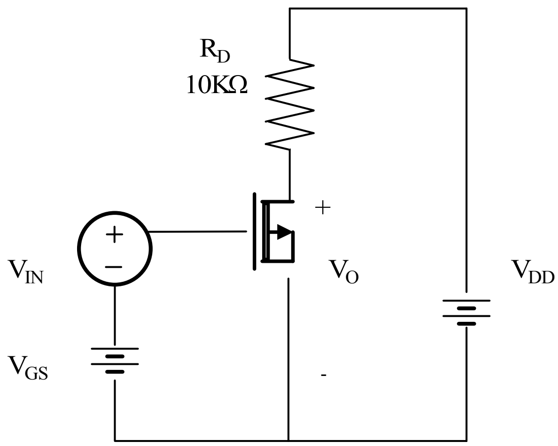
+
$V_{in}(t)$
+
−
V_O
-



+
$V_{in}(t)$
+
−
V_O
-

Problem 16.6

The MOSFET in fig, has Gm = 1mS, and Rds = 100kohm. Draw the small signal circuit and find out the voltage gain Vo/Vin.



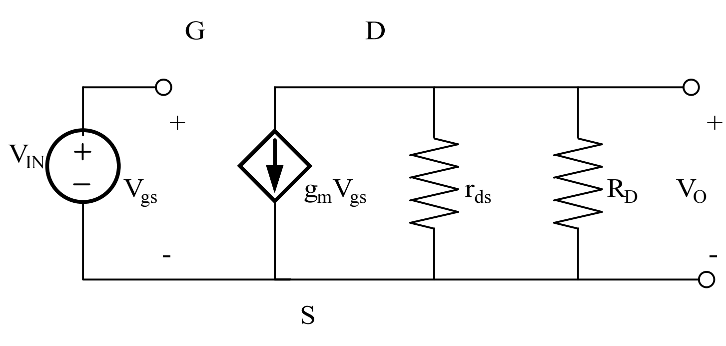
Suggested Solution

$$V_{GS} = V_{IN}$$

$$V_0 = -g_M v_{gs}[r_{ds} \parallel R_D]$$

$$\frac{V_0}{V_{IN}} = -9.1$$

$$\frac{V_O}{V_{GS}} = -g_m[r_{ds} \parallel R_D] = -(1m)[100K \parallel 10K] = -9.1$$



Problem 16.7

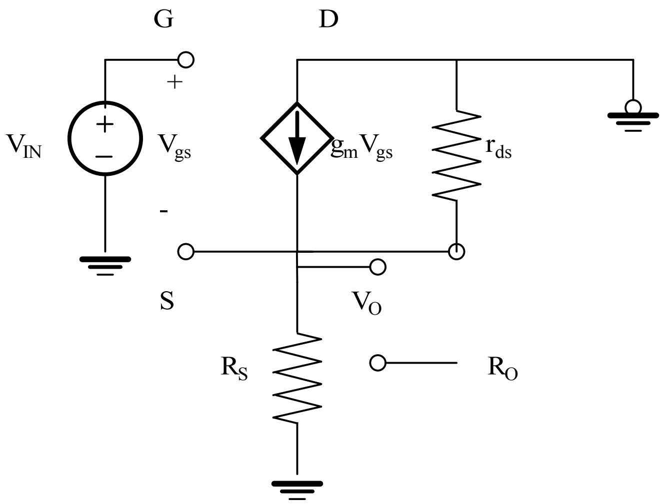
For the amplifier in problem 16.6 plot the gain V0/Vin verses Rd for Rd between 1Kohm and 1Mohm. Why doesn't the gain increase linearly with Rd.

Suggested Solution

The small-signal model resistance Rds is in parallel with Rd. For small values of Rd, it will dominate the parallel combination and the gain will be nearly linear with Rd. As Rd increases, the affect of Rds becomes stronger until it dominates the combination and thus the gain.

Problem 16.8

The amplifier in fig is a comman drain configuration. Find Vo/Vin and the output resistance, Ro in terms of Gm,Rds,Rs. If Gm = 10mS and Rds=1Kohm. What is the value of Ro. What possible utility could this configuration have?



Suggested Solution

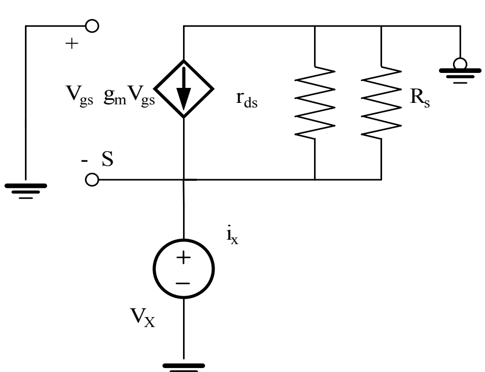
$$V_{GS} = V_{IN} - V_O$$

$$G_M V_{GS} = \frac{V_0}{R_{DS}} + \frac{V_0}{R_S}$$

$$R_S \parallel R_{DS}$$

$$G_M (V_{IN} - V_O) = \frac{V_0}{R_{DS} \parallel R_S}$$

$$G_M V_{IN} = V_0 [G_M + \frac{1}{R_{DS} \parallel R_S}] \Rightarrow \frac{V_0}{V_{IN}} = \frac{G_M}{G_M + \frac{1}{R_{DS} \parallel R_S}}$$



$$A = \frac{V_0}{V_{IN}} = \frac{G_M(R_S \parallel R_{DS})}{G_M(R_S \parallel R_{DS}) + 1}$$

Gain is positive and less than 1.

Rout : Eliminate Vin and apply a test source Vx at point of interest.

$$R_O = \frac{V_X}{I_X}$$

$$V_X = V_{GS} = -V_{GS}$$

ByKCL

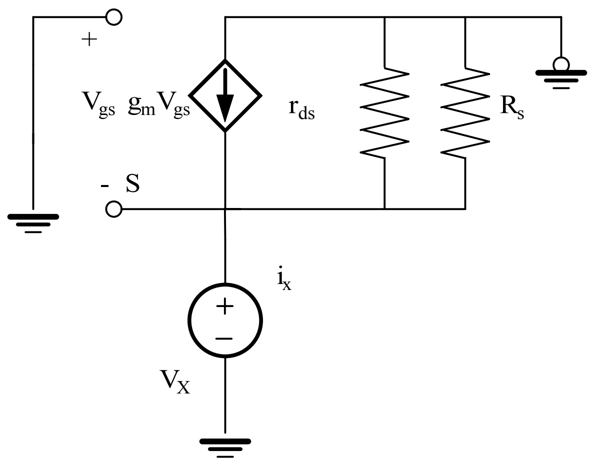
$$I_X + G_M V_{GS} = \frac{V_X}{R_S \parallel R_{DS}}$$

$$I_X = V_X[G_M + \frac{1}{R_S \parallel R_{DS}}]$$

$$R_0 = \frac{V_X}{I_X} = \frac{R_{EG}}{G_M R_{EG} + 1}$$

Where

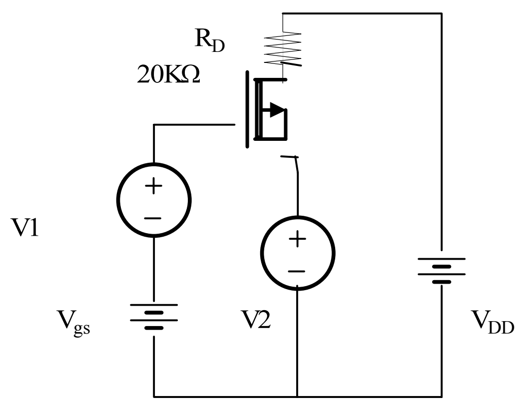
$$R_{EG} = R_S \parallel R_{DS}$$



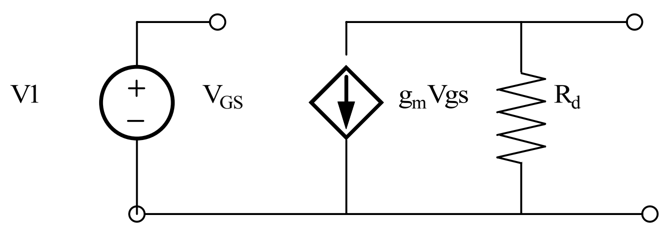
If GmReg >>1, then Ro= 1/Gm, which is generally small.
For Gm=10mS, Rds = 1k-ohm and Rs = 1 K-ohm
R0 = 83.3 ohm
The comman drain configuration has a voltage near 1 and a low output resistance. It can serve as a voltage buffer much like the unity gain buffer op-amplifier.

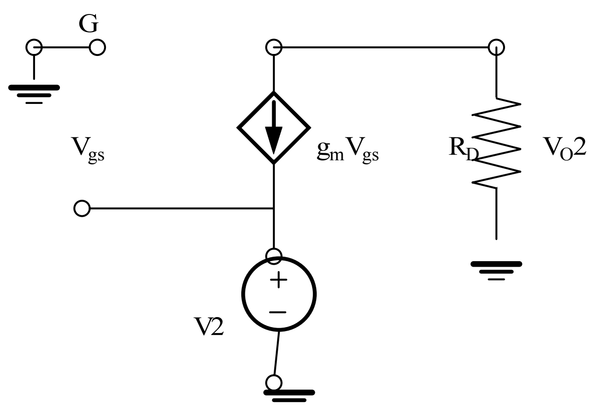
Problem 16.9

For MOSFET in fig, Gm= 2mS and assume Rds is infinity. Using superposition find Vo/(V2-V1)



Suggested Solution





$$For V1$$

$$V_{GS} = V_1$$

$$V_{01} = -G_M R_D$$

$$V_{01} = -V_1 G_M R_D = -40V_1$$

$$For V2$$

$$V_{GS} = -V_2$$

$$G_M V_{GS} + \frac{V_{02}}{R_D} = 0$$

$$V_{02} = V_2 G_M R_D = 40V_2$$

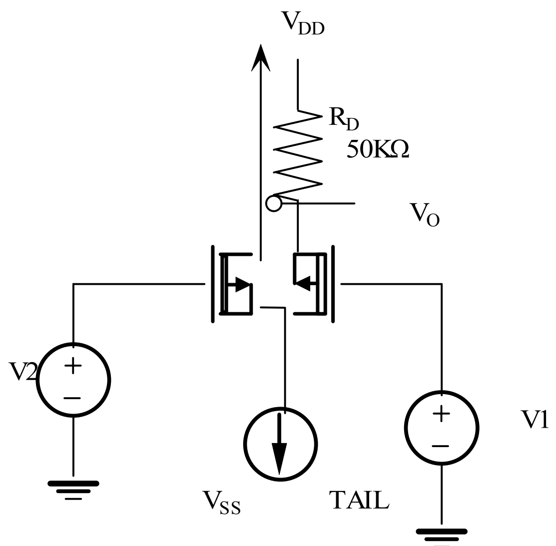
$$V_0 = V_{01} + V_{02} = 40(V_2 - V_1)$$

$$\frac{V0}{V_2 - V_1} = 40$$

$$\frac{V0}{V_2 - V_1} = 40$$

Problem 16.10

The amplifier in Fig is called as a differential amplifier. MOSFETs M1 and M2 are identical devices with Gm values and V1 and V2 small signal inputs. Find Gain V0/V2-V1 in terms of Gm and Rd. Assume Rds is infinite.



Suggested Solution

Use Superposition

$$V_2 = V_{GS2} - V_{GS1}$$

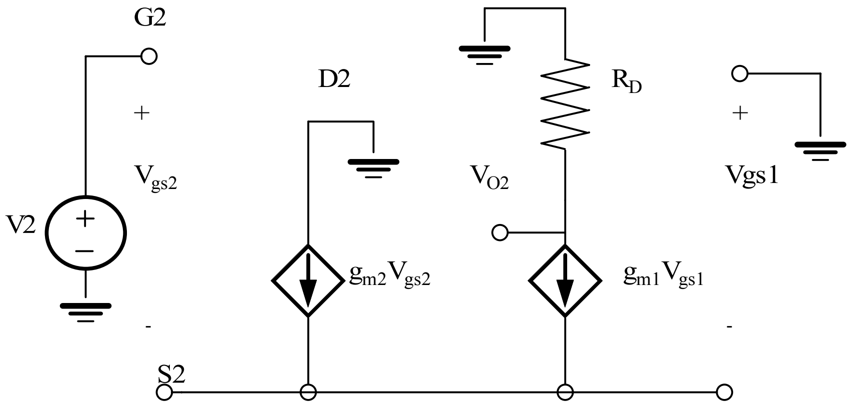
$$G_{M1} = G_{M2} = G_M$$

$$G_M V_{GS2} + G_M V_{GS1} = 0$$

$$V_{GS2} = -V_{GS1}$$

$$V_2 = 2V_{GS2} \Rightarrow V_{GS2} = \frac{V_2}{2} = -V_{GS1}$$

$$V_{02} = -G_{M1} V_{GS1} R_D = \frac{G_M R_D}{2} V_2 \Rightarrow V_{02} = V_2 \left(\frac{G_M R_D}{2}\right)$$



$$V_1 = V_{GS1} - V_{GS2}$$
$$G_{M1} = G_{M2} = G_M$$
$$G_M V_{GS2} + G_M V_{GS1} = 0$$
$$V_{GS2} = -V_{GS1}$$

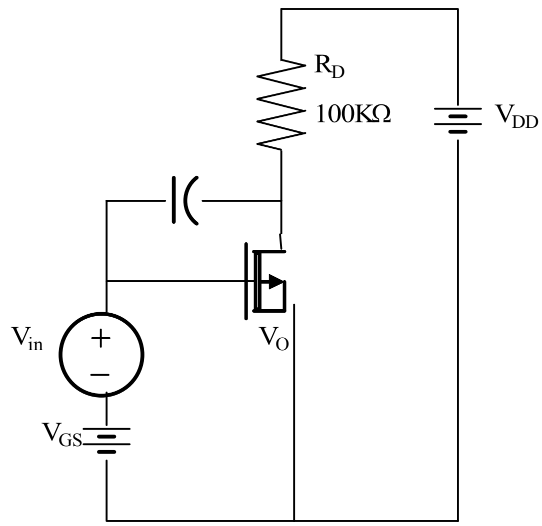
$$\frac{G_M R_D}{2}(V_2 - V_1) = V_0$$

$$V_{01} = -G_M V_{GS1} R_D = -V1(\frac{G_M R_D}{2})$$

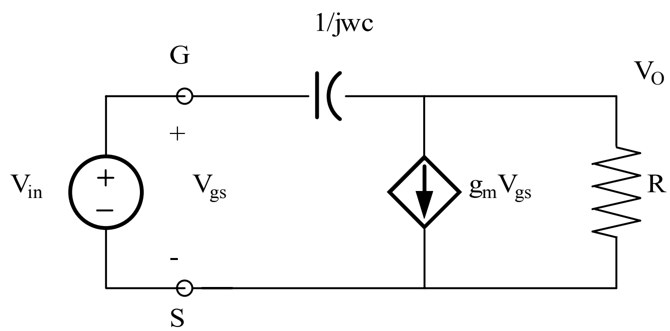
$$V_0 = V_{01} + V_{02} = \frac{G_M R_D}{2}(V_2 - V_1) = V_0$$

Problem 16.11

Find an expression for the complex gain A(jw) = Vo/Vin for the amplifier in terms of Gm, Rds, Rd and C. Find expression for the gain at dc and pole zero location. For Gm=500uS and Rds=400kohm, graph bode plot over frequency range 1Hz-100KHz.



Suggested Solution



$$R = R_{DS} \,||\, R_D = 80k\Omega$$
$$V_{GS} = V_{IN}$$

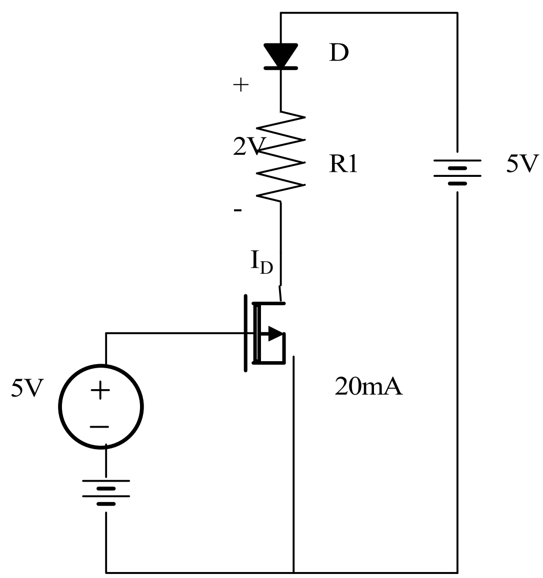
$$KCL:$$

$$(V_{IN} - V_0)\, j\omega C = G_M V_{GS} + \frac{V_0}{R} \Rightarrow V_{IN}(j\omega C - G_M) = V_0(j\omega C + \frac{1}{R})$$

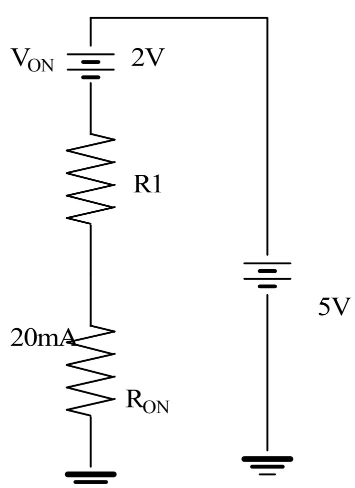
$$A(j\omega) = \frac{V_0}{V_{IN}} = \frac{(\frac{j\omega C}{G_M} - 1)}{(j\omega CR + 1)}(G_M R)$$

Problem 16.12

The MOSFET in fig, acts as a switch to turn red LED on or off. On-resistance of MOSFET is given as Ron=2/(Vgs-Vth) where Vth=1V. A LED is nothing more than a diode which glows when forward biased. The desired operating point for the LED is 2.0V at 20mA. Find Ron and the required value of R1.



Suggested Solution



Equivalent Circuit

KCL:

$$5 = 2 + 0.02(R_1 + R_{ON})$$

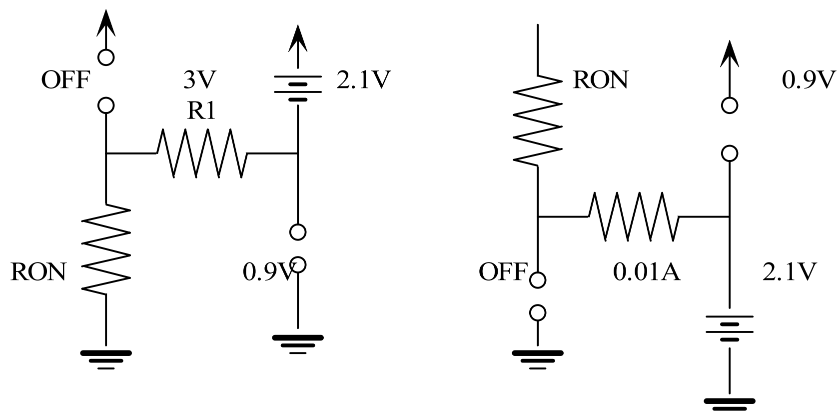
$$R_{ON} = \frac{2}{V_{GS} - V_T} = \frac{2}{5-1} = 0.5\Omega$$

$$R_{ON} = 0.5\Omega$$
$$R_1 = 149.5\Omega$$

Problem 16.13

The MOSFET in fig are used to turn on a green ar red LED. Vgs is switched to either 0 or 3 V. Which voltage level turns on which LED? Use the constant voltage model for diodes with Von = 2.1 V model the ON resistance of the MOSFET as Ron=17/(Vgs-0.75). Find the values of R1 such that diode current never exceeds 10mA.

Suggested Solution



$$V_{GS} = 3V, V_{GS1} = 3V, V_{GS2} = 0V,$$
$$M_1 = ON, M_2 = OFF$$
$$R_{ON} = \frac{17}{3-0.75} = 7.55\Omega$$
$$KVL:$$
$$3 = 2.1 + 0.01(R_1 + R_{ON}) \Rightarrow R1 \geq 82.4\Omega$$

$$V_{GS} = 0V, V_{GS1} = 0V, V_{GS2} = -3V,$$
$$M_1 = OFF, M_2 = ON$$
$$R_{ON} = 7.55\Omega$$
$$KVL:$$
$$3 = 0.01(R_1 + R_{ON}) + 2.1 \Rightarrow R_1 \geq 82.4\Omega$$

$$R_{ON} = 7.55\Omega$$
$$R_1 \geq 82.4\Omega$$

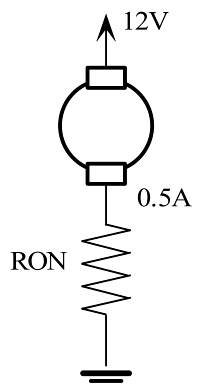
Problem 16.14

The MOSFETin fig, is used as a switch to turn on motor. It is desired that circuit operate at an efficiency at least 95% where efficiency is defined as

$$n = \frac{\text{Motor Power (W)}}{\text{Total Power (W)}}$$

Find the maximum allowable value of Ron.

Suggested Solution



$$\text{Total Power} = 12(\frac{1}{2}) = 6W$$

$$\text{Power in FET} = P_{FET} = I_2 R_{ON} = \frac{R_{ON}}{4}$$

$$R_{ON} \leq 1.2\Omega$$

$$\text{Total Power} = P_{MOTOR} + P_{FET} \ 6W$$

$$\eta = \frac{6 - P_{FET}}{6} > 0.95 \Rightarrow P_{FET} < 0.3W = \frac{R_{ON}}{4}$$

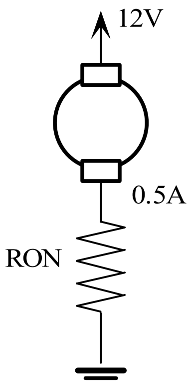
Problem 16.15

Fig shows an electric car motor can be controlled by a MOSFET. For the conditions shown find the efficiency of the circuit and MOSFET power when Ron = 5mohm.
Efficiency is defined as

$$n = \frac{\text{Motor Power (W)}}{\text{Total Power (W)}}$$

Find the values of Ron required for 99% efficiency. As mentioned in the text, MOSFETs, with Ron values as low as 2mohm are commercially available (circa 2000). How can you produce the required Ron value.

Suggested Solution



$$\text{Total Power} = P_{\text{MOTOR}} + P_{\text{FET}}$$

$$= 50(600) + I^2 R_{ON}$$
$$= 30000 + 600^2(5M)$$
$$= 31800W$$

$$\eta = \frac{30000}{31800} = 94.3\%$$

for

$$\eta = 99\% = \frac{30k}{30k + P_{FET}} \Rightarrow P_{FET} = 303W$$

$$R_{ON} = \frac{P_{FET}}{I^2} = \frac{P_{FET}}{600^2} = 0.84m\Omega$$

$$\eta = \frac{30000}{31800} = 94.3\%$$

$$R_{ON} = \frac{P_{FET}}{I^2} = \frac{P_{FET}}{600^2} = 0.84m\Omega$$

Less than currently available 2mΩ units.

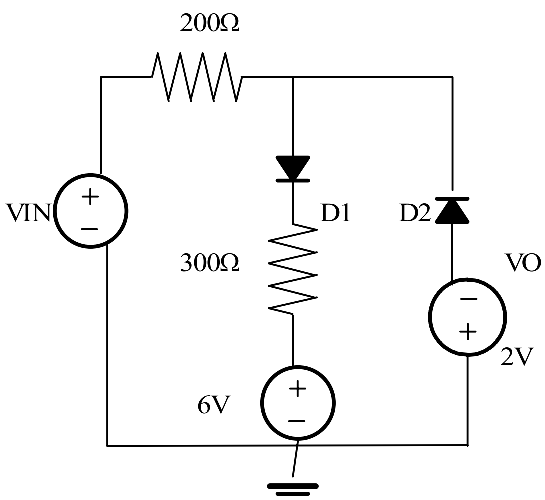
Solution:

Use several identical devices in parallel. If 5 are used with Ron = 4 mΩ, then the combined

$$R_{ON} = \frac{4}{5}m = 0.8m\Omega$$

Problem 16FE-1

For the circuit sketch voltage Vo verses Vin, over the range –10V to 10V. What are the bias conditions for the diodes at Vin=-10, 0 and +10 V. Assume diodes are ideal.



Suggested Solution

When Vin is greater than 6 V. D1 is forward biased and D2 is reversed biased. The circuit reduces to that where

$$V_{IN} = 500I + 6$$
$$V_0 = 300I + 6$$

$$V_0 = 6 + 0.6(V_{IN} - 6)$$

When Vin is less than –2 V, D2 is forward biased and D1 is reverse biased. Under these conditions V0= -2V
For Vin between –2 V and 6 V, both diodes are reverse biased, no flows anywhere. And Vo= Vin.
A plot is shown

Problem 16FE-2

The MOSFET in the circuit acts as a switch with an resistance of 1ohm and diode is ideal.

a. Assume the MOSFET is on the network ahs reached steady state. What is the value of Inductor current.
b. With the current operating as described in a) the MOSFET is turned off at time 0. Find expression for iL(t) and VL(t), for t>=0.

Suggested Solution

A. The figure shown models the circuit when MOSFET is ON. In steady state, the inductor voltage is 0, yielding,

20=5IL
IL = 4 A

B. The circuit models transients in the circuit when the MOSFET turns OFF. The initial condition of the inductor current is found a) to be 4A. When MOSFET turns off, the inductor current cannot go to 0 instantaneously. It flows through diode and discharges inductor. The inductor current and voltage can be given as

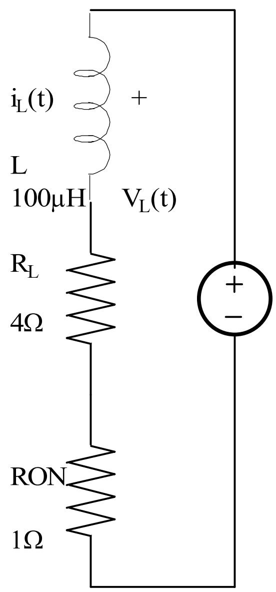
$$i_L(t) = K_1 + K_2 e^{\frac{-t}{\tau}}$$

$$V_L(t) = K_3 + K_4 e^{\frac{-t}{\tau}}$$

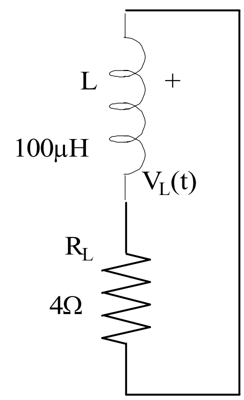
where iL (0) = 4 A and iL(∞) = 0 A. This K1 = 0 and K2 = 4A. Similarly, Vl(0) = -Il(0)Rl = -16Vand VL(∞)=0V. Thus K3 = 0 and K4 = -16V. The time constant =25micro sec. The resulting expressin for current and voltage are

$$i_L(t) = 4e^{-40000t}$$

$$V_L(t) = -16e^{-40000t}$$



iL(t)
+
L
100μH
VL(t)
RL
4Ω
+
−
RON
1Ω



L
+
100μH
VL(t)
RL
4Ω